

## **Integrating Renewables with Electricity Storage**

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Fossil fuel, hydro, nuclear, and geothermal electricity generating plants match controllable output with uncontrollable demand. Short term demand for electricity can be estimated with a fair degree of confidence. Generating plants are brought on line or taken off line in anticipation of electricity demand growing in the morning, peaking in the afternoon and early evening, and declining late in the evening. Some generating plants (nuclear and coal) operate at full capacity to satisfy base load demand while others (natural gas and hydro) are ramped up and down in response to changing variable load demand. Hydro in Canada, Norway, Brazil and a host of developing nations and nuclear in France serve both base and variable needs. This paper focuses on how simulation of electricity storage performance can be a planning tool to transform uncertain or uncontrollable supply to reliable and controllable supply.

While hydro and geothermal are controllable renewable energy sources, far more challenging are solar and wind. Yes, the sun shines every day, but what about cloud cover? Yes, the wind blows every day, but what about wind speed? Thus solar and wind outputs are uncertain; therefore uncontrollable. With continuing growth of solar and wind power, matching uncontrollable supply to uncontrollable demand is a growing challenge for utility operators. It may become a daunting task as solar and wind gain in relative importance to controllable conventional supply if no large scale means of storing electricity is available.

Solar and wind can be transformed to a controllable source of power if there is ample storage of electricity from which a dispatcher can compensate for falling solar and wind output in the same sense as ramping up a fossil fuel plant. Electricity storage can be likened to traditional inventory of goods for storing excess production during times of depressed demand and drawing down during times of heightened demand. This allows for more or less level production with inventory absorbing fluctuations in sales. In like manner, if vagaries of solar and wind output can be directed to and from electricity storage of sufficient capacity, then solar and wind can be transformed to a controllable supply.

A pumped storage plant or gravity battery can store and supply electricity to cover the mismatch between electricity supply and demand. A pumped storage plant or gravity battery consists of two water reservoirs at different heights fitted with reversible pump-turbines. Surplus electricity is consumed pumping water from lower to upper reservoirs and electricity is generated by gravity flow of water from upper to lower reservoirs. Pumps and turbines are the same equipment whereby an electric motor that drives a turbine to pump water to a higher elevation becomes a generator powered by the flow of water through its turbine to the lower elevation. A utility-sized electricity battery performs the same function of a pumped storage plant of storing surplus electricity for dispatching to cover shortfalls. Today only gravity batteries have the necessary storage capacity to serve utilities. Utility-sized electricity batteries are under development, but a technological breakthrough is necessary in battery design to identify a low-cost material that can store enormous quantities of electricity while accommodating rapid rates of charge and discharge.

The purpose of this paper is to illustrate how the inherent uncertainty of renewables can be handled relying on @RISK simulation software to model the output of a system of solar and wind farms located in different sites.<sup>1</sup> System output is then compared to uncertain demand to obtain a probability distribution of the mismatch between supply and demand. This is then used to size a gravity battery to compensate for the vagaries in supply and demand, thus transforming uncertain supply into controllable supply to meet changes in demand. Sizing a utility-sized battery would follow the same general format.

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<sup>1</sup> @RISK simulation software is available from Palisade Corporation ([www.palisade.com](http://www.palisade.com)). Subject matter in this paper is from *Energy Risk Modeling* available at [www.palisade.com/books/energy.asp](http://www.palisade.com/books/energy.asp). The author, a professor at Monmouth University ([rnersesi@monmouth.edu](mailto:rnersesi@monmouth.edu)), also wrote *Energy for the 21<sup>st</sup> Century* (2010) and its updated version *Energy Economics: Markets, History and Policy* to be released in 2016 by Routledge Publishing ([www.routledge.com](http://www.routledge.com)).

## Description of Contents

This paper begins with modeling solar output incorporating the impact of cloud cover followed by modeling wind output incorporating the impact of wind speed. Three 1-megawatt solar and three 1-megawatt wind units are located at different sites. Solar units are statistically linked incorporating the Copula function available in @RISK7 as a means for correlating variables while the wind sites are statistically linked using conditional probabilities. The analysis is then expanded from solar and wind units to farms with larger outputs to model a utility sized enterprise. A utility may have entirely solar or wind farms or a combination where one source of renewable energy dominates. Here the farms are 60% solar and 40% wind in order to illustrate the nature of uncertainty of matching variable solar and wind output with variable demand.

Simulation results of daily production of three solar farms and three wind farms are modeled using the Fit Distribution feature of @RISK. Since a single Fit Distribution probability function does not adequately model the output, a methodology is proposed to assemble two or more Fit Distribution probability functions for a better fit. System performance of matching fossil fuel and renewables to demand is first analyzed in terms of the degree of excess and shortfalls in electricity generation. Base load is covered by fossil fuel and nuclear plants. Under the circumstances, natural gas plants must be added to balance supply and demand with RISKOptimizer determining the desired capacity. A thought experiment is conducted on sizing a storage battery, but while this approach appears reasonable, the necessary storage capacity caused by the significant swing in seasonal demand is far greater than anticipated. @RISK simulation provides guidance on the capacity of a pumped storage or gravity battery needed to transform renewables into a reliable source of electricity. The natural gas plants have to be transformed from base to variable load to reduce the size of electricity storage with RISKOptimizer establishing the set points for full and partial utilization. The paper ends with an analysis of the fluctuations in depth of the upper reservoir to ensure reliability and thoughts on similar problems that can be handled by simulation.

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## Section 1

### Modeling Multiple Solar Unit Sites

Modeling multiple solar power units generally follows the descriptions in Sections 5 and 7 of *Energy Risk Modeling*. Referring to the Solar tab in spreadsheet ReNew1, the probability for output of a solar panel in half-hour increments was obtained from a @RISK Pert function, substituting for a sine wave, where a minimum of 6, the mean of 12, and a maximum of 18 mimicking twelve hours of daylight. The probability of readings between 6 and 6.5, which can correspond to 6 am to 6:30 am, is the difference between cumulative probabilities of being less than 6.5 and being less than 6.0. In Figure 1, time in column A for Unit 1 was arbitrarily set back by half an hour for the sun to rise at 5:30 with the probability values in column B derived as just described. Unit 2 is moved ahead half an hour with respect to Unit 1 and Unit 3 is moved ahead yet another half an hour with respect to Unit 2. Thus sunrise for Unit 1 is 5:30 am, Unit 2, 6:00 am, and Unit 3, 6:30 am with 12 hours of daylight. The Pert function can be modeled for eight hours of day in half-hour or even quarter-hour increments for the winter season with shorter days. A lower peak output compensates for the sun not rising as high above the horizon as in summer. Summer can be modeled with fourteen hour days by using a Pert function with a minimum of 5, mean of 12, and a maximum of 19 with a larger peak output to reflect the sun being higher above the horizon. Placement of solar output on an Excel spreadsheet can model sunrise and sunset being at different times for various sites. Thus seasonal solar adjustments can be made as appropriate to simulate an entire year.

Column B below is the share of solar output in terms of daily output in half-hour increments. Column C is the power output of a 1 megawatt solar installation where cell C7 has a value of 1 in Figure 1 and the formula in cell 30 is:  $=\$C\$7*B30/\$B\$30$ . Thus the highest output of a 1 megawatt facility occurs during the midday hour when the share of daily solar output is 7.8%. The remaining outputs are proportional to the midday output. Cell C7 can be changed to accommodate any solar output.

	A	B	C	D	E	F	G	H
1	Three Solar Installations			Clear/Cloudy		Temperature		
2			Unit 1	0		24.9		
3			Unadjusted					
4		Discrete	Power		Solar	Solar	Unit 1	Unit 1
5		Probability	Output		Output with	Output with	Power	Energy
6		Distribution	mW	Cloud	Cloud	Temperature	Output	Output
7		Half-Hour Intervals	1	Cover	Correction	Correction	mW	mWh
8	00:00-00:30	0.0%					0.00	0.00
9	00:30-01:00	0.0%					0.00	0.00
10	01:00-01:30	0.0%					0.00	0.00
11	01:30-02:00	0.0%					0.00	0.00
12	02:00-02:30	0.0%					0.00	0.00
13	02:30-03:00	0.0%					0.00	0.00
14	03:00-03:30	0.0%					0.00	0.00
15	03:30-04:00	0.0%					0.00	0.00
16	04:00-04:30	0.0%					0.00	0.00
17	04:30-05:00	0.0%					0.00	0.00
18	05:00-05:30	0.0%					0.00	0.00
19	05:30-06:00	0.1%	0.013	0.0%	0.013	0.013	0.01	0.01
20	06:00-06:30	0.4%	0.051	0.0%	0.051	0.051	0.05	0.03
21	06:30-07:00	1.1%	0.141	0.0%	0.141	0.141	0.14	0.07
22	07:00-07:30	1.9%	0.244	0.0%	0.244	0.243	0.24	0.12
23	07:30-8:00	2.9%	0.372	0.0%	0.372	0.372	0.37	0.19
24	08:00-08:30	3.9%	0.500	0.0%	0.500	0.500	0.50	0.25
25	08:30-09:00	4.9%	0.628	0.0%	0.628	0.628	0.63	0.31
26	09:00-09:30	5.8%	0.744	0.0%	0.744	0.743	0.74	0.37
27	09:30-10:00	6.5%	0.833	0.0%	0.833	0.833	0.83	0.42
28	10:00-10:30	7.1%	0.910	0.0%	0.910	0.910	0.91	0.45
29	10:30-11:00	7.6%	0.974	0.0%	0.974	0.974	0.97	0.49
30	11:00-11:30	7.8%	1.000	0.0%	1.000	1.000	1.00	0.50
31	11:30-12:00	7.8%	1.000	0.0%	1.000	1.000	1.00	0.50
32	12:00-12:30	7.6%	0.974	0.0%	0.974	0.974	0.97	0.49

Cell D2 is a RiskDiscrete distribution where there is a 60% chance for a clear day (0) and a 40% chance of a cloudy day: =RiskDiscrete({0,1},{0.6,0.4}). Below shows the start of day cloud cover in cell D19. Subsequent changes are in cell D20 and the impact of the degree of cloud cover on solar output in cell E19.

	D	E
19	=IF(D2=0,0,RiskUniform(0,1))	=(-0.7*D19+1)*C19
20	=MIN(D19*RiskUniform(0.95,1.05),1)	=(-0.7*D20+1)*C20

If there is a clear day signified by cell D2=0, then the degree of cloud cover in cell D10 is zero with no correction in cell E19. A clear day, as determined in cell D2, remains clear for the entire day. If cell D2=1, then cloud cover in cell D19 is a random value between 0 and 1. Subsequent cloud cover can change +/- 5% on a half-hourly basis with a proviso of not exceeding 100%. Cell E19 reduces solar output to 30% for 100% cloud coverage increasing linearly to 0% reduction for no cloud coverage as derived in Section 5 of *Energy Risk Modeling*. Column F is the correction for temperature also derived in Section 5 reflecting the output of a solar panel being higher with lower temperatures. Solar panels are more efficient in polar regions, not torrid deserts. Column G is corrected output in terms of power in megawatts (mW) while electricity (energy) output in megawatt-hours (mWh) in column H for each half-hour segment are values in column G divided by 2.

The following spreadsheet portion covers units 2 and 3 for the solar installations. Each are set off by half an hour to take into account east-west distances between these units. Since the sun rises at 5:30 am for unit 1, 6:00 am for unit 2, and 6:30 am for unit 3, the east-west distance between Units 1 and 3 is about one time zone. This effectively widens the peak power period. For shorter east-west distances, it would be necessary to use smaller time increments.

	J	K	L	M	N	O	P	Q	R	S	T	U	V
1		Clear/Cloudy		Temperature					Clear/Cloudy		Temperature		
2	Unit 2	0		24.5					Unit 3	1	24.9		
3	Unadjusted												
4	Power		Solar	Solar	Unit 2	Unit 2		Unit 3		Solar	Solar	Unit 3	Unit 3
5	Output	1	Output with	Output with	Power	Energy		Power	2	Output with	Output with	Power	Energy
6	mW	Cloud	Cloud	Temperature	Output	Output		Output	Cloud	Cloud	Temperature	Output	Output
7	1	Cover	Correction	Correction	mW	mWh		1	Cover	Correction	Correction	mW	mWh
8					0.00	0.000						0.00	0.00
9					0.00	0.000						0.00	0.00
10					0.00	0.000						0.00	0.00
11					0.00	0.000						0.00	0.00
12					0.00	0.000						0.00	0.00
13					0.00	0.000						0.00	0.00
14					0.00	0.000						0.00	0.00
15					0.00	0.000						0.00	0.00
16					0.00	0.000						0.00	0.00
17					0.00	0.000						0.00	0.00
18					0.00	0.000						0.00	0.00
19		0.0%			0.00	0.000			89.2%			0.00	0.00
20	0.013	0.0%	0.013	0.013	0.01	0.006			89.2%			0.00	0.00
21	0.051	0.0%	0.051	0.051	0.05	0.026		0.013	93.2%	0.004	0.004	0.00	0.00
22	0.141	0.0%	0.141	0.141	0.14	0.070		0.051	89.3%	0.019	0.019	0.02	0.01
23	0.244	0.0%	0.244	0.243	0.24	0.121		0.141	81.4%	0.061	0.061	0.06	0.03

The chief difference between units 2 and 3 is in the determination of cloud cover. Cloud cover for unit 2 is based on cloud cover for unit 1. In cell K2, there is an 70% chance that weather will be clear or cloudy for unit 2 as indicated in cell D2 for unit 1:

=IF(RAND()>0.3,D2,IF(D2=1,0,1))

There is a 70% chance that unit 2 will have a clear or cloudy day as in unit 1. For the remaining 30% probability, if the weather is cloudy for unit 1, it will be clear for unit 2; and if the weather is clear in for unit 1, it will be cloudy for unit 2.

Cell K5 compares the output of cells D2 and K2. If they are the same, a “1” is generated, if the values are D2=0 and K2=1, a “2” is generated, otherwise a “3” is generated:

=IF(AND(D2=K2),1,IF(AND(D2=0,K2=1),2,3))

The formula in K19 is:

=IF(K5=1,MIN(D19\*RiskUniform(0.7,1.3),1),IF(K5=2,RiskUniform(0,1),0))

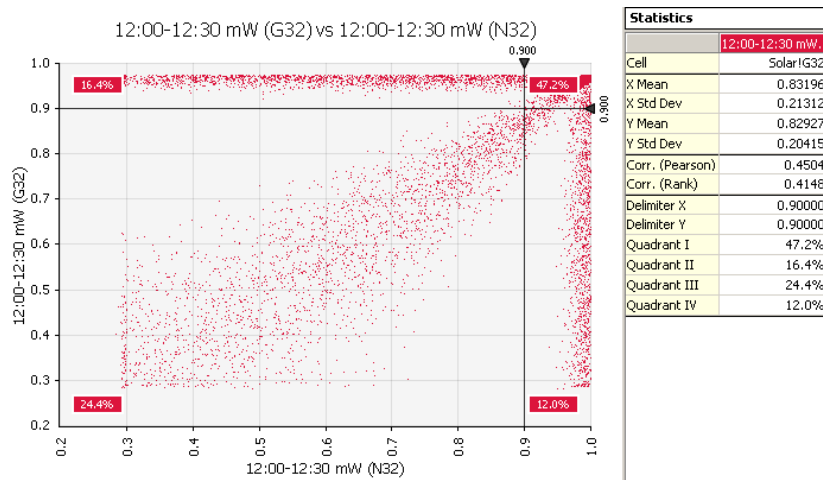
If a “1” is generated in cell K5, then the initial cloud cover for unit 2 will be +/-30% of unit 1 not exceeding 100%. If unit 1 happens to have a clear day, then the formulation assures that unit 2 will remain a clear day (+/-30% of zero). If a “2” is generated in cell K5 meaning that unit 1 has a clear day and unit 2 a cloudy day, then the initial cloud cover will be between 0% and 100%. A “3” can only be generated if the prior conditions are not true; the only one left is unit 1 being a cloudy day and unit 2 a clear day. For this condition denoted by generating a “3”, then unit 3 will have a clear day. The temperature for unit 2 is +/-10% of unit 1. Unit 3 is constructed similar to unit 2 except dependency for cloudiness and temperature is keyed to unit 2, not unit 1.

### Simulating Impact of Cloud Cover

A simulation shows that solar unit 1 is clear 60% of the time as would be expected from the Risk Discrete function. With the probability linkages as constructed herein, unit 2 is clear 53% of the time and unit 3 51% of the time.

Bearing in mind that the minimum output is 0.3 mW when there is 100% cloud cover and 1 mW with no cloud cover, Figure 1a shows that both solar units will simultaneously have a high output (0.9 mW or more) 47.2% of the time.

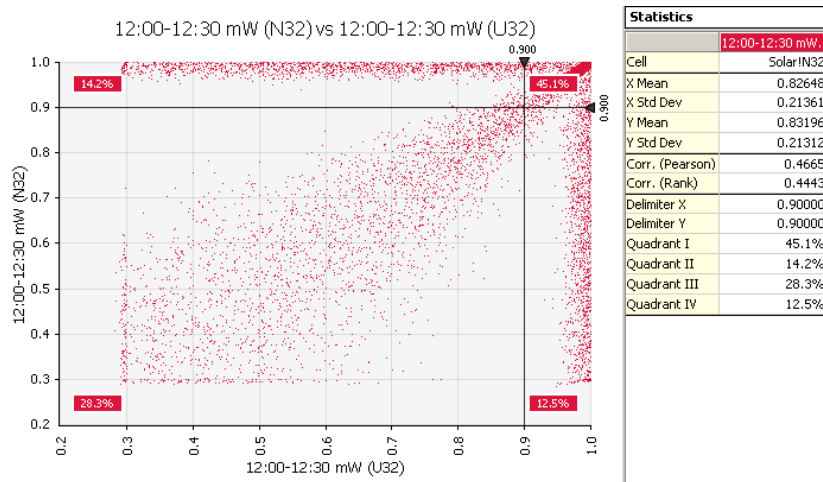
**Figure 1a Statistical Relationship Between Solar Units 1 and 2**



Both share low outputs (less than 0.9 mW) 24.4% of the time. Unit 1 (cell G32) has a high output while unit 2 (cell N32) has a low output 16.4% of the time. Similarly unit 2 has a high output with unit 1 having a low output 12% of the time. Unit 1 has a high output 63.6% of the time and unit 2, 59.2% of the time.

## Figure 1b Statistical Relationship Between Solar Units 2 and 3

Basically the same statistical relationship holds as between units 1 and 2 in Figure 1a.



Both share high outputs 45.1% of the time and low outputs 28.3% of the time. Unit 2 (cell N32) has a high output while unit 3 (cell U32) has a low output 14.2% of the time. Unit 3 has a high output with unit 2 having a low output 12.5% of the time. Unit 2 has a high output 59.3% of the time and unit 3, 57.6% of the time.

## Solar System Output

The power output in mW and energy output in mWh output for each half-hour segment of the system of three solar units are in columns Y and Z respectively as follows (the zero entries reflect night time) along with total daily megawatt-hour output of the three solar installations.

	X	Y	Z
4		Solar	Solar
5		Power	Energy
6		Output	Output
7		mW	mWh
8	00:00-00:30	0.00	0.00
9	00:30-01:00	0.00	0.00
10	01:00-01:30	0.00	0.00

	V	W	X	Y	Z
58	Total Solar Power mW output				34.8
59					
60	Total Solar Energy mWh Output				17.1
61					
62	Fossil Fuel Equivalent (95% for 24 hours)				68.4
63					
64	Relative Effectiveness				25%
65					
66	In competition with fossil fuel daylight hours only				
67					
68	Fossil Fuel Equivalent (95% for 12 hours)				34.2
69					
70	Relative Effectiveness				50%

Cell Z60 is the daily combined output of the three solar units in megawatt-hours with a peak system output of 3 megawatt-hours. Cell Z62 is the megawatt-hour energy output of a 3 megawatt fossil fuel plant operating at 95% of capacity for 24 hours:  $=(C7+J7+Q7)*0.95*24$

On a 24-hour basis, the solar installation for this iteration of a simulation is only 25% of rated capacity. The relative effectiveness of solar power is restricted to the sun being above the horizon with maximum power

output available only about 2 or so hours per day, which is also affected by cloud cover and temperature. Thus a 1 megawatt solar power plant does not represent the power output from a 1 megawatt fossil fuel plant except for a few hours around mid-day. Its megawatt-hour output is much lower than a fossil fuel plant with the caveat that solar power is available during peak demand.

One can argue that solar power should not be compared to a nuclear or coal plant operating 24/7 to satisfy base load demand, but to a natural gas plant running during daylight hours to satisfy variable demand. On the basis of comparing performance to a natural gas plant whose output is primarily restricted to daylight hours, solar performance is much better delivering 50% of rated power versus 95% for a comparable a natural gas plant based on 12 hours of operation. Pricing electricity of a natural gas plant limited to operating 12 hours per day provides a realistic measure for comparison with solar power. Comparing electricity rates between solar and a natural gas plant operating 24/7 would give an unwarranted advantage to the natural gas plant as natural gas plants are mainly built for variable, not base, demand. For natural gas plants serving base load demand, the economic analysis for solar should still be based on the cost of electricity from natural gas plants serving variable load periods. While solar should be compared to a natural gas plant operating 12 hours per day, some may argue that natural gas plants operating 8 hours a day should be used as a comparative measure. Peaking generators operating for a few hours a day for a few weeks per year are not an appropriate measure for conducting a comparative economic analysis with solar. However solar power still requires peaking generators for those days when solar power is less than optimal. Gravity batteries and utility-sized electricity batteries are the sole means to get rid of peaking generators and natural gas plants that only run a few hours a day year-round.

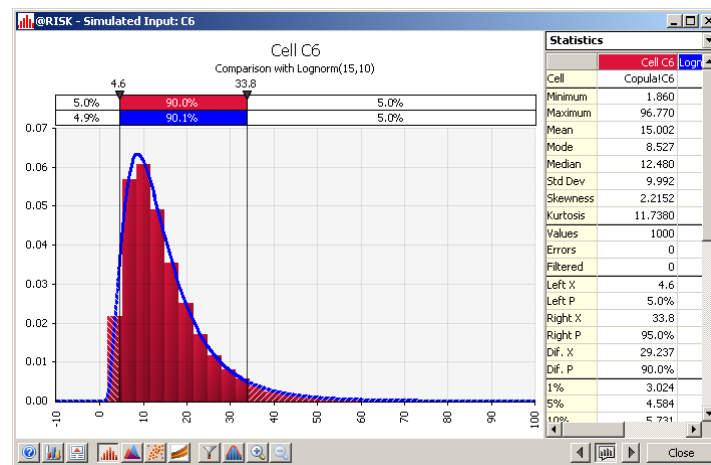
## Modeling Multiple Wind Unit Sites

Modeling wind power units generally follows the description in Section 7 of *Energy Risk Modeling* with the change of incorporating a Copula function rather than a standard correlation to statistically link wind speed at multiple locations.

### Copula Function

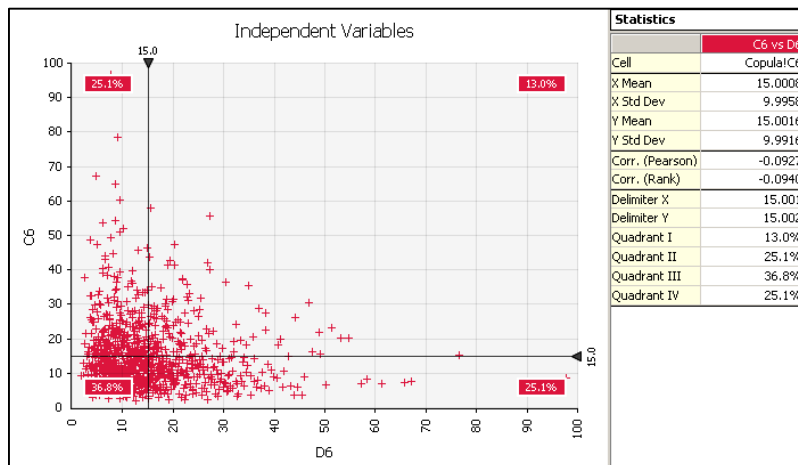
The copula function is new in @RISK 7. Suppose that there are two locations proposed for a wind farm where the wind speed can be modeled by a lognormal distribution with a mean speed in 15 miles per hour (mph) with a standard deviation 10 mph. Put RiskLognorm(15,10) in two adjacent cells and run a simulation of 1,000 iterations (these cells do not have to be designated as output cells). Select either cell or the Browse Results icon to obtain Figure 2.

**Figure 2 – LogNormal Distribution**



Press the sixth icon from left at the bottom of the graph, Create Scatter Plot, to get a scatter plot and input the adjacent cell (two cells are necessary to draw a scatter plot) to obtain Figure 3.

**Figure 3 – Scatter Plot with Independent Variables**

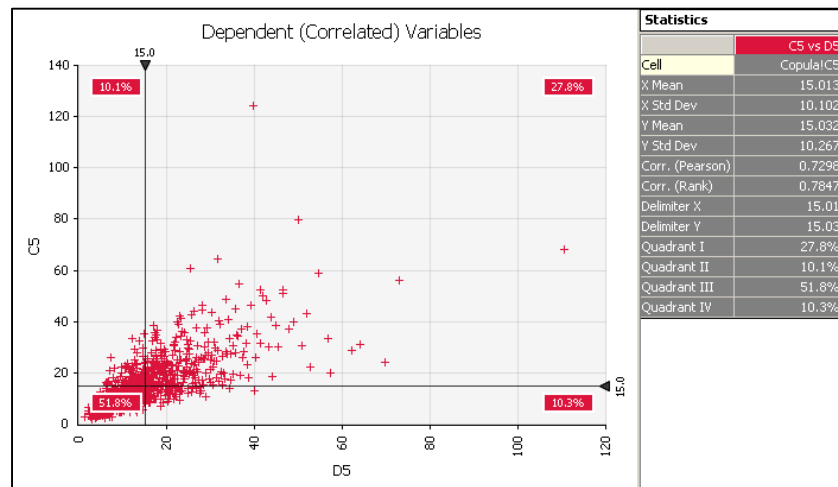




The scatter plot pairs up 1,000 iterations of the two variables. Quadrant I holds pairs of wind speed for both cells above the means of 15 mph, which accounted for 13% of the 1,000 pairs of results. Quadrant III holds pairs for wind speed for both pairs below 15 mph, which accounted for 36.8% of the iterations.

If two variables are not independent, then the correlation function can link the two with an indicated degree of statistical strength. Figure 4 shows the results of correlating the two lognormal distributions by selecting the two independent lognormal distributions and the Define Correlations icon and inserting a correlation value of 0.8 into the Define Correlation Matrix.

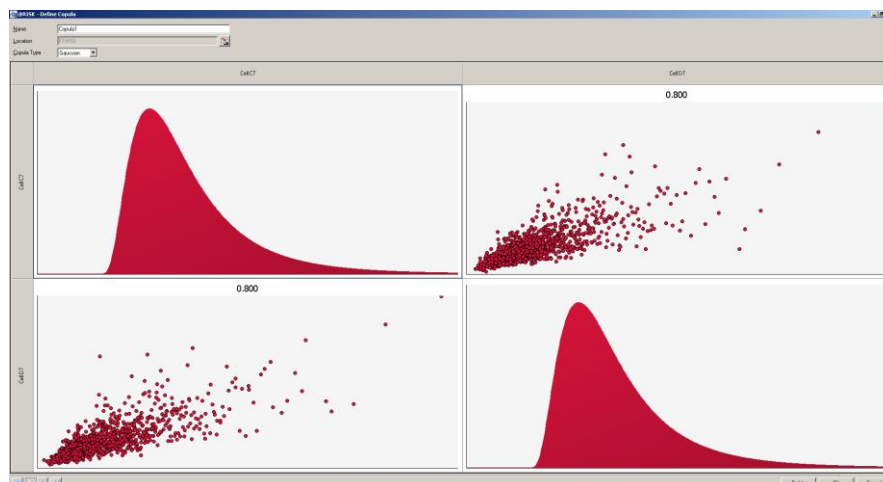
**Figure 4 – Scatter Plot with Correlated Variables**



@RISK Copula: Copula1		
Type	Gaussian	
Matrix	1.000	
	0.800	1.000

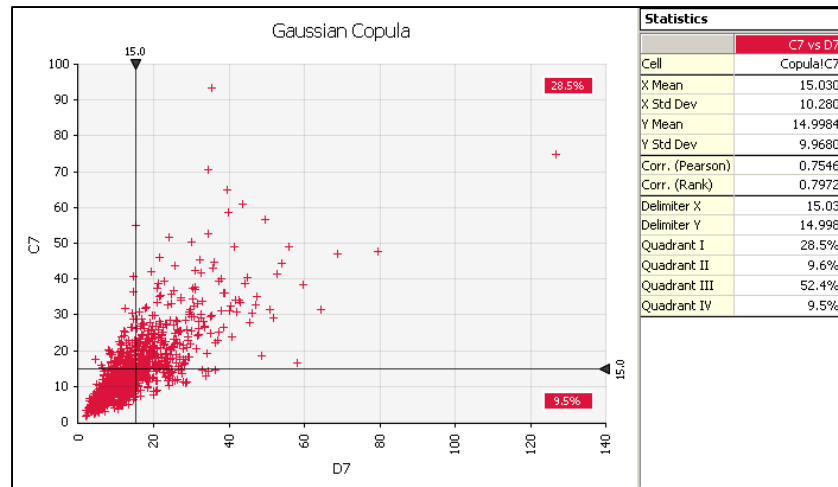
The pairs are much more “centralized” with Quadrant I making up 27.8% and Quadrant III 51.8% of all pairs. Define Correlation has another option to Define Copula. Two other lognormal variables were selected, then Define Copula, and Gaussian with an entered value of 0.8 to obtain Figure 5.

**Figure 5 – Gaussian Correlation with Value of 0.8**



Run a simulation of 1,000 iterations and select the two cells linked by the Gaussian correlation to obtain the scatter plot in Figure 6.

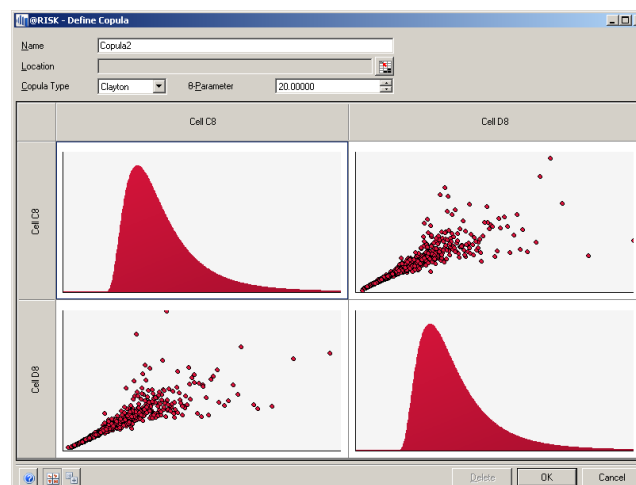
**Figure 6 – Scatter Plot for Gaussian Copula**



The distribution of paired points is nearly identical with Figure 4, which was correlated with a correlation value of 0.8. Hence the Gaussian Copula obtained from Define Correlations/Define Copula can be used as a substitute for correlating variables via Define Correlation Matrix.

Suppose that data were collected while evaluating two sites for building a wind farm. At given dates and times, weather data were simultaneously collected and wind speeds at the two locations were paired in a scatter diagram. Select the paired data in the worksheet Copula and Define Correlations/Fit Copula to determine the best copula that describes the linkage between the data. Suppose the result of this search is a Clayton copula with a parameter value of 20 as in Figure 7.

**Figure 7 – Two Sites with a Clayton Copula**

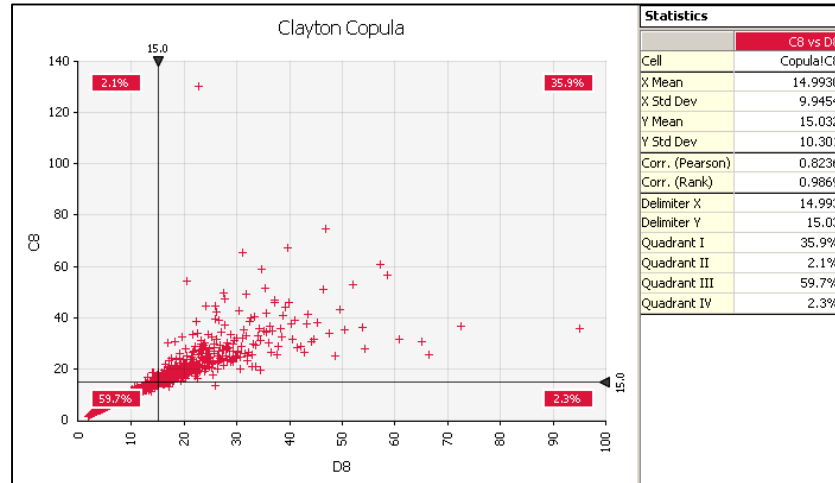


@RISK Copula: Copula2	
Type	Clayton
Dimension	2
Parameter	20.000

Pressing OK and selecting an Excel range location creates the formula: =RiskLognorm(15,10,RiskCopula(Copula2,1)) where Copula2 is the name of the Excel range. The procedure for defining Copula correlation and defining Correlation Matrix is intentionally similar.

Figure 8 results from running a simulation followed by a Scatter Plot for the Copula correlation formula.

**Figure 8 – Scatter Plot for Clayton Copula**



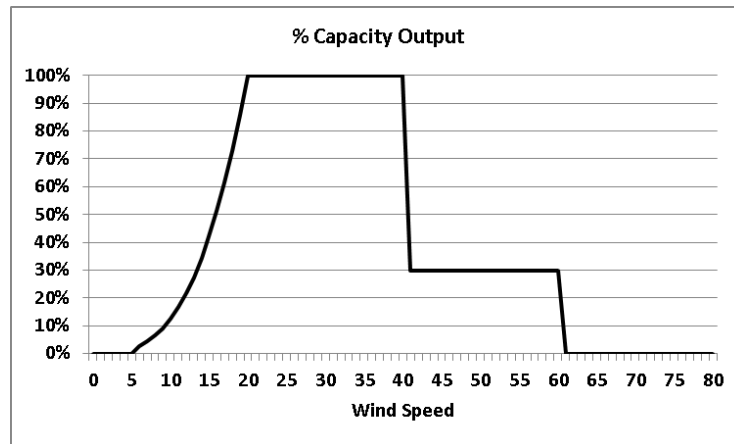
This is a distinctly different paired plot of wind speed data than in Figures 4 and 6. Nearly 36% of paired data points are in Quadrant I, 60% of data is in Quadrant III, and only 4% in quadrants II and IV. This means that wind speeds are very closely related to one another at both sites for wind speeds less than 15 mph, but are quite dispersed above 15 mph. It is assumed that the Clayton copula with a parametric value of 20 fits both sites 1 and 2 and also sites 2 and 3.

### Modeling Wind Speed

As described in Section 5 of *Energy Risk Modeling*, wind output is related to wind speed as follows and illustrated in Figure 9.

Wind Speed (mph)	Power Output	Comments
0-5	None	Wind speed too low to produce power
5-20	Increases by the cube of wind speed up to 100% at 20 mph	Power output = (wind speed/20) <sup>3</sup> X 100%
20-40	100% rated output	Turbine operating at maximum performance
40-60	30% rated output	Power is reduced to protect the integrity of the wind turbine from high winds
Over 60	0% rated output	Objective is to preserve the wind turbine structure from damage, not generate power

**Figure 9 – Capacity Output as % Power versus Wind Speed**



Cells E6, I6, and M6 in the Wind tab of spreadsheet ReNew1 shown below originally contained the wind speed formula:

=RiskLognorm(15,10).

@RISK Copula: Copula 4	
Type	ClaytonR
Dimension	3
Parameter	20.000

Using the Control key, select all three cells, then Define Correlations/Define Copula and select ClaytonR for Copula Type with a parametric value of 20. The formulas for all three cells will become as follows. (The numeric value 4 means that this was the fourth copula formed in this spreadsheet.)

=RiskLognorm(15,10,RiskCopula(Copula4,1))

	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																
2																
3			Unit 1	Unit 1			Unit 2	Unit 2			Unit3	Unit 3				
4		Wind	Power	mWh		Wind	Power	mWh		Wind	Power	mWh				
5		Speed	Output	Output		Speed	Output	Output		Speed	Output	Output			Wind Power	Wind Energy
6		2.79	1			29.59	1			50.27	1				Output	Output
7															mW	mWh
8	00:00-00:30	2.79	0.000	0.000		29.59	1.000	0.500		50.27	0.300	0.150		00:00-00:30	1.300	0.650
9	00:30-01:00	2.90	0.000	0.000		29.90	1.000	0.500		52.16	0.300	0.150		00:30-01:00	1.300	0.650
10	01:00-01:30	2.78	0.000	0.000		31.07	1.000	0.500		51.94	0.300	0.150		01:00-01:30	1.300	0.650

Some locations experience higher wind speeds at night than during the day and this can be handled by multiplying night time speed by, say, 1.05 or 1.1 and day time speed by 0.95 or 0.9. Of course values for other cells would be smoothed in to avoid abrupt changes.

Cell E6 is the start of day wind speed defined by the lognormal distribution. Cell E8 is equal to cell E6 and all half-hour cells thereafter are +/- 5% the previous cell; e.g. cell E9 contains the formula:

=E8\*(RiskUniform(0.95,1.05))

Cell F8 contains the formula, derived in Section 5, and illustrated in Figure 8 linking wind speed in column E to power output in cell F6:

=F\$6\*IF(E8<=5,0,IF(E8<=20,(E8/20)^3,IF(E8<=40,1,IF(E8<=60,0.3,0))))

Cells F6, J6, and N6 are the power output for each unit in megawatts.

	Q	R	S
4			
5		Wind Power	Wind Energy
6		Output	Output
7		mW	mWh
8	00:00-00:30	1.323	0.662
9	00:30-01:00	1.358	0.679
10	01:00-01:30	1.378	0.689

Columns F and G are the power and energy output by half-hour intervals. Column G, the energy output, is one-half of column F to reflect one-half hour intervals. The same relationship holds for columns J and K and columns N and O. Columns R and S aggregate the daily output for the three wind units in half-hourly segments.

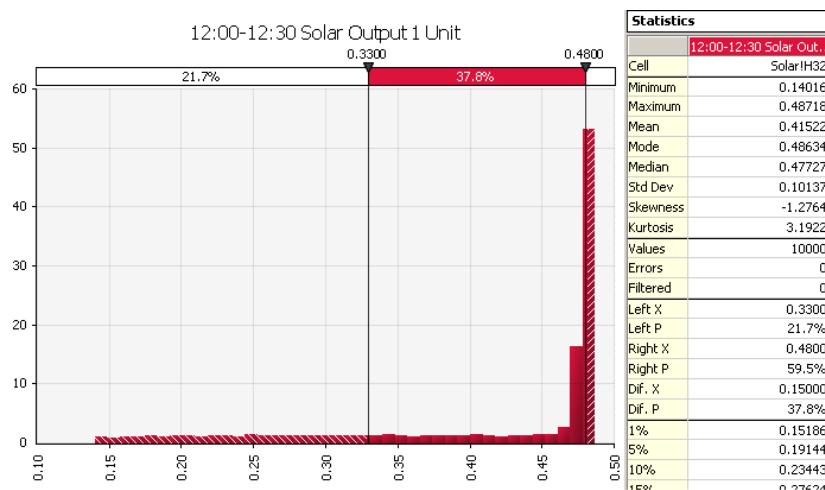
	P	Q	R	S
58	Total Wind Power mW Output			60.2
59				
60	Total Wind Energy mWh Output			30.1
61				
62	Fossil Fuel Equivalent (95% for 24 hours)			68.4
63				
64	Relative Effectiveness			44%

Row 56 totals power and electricity output in megawatt-hour output for all three wind units. Cell S58 is total power output and cell S60 is total energy output. This is compared to a fossil fuel plant operating 24/7 in cell S62 to derive its relative effectiveness in cell S64.

### Multiple Locations Reduce Risk of Experiencing Extreme Outputs

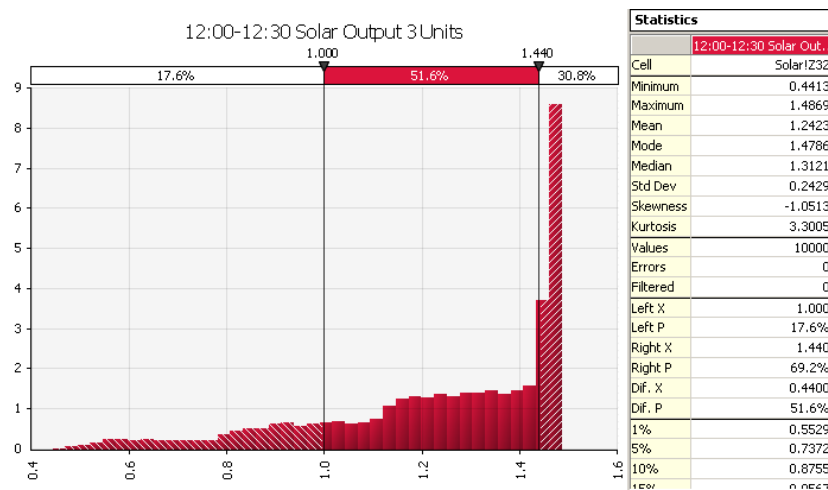
Dealing with solar power first, Figure 10a is the 12 pm-12:30 pm energy output for a single solar unit.

**Figure 10a – Single Solar Unit Output at Noon (mWh)**



The probability of energy output in mWh during this half-hour being less than 0.33 mWh is 21.7% and 40.5% chance of being greater than 0.48 mWh. Figure 10b shows the probability of energy output for three units.

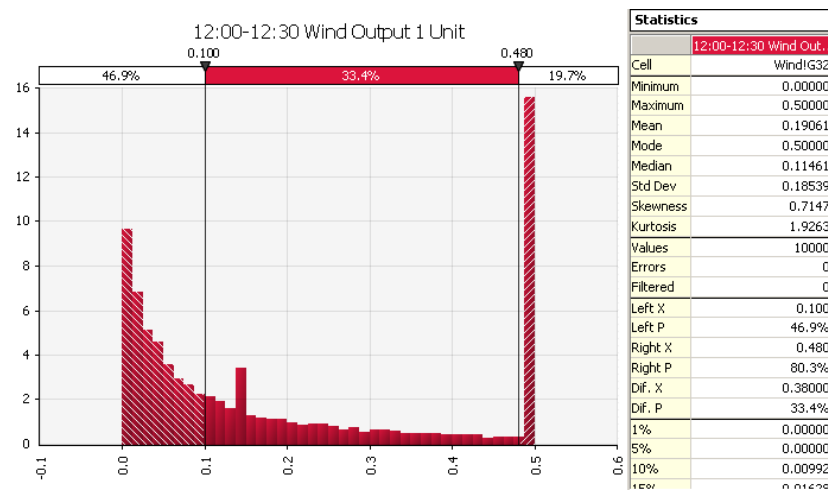
**Figure 10b – Three Solar Units Output at Noon in mWh**



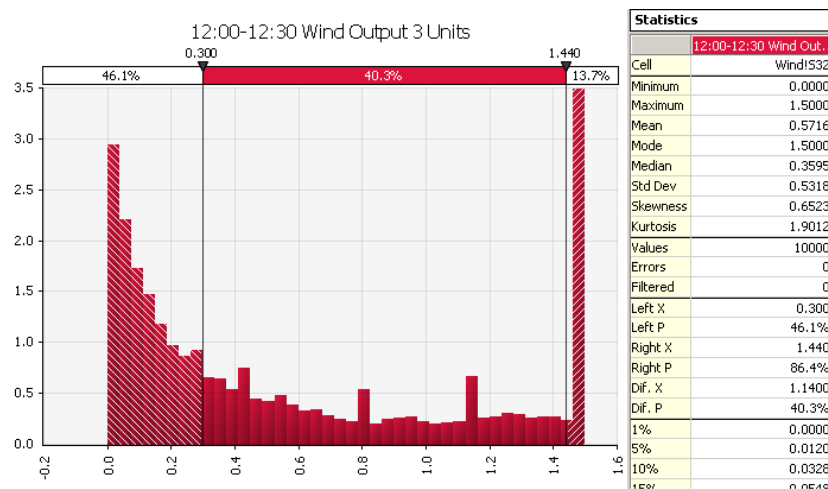
To put this on an equal basis, an output of 1 mWh in Figure 10b of 3 units would correspond to 0.33 mWh in Figure 10a for a single unit; similarly 1.44 mWh in Figure 10b would correspond to 0.48 mWh in Figure 10a. Thus these demarcations are for the equivalent energy output of one versus three solar units. The probability of low outputs has declined from 21.7% (Figure 10a) to 17.6% (Figure 10b) by increasing the number of location sites. The probability of high outputs has also declined from 40.5% (Figure 10a) to 30.8% (Figure 10b). This is caused by having three sites in different east-west locations spreading out peak output over a longer period of time. The lowering of probabilities at the extremes “pushes” more outcomes into the middle range reducing risk associated with extreme values. Thus the system becomes more manageable from a dispatcher’s viewpoint for multiple locations in the east-west direction. A dispatcher would be more amenable for the probability of low outputs to decline over high outputs. If extreme values represent risk, a dispatcher does not view risk associated with high and low outputs through the same operational lens.

Figures 11a and 11b do the same comparison for wind units. The peak spike at 0.5 mWh in Figure 11a reflects maximum 1 mW power output when wind speeds are in their optimal range of 20 and 40 mph. The smaller spike at 0.15 mWh (30% of 0.5 mWh) reflects wind speeds between 40 and 60 mph where the probability of being in this range is lower than wind speeds of 20-40 mph. There is a 46.9% chance that the 1 mW unit will generate less than 0.1 mWh and a 19.7% chance of more than 0.48 mWh (maximum energy output of 1 mW in power would be 0.5 mWh for a half-hour time segment). The major risk in operating a wind farm is low output from low wind speeds. The chance of a single wind unit having an output of 0.1 mWh or less of 46.9% was only slightly reduced to 46.1% by having 3 wind units in different locations shown in Figure 11b. This is a direct effect of the Cupola function incorporated in the spreadsheet where speeds below average were very tightly correlated shown in Figure 8. The high end probability of high wind speeds, which is not a risk that bothers dispatchers, falls from 19.7% for a single unit (Figure 11a) to 13.7% for triple units (Figure 11b).

**Figure 11a – Single Wind Unit Output at Noon in mWh**



**Figure 11b – Three Wind Units Output at Noon in mWh**

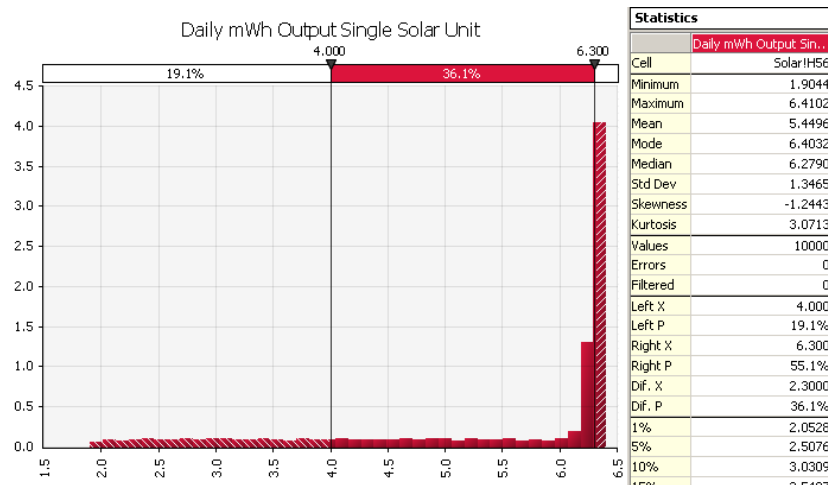


There is a valuable lesson to be learned that locating wind turbines with a high correlation where all the sites having more or less the same wind speed will do little to lessen the risk of low power output during calm weather. Dispersion of locations where the degree of correlation is far less than assumed here would more effectively reduce risk of all wind turbines being exposed to calm winds. But in Germany, where one would expect a looser association between wind speeds in different locals, there is a two-week hiatus when a calm overtakes the nation adversely affecting wind turbine output everywhere. A looser correlation above average speeds had a more significant reduction in the risk of high speeds from 19.7% for a single unit to 13.7% for three units. Higher wind speeds either reduce wind turbine to 30% of rated capacity or the turbine must be shut down to prevent wind damage. Reducing the probability of being above a high output and below a low output increases the probability of being between the two extremes reducing risk. But here, risk of low speeds, which is of primary concern, was only marginally reduced because of the nature of the selected Cupola function. However the selected Cupola function may be more appropriate than the Gaussian for locations that have consistent wind patterns such as mountain passes, coastal areas, and alongside mountains or in broad areas such as northern China. Other locations such as Midwest or northeast US and continental Europe have more diverse wind patterns that lessen the risk of all locations having calm winds.

## Comparing Risk Between Multiple Units on a Daily Basis

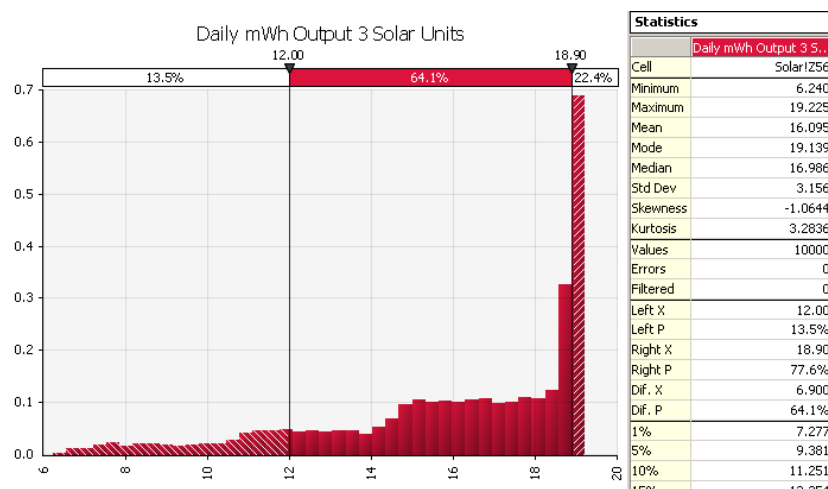
Figure 12a shows the impact of clouds on solar power for a single and three solar units respectively on a daily basis.

**Figure 12a – Daily Output for a Single Solar Unit**



The maximum output of a 1 mW solar unit on a clear day is 6.41 mWh. The probability of output being less than 4 mWh is 19.1% and 54.8% being above 6.3 mWh. Figure 12b shows the daily output for 3 solar units dispersed in different locations.

**Figure 12b – Daily Output for Three Solar Units**

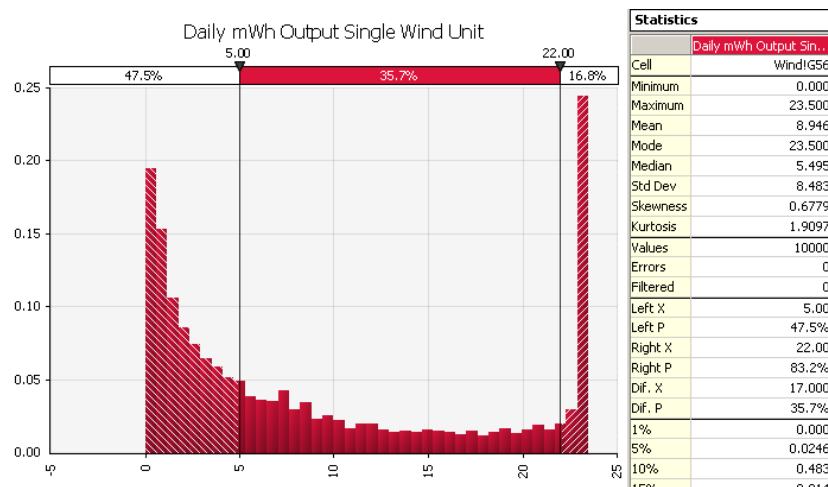


For three solar units widely dispersed, the probability of being less than 12 mWh, equivalent to 4 mWh for each unit in Figure 12a, is 13.5% down from 19.1%. The probability of being above 18.9 mWh outputs, equivalent to 6.3 mWh for each solar unit, is 22.4%, down from 54.8%. The risk of cloudy weather adversely affecting performance has been reduced by having different sites. The probability of output being at high levels has also been reduced, which is not an operational risk. The mean for both distributions are



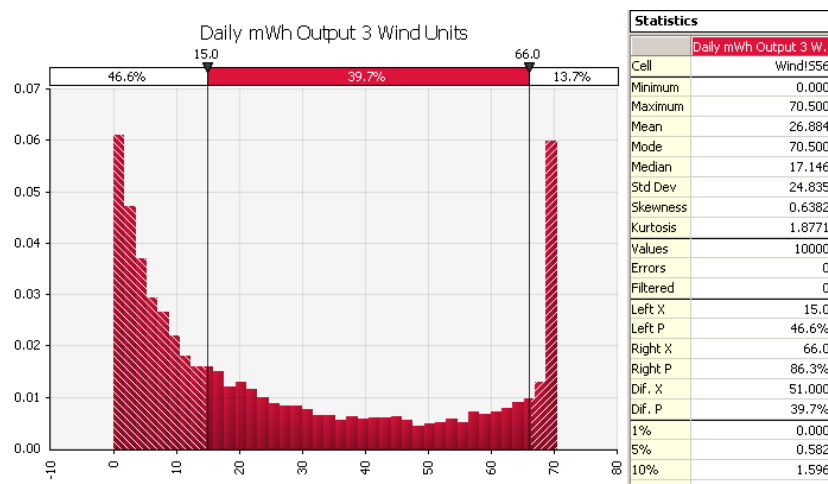
the same (three solar system has three times the energy output as the single solar system). In the aggregate both are delivering the same energy output but the risk of having low-powered days has been reduced by diversifying solar site. Figures 13a and 13b perform the same functions for wind as Figures 12a and 12b.

**Figure 13a – Daily Output for a Single Wind Unit**



The maximum output for a 1 mWh wind turbine over 24 hours is 24 mWh. The chance of output being less than 5 mWh is 47.5%, essentially the same in Figure 13b for being an equivalent 15 mWh for three wind units. This, again, is the consequence of linking the three wind units with a Cupola function with little dispersion when wind speed is below average. The chance of output being greater than 22 mWh for a single wind unit is 16.8% versus 13.7% for a three wind units being greater than 66 mWh.

**Figure 13b – Daily Output for Three Wind Units**



The mean output of the single wind unit is 8.95 mWh versus a possible 24 mWh or an average capacity output of 37% and an expected 26.88 mWh for three wind units with the same average capacity output.

## Combined Solar and Wind Units Output

	C	D	E
5		Solar&Wind	Solar&Wind
6		Power Output	Energy Output
7		mW	mWh
8	00:00-00:30	2.05	1.02
9	00:30-01:00	1.99	0.99
10	01:00-01:30	1.90	0.95
11	01:30-02:00	1.83	0.92
12	02:00-02:30	1.84	0.92

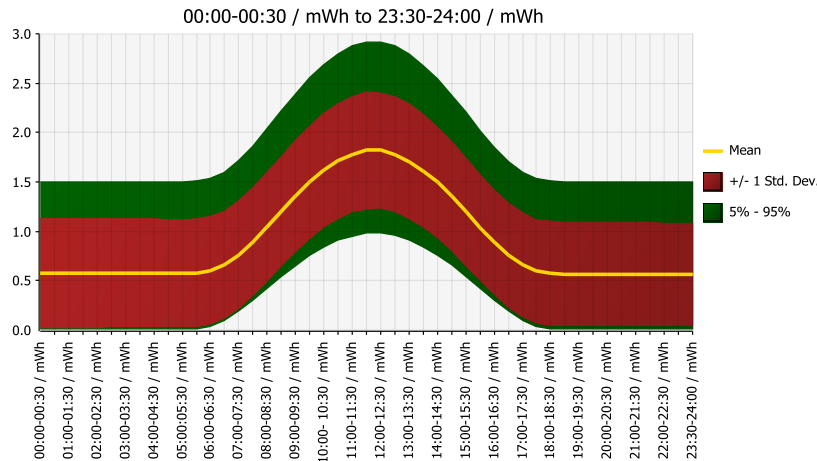
The Combined tab simply totals the output from the Solar and Wind tabs. Each half-hourly cell in column D represents three solar farms of 1 megawatt each and three wind farms of 1 megawatt each.

	B	C	D	E
56			mW	mWh
57	Total Solar & Wind Output		85.1	42.6
58				
59	Fossil Fuel Equivalent			102.6
60				
61	Relative Effectiveness			41%

Total of column D is daily power output in mW and total column E is daily electricity (energy) output in mWh. These cells along with each half-hourly cell, were assigned as output cells for a simulation of 10,000 iterations.

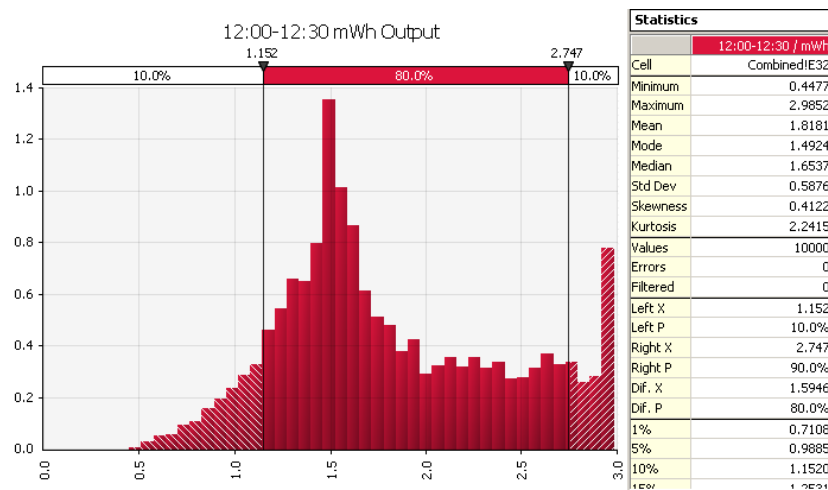
Figure 14 is the summary graph of each half-hour interval in terms of electricity output in mWh for a system consisting of three 1-megawatt solar units and three 1-megawatt wind units.

**Figure 14 – Summary Graph for mWh Per Half-Hour**



The maximum output in terms of megawatt-hours is 1.5 for both solar and wind units on a half-hourly basis. Wind output for the early and late hours of each day varies from a maximum of 1.5 mWh to a minimum of 0 mWh when the wind is calm at all three locations. Average output for wind of 0.57 mWh was also seen in Figure 13.b (daily mean of 26.884 mWh divided by 48 half-hour time intervals is 0.56 mWh per half hour interval) for three 1 mW wind units. System output increases during the day by the addition of solar energy. Figure 15 is the combined output for noon of both wind and solar at 12 noon, which has a maximum of 1.5 mWh for solar and 1.5 mWh for wind for a total of 3 mWh.

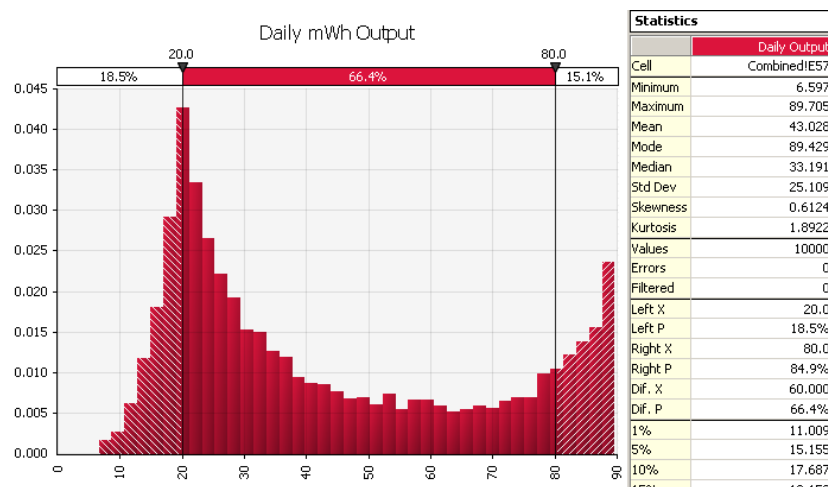
**Figure 15 – Combined Output Solar and Wind at Noon**



The maximum was 2.985 mWh, just shy of 3 mWh with a minimum of 0.4 mWh and a mean of 1.8 mWh. There is a 10% chance that the output would be over 2.747 mWh and a 10% chance it would be lower than 1.152 mWh.

On a daily basis, the maximum output for wind in mWh is 3 mWh/2 per half-hour intervals multiplied by 48 intervals or 72 mWh and for solar, let's say 6 hours at maximum output, or 3 mWh/2 multiplied by 12 half-hour intervals or 18 mWh for a total of 90 mWh. Figure 16 is the simulation results of a system of 3 mW of solar units and 3 mW of wind units on a daily basis.

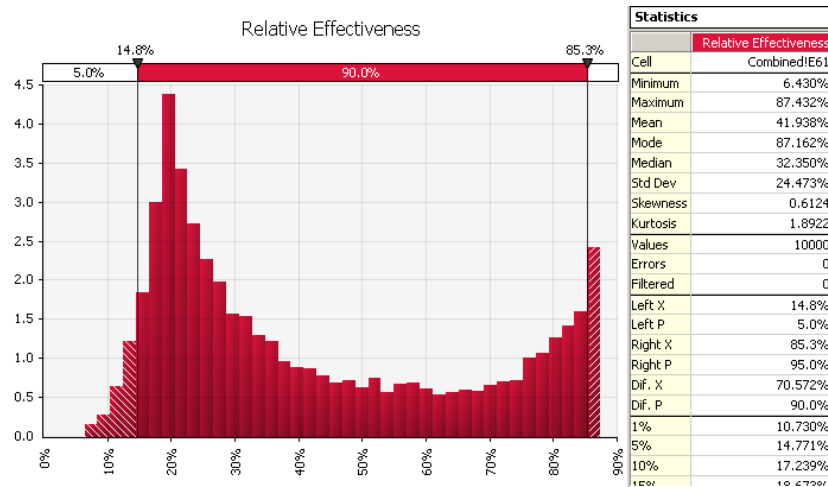
**Figure 16 – Daily mWh Output for a System of 3 mW Solar and 3 mW Wind Units**



The maximum in the simulation was 89.7 mWh and a minimum of 6.6 mWh. There is a 18.5% chance of electricity output being less than 20 mWh and 15.1% chance of being greater than 80 mWh – a significant variation of output posing an imposing challenge for dispatchers attempting to line up supply with demand.

Figure 17 is the relative efficiency of the wind and solar system compared to an equivalent 3 mW fossil fuel plant operating for 24 hours for base load plus another 3 mW fossil fuel plant operating for 12 hours for variable load.

**Figure 17 – Relative Effectiveness**



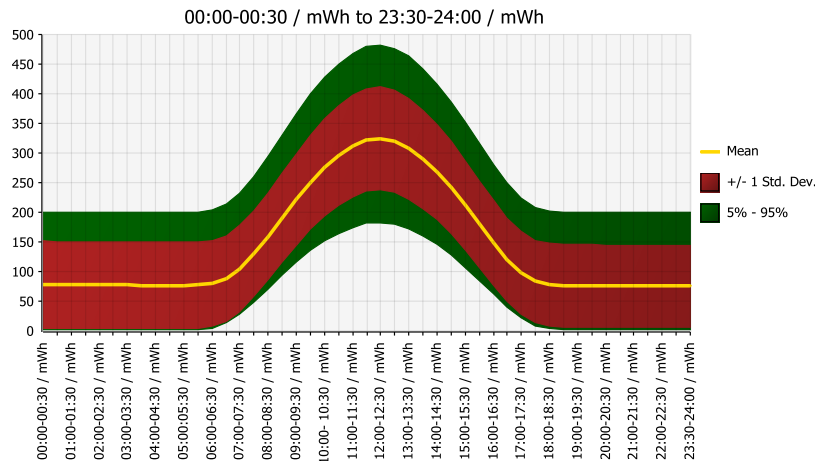
Five percent of the time the relative effectiveness is over 85.3% and less than 14.8% and 90% between these two limits. The mean is 41.9%, thus the rated capacity of the solar and wind system output has to be nearly 250% of the desired output to be confident that the system can produce the equivalent of a fossil fuel plant on an average basis. But even here, this “overcapacity” would still not be reliable in that the relative effectiveness of the solar and wind system has a median of 32.5% meaning that the relative effectiveness will be less than 41.9% over half the time.

## Section 2

### Analyzing the Output of Solar and Wind Farms

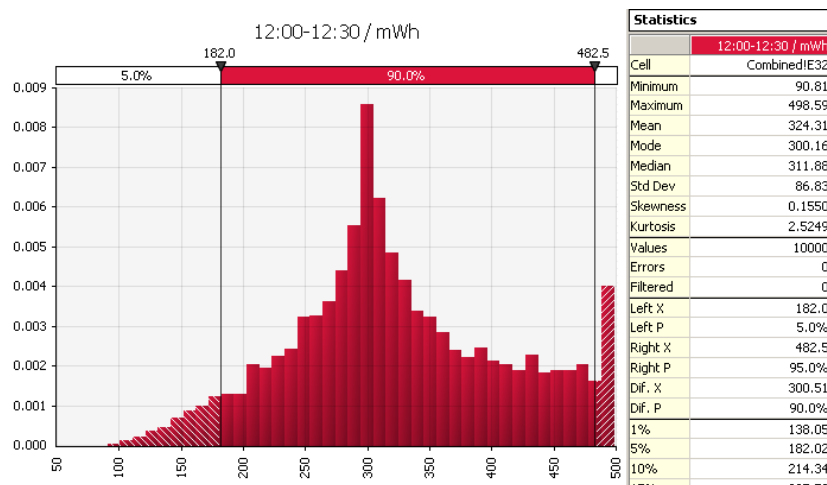
Suppose that a utility has 1 gigawatt (1000 megawatts) of renewable energy of which 600 megawatts are solar in three farms of 100, 200, 300 megawatts and 400 megawatts are wind at three farms of 100, 125, and 175 megawatts. Hence power capacity is 60% solar and 40% wind. Spreadsheet ReNew2 is a copy of ReNew1 with new outputs for the solar and wind farms. Figure 18 is the half-hourly summary performance of the solar and wind farms. The prominence of the day time contribution of solar power reflects its 60% contribution to renewable capacity.

**Figure 18 – Summary Graph for mWh Per Half-Hour**



There is a small probability that night time output can be essentially zero. The peak output at night of 200 mWh per half hour reflects the 400 mWh per hour total output of the wind farms. The peak of about 480 mWh per half hour at noon is less than the expected 500 mWh per half hour of wind and solar totaling 1000 mW. But the green band includes essentially two standard deviations (95% of all data). The discrete probability distribution for 12:00-12:30 in Figure 19 shows that 500 mWh was almost reached (maximum value of 498.59) and that the probability of being above 482.5 mWh per half hour was 5%.

**Figure 19 – Peak System Output at 12 Noon**



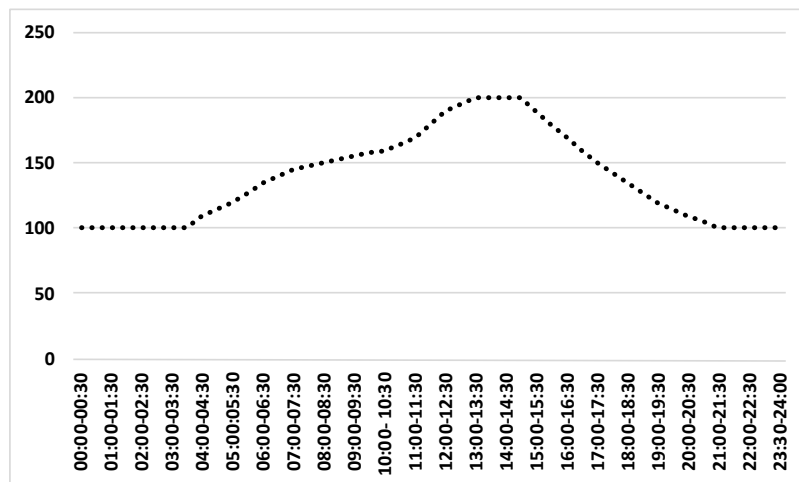
## Solar and Wind Farm System Performance

Section 10 of Energy Risk Modeling describes the use of beta function for mathematically describing the daily load of a utility in half-hour increments. The general formula for a beta distribution is:

$$y = k(x-a)\alpha(b-x)\beta$$

Figure 20 shows the actual load daily demand to be modeled, which can cover an entire year or apply for seasons or months. To ease modeling requirements to illustrate the methodology, it is assumed that this applies for the entire year on a daily basis, but seasonal adjustments will be made to reflect greater electricity demand for portions of the year.

**Figure 20 – Desired Daily Load Demand to be Modeled**

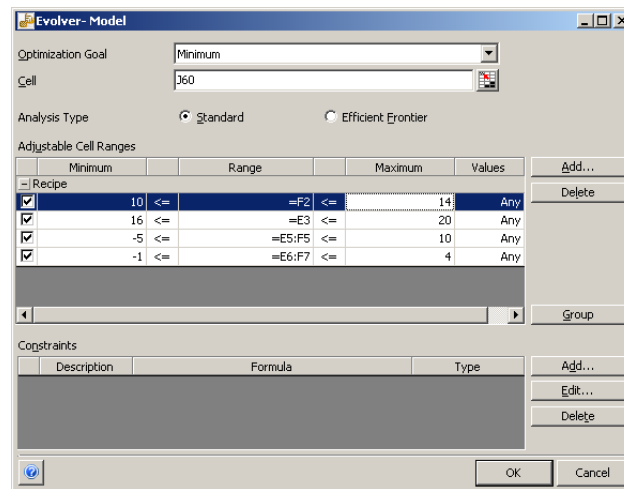


Two beta distributions are necessary to model the bimodal nature of daily demand. Each one needs start and end times. As seen below, the AM/Beta1 distribution starts at 4 am selected arbitrarily and an Evolver run determined the end time of 11.74 (11:44). The PM/Beta2 distribution starts at 17.85 (17:51 or 5:51 pm) as determined by an Evolver run and arbitrarily ends at 22 (10 pm). These are the “a” and “b” parameters in the beta general formula where x values (time) is limited between these minima and maxima.

	D	E	F
1	Time	AM Beta	PM Beta
2	Start	4	11.74
3	End	17.85	22
4		Beta 1	Beta 2
5	Constant	0.76	0.98
6	Alpha	1.26	1.00
7	Beta	1.00	1.47

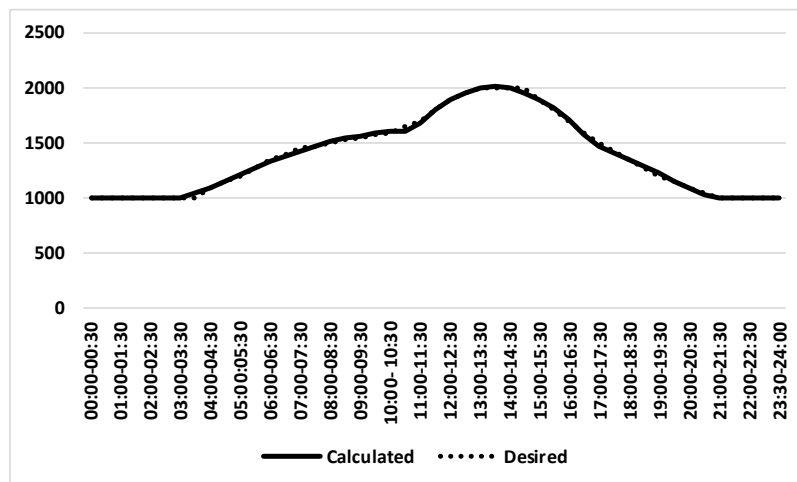
$\alpha$  and  $\beta$  values in the beta general distributions must have values greater than -1. For 100 as a scalar factor denoting the base load and 200 for the peak load occurring at the time indicated in Figure 20, profile beta distribution parameters to the left were developed with the Evolver menu set up in Figure 21.

**Figure 21 – Evolver Menu for Determining Parametric Values for the Two Beta Distributions**



Referring to spreadsheet Renew3 (same as Excel spreadsheet Section 10 of *Energy Risk Modeling*), Evolver is to minimize cell J60, which is the square of the differences between the data points and the generated beta distribution. To achieve this objective, the variables to be adjusted are in cells F2, the starting time of the second beta distribution and E3, the ending time of the first beta distribution. Other variables are the constant  $k$  and  $\alpha$  and  $\beta$  values associated with each beta distribution. The resulting curve was scaled by a factor of ten to yield Figure 22.

**Figure 22 – Daily Load Curve for a Utility**



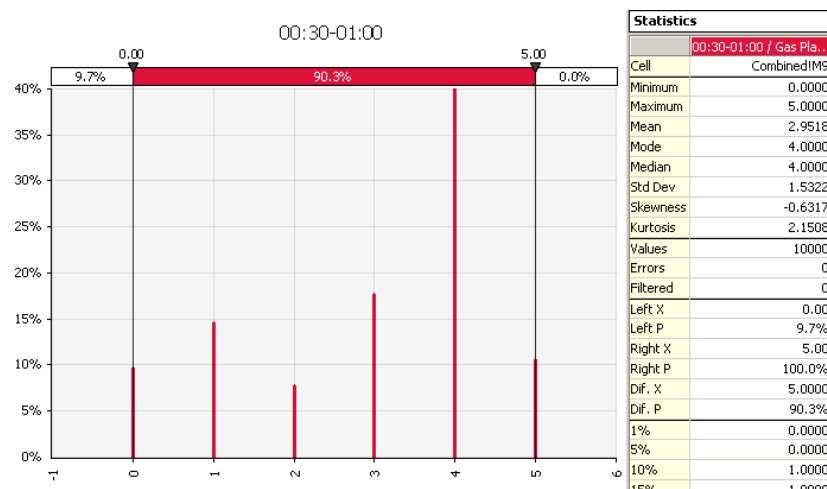
The dotted desired curve is barely observable in Figure 22 and shows how well Evolver performed in selecting parametric values for the two beta distributions to model the desired output. From experience, it is better to fashion the load curve using 100 as the base load and the appropriate peak load time or times as a factor of base load and then simply scale the results to model the desired load profile. Another peak load later in the day would require three beta distributions. Thus a set of daily load curves with a base load denoted by 100 and peak loads in terms of scalar factors of base load at appropriate times can be used to model profiles, which can then be scaled to fit the actual situation.

The question comes up on how well renewables would fare if combined with fossil fuel to satisfy the daily load curve. Referring to the Combined tab in spreadsheet Renew2, column G contains the calculated power demand in mW using Paste Values of the beta distributions derived in spreadsheet ReNew3. Column H introduces uncertainty in demand with variations +/- 5% of values in column G. With wind farm capacity at 400 mW and 600 mW of fossil fuel plants, superficially there is 1 gigawatt of capacity to meet load demand, but of course this is not the case in reality. Column I has fossil fuel/nuclear plants providing a contribution of 600 mW to base load with the remainder being night time wind power.

	C	D	E	F	G	H	I	J	K	L	M
3											# 100 mW
4									mWh		@50mWh
5		Solar&Wind	Solar&Wind				Base	mW	per half hr	Electricity	per half hr
6		Power Output	Energy Output			+/- 5%	Load	Renewable	Renewable	Output	Natural
7		mW	mWh/half hr		Demand	Demand	Fossil	Demand	Demand	Deficit	Gas Plants
8	00:00-00:30	268.89	134.44		1000	957	600	357	178	44	1
9	00:30-01:00	290.93	145.46		1000	1020	600	420	210	64	2
10	01:00-01:30	298.95	149.47		1000	995	600	395	197	48	1
11	01:30-02:00	305.09	152.54		1000	959	600	359	180	27	1

Column J is the difference in mW between column H and J to be filled by renewables. Column K is the megawatt-hour equivalent of column J. Column K translates column J to megawatt-hours on a half-hourly basis by dividing by two. Column L nets column K of the solar and wind output in column E. Any deficit in column L is satisfied by 100 mW natural gas plants whose half-hourly megawatt-hour output is 50. The number of natural gas plants is rounded up. The same result would have occurred by dealing in terms of power versus energy output. Figure 23 shows the number of 100 mW natural gas plants needed at night.

**Figure 23 – Number 100 mW Natural Gas Plants Required at Night**

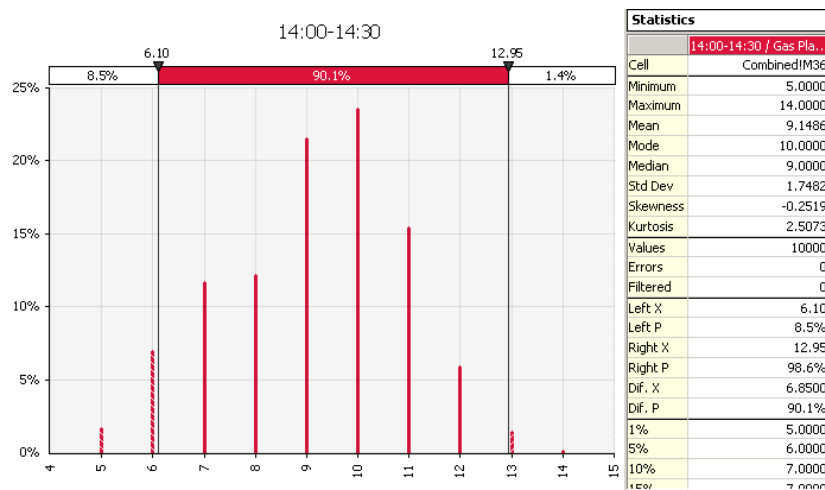


There is a 10% chance that none would be needed reflecting full output of the wind farm. There is a near-40% chance that four natural gas plants would be needed reflecting essentially no output from the 400 mW wind farm installation. The need for five is the result of rounding up on a small incremental demand above 400 mW. Thus system performance suggests that 100% fossil fuel backup is required for wind power to handle base load demand. This makes wind power rather expensive because one can simply rely on fossil fuel plants and dispense with the wind farm for base load demand!

Figure 24 is the peak time for demand for fossil fuel plants. It occurs at 2 pm when peak electricity demand occurs later than peak solar supply.

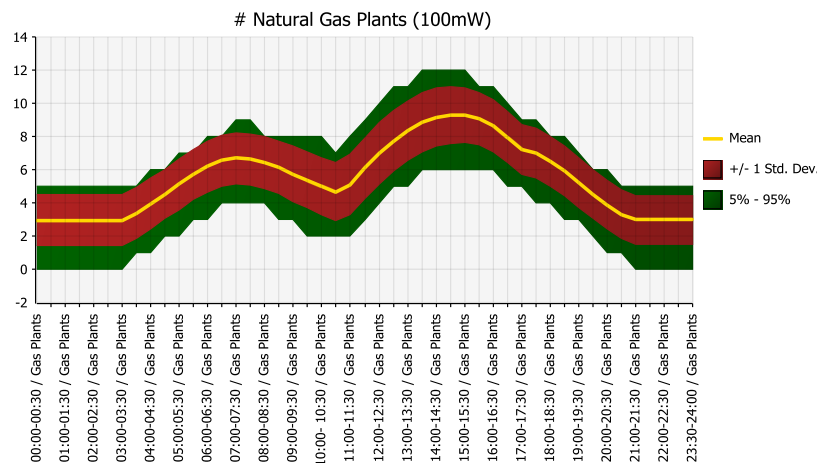


**Figure 24 – Number 100 mW Natural Gas Plants Required at Peak Demand**



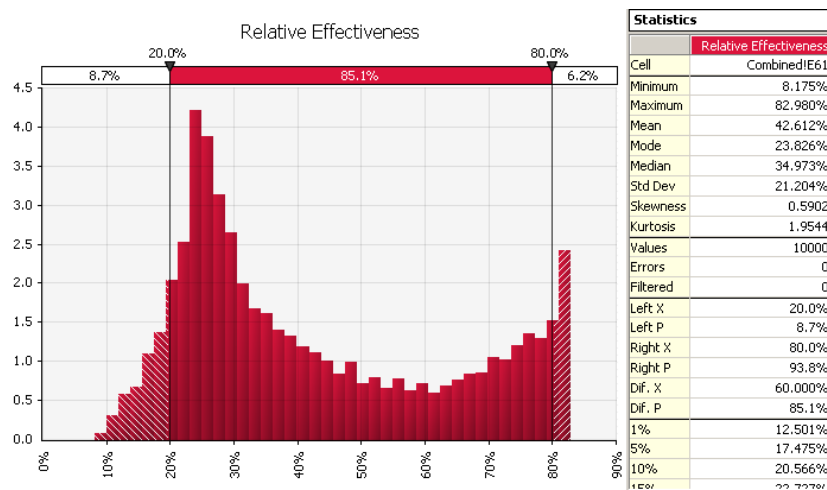
There is an 8.5% chance that 5 or 6 fossil fuel plants would be required with a 23% discrete probability of 10 plants being required read from the y-axis. During times of cloudiness and calm winds, 11 or 12 plants may be needed with a slim 1.4% chance that 13 or 14 plants may have to be on the line. Figure 25 is the summary chart showing the number of fossil fuel plants that would be necessary to ensure that supply meets demand.

**Figure 25 – Number of Natural Gas Plants Necessary to Assure Uninterrupted Service**



It is clear that peaking generators are necessary to meet demand for 2-3 hours between 1 and 4 pm. Peaking generators provide very expensive electricity because amortization of the investment is not over 24 hours per day for base load supplies of electricity or 12 hours for variable load over a 360-day operating year, but only 2-3 hours over a few weeks for a month or so during the peak season for electricity demand. This is a consequence of the poor relative performance of solar and wind farms vis-à-vis fossil fuel plants of equivalent capacity reflected in Figure 26 (output cell E61 in Combined tab of spreadsheet ReNew2).

**Figure 26 – Relative Effectiveness of Renewables to Comparably Sized Fossil Fuel Plants**

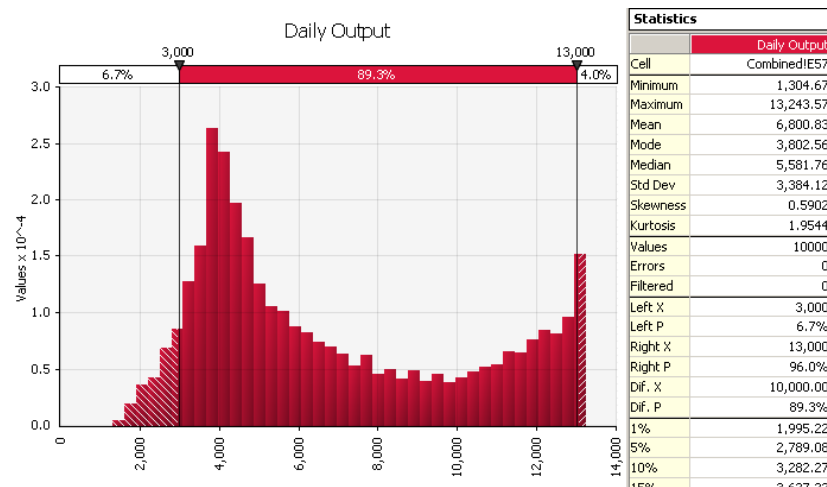


There is only a 6.2% chance that the output of the renewable plants will be 80% or higher of fossil fuel plants of equal capacity and an 8.7% chance that the output of the renewable plants will be 20% or lower of fossil fuel plants.

### ***Fitting a Distribution***

Daily output of the solar and wind farms is in Combined tab of spreadsheet ReNew2. Figure 27 is the results of a simulation with E57 as the output cell.

**Figure 27 – Daily mWh Output for a System of 3 Solar Farms and 3 Wind Farms**

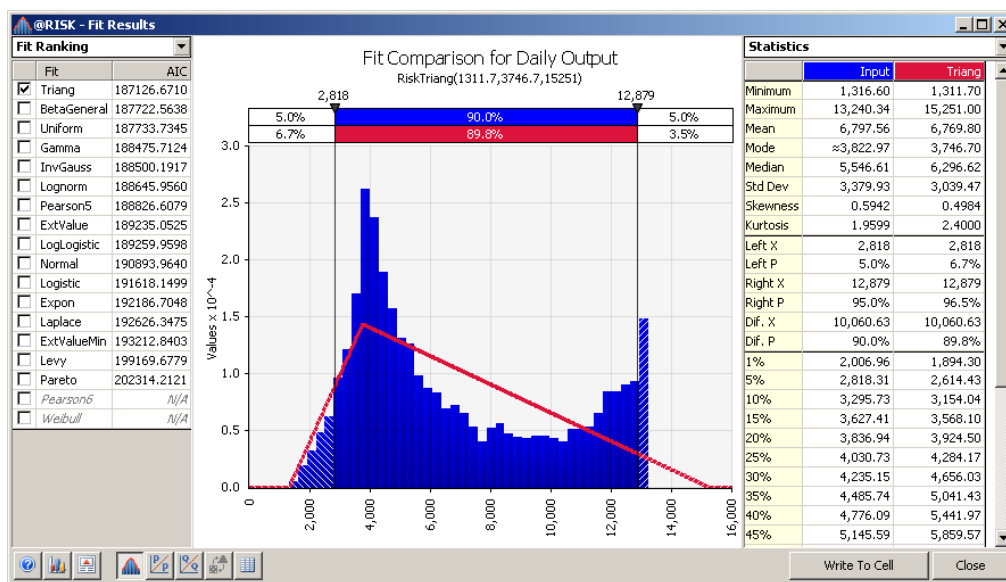


Total system capacity is 400 megawatts of wind plus 600 megawatts of solar. Fossil fuel equivalent megawatt-hour output would be 95% of 1,000 megawatts X 24 hours or 21,888 megawatt-hours (mWh). On a daily basis, the maximum for wind would be 400 mW for 24 hours or 9,600 mWh. Solar is probably near capacity for 6 hours a day or 600 mW X 6 hours or 3,600 mWh for a total of 13,200 mWh, a very close estimate to the actual maximum of 13,243 mWh. The daily average of 6,800 mWh is only 31% of an equivalent sized fossil fuel plant and the maximum output of 13,243 mWh is only 60% of the sustained

output of fossil fuel plant. Increasing the portion of wind power would increase average output because wind power is a 24-hour per day phenomenon whereas solar power is available at equivalent full power for only about 6 hours a day. The problem with wind power is that night time output has limited value when utilities normally cover their entire base load with dependable fossil, nuclear, and hydro sources. Wind can only become a more desirable choice if night time output can be stored for daytime dispensing.

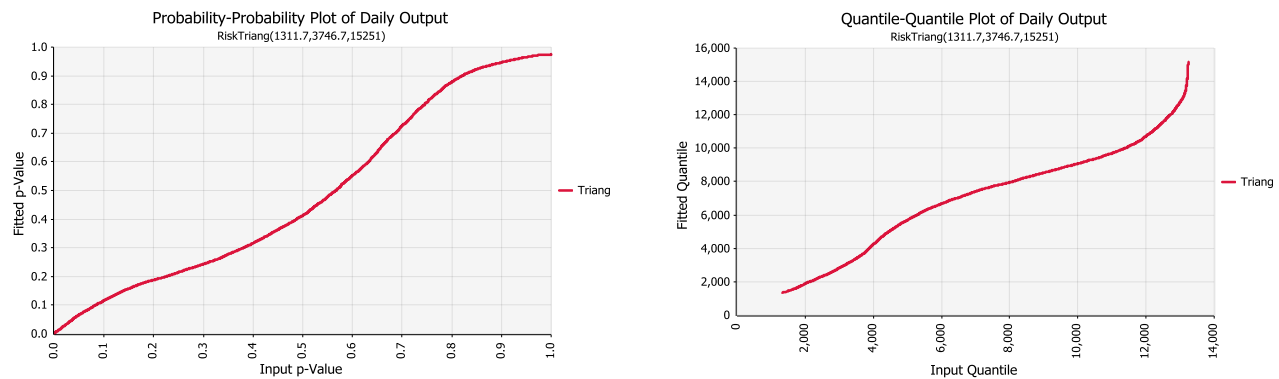
Section 6 in *Energy Risk Modeling* describes the process of obtaining a series of best fitting curves by segmenting data. Figure 28 is the first attempt to model the probability distribution for daily output shown in Figure 27 using the @RISK Fit Comparison feature invoked by pressing the fourth icon from the left. The fit rankings of other probability distributions are listed to the left and these can be selected for visual comparisons. Statistical comparison data is provided on the right. The selected fit comparison formula shown in the heading to the chart can be written directly to an Excel cell using the Write to Cell icon at the bottom right of the chart.

**Figure 28 – Fit Comparison for Daily Output for Renewable Supply**



Clearly a triangle distribution is not desirable for modeling purposes. Besides visual inspection, @RISK provides two means to determine the quality of fit for a selected distribution: the P-P and Q-Q graphs. P-P (Probability-Probability) graph plots the probability of a given value of the fitted distribution versus the probability of the same value in the input data. If the fit is good, the plot will be nearly linear. Q-Q (Quantile-Quantile) graph plots the percentile values of the fitted distribution versus the percentile values of the input data. One can look at these as a series of buckets containing data points for a given segment. If the number of data points in the buckets for the best fitting curve and the actual data in each segment are about the same, then the resulting Q-Q curve will also be nearly linear. Figure 29 shows that the P-P and Q-Q curves are far from linear indicating a poor fit between the triangle distribution and the simulation output. Note that the axes of the P-P curve are in terms of probabilities between the fitted and actual data whereas the axes of the Q-Q curve are in terms of the range between minimum and maximum values of fitted and actual data.

**Figure 29 – P-P and Q-Q Curves for the Best Fitting Triangle Distribution**



To attempt to get a better fit using multiple best fitting curves, the Simulation Data icon was used to copy and paste the data in output cell E57 for a simulation of 10,000 iterations to tab BestFit in spreadsheet ReNew2. BestFit tab was set up for obtaining four best fitting curves, but here, only three are used.

	A	B	C	D	E	F
1						
2						Total
3	Count	3533	3847	2620	0	10000
4	Dis Probability	35.3%	38.5%	26.2%	0.0%	100.0%
5	Cum Probability	35.3%	73.8%	100.0%	100.0%	
6						
7						
8	Daily Output	Less than	Between		Greater than	
9	Output/ReNew2.xlsx	4,500	4,500	9,500	15,000	
10	CombinedIE57		9,500	15,000		
11	4388	4388				
12	3940	3940				
13	12606			12606		
14	5598		5598			
15	11823			11823		
16	2510	2510				
17	4405	4405				
18	4030	4030				
19	3179	3179				
20	5698		5698			
21	9224		9224			
22	12994			12994		

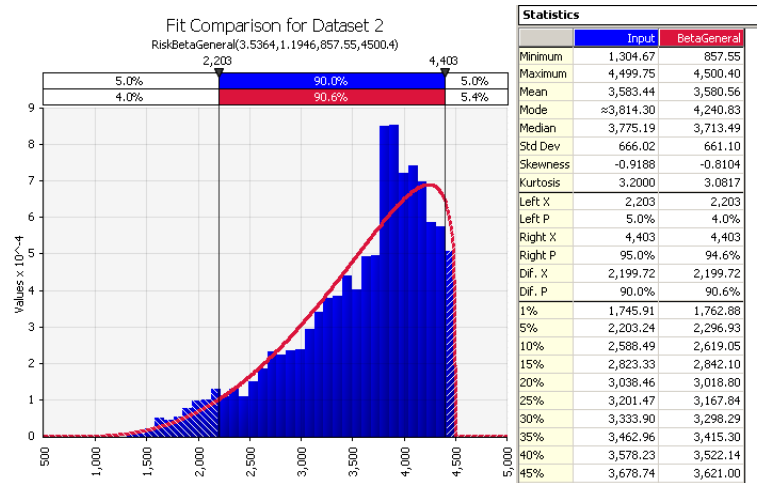
Column A is the simulation data and columns B, C, D, and E divide the data into four segments where the first segment is all values less than 4,500, the second between 4,500 and 9,500, and the third values between 9,500 and 15,000. The fourth is all values above 15,000. As the maximum value is 13,244 in Figure 30, this last segment will be empty of contents. The selected segment values are the end result of experimentation to obtain the best fitting curves. Row 3 is the count of the entries in the various segments, row 4 the discrete probabilities and row 5 the cumulative probabilities with totals in column F.

Formulas in row 11 assign simulation results to the appropriate segment:

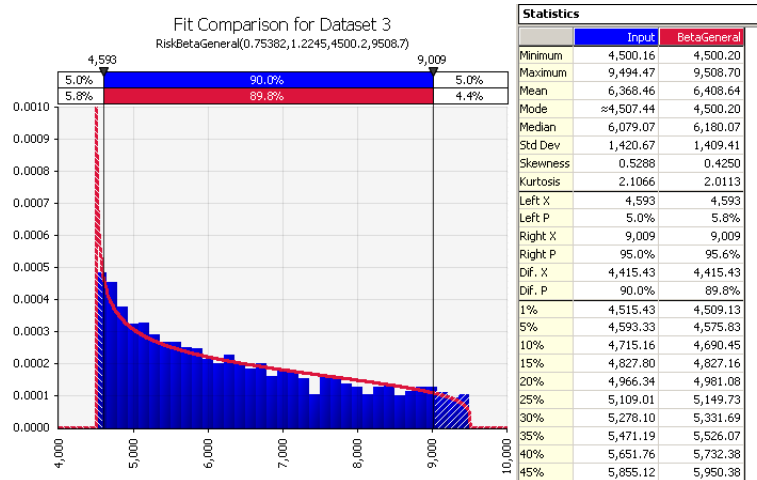
B11: =IF(A11<\$B\$9,A11,"")	C11: =IF(AND(A11>=\$C\$9,A11<=\$C\$10),A11,"")
D11: =IF(AND(A11>\$D\$9,A11<=\$D\$10),A11,"")	E11: =IF(A11>\$E\$9,A11,"")

Best fitting curves are obtained for each segment in Figures 30-32. The appropriate best fitting curve formulas are listed within each chart and these can be written directly to a cell address.

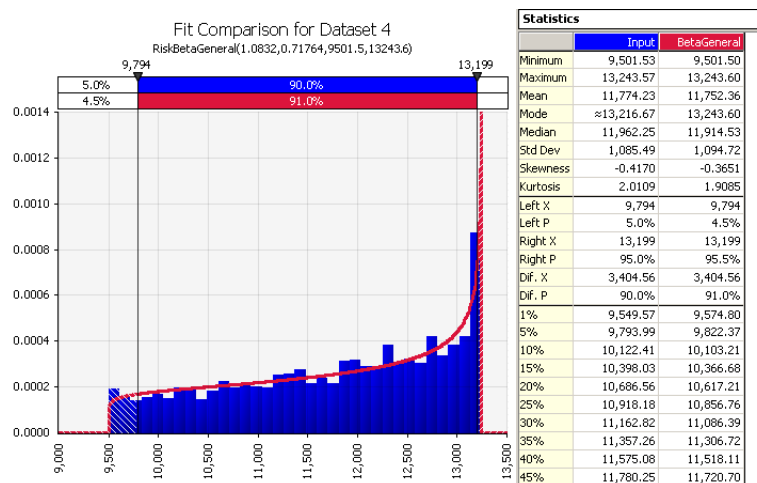
**Figure 30 – Best Fitting Distribution Function for Values Less Than 4,500**



**Figure 31 – Best Fitting Distribution Function for Values Between 4,500 and 9,500**



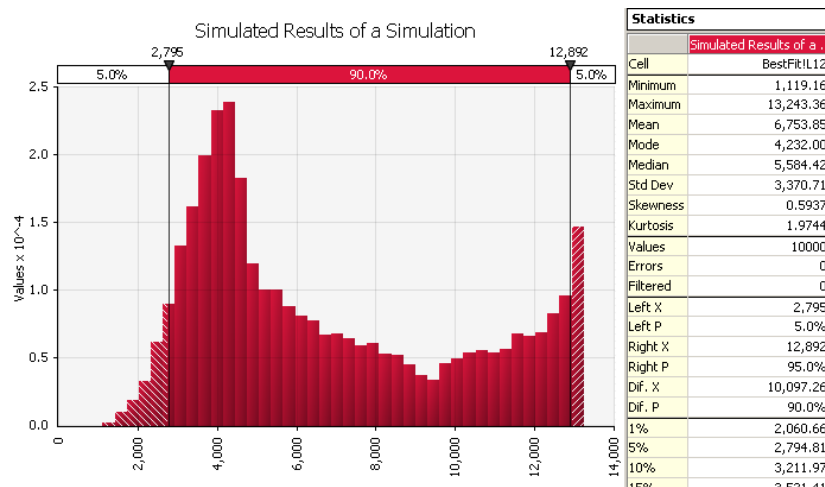
**Figure 32 – Best Fitting Distribution Function for Values Between 9,500 and 15,000**



	H	I	J	K	L	M
5					Random #	
6					0.205195	
7			Less than	4,500	1	4114
8	Between	4,500	and	9,500	0	7762
9	Between	9,500	and	15,000	0	9909
10			Greater than	15,000	0	
11						
12						4114
13						
14				Adjusted		4114

Cell L6 draws a random number and using the previously shown cumulative probability values in row 5, a “1” is assigned to the appropriate segment. Column M contains the best fitting distributions listed at the top of Figures 30-32 using the Write to Cell function. Cell M12 is the sumproduct of cells L7 to L9 and cells M7 to M9. Figure 33 is the simulated results of a simulation contained in cell M12.

**Figure 33 – Simulated Results of a Simulation**



The following table compares the actual simulation results in Figure 27 with the derived simulated results in Figure 33 incorporating the best fitting distribution functions depicted in Figures 30-32.

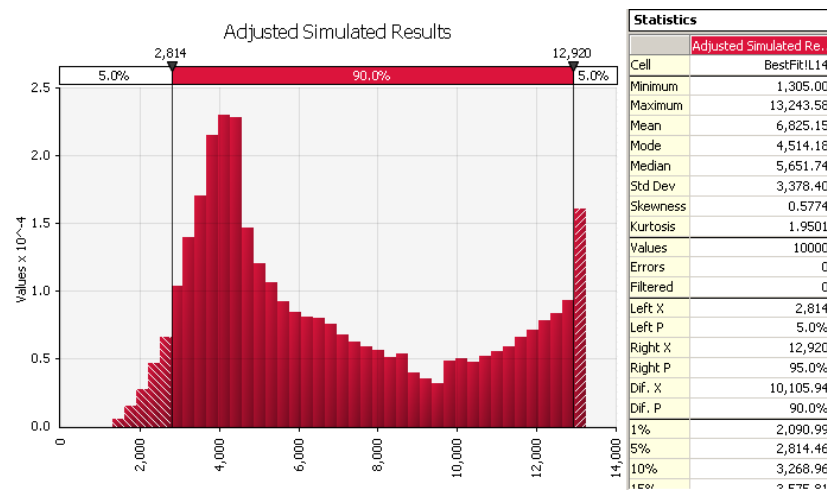
Parameter	Actual Simulation Results	Simulated Results	Adjusted Simulated Results
Minimum	1,305	1,119	1,305
Maximum	13,244	13,243	13,214
Mean	6,800	6,753	6,825
Mode	3,803	4,232	4,514
Median	5,582	5,584	5,652
Std Dev	3,384	3,370	3,378
Skewness	0.59	0.59	0.58
Kurtosis	1.95	1.97	1.95

Other than the minimum, the values of the various parameters in Figures 28 and 34 are reasonably close. The adjusted simulation results in cell L14 forces a minimum value of 1,305:

=RiskOutput("Adjusted Simulated Results")+MAX(1305,L12)

Figure 34 is the chart of the adjusted simulated results summarized in the right column of the above table.

**Figure 34 – Adjusted Simulated Results**



The adjusted simulated results in Figure 34 can be a substitute for the daily output of a system of 600 mW of wind farm capacity and 400 mW of solar farm capacity. It may be that using other sized segments or changing the number of segments would provide a better model, but the methodology described herein remains the same.

## Section 3

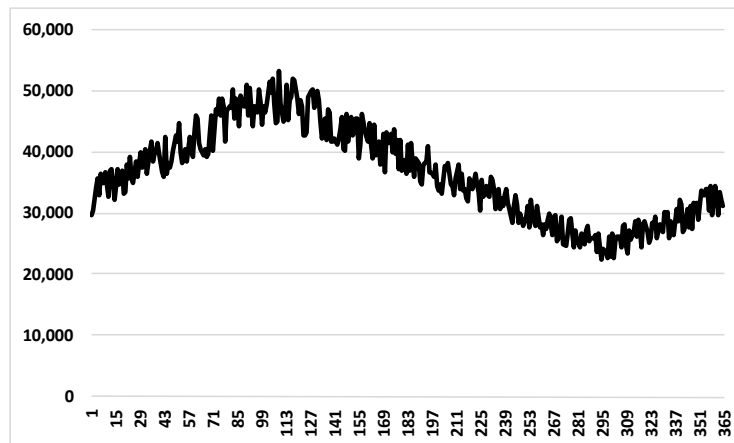
### System Performance

Referring to spreadsheet ReNew4, an adapted version appearing in Section 6 of *Energy Risk Modeling*:

	A	B	C	D	E	F	G	H	I	J	K	L	M
1				Stochastic									
2		Deterministic		+/-5%				Between					
3		Daily	Seasonal	Seasonal				4500 &					
4		Demand	Demand	Demand				9500					
5	Day	mWh	Factor	mWh	Random #			<=4500	9500	>9500			
6	1	32612	1.000	33592	0.364948			35.3%	73.8%	100.0%			
7	2	32612	1.005	31961	0.159031			1	0	0		3746	7773
8	3	32612	1.010	34251	0.448692			0	1	0		5878	11974
9	4	32612	1.015	32573	0.353847			0	1	0		4045	5516
10	5	32612	1.020	32400	0.090678			1	0	0		2722	7111
												12159	9889

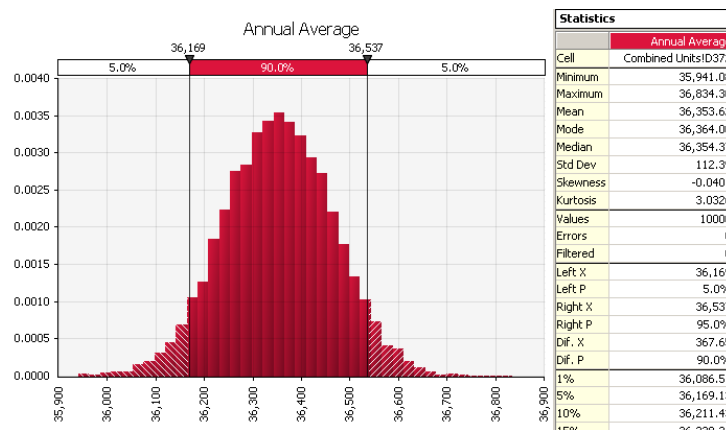
Column A covers a 365-day year, column B of 32,612 megawatt-hours is daily expected demand generated by the two beta general distributions, column C is a seasonal factor, and column D applies the seasonal factor to the values in column B with a daily variance of +/-10% illustrated in Figure 35.

**Figure 35 – Seasonally Adjusted Daily Demand in mWh**



Daily demand cycles between 25,000 and 50,000 mWh per day with an average annual daily demand (cell D372) of 36,400 mWh with a maximum of 36,800 and a minimum of 36,000 as depicted in Figure 36.

**Figure 36 – Average Annual Daily Demand**





	F	G	H	I	J	K	L	M	N	O	P
1											
2		Between									
3		4500 &									
4		<=4500	9500	>9500						Adjusted	
5	Random #	35.3%	73.8%	100.0%						Output	Output
6	0.675376	0	1	0		2868	5273	11083		Per Unit	Per Unit
7	0.135199	1	0	0		3649	5085	11972		5273	5273
8	0.062563	1	0	0		3842	8561	13078		3649	3649
9	0.988913	0	0	1		3379	5267	11613		3842	3842
10	0.414952	0	1	0		3379	5267	11613		11613	11613
11	0.471122	0	1	0		3382	7776	12738		7776	7776
12	0.573788	0	1	0		3547	4565	10822		4565	4565
13	0.528061	0	1	0		3667	5412	12530		5412	5412
						3203	6515	10786		6515	6515

For each day of the year a random number is drawn (column F) and a “1” is assigned to a specific segment of values in columns G-I as per the cumulative probabilities derived in the BestFit tab of spreadsheet ReNew2. Columns K-M are the best fitting probability distributions for each segment from the BestFit tab. Column O is the sumproduct of columns G-I and columns K-M and column P is the adjusted output contained in cell L14 of the BestFit tab.

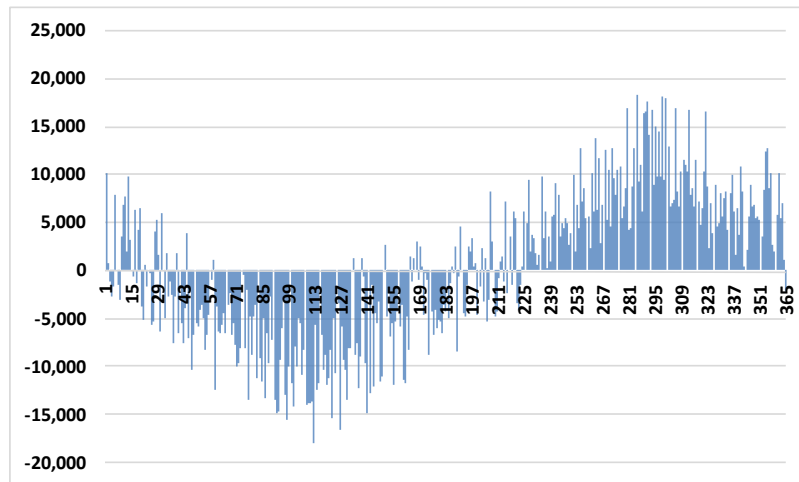
	Q	R	S	T	U	V	W	X	Y	Z
1	1 Gigawatt	Number								
2	Fossil Fuel	100 mW								
3	Base Load	NatGas	Total	Total					Shortfall	1,278,050
4	Supply	Plants	Production	Demand		mWh	mWh		Excess	1,288,494
5	mWh	3	mWh	mWh	Difference	Shortfall	Excess		Balance	10,444
6	22800	6840	35163	32482	2681		2681			
7	22800	6840	32904	35065	-2162	2162				
8	22800	6840	34216	33079	1137		1137			
9	22800	6840	33390	34537	-1147	1147				
10	22800	6840	32642	34819	-2177	2177				
11	22800	6840	34006	35020	-1014	1014				

Column Q is the 1 gW base load fossil fuel plants (probably coal, but could include nuclear) operating at 95% utilization, 24 hours/day, for 360 days per year (5 days for planned maintenance). It is necessary to have additional 100 mW natural gas plants that will run as needed and are set up in column R. Natural gas plants run the same as base load plants for preliminary planning purposes. Column S is the total of fossil fuel and renewables, column T is the same as column D, column U is the difference between electricity generation and consumption whereas column V keeps track of electricity shortfalls and column W excess electricity generation.

The need for additional natural gas plants (cell R5) is necessary in order to have a positive balance between electricity generation and consumption on an annual basis. This is necessary either to keep a storage battery adequately charged or a gravity battery with sufficient water in the upper reservoir. Figure 37 shows the times of negative and positive generation of electricity with regard to consumption with three 100 mW natural gas plants.

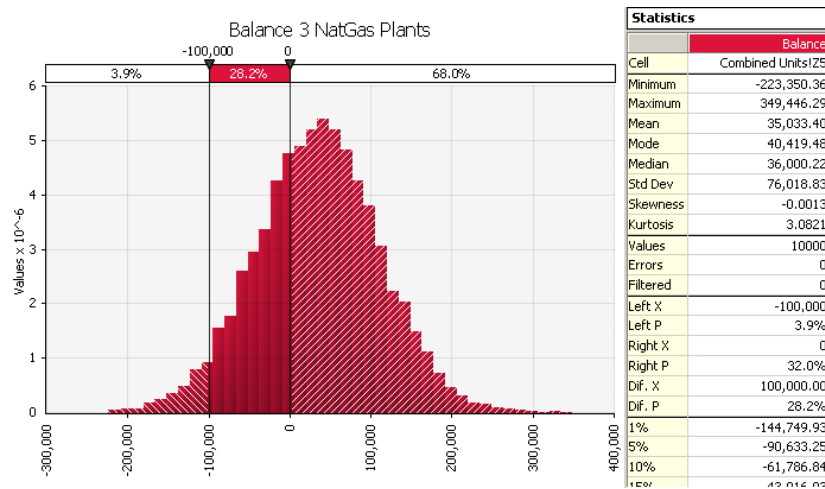
There is a net shortfall in electricity generation during the first half of the year with net excess generation during the second half of the year. From the point of view of a utility sized storage battery, its capacity has to be large enough to satisfy the shortfalls starting from a fully charged state at the beginning of the year. Excess electricity during the second half of the year has to be sufficient to recharge the storage battery in anticipation of the next annual shortfall. For a gravity battery, the volume of water in the upper reservoir has to be sufficient to supply shortfalls during the first half of the year with water in the upper reservoir restored during the second half of the year when excess electricity is available.

**Figure 37 – Shortfalls and Excess Generation of Electricity with 3 NatGas Plants**



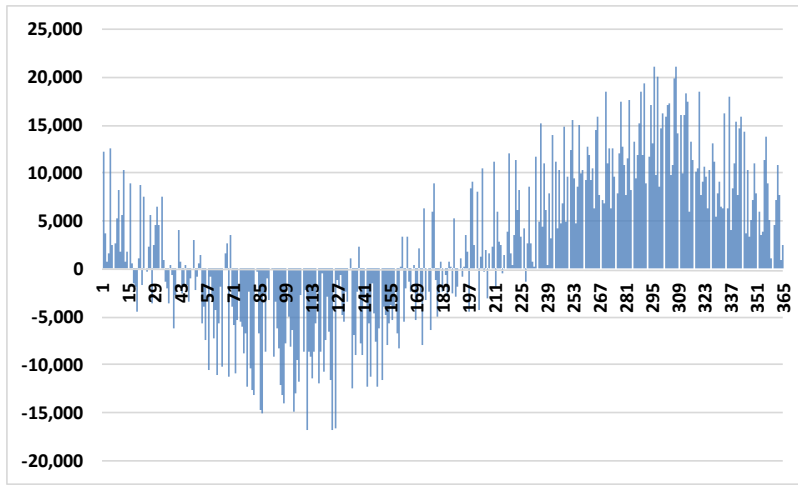
The annual shortfall and excess generation are in cells Z3 and Z4 with the difference in cell Z5. Cell Z5 must be positive on balance to prevent a storage battery from becoming depleted or the upper reservoir becoming dry. Figure 38 reflects the nature of annual balances for three natural gas plants.

**Figure 38 – Annual Average Balance for 3 NatGas Plants**



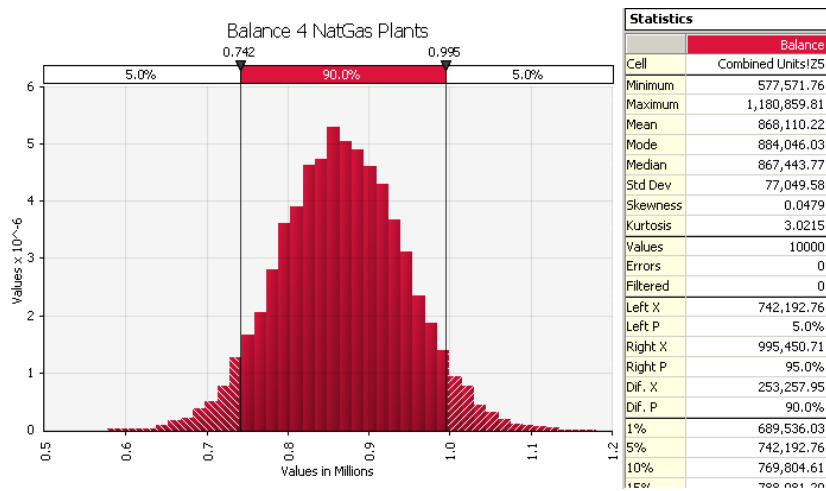
While the mean is positive at 35,000 mWh, there is a 32% chance that the balance will be negative meaning that the storage battery may be fully discharged or the water in the upper reservoir of a gravity battery completely drained. Figures 39 and 40 are the corresponding charts for 4 natural gas plants.

**Figure 39 – Shortfalls and Excess Generation of Electricity with 4 NatGas Plants**



Comparing Figure 39 with Figure 37 shows less shortfalls and more excess electricity being generated with four natural gas plants. Figure 40 has zero probability of having a shortfall for the annual average balance and a larger excess electricity balance between having 3 or 4 natural gas plants. Thus somewhere between 3 and 4 natural gas plants of 100 mW are necessary to maintain a positive annual average balance.

**Figure 40 – Annual Average Balance for 4 NatGas Plants**



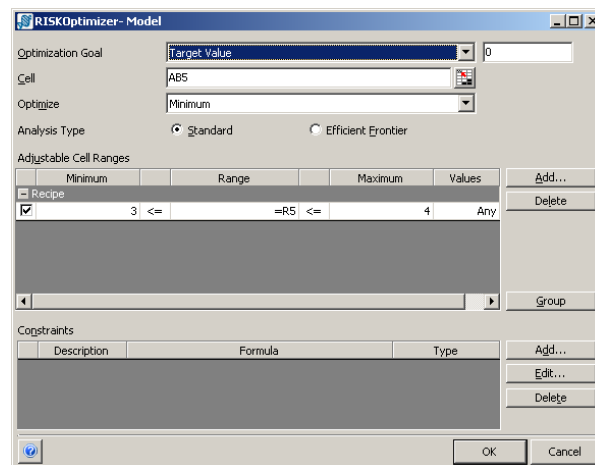
## Determining Optimal Number Natural Gas Plants

Although in this case, trial and error with different number of natural gas plants could determine which choice minimizes the chances of having a negative balance, this situation can also be addressed by RISKOptimizer. RISKOptimizer would have to be used if there were two or more variables that would make trial and error exceedingly time consuming and tedious. Evolver is used when variables have discrete values while RISKOptimizer handles variables that are probability distributions. One correction to be made is that electricity stored and discharged is not equal in the aggregate to the input. There are inefficiencies associated with electricity batteries. Suppose that 80% of the electricity stored in a gravity battery is retrievable (an electricity battery would probably have a higher efficiency).

	Y	Z	AA	AB
1			Inefficiency	
2		W/O Factor	Factor	With Factor
3	Shortfall	1,151,270		1,151,270
4	Excess	1,443,018	0.8	1,154,414
5	Balance	291,747		3,144

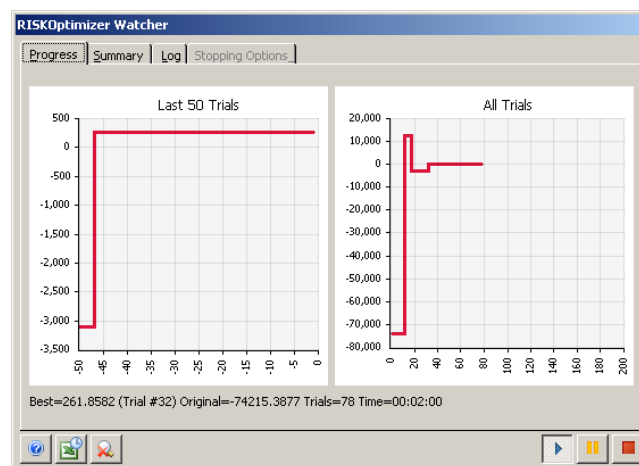
Cell AB4 reduces excess generation by 80% to take into account the inefficiency or energy lost in a gravity battery both in pumping water to the upper reservoir and in generating electricity by reverse flow. Figure 41 designates cell AB5 as the target cell whose minimum value is to be set to zero by varying the number of natural gas plants in cell R5 between 3 and 4.

Figure 41 – RISKOptimizer Menu



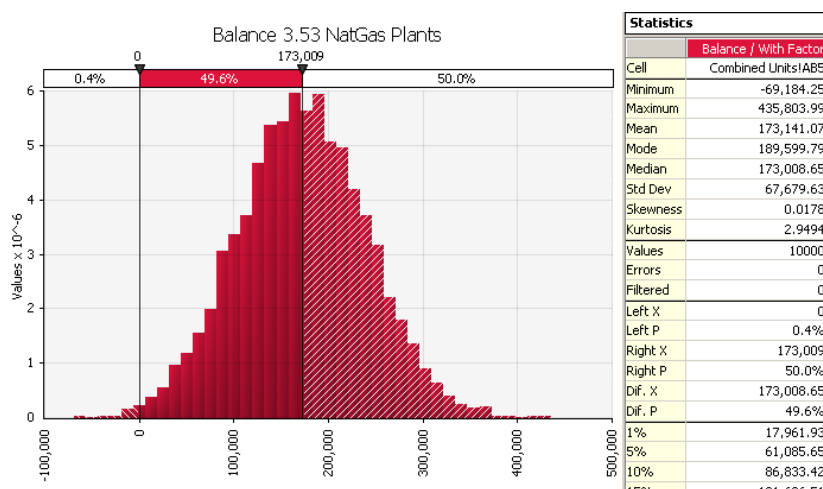
RISKOptimizer Watcher in Figure 42 keeps track of the progress being made to achieve a target value of zero.

Figure 42 – RISKOptimizer Watcher



The number of iterations for RISKOptimizer trial runs was set at 100 iterations on the @RISK ribbon. RISKOptimizer was run until there were no further improvements in achieving a target value of zero. The optimal number of natural gas plants is 3.53 meaning that 353,000 mW of capacity would be sufficient to contain the minimum balance at or above zero. Figure 43 is the ending balance with 3.53 natural gas plants.

**Figure 43 – Annual Average Balance for 3.53 NatGas Plants**



The chance of a negative balance for 3.53, or 353,000 mW of natural gas plant capacity, is 0.4%. Rounding up to 3.6, or 360,000 mW of natural gas plants would reduce the chance of a negative balance to 0.1% but with an increase in the mean to 221,300 from the above 173,000, or presumably an increase in the capital cost for sufficient storage capacity of 28%. This may represent a large incremental capital cost to achieve a modest decline in the probability of a negative balance. Just to complete the picture, the situation 3.5 natural gas plants is a risk of 1.5% of a negative balance with a mean of 147,700, a diminution of capital costs. This interplay of risk versus cost of risk mitigation via various sized storage batteries can be analyzed through successive @RISK simulation runs. If purchase of outside sources of electricity were possible with the requisite reliability, then a higher risk of a negative balance can be tolerated. In theory, if sales and purchases were always available to compensate or absorb for a deficit or surplus of electricity generation, then no battery would be necessary. In order to illustrate a methodology for sizing a storage battery, either electricity or gravity, it is assumed that opportunities for buying and selling electricity are limited.

### ***Thoughts on Sizing a Storage Battery***

One could take the position that having sufficient electricity storage to cover 100% wind capacity during the night might be a first cut on estimating storage. The stored electricity would be expended during the day when load exceeds fossil fuel and renewables capacity. Battery capacity needed would be 400 mW of wind power stored for 12 hours during night time. Suppose that 5 mW wind turbines are used and an installed cost of a \$2,000 per kW. Thus a 5mW wind turbine would cost 5,000 kW X \$2,000/kW or \$10 million.<sup>2</sup>

<sup>2</sup> *Renewables Energy Technologies: Cost Analysis Series*, International Renewable Energy Agency, Web site [www.irena.org/documentdownloads/publications/re\\_technologies\\_cost\\_analysis-wind\\_power.pdf](http://www.irena.org/documentdownloads/publications/re_technologies_cost_analysis-wind_power.pdf).

Suppose that a 5 mW wind turbine can be made 100% reliable by having 5 mW of battery capacity for 12 hours or 60 mWh, which would then be available for discharge during the day. A commercial sized lithium battery currently costs about \$500 per kWh, but is expected to fall to \$200, and possibly as low as \$150 per kWh.<sup>3</sup> Taking the lower estimate of \$150 per kWh, the cost of 60 mWh of battery capacity would be 60,000 kWh X \$150 per kWh or \$9 million, roughly in line with the installed cost of the wind turbine. While wind turbines are considered to be competitive with fossil fuel plants, including a storage battery to assure reliability, clearly makes wind turbines noncompetitive with fossil fuel plants. In today's world with battery costs double to triple this level, a totally reliable wind turbine becomes economically prohibitive.

If we rule out electricity batteries for now until further technological advances are made in reducing costs, the first question to be addressed whether the required pumping power for a pumped storage facility to meet maximum wind turbine output of 400 mW is available. With 1,341 horsepower in a megawatt, the required lifting horsepower for 400 mW would be 536,400 horsepower. There are wind tunnel AC motors built in excess of 60,000 hp.<sup>4</sup> With regard to electric powered pumping stations to move water, Hitachi commissioned in 2004 the world's largest lift pumping plants, the Edmonston Pumping Plant, to pump water vertically 1,926 feet from the California Aqueduct across the Tehachapi Mountain Range to connect northern and southern California. The massive pumps are powered by 80,000 horsepower (60,000 kW) synchronous motors.<sup>5</sup> Thus there is sufficiently sized electric motors to support a gravity battery, but the cost will be significant. Of course multiple motors of a different horsepower may be selected depending on the economics of the situation.

Four hundred mW for 12 hours would require 4,800 mWh of electricity storage capacity of electricity or gravity storage. But a look at Figure 42 shows peak excess electricity output of 20,000 mWh, far in excess of the initial scoping of 4,800 mWh. Moreover 20,000 mWh over a 24 hour period implies an average power output of 833 mW, over twice that initially contemplated. The reason for this is the highly variable nature of seasonal demand depicted in Figure 38 which cycles between 25,000 and 50,000 mWh on a daily basis over the course of a year. Peak excess electricity generation occurs during times of low electricity demand with near-maximum output of renewable energy. Peak shortfalls occur during times of high electricity demand with near-minimum output of renewable energy. While a non-negative balance is necessary to ensure reliable performance, the unintended consequence is that the system is generating too much electricity for a lengthy duration over the course of a year. An electricity or gravity battery would have to be large to store this amount of electricity. It is necessary that the natural gas plants, originally set up as though they were base load plants, become variable load plants where at times they operate at full capacity, but other times are cut back to better balance supply and demand. This is necessary to reduce the size of a storage battery while maintaining a sufficient backup in storage to meet any potential shortfalls.

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<sup>3</sup> "Energy Storage Could Reach Big Breakthrough Price Within Five Years," CleanTechnica Web site <http://cleantechnica.com/2015/03/04/energy-storage-could-reach-cost-holy-grail-within-5-years>.

<sup>4</sup> World's Largest AC Motors, Web site [www.eng-tips.com/viewthread.cfm?qid=179877](http://www.eng-tips.com/viewthread.cfm?qid=179877).

<sup>5</sup> "Hitachi's Massive Pumps Supports the World's Water Environment," Hitachi Web site [www.hitachi.com/businesses/innovation/technology/water/water\\_pump.html](http://www.hitachi.com/businesses/innovation/technology/water/water_pump.html).

## Converting Base Load Natural Gas Plants to Variable Load

	P	Q	R	S	T	U	V	W	X	Y	Z
1		1 Gigawatt	Number						Inefficiency		
2		Fossil Fuel	100 mW						Factor		
3	Adjusted	Base Load	NatGas		Total	Total			0.8	Cumulative	
4	Output	Supply	Plants		Generation	Demand			mWh	Storage	Shortage
5	Per Unit	mWh	3.5	Utilization	mWh	mWh	Difference	Shortfall	Excess	1043136	
6	11276	22800	7980	1.00	42056	29428	12629	0	10103	1053239	0
7	3850	22800	7980	0.67	31997	34148	-2151	2151	0	1051087	0
8	12602	22800	7980	0.67	40749	31323	9426	0	7541	1058629	0
9	11060	22800	7980	0.67	39206	31102	8104	0	6483	1065112	0

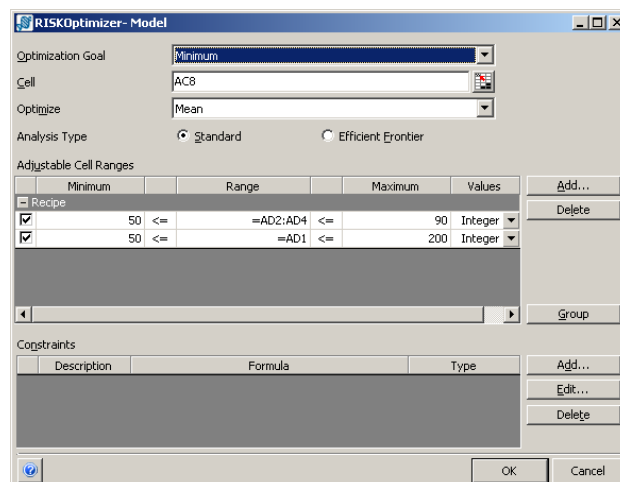
Spreadsheet Renew5, initially a copy of Renew4, was modified with a new column S to cover utilization of the natural gas plants, whose output can be cut back. Column V is the difference between supply and demand, which is segmented as positive or negative values to depict a shortfall or an excess in generating electricity. Excess electricity is multiplied by 80% to reflect the inefficiency associated with the operation of a pumped storage facility. The cumulative state of the battery is in column Y while column Z contains any negative values in column Y.

	AA	AB	AC	AD
1	Size of Battery (mWh)		1220000	122
2	Start Point to Cut Generation		0.85	85
3	Degree of Generation Decrease		0.69	69
4	Initial Inventory as % Storage		0.82	82
5	Total Shortfall		0	
6	Maximum		1159133	
7				
8	Objective		2379133	
9				
10	Difference Start and End Invent		158733	

A new section was added depicted at the left. Cell AC1 is the size of the storage battery to be minimized. Cell AC2 is the point above which the generation capacity of the natural gas plants will be cut back – here it is 85% of 122,000 mWh. Cell AC3 is the degree of cutback – here 69% of full load output. Cell AC4 is the starting point of inventory – here 82% of 122,000 mWh.

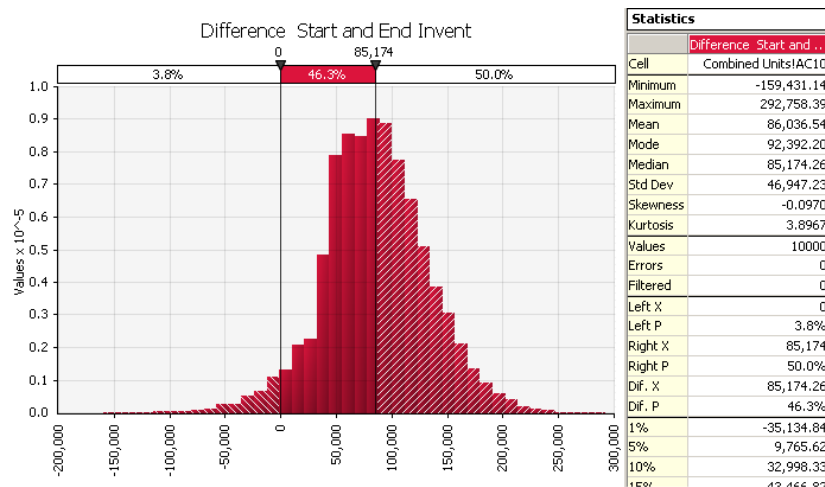
The formula in cell S7 is: =IF(Y6>\$AC\$2\*\$AC\$1,\$AC\$3,1) – if battery storage is above the start point to cut generation multiplied by battery capacity, then natural gas electricity generation will be reduced by the degree of cutback, otherwise the natural gas generation will run at capacity. Figure 44 is the RISKOptimizer menu where the objective is to minimize the mean of cell AC8, which is the sum of the size of the storage battery, the maximum value of the cumulative electricity in storage (column Y), and the total deficit electricity in storage (column Z).

**Figure 44 – RISKOptimizer Menu**



The variables, cells AD1 through AD4, are randomly selected integers within the indicated ranges. Cell AC1 sets the range on battery storage as 10,000 times the value in cell AD1 and the integer selection limits only 150 permissible values for cell AC1 to be used by RISKOptimizer. Cells AC2 through AC4 are one hundredth of the values in cells AD2:AD4 and there are only 40 permissible values for each cell. The purpose of doing this is to shorten RISKOptimizer run time by reducing the number of possible combinations. Cell AC10 is the difference between the starting and ending “inventory” in column Y. Figure 45 is the probability distribution of the difference between starting and ending storage (inventory) of electricity in mWh over the course of a year.

**Figure 45 – Difference in mWh Between Starting and Ending Position of Battery Capacity**



	AA	AB	AC	AD	AE
1	Size of Battery (mWh)		1160000	116	
2	Start Point to Cut Generation		0.89	89	
3	Degree of Generation Decrease		0.67	67	
4	Initial Inventory as % Storage		0.84	84	
5	Total Shortfall		0		
6	Maximum		1059623		
7					
8	Objective		2219623		
9					
10	Difference Start and End Invent		21591		
11					
12	Best Fitting Distribution		63633		

The addition of aggregate control over natural gas plant output was sufficient to reduce the optimal inventory from 1.22 million mWh to 1.16 mWh. The best fitting probability distribution in cell A12 is:

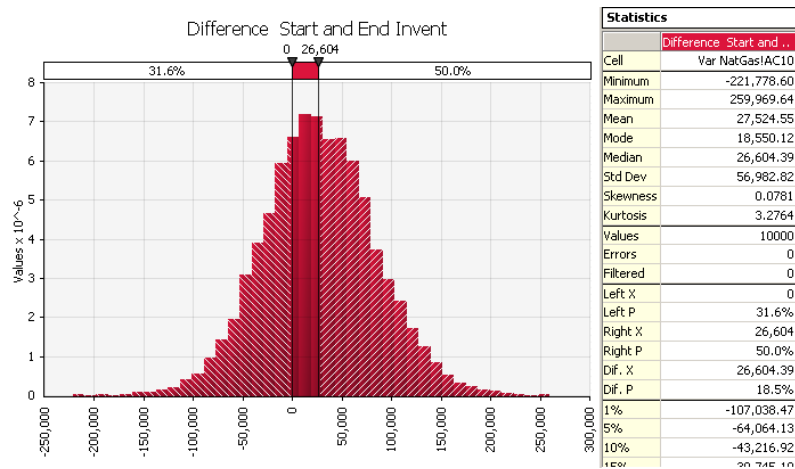
=RiskLogistic(86502,25695)

This was added to cell Y5: =AC4\*AC1+AC12 to model the starting inventory based on a non-zero balance at the end of the year in order to essentially model a multi-year simulation.

Figure 46 is the difference between starting and ending inventory (mWh) incorporating a starting position with a best fitting distribution of the ending inventory. This is a marked difference from Figure 45 where the chances of a negative balance with regard to the initial storage capacity was 3.8%, but is 31.6% in Figure 46. This does not mean an initial storage being negative, but a deviation from its starting position of 1.16 million mWh.



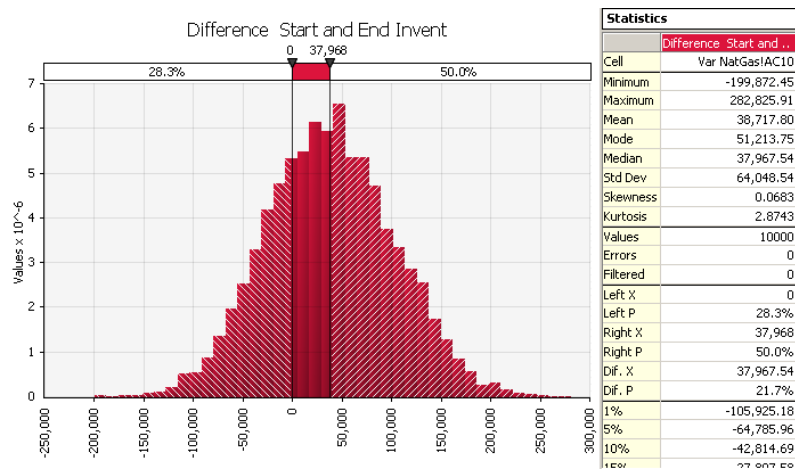
**Figure 46 – Modified Difference Between Starting and Ending Position of Battery Capacity**



A new best fitting distribution was obtained and cell C12 was changed to:

=RiskNormal(27189,57899,RiskTruncate(-200000,250000)). The normal distribution was truncated to model the maximum and minimum of the actual results. Figure 47 was the modified difference between starting and ending inventory.

**Figure 47 – Modified Difference Between Starting and Ending Position of Battery Capacity**

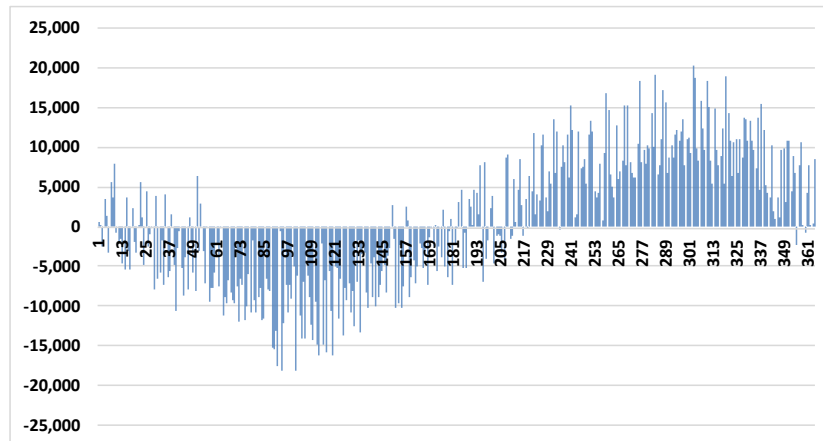


	AA	AB	AC	AD
1	Size of Battery (mWh)		1160000	116
2	Start Point to Cut Generation		0.9	90
3	Degree of Generation Decrease		0.67	67
4	Initial Inventory as % Storage		0.89	89
5	Total Shortfall		0	
6	Maximum		1137430	
7				
8	Objective		2297430	
9				
10	Difference Start and End Invent		76765	

The size of the battery remained the same, but there were small changes in the values of cells AC2 through AC4.

Figure 48 shows the daily shortfalls and excess generation of electricity. Readings above 15,000 mWh and below -15,000 mWh are relatively infrequent.

**Figure 48 – Shortfalls and Excess Generation of Electricity with 3.5 NatGas Plants**



An extreme event is the coupling of low demand for a given day with high output of renewables. Figure 49 shows supply totaling 43,707 (base load 22,800, natural gas 7,980, and renewables of 12,927 mWh) with demand of 23,345 mWh. On this day, surplus or excess electricity generation was 20,362 mWh, which would have entailed eliminating natural gas generation and even cutting back base load generation to reduce the magnitude of the excess generation. But doing this would reduce the electricity potential contained in the gravity battery to supply times of deficit generation.

**Figure 49 – Supply and Demand for Electricity During Excess Electricity Generation**

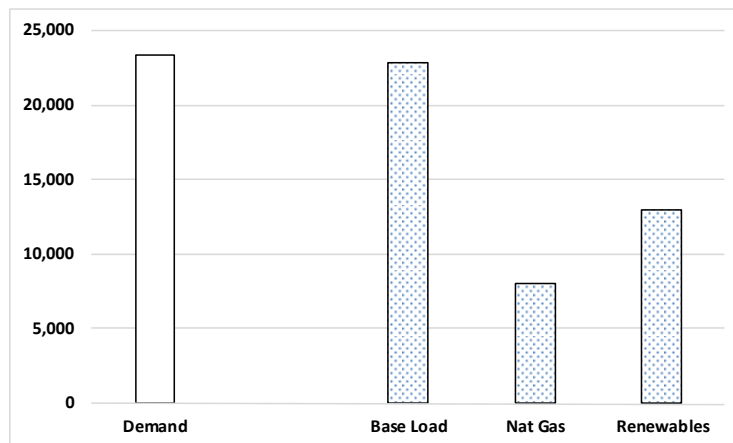
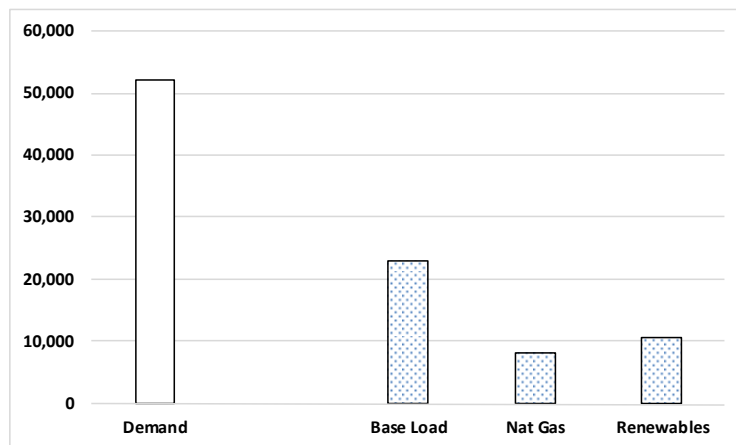


Figure 50 shows the opposite – a significant shortfall where demand is 52,223 mWh with base load of 22,800, natural gas 7,980, and renewables 10,652. The resulting shortfall of 17,149 mWh would have to be satisfied by electricity storage, which would have to be capable of delivering this amount of electricity. Being capable of delivery infers that the gravity battery would have to be filled during times of excess electricity generation.

**Figure 50 – Supply and Demand for Electricity During Shortfall of Electricity Generation**



The aggregate control mechanisms built into the model does not work. Aggregate control must be replaced by daily control. Each day dispatchers have to carefully monitor the situation cutting back on natural gas plants and base load plants if necessary to reduce excess production, or selling electricity were this option available. But cutting back on the degree of excess electricity transmitted to the storage battery would threaten its effectiveness to satisfy shortfalls. The state of charge of the electricity battery or level of water in the upper reservoir has to be carefully monitored to ensure that cutting back on generation does not hamper system reliability. It may be necessary at times to run the natural gas plants to restore the amount of stored electricity to meet future demand needs.

## Section 4

### Scoping the Size of a Gravity Battery



Pictured here is the Yanbaru Okinawa pumped hydro energy storage. Using this as a mental image, the head of water to operate the turbines at the lower reservoir, which here happens to be the ocean, is the vertical height from sea level to the bottom of the upper reservoir plus the depth of water in the upper reservoir. Reversible pump-turbines are sized to meet the maximum fluctuations in excess generation of electricity and shortfalls in meeting demand.

In the following analysis, rainfall compensates for losses from evaporation and internal leakages to the environment. The inefficiency of the system has already been included by reducing the amount of electricity to be stored by 20%.

The question is what should be the area of the surface of the upper reservoir and its depth in order to handle the fluctuations in excess and shortfall electricity generation as set forth in this paper.<sup>6</sup>

One megawatt-hour is equivalent to 2.66 billion foot-pounds of energy (2,655,223,737.19 to be exact). A cubic foot of water weighs 62.4 pounds for temperatures between 40 and 60 degrees Fahrenheit, thus energy required to lift one cubic foot of water one foot is 62.4 foot-pounds. The optimal sized battery capacity has already been assessed at 1,160,000 mWh to handle both primarily seasonal fluctuations in supply and demand. Thus the reservoir has to be at a height and capacity to store 2.66 billion foot-pounds/mWh X 1,160,000 mWh or  $2.66 \times 10^9 \times 1.16 \times 10^6$  or  $3.086 \times 10^{15}$  foot-pounds.

Suppose that the bottom of the top reservoir is at a vertical height of  $H_1$  feet above the turbines and the depth of the reservoir is  $H_2$  feet. The head of water is determined by the vertical height from the turbines to the reservoir, which is  $H_1 + H_2$  feet when full and  $H_1$  feet when empty. The volume of the reservoir for one foot of depth is its cross sectional area,  $A$ , measured as cubic feet.

Further suppose that the reservoir can be approximated as a cylinder where  $A$  is not dependent on reservoir depth. The potential energy, or stored mWh, is less per foot of depth when the reservoir is low compared to being full. The potential energy represented by one foot of depth in the reservoir is  $(H_1 + h)$  feet X  $A$  square feet X 62.4 pounds/cubic feet or  $62.4 (H_1 + h) (A)$  pounds per foot of depth where  $h$  can vary between 0 and  $H_2$  feet.

The potential energy at the bottommost one-foot layer of the reservoir is  $62.4 (H_1) (A)$  per foot of depth and  $62.4 (H_1 + H_2) (A)$  per foot of depth when it is full. Hence the average potential energy per foot between the reservoir being empty and full would be  $62.4 (H_1 + H_2/2) (A)$ . The dimensions of this quantity is foot-pounds per foot of reservoir debt. This when multiplied by the height of the reservoir ( $H_2$ ) results in total foot-pounds and this must equal  $3.086 \times 10^{15}$  foot-pounds.

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<sup>6</sup> Section 8 in *Energy Risk Modeling* provided guidance in constructing the model.

$$62.4 (H1+H2/2) (A) (H2) = 3.086 \cdot 10^{15} \text{ foot-pounds}$$

$$A = 3.086 \cdot 10^{15} / [62.4 (H1+H2/2) (H2)]$$

Since  $A = \pi R^2$ , the radius of the cylindrical reservoir would be:

$$R^2 = 3.086 \cdot 10^{15} / [62.4 \pi (H1+H2/2) (H2)] = 3,086 \cdot 10^{12} / [196 (H1+H2/2) (H2)]$$

$$R^2 = 15.74 \cdot 10^{12} / [(H1+H2/2) (H2)]$$

$$R = 3.97 \cdot 10^6 / [(H1+H2/2) (H2)]^{.5}$$

The diameter of the reservoir would be  $D = 2R$  or  $7.94 \cdot 10^6 / [(H1+H2/2) (H2)]^{.5}$

Suppose  $H1$  is 1000 feet and  $H2$  is 300 feet, the diameter in feet of the reservoir would be:

$$7,940,000 / [(1000 + 300/2) (300)]^{.5} = 7,940,000 / (345,000)^{.5} = 7,940,000 / 587.4 \text{ or } 13,517 \text{ feet or } 2.56 \text{ miles across.}$$

If the reservoir depth were 200 feet, then the reservoir diameter would be:

$$7,940,000 / [(1000 + 200/2) (200)]^{.5} = 7,940,000 / (220,000)^{.5} = 7,940,000 / 469 \text{ or } 16,929 \text{ feet or } 3.21 \text{ miles across.}$$

If  $H1$  is 500 feet and  $H2$  200 feet, then the diameter would be:

$$7,940,000 / [(500 + 200/2) (200)]^{.5} = 7,940,000 / (120,000)^{.5} = 7,940,000 / 346.4 \text{ or } 22,921 \text{ feet or } 4.34 \text{ miles across.}$$

If  $H1$  is 500 feet and  $H2$  300 feet, then the diameter would be:

$$7,940,000 / [(500 + 300/2) (300)]^{.5} = 7,940,000 / (195,000)^{.5} = 7,940,000 / 441.6 \text{ or } 17,980 \text{ feet or } 3.40 \text{ miles across.}$$

### ***Is Size a Problem?***

While it is true that circumstances were selected to handle a great deal of seasonal demand that upped the requirement for electricity storage capacity, renewables were still less than half of total system output. Yet the required storage capacity to ensure 100% reliability of 1.16 million megawatt-hours of capacity is a large amount of stored electricity. How does this compare with existing gravity batteries?

Gravity batteries are the largest means for storing electricity by far. In 2010, global gravity battery capacity was 140,000 mW compared to 976 mW (only 0.7 percent!) for other storage alternatives such as compressed air energy storage and batteries of various types. The largest pumped storage facility in the world is Bath County Pumped Storage Station (US) rated at 3,003 mW followed by Huizhou Pumped Storage Station (China) at 2,448 mW; Guangdong Pumped Storage Station (China) at 2,400 mW; and Okutataragi Pumped Storage Station (Japan) at 1,932 mW. Of a total of 51 pumped storage facilities in the world over 1,000 mW, 14 are in China, 11 in Europe, 10 in the US, 8 in Japan, and remaining 8 in other nations. Thirteen are under construction of which 7 are in China including one of 3,600 mW.<sup>7</sup>

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<sup>7</sup> "List of Pumped-Storage Hydroelectric Power Stations," Wikipedia, The Free Encyclopedia, Web site [http://en.wikipedia.org/wiki/List\\_of\\_pumped-storage\\_hydroelectric\\_power\\_stations](http://en.wikipedia.org/wiki/List_of_pumped-storage_hydroelectric_power_stations).

We're mixing apples and oranges in that megawatt-hours of capacity are not the same as megawatts of power. If a 1,000 mW gravity battery operates at 10% of its capacity for a year, its megawatt-hour output would be  $1,000 \text{ mW} \times 10\% \times 24 \text{ hours per day} \times 365 \text{ days}$  or 876,000 mWh of energy capacity, close to the 1.16 million mWh required here. It is difficult to judge megawatt-hour capacity given that electricity is flowing both in and out of a battery. In terms of 20,000 megawatt-hours of peak inflow and outflow per day, average power is 833 megawatts, appears to be within the limits of a large sized gravity battery. Required pumping capacity, which has been previously shown to be significant, also appears to be feasible. If we take the view that a gravity battery with sufficient megawatt-hour capacity and pumping capacity are both technically and economically doable, then we can judge the performance of the gravity battery in terms of changing water depths in the upper reservoir.

### ***Gravity Battery Performance***

Water level will change differently for the same energy input/output depending on the depth of water in the gravity battery. Taking the case for an upper reservoir 300 feet deep whose penstock opening at the working bottom of the reservoir is 500 feet above the pump-turbines, storage capacity for one foot of reservoir depth is  $(H_2+H_1) \text{ feet} \times 62.4 \text{ pounds per cubic foot} \times \text{Area in square feet}$  for a circle with a diameter of 17,980 feet (radius 8,990 feet) or  $1.27 \times 10^{13}$  foot-pounds or  $12.7 \times 10^{12}$  foot-pounds.

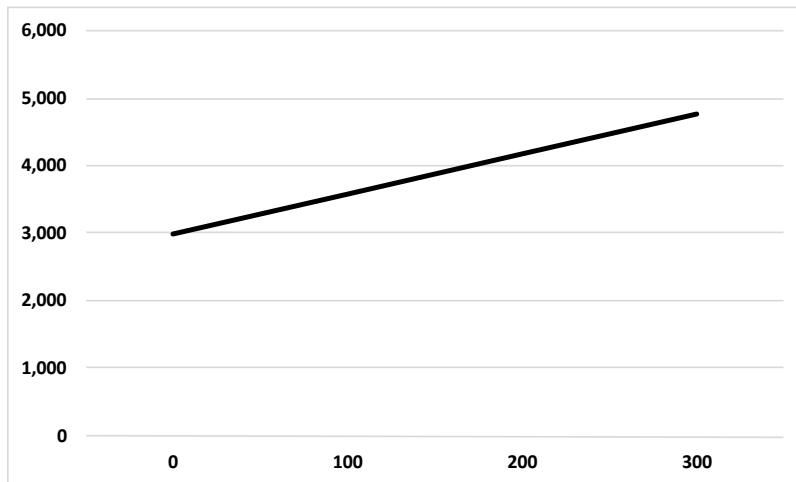
With one megawatt-hour equivalent to  $2.66 \times 10^9$  foot-pounds of energy, one foot of reservoir depth when full can hold 4,774 megawatt-hours. Thus maximum outflow of 20,000 megawatt-hours in a day's time will cause the level of the reservoir to fall 4.2 feet.

On the other extreme when the reservoir is at its lowest level with one foot of water depth, then the calculation, based only on  $H_2$  as the head of water, is  $7.92 \times 10^{12}$  foot-pounds per foot of reservoir depth, which at  $2.66 \times 10^9$  foot-pounds per megawatt-hour, one foot of reservoir depth can hold 2,977 megawatt-hours.

An inflow of 20,000 megawatt-hours when the reservoir is near empty will cause a level increase of 6.7 feet. Using these energy capacity assessments at extreme levels of reservoir capacity, a linear relationship between megawatt-hours and water depth can be obtained.

Let  $x$  be the depth of water in the reservoir which can vary from 0 to 300 feet and  $y$  be the associated energy storage capacity per foot depth of water which can vary from 2,977 mWh to 4,774 mWh at 0 and 300 feet respectively. The resulting linear relationship, derived in the Reservoir tab of spreadsheet Renew5, is  $y = 2,977 + 5.99 x$  for the energy storage at any depth between 0 and 300 shown in Figure 51.

**Figure 51 – Megawatt-hour Capacity per Foot Versus Reservoir Water Depth**



With this relationship and megawatt-hours to be added or withdrawn from the reservoir on any given day, one can obtain a daily record of reservoir level over a year period by incorporating the following changes to spreadsheet ReNew5.

	AE	AF
2	Incremental	Reservoir
3	Change	Water
4	in Feet	Depth
5		268.9
6	1.3	270.2
7	-0.2	270.0
8	1.8	271.8

Cell AF5: =300\*Y5/1160000 changes initial mWh storage to initial water depth.

Cell AE6: =(X6-W6)/(2977+5.99\*AF5) converts excess/shortfall electricity generation/discharge to equivalent incremental water depth change.

Cell AF6: =AF5+AE6 applies incremental daily change to previous water depth.

Figure 52 is a single iteration of how reservoir water depth varies over a year.

**Figure 52 – Iteration of Reservoir Water Depth Over a Year Period**

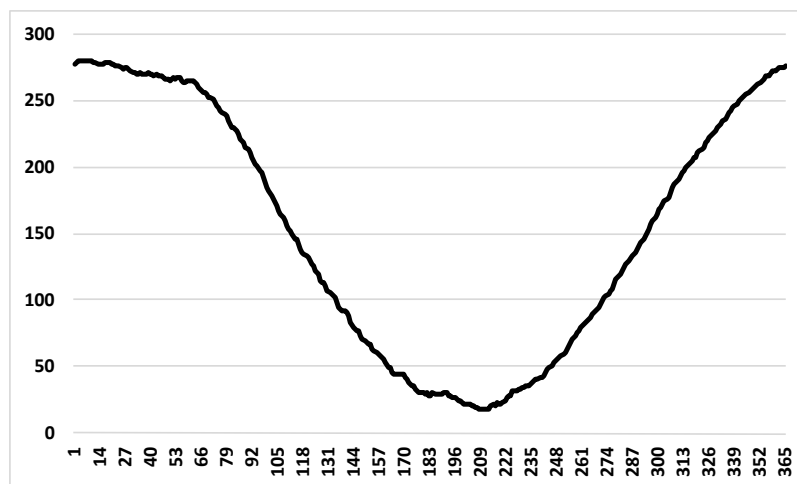
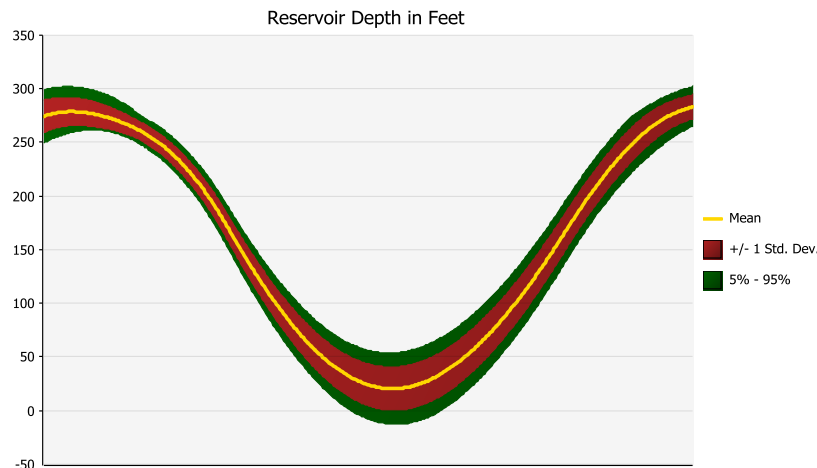


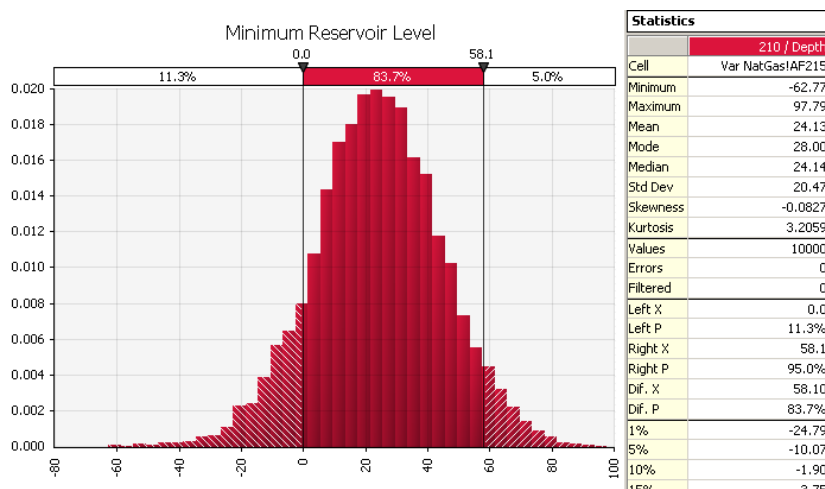
Figure 53 shows a simulation where all cells in column AF were designated output cells by first selecting column AF cells and then the Add Output icon.

**Figure 53 – Simulation Output of Annual Reservoir Depth**



Variation is within a relatively narrow band because the seasonal factor remained the same for all simulations with +/-10% variation on a daily basis. Allowing variation in the seasonal factor would increase the degree of uncertainty. Note that the minimum level occasionally dips below zero. The probability of this happening can be assessed by examining the output for some arbitrary day during this low-water period as in Figure 54.

**Figure 54 – Reservoir Level for Day 210**



The probability of the reservoir going dry is 11%, about once in nine years. It would be very expensive to handle these occasions by having installed peak generators that work this infrequently. Other possibilities are the purchase of electricity if available or demand management (having heavy users switch to emergency power supplies), or controlled blackouts, which would not be well-received by consumers. Another more practical way is to build the gravity battery with a greater depth of water. Figure 54 suggests that most of the infrequent exhaustion of the gravity battery could be taken care of with another 20 feet of depth. Yet even so, a utility must have plans to cope on rare occasions when reservoir capacity is essentially depleted.



## What Has Been Accomplished?

This paper demonstrates that @RISK simulation can handle analysis of matching uncontrollable supply to uncontrollable demand using, in this case, a gravity battery as a buffer between the two. The methodology employed in this report can be a paradigm for analyzing the mixing of renewables with fossil fuel and nuclear plants. Hydropower, which can be part of base load, is often considered to be as dependable as fossil fuel plants, but this is not true as hydropower is subject to rainfall fluctuations in the collection basin supplying a reservoir. An interesting exercise is to incorporate a seasonal flowing stream feeding a gravity battery reservoir. During times of high electricity demand, the additional water would reduce the required reservoir volume. However, during times of low electricity consumption, this additional water along with water being pumped into the gravity battery from excess electricity generation will fill the reservoir to its brim. At some point, water from the reservoir will have to be diverted to flow through the pump-turbines to generate electricity. This electricity could act as a buffer between night time wind power electricity generation and base load demand. This allows the output of base load fossil fuel plants to be reduced to allow wind power, stabilized by the gravity battery turned hydropower plant, to become dependable for base load demand. Thus night time wind energy does not have to be pumped into the gravity battery enhancing system efficiency and cutting back on fossil fuel consumption. This would occur during the seasonal lull in electricity demand. As seasonal demand increases, base load fossil fuel plants would resume full power operation and night time wind power would be fed to the gravity battery. Hence a combination of wind power and a gravity battery sometimes acting as a hydropower plant would allow a reduction of base load fossil fuel consumption during seasonal lulls in electricity demand. Actually a gravity battery could be eliminated if a hydropower plant of sufficient capacity can fill the varying gap between electricity demand less renewables and conventional sources of electricity generation. These alternatives can be handled by @RISK with its host of useful tools to address the growing challenge of solar and wind farms playing a more important role in generating electricity at the expense of conventional plants.

A gravity battery may not be possible for utilities located where no natural geological formations are available for transformation into a gravity battery. If and when super batteries at five times the capacity at one fifth the cost of existing batteries come into being, then a distributive storage battery system can be built to capture excess night time electricity generation to meet daytime demand. They could also store peak solar and wind generation in excess of variable load and be available to serve shortfalls in electricity generation. It might be necessary to reduce base load fossil fuel output during the day to discharge the storage batteries in anticipation of absorbing wind power output during the night. A cutback in fossil fuel base load generation to discharge the batteries would be constant and not fluctuate with wind conditions as coal and nuclear plants are not amenable to rapid shifts in output. Fluctuations would be satisfied by varying battery power output. Semi-dependable forecasts on wind speed and cloud cover would be necessary for dispatchers to plan daily operations. This general type problem of managing large scale storage battery capacity can also be handled by @RISK simulation.

It appears to the author that significant dependence on solar and wind would place consumers at risk of interrupted service given the state of today's battery technology. Large scale electricity storage is an absolute necessity, which, unfortunately, will not be available until a technological breakthrough occurs in material design to bring about the super battery. Until that time, and in the absence of large scale gravity batteries, there may be an upper limit, say 30%, for renewables to penetrate electricity generating capacity in order for dispatchers to maintain system stability by varying the output of natural gas or possibly hydropower plants. Maintaining system stability where solar and wind play a significant role in generating electricity is a growing challenge facing utility operators. Hopefully this paper spurs interest in the described methodology for planning purposes when utilities think about making major investments in renewable power sources.