Introduction to Engineering Reliability

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Topics

• Reliability

• Basic Principles of Reliability Analysis
  – Non-Probabilistic Methods
  – Probabilistic Methods
    • First Order Second Moment
    • Point Estimate Method
    • Monte Carlo Simulation
    • Response Surface Modeling
• Reliability

- "Probability that a system will perform its intended function for a specific period of time under a given set of conditions"

\[ R = 1 - P_f \]

- Reliability is the probability that unsatisfactory performance or failure will not occur
Reliability

Reliability and Probability of Failure

- Blue line: Reliability $R(t)$
- Pink line: Probability of Failure $F(t)$

Graph shows the relationship between time (years) and reliability/probability of failure.
• **Probability of Failure, “Pₚ”**
  – Easily defined for recurring events and replicate components (e.g., mechanical and mechanical parts)

• **Probability of Unsatisfactory Performance, P(u) “Pₚ”**
  – Nearly impossible to define for non-recurring events or unique components (e.g., sliding of gravity structures)
Reliability

pdf

Demand  Capacity

value

P_f or P_{up}
Delivering Integrated, Sustainable, Water Resources Solutions

Reliability

\[
\mu_d \quad \mu_c
\]

\[
P_f \text{ or } P_{up}
\]
Reliability

Safety Margins

If limit at SM = C - D = 0
If limit at SM = C / D = 1

E(SM), σ(SM)

SM = Safety Margin

Pup

0 or 1
• **Basic Principles of Reliability Analysis**

  • Identify critical components
  • Use available data from previous design and analysis
  • Establish base condition for component
  • Define performance modes in terms of past levels of unsatisfactory performance
  • Calibrate models to experience
  • Model reasonable maintenance and repair scenarios and alternatives
• **Non-Probabilistic Reliability Methods**
  
  – Historical Frequency of Occurrence
  – Survivorship Curves (hydropower equipment)
  – Expert Opinion Elicitation
• **Probabilistic Reliability Methods**
  - Reliability Index ($\beta$) Methods
    - First Order Second Moment (Taylor Series)
    - Advanced Second Moment (Hasofer-Lind)
    - Point Estimate Method
  - Time-Dependent (Hazard Functions)
  - Monte Carlo Simulation
  - Response Surface Modeling
• **Historical Frequencies**

  – Use of known historical information for records at site to estimate the failure rates of various components

  – For example, if you had 5 hydraulic pumps in standby mode and each ran for 2000 hours in standby and 3 failed during standby. The failure rate during standby mode is:

    \[
    \text{Total standby hours} = 5(2000 \text{ hours}) = 10,000 \text{ hours} \\
    \text{Failure rate in standby mode} = 3 / 10,000 \\
    = 0.0003 \text{ failures per hour}
    \]
• Manufacturers’ survivorship/mortality curves
  – Curves are available from manufacturers’ for different motors, pumps, electrical components, etc...
  – Curves are developed from field data and “failed” components
    • Caution is to be exercised on mode of failure
    • Failure data may have to be censored
  – However, usually this data id not readily available for equipment at Corps projects except mainly hydropower facilities
  – Report available at IWR on hydropower survivorship curve as well as many textbooks on the subject
• **Expert Opinion Elicitation (EOE)**
  
  – Solicitation of “experts” to assist in determining probabilities of unsatisfactory performance or rates of occurrence.
  
  – Need proper guidance and assistance to solicit and train the experts properly to remove all bias and dominance.
  
  – Should be documented well for ATR/IEPR
  
  – Some recent projects that used EOE
    
    • John Day Lock and Dam – Dam Anchorage, NWP
• **Probabilistic Methods**

  – Reliability models are:
    • defined by random variables and their underlying distributions
    • based on limit states (analytical equations) similar to those used in the design of engineering components
    • based on capacity/demand or factor of safety relationships

  – One method is the Reliability Index or $\beta$ method
Reliability

β Method - Normal Distribution

$\beta = \frac{E(SM)}{\sigma(SM)}$

- If limit at SM = C-D = 0

$\beta = \frac{(E(SM) - 1)}{\sigma(SM)}$

- If limit at SM = C/D = 1

$E(SM)$ is the expected value of Safety Margin (SM).

$\sigma(SM)$ is the standard deviation of Safety Margin (SM).

$\beta$ is the reliability index.

Pup at SM = 0 or 1

SM = Safety Margin
• **Reliability Index (β) Methods**
  
  – **Taylor Series Finite Difference**  
    (Cornell, 1969 and Rosenblueth, 1972)
    • First-order expansion about mean value
    • **Linear** approximation of second moment
    • Uses analytical equations (deflection, moment, stress/strain, etc…)
    • Easy to implement in spreadsheets
    • Requires 2n+1 sampling (n = number of variables)
    • Results in a Reliability Index value (β)
      – Based on E(SM) and σ (SM)
    • Problem: caution should be exercised on non-linear limit states
Taylor Series Finite Difference

Variable C

\[ \mu_C + \sigma_C \]

\[ \mu_B - \sigma_B \]

\[ \mu_C - \sigma_C \]

FS = 0

Safe

Fai

Variable B
Reliability Example

• Determine the reliability of a tension bar using the TSFD reliability index ($\beta$) method

\[ \text{Limit State} = \frac{F_t A}{P} \]
Reliability Example

- **Random Variables**

  - **Ultimate tensile strength**, $F_t$
    - mean, $\mu = 40$ ksi; standard deviation, $\sigma = 4$ ksi
    - assume normal distribution
  - **Load**, $P$
    - mean, $\mu = 15$ kips; standard deviation, $\sigma = 3$ kips,
    - assume normal distribution
  - **Area**, $A$
    - constant (no degradation) circular cross section, $A = 0.5$ in$^2$
Reliability Example

• **Mean FS**
  - \( \mu_{FS} = \frac{40(0.50)}{15} = 1.333 \)

• **Standard Deviation FS**
  - \( \sigma_{FS} = \sqrt{\left(\frac{1.467 - 1.200}{2}\right)^2 + \left(\frac{1.111 - 1.667}{2}\right)^2} \)
  - \( \sigma_{FS} = \sqrt{0.134^2 + 0.278^2} \)
  - \( \sigma_{FS} = 0.309 \)
Reliability Example

• Reliability Index

\[ \beta = \frac{E[SM] - 1}{\sigma[SM]} = \frac{0.333}{0.309} = 1.06 \]

\[ P(u) = 0.14 \]

\[ R = 1 - P(u) = 0.86 \]
• **Reliability Index** ($\beta$) **Methods**

• **Point Estimate Method**
  
  (Rosenblueth (1975))
  
  – Based on analytical equations like TSFD
  – Quadrature Method
  – Finds the change in performance function for all combinations of random variable, either plus or minus one standard deviation
    • For 2 random variables - ++, +-, -+, -- (+ or – is a standard deviation)
  – Requires $2^n$ samplings ($n =$ number of random variables)
  – Results in a Reliability Index value ($\beta$)
    • Based on $E(SM)$ and $\sigma$ (SM)
Point Estimate Method

Figure from Baecher and Christian (2003)
Point Estimate Method

Figure from Baecher and Christian (2003)
• **Reliability Index (β) Methods**
  
  – **Advanced Second Moment**  
    (Hasofer-Lind 1974, Haldar and Ayyub 1984)
    • Based on analytical equations like PEM
    • Uses directional cosines to determine shortest distance (β) to multi-dimensional failure surface
    • Accurate for non-linear limit states
    • Solved in spreadsheets or computer programs
Reliability

Random Variable, X2

C-D >0  Safe

C-D < 0  Fail

Random Variable, X1
• Reliability Index ($\beta$) Methods

• Shortcomings
  – *Instantaneous* - capture a certain point in time
  – Index methods *cannot* be used for time-dependent reliability or to estimate hazard functions even if fit to Weibull or similar distributions
  – Incorrect assumptions are sometimes made on underlying distributions to use $\beta$ to estimate the probability of failure
• **Monte Carlo Simulation**
  
  – “Monte Carlo” is the method (code name) for simulations relating to development of atomic bomb during WWII
    
      • Traditional – static not dynamic (not involve time), U(0,1)
      • Non-Traditional – multi-integral problems, dynamic (time)
  
  – Applied to wide variety of complex problems involving random behavior
  
  – Procedure that generates values of a random variable based on one or more probability distributions
  
  – Not simulation method per se – just a name!
• **Monte Carlo Simulation**
  – Usage in USACE
    • Development of numerous state-of-the-art USACE reliability models (structural, geotechnical, etc.)
    • Used with analytical equations and other advanced reliability techniques
    • Determines $P_f$ directly using output distribution
    • Convergence must be monitored
      – Variance recommend
Monte Carlo Simulation

- Reliability
  - Determined using actual distribution or using the equation:

\[ R = 1 - P(u) \]

where, \( P(u) = \frac{N_{pu}}{N} \)

\( N_{pu} = \) Number of unsatisfactory performances at limit state < 1.0

\( N = \) number of iterations
• **Hazard Functions**
  – **Background**
    • Previously used reliability index (\( \beta \)) methods
    • Good estimate of relative reliability
    • Easy to implement
    • Problem: “Instantaneous” - snapshot in time
Hazard Functions/Rates

- Started with insurance actuaries in England in late 1800’s
  - They used the term mortality rate or force of mortality
- Brought into engineering by the Aerospace industry in 1950’s
- Accounts for the knowledge of the past history of the component
- Basically it is the rate of change at which the probability of failure changes over a time step
- Hazard function analysis is not snapshot a time (truly cumulative)
  - Utilizes Monte Carlo Simulation to calculate the true probability of failure (no approximations)
- Easy to develop time-dependent and non-time dependent models from deterministic engineering design problems
• **Typical Hazard Bathtub Curve**

![Bathtub Curve Diagram]

- **Burn-in Phase**
- **Constant failure rate phase**
- **Wear-out phase**

- **h(t)** vs. **Lifetime, t**

**Lines:**
- **Electrical**
- **Mechanical**
• Ellingwood and Mori (1993)

\[ L(t) = \exp \left[ -\lambda t \left[ 1 - \frac{1}{t} \int_{0}^{t} F_S(g(t)\,r) \, dt \right] \right] f_R(r) \, dr \]

- \( F_S \) = CDF of load
- \( g(t)\,r \) = time-dependent degradation
- \( f_R(r)\,dr \) = pdf of initial strength
- \( \lambda \) = mean rate of occurrence of loading

Closed-form solutions are not available except for few cases

Solution: Utilize monte carlo simulations to examine the “life cycle” for a component or structure
• **Hazard Functions**
  – Degradation of Structures
    • Relationship of **strength (R) (capacity)** vs. **load (S) (demand)**
• **Life Cycle**

![Diagram showing the Life Cycle process]

- **Initial Random Variables for Strength and Load**
- **Propagate variables for life cycle (e.g. 50 years)**
- **Run life cycle and document year in which unsatisfactory performance occurs**
- **Develop L(t), h(t)**

For I = 1 to N where N = number of MCS

Reinitialize with new set of random variables
Hazard Function (conditional failure rate)

- Developed for the ORMSSS economists/planners to assist in performing their economic simulation analysis for ORMSSS investment decisions

- \( h(t) = P[\text{fail in } (t,t+dt) | \text{survived } (0,t)] \)

- \( h(t) = \frac{f(t)}{L(t)} \)

= \( \frac{\text{No. of failures in } t}{\text{No. of survivors up to } t} \)
• Response Surface Methodology (RSM)

  Reliability is expressed as a limit state function, $Z$ which can be a function of random variables, $X_n$, where

  $$Z = g(X_1, X_2, X_3, \ldots)$$

  and the limit state is expressed as

  $$Z = C - D > 0$$

  where $D$ is demand and $C$ is capacity.
Response Surface Methodology (RSM)

Reliability (in 2 variable space)

Safe  \( C > D \)

Limit State Surface

Fail  \( C < D \)
Response Surface Methodology (RSM)

– Utilizes non-linear finite element analysis to define to the response surface
– Not closed form solution but close approximation
– Constitutive models generally not readily available for performance limit states
  • Typical design equations generally are not adequate to represent limit state for performance
Response Surface Methodology (RSM)

- Accounts for variations of random variables on response surface
- Reflects realistic stresses/strains, etc. that are found in navigation structures
- Calibrated to field observations/measurements
- Develop response surface equations and use Monte Carlo Simulation to perform the reliability calculations

- Recent USACE Applications
  - Miter Gates (welded and riveted)
  - Tainter Gates
  - Tainter Valves (horizontally and vertically framed)
  - Alkali-Aggregate Reaction
Response Surface Methodology (RSM)
Response Surfaces

Concrete Strain in Anchorage Region vs Compressive Strength

- Strain (%)
- Time (years)
- 3600 psi
- 5000 psi
- 6400 psi
• Response Surface Methodology
  – Proposed Methodology for I-Wall Reliability

  • Assumptions
    – Poisson ratio – constant
    – Random variables – E, Su (G, K)

  • Limit state based on deflection (Δ) at ground surface

  • \( g(\Delta) = f( E, Su) = \frac{\Delta_{cr}}{\Delta} < 1.0 \)
Response Surface Modeling Concept (under development)

\[
\begin{align*}
(E_{\text{max}}, S_{\text{u min}}, \Delta 1) & \\
(E_{\text{max}}, S_{\text{u max}}, \Delta 2) & \\
(E_{\text{BE}}, S_{\text{uBE}}, \Delta 5) & \\
(E_{\text{min}}, S_{\text{u max}}, \Delta 4) & \\
(E_{\text{min}}, S_{\text{u min}}, \Delta 3) &
\end{align*}
\]

☆ Point of \( \Delta = \Delta_{cr} \)
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Reliability

• **Preferred Methods**
  – For non-time dependent reliability problems
    • Linear – Taylor Series Finite Difference, Point Estimate or Monte Carlo Simulation
      – Assume normal distributions for TSFD
      – Assume any distributions for MCS
    • Non-Linear – Advanced Second Moment or Monte Carlo Simulation
  – For time-dependent reliability problems
    • Hazard Function/Rates using Monte Carlo Simulation