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Chapter 1: Welcome to @RISK for Excel

Introduction

Welcome to @RISK, the revolutionary software system for the analysis of business and technical situations impacted by risk! The techniques of risk analysis have long been recognized as powerful tools to help decision makers successfully manage situations subject to uncertainty. Their use has been limited because they have been expensive, cumbersome to use, and computationally intensive. However, the growing use of computers in business and science has offered the promise that these techniques can be commonly available to all decision makers.

That promise has been realized with @RISK (pronounced “at risk”), a system that brings these techniques to the industry standard spreadsheet package, Microsoft Excel. With @RISK and Excel, any risky situation can be modeled, from business to science to engineering. You are the best judge of your analysis requirements, and @RISK, combined with the modeling capabilities of Excel, allows you to design models that best satisfy requirements. Any time you face a decision or analysis that involves uncertainty, you can use @RISK to gain insight into how the future might unfold.

Why You Need Risk Analysis and @RISK

Traditional analyses combine single “point” estimates of a model's variables to predict a single estimate of results. Estimates of model variables must be used because future values are not known with certainty. In reality, however, many things don't turn out the way you planned. Maybe you were too conservative with some estimates and too optimistic with others. The combined errors in each estimate often lead to real-life results that are significantly different from estimated results. The decision you make based on your “expected” result might be an unwise decision, a decision you would not have made if you had a more complete picture of all possible outcomes. Business decisions, technical decisions, and scientific decisions all use estimates and assumptions. With @RISK, you can explicitly include the uncertainty present in your estimates to generate results with a wide range of possible outcomes.
@RISK uses a technique called “simulation” to combine all uncertainties you identify in your situation. You are no longer forced to reduce what you know about a variable to a single number. Instead, you can include all you know about the variable, including its full range of possible values and the likelihoods of their occurrence. @RISK uses all of this information, along with your Excel model, to generate a range of possible outcomes. It’s just as if you ran hundreds or thousands of “what-if” scenarios all at once. In effect, @RISK lets you see the full range of what could happen in your situation. It enables you to “live” through your situation many times, each time under a different set of conditions with a corresponding set of results occurring.

All this added information might sound like it would complicate your decisions, but in fact, one of simulation’s greatest strengths is its power of communication. @RISK provides results that graphically illustrate the risks you face. This graphical presentation is easily understood by you, and it is easily explained to others.

So when should you use @RISK? Any time you perform an analysis in Excel that involves uncertainty, you can and should use @RISK. Applications in business, science, and engineering are practically unlimited, and you can build on your existing base of Excel models. Consider the decisions and analyses you make every day. If you have ever been concerned with the impact of risk in these situations, you have just found a good use for @RISK!

**Modeling Features**

As an add-in to Microsoft Excel, @RISK links directly to Excel to add risk analysis capabilities. The @RISK system provides all the necessary tools for setting up, executing, and viewing the results of Risk Analyses. And @RISK works in a style you are familiar with: Excel-style menus and functions.

@RISK allows you to use functions to define uncertain cell values as probability distributions. @RISK adds a set of new functions to the Excel function set, each of which allows you to specify a different distribution type for cell values. Distribution functions can be added to any number of cells and formulas throughout your worksheets, and they can include arguments, possibly cell references and expressions, that allow extremely sophisticated specification of uncertainty. To help you assign distributions to uncertain values, @RISK includes a graphical window where distributions can be previewed and added to formulas.
The probability distribution functions provided by @RISK allow you to specify nearly any type of uncertainty in cell values in your model. A cell containing the distribution function RiskNormal(100,10), for example, would return samples during a simulation drawn from a normal distribution with mean 100 and standard deviation 10. These distribution functions are invoked only during a simulation. In normal Excel operations, they show a single cell value, just as in Excel without @RISK.

All distributions can be truncated to allow only samples within a specified range of values within the distribution. Also, many distributions can be specified with alternate percentile parameters instead of the distribution’s traditional parameters.

@RISK has sophisticated capabilities for specifying and executing simulations of Excel models. Both Monte Carlo and Latin Hypercube sampling techniques are supported, and distributions of possible results can be generated for any cell or range of cells in your spreadsheet model. Both simulation options and the selection of model outputs are entered with Windows-style menus, dialog boxes, and the use of the mouse.

High resolution graphics are used to present the distributions of outputs from your @RISK simulations. Histograms, cumulative curves, and summary graphs for cell ranges all lead to a powerful presentation of results. All graphs can be displayed in Excel for further enhancement and hard copy. An essentially unlimited number of output distributions can be generated from a single simulation, allowing for the analysis of even the most complex spreadsheet models.

The options available for controlling and executing a simulation in @RISK are among the most powerful ever available. They include:

- Latin Hypercube or Monte Carlo sampling
- Any number of iterations per simulation
- Any number of simulations in a single analysis
- Animation of sampling and recalculation of the spreadsheet
- Seeding of the random number generator
- Real-time results and statistics during a simulation
@RISK graphs a probability distribution of possible results for each @RISK output cell. @RISK graphics include:

- Relative frequency distributions and cumulative probability curves
- Summary graphs for multiple distributions across cell ranges (for example, a row of time series values)
- Statistical reports on output distributions
- Probabilities for target values in a distribution
- Export of graphics as Windows metafiles for further enhancement

Execution time is of critical importance because simulation is extremely calculation intensive. @RISK uses advanced sampling techniques for the fastest possible simulations.
Chapter 2: Getting Started with @RISK

Introduction

About This Version
This version of @RISK can be used with Microsoft Excel 2007 or higher.

Working with your Operating Environment
This manual assumes that you have a general knowledge of the Windows operating system and Excel. In particular:

- You are familiar with your computer and using the mouse.
- You are familiar with terms such as icons, click, double-click, ribbon, menu, and window.
- You understand basic concepts such as directory (or folder) structures and file naming.

Standard, Professional, and Industrial Versions
@RISK is available in three versions: Standard, Professional, and Industrial. You can find the features included in each of these by visiting www.palisade.com/risk.

If You Need Help
Palisade provides free technical support to all registered users of @RISK with a current maintenance plan. Technical support is also available on a per incident charge. To ensure that you are a registered user of @RISK, please register online at www.palisade.com/support/register.asp

If you contact Palisade by telephone, please have your serial number and this online manual handy. We can offer better technical support if you are at your computer and ready to work.
Before calling technical support, please review the following checklist:

- Have you consulted the relevant sections of this online manual?
- Have you watched the online Quick Start videos available from the @RISK Welcome screen?
- Have you read the README file? It contains current information on @RISK that might not be included in this manual.
- Can you duplicate the problem consistently? Can you duplicate the problem on a different computer or with a different model?
- Have you looked at our Web site, http://www.palisade.com? This Web site contains the latest FAQ (a searchable database of tech support questions and answers). We recommend visiting our Web site regularly for all the latest information on @RISK and other Palisade software.

Contacting Palisade

Palisade Corporation welcomes your questions, comments or suggestions regarding @RISK. Contact our technical support staff using any of the following methods:

- Email us at support@palisade.com.
- Telephone us at (607) 277-8000 any weekday from 9:00 AM to 5:00 PM, EST. Follow the prompt to reach Technical Support.
- Fax us at (607) 277-8001.
- Mail us a letter to:

  Technical Support  
  Palisade Corporation  
  798 Cascadilla St  
  Ithaca, NY 14850  
  USA
If you want to contact Palisade Europe:

- Email us at support@palisade-europe.com.
- Telephone us at +44 1895 425050 (UK).
- Fax us at +44 1895 425051 (UK).
- Mail us a letter to:

Palisade Europe
31 The Green
West Drayton
Middlesex
UB7 7PN
United Kingdom

If you want to contact Palisade Asia-Pacific:

- Email us at support@palisade.com.au.
- Telephone us at +61 2 9252 5922 (AU).
- Fax us at +61 2 9252 2820 (AU).
- Mail us a letter to:

Palisade Asia-Pacific Pty Limited
Suite 404, Level 4
20 Loftus Street
Sydney NSW 2000
Australia

Regardless of how you contact us, please include the product name, exact version, and serial number. The exact version can be found by selecting About @RISK from the Help menu on the @RISK ribbon.

Student Version

Telephone support is not available with the student version of @RISK. If you need help, we recommend the following alternatives:

- Consult with your professor or teaching assistant.
- Go to http://www.palisade.com for answers to frequently asked questions.
- Contact our technical support department via e-mail or fax.
@RISK System Requirements

System requirements for @RISK include:

- Microsoft Windows XP or higher.
- Microsoft Excel 2007 or higher.
Installation Instructions

General Installation Instructions

The Setup program copies the @RISK system files into a folder you specify on your hard disk. To run the Setup program in Windows XP or higher:

Double-click the RISK Setup.exe (or the DTSuite Setup.exe) from your download or installation CD

Follow the Setup instructions on the screen.

If you encounter problems while installing @RISK, verify that there is adequate space on the drive to which you’re trying to install. After you have freed up adequate space, try rerunning the installation.

If you want to remove @RISK from your computer, use the Control Panel’s Add/Remove Programs utility and select the entry for @RISK (or the DecisionTools Suite).

The DecisionTools Suite

@RISK for Excel is a member of the DecisionTools Suite, a set of Excel add-ins for risk and decision analysis described in Appendix D: Using @RISK With Other DecisionTools. The default installation procedure of @RISK puts @RISK in a subfolder of a main Program Files\Palisade (or Program Files (x86)\Palisade) folder.

One subfolder of this Palisade folder will be the @RISK folder (by default called RISK7). This folder contains the program files, plus example models and other files necessary for @RISK to run. Another subfolder of the Palisade folder is the SYSTEM folder. This contains files required by every add-in in the DecisionTools Suite, including common help files and program libraries.
Software Activation

Activation is a one-time license verification process that is required for your Palisade software to run as a fully licensed product. An activation ID such as DNA-6438907-651282-CDM is on your printed/emailed invoice. If you enter your activation ID during installation, your software is activated at the end of the installation process and no further user action is required. If you need to activate your software after installation, you should select License Manager on the @RISK Help menu.

The License Manager can be used to activate, deactivate, or move software licenses. It can also be used to manage licenses for network installations. You can follow the prompts and dialogs in the License Manager to perform the desired licensing operation.
Instructional Materials

The @RISK package includes a number of instructional materials to help you learn @RISK features and how to model with @RISK.

Example Spreadsheets

@RISK includes more than 170 example spreadsheets (and even more can be found at the Palisade Web site). Some of these examples illustrate specific features of @RISK, whereas many others illustrate potential uses of @RISK in various fields: finance, insurance, oil and gas, supply chain, marketing, and others. These examples not only help you learn how to use @RISK, but they illustrate how extensively @RISK can be applied.

You can find these example spreadsheets from the @RISK Help menu. When you click its Example Spreadsheets command, an “example file list” file opens in Excel. This file contains links to all of the example files.

Quick Start Tutorials

From the @RISK Welcome screen, which you see when you launch @RISK or you can access at any time from the @RISK Help menu, you can click the Quick Start link to see a series of short videos that lead you through the basic features of @RISK. These Quick Start videos are intended for beginners, but they are sufficient to get you started creating your own @RISK models.

Guided Tour Videos

A series of Guided Tour videos is also available from the @RISK Welcome Screen. These are more in-depth videos, and they lead you through practically all of the @RISK features, from simple to more complex.

Other Videos

You can also click the Videos command on the @RISK Help menu. This takes you to a Palisade Web site with links to a number of videos, including the Quick Start and Guided Tours videos.
Examples and Videos for the XDK

Once you are comfortable using @RISK, you might want to learn how to use the macro language of Excel, Visual Basic for Applications (VBA), to automate @RISK procedures. @RISK provides an Excel Developer Kit (XDK) for doing this. You can learn about the XDK from the Developer Kit (XDK) command on the @RISK Help menu. There you will see links to example spreadsheet models of @RISK automation, as well as additional instructional videos on using the XDK.
Compatibility with Previous Versions

Using @RISK 7 Spreadsheets in @RISK 3.5 or earlier

@RISK 7 spreadsheets can only be used in @RISK 3.5 or earlier when the simple forms of distribution functions are used. In the simple distribution function format, only required distribution parameters can be used. No new @RISK 5, 6, or 7 distribution property functions can be added. In addition, RiskOutput functions must be removed and outputs reselected when simulating in @RISK 3.5.

Using @RISK 7 Spreadsheets in @RISK 4.0

@RISK 7 spreadsheets can be used directly in @RISK 4.0 with the following exceptions:

- **Alternate Parameter functions**, such as RiskNormalAlt, will not work and will return an error.
- **Cumulative Descending functions**, such as RiskCumulD, will not work and will return an error.
- **Distribution property functions** specific to @RISK 5, 6 and 7 (such as RiskUnits) will be ignored in @RISK 4.0.
- **Statistics functions** specific to @RISK 5 and 6 (such as RiskTheoMean) will return #NAME? in @RISK 4.0.
- **Other new functions** specific to @RISK 5, 6, and 7 such as RiskCompound, RiskSixSigma statistics functions, RiskConvergenceLevel, and supplemental functions such as RiskStopRun will return #NAME? in @RISK 4.0.
Using @RISK 7 Spreadsheets in @RISK 4.5

@RISK 7 spreadsheets can be used directly in @RISK 4.5 with the following exceptions:

- **Distribution property functions** specific to @RISK 5, 6, and 7 (such as RiskUnits) will be ignored in @RISK 4.5. However, functions that contain them will sample properly.

- **Statistics functions** specific to @RISK 5, 6, and 7 (such as RiskTheoMean) will return #NAME? in @RISK 4.5.

- **Other new functions** specific to @RISK 5, 6 and 7 such as RiskCompound, RiskSixSigma statistics functions, RiskConvergenceLevel, and supplemental functions such as RiskStopRun will return #NAME? in @RISK 4.5.

Using @RISK 7 Spreadsheets in @RISK 5

@RISK 7 spreadsheets can be used directly in @RISK 5 with the following exceptions:

- **The Distribution property function RiskIsDate** specific to @RISK 5, 6, and 7 will return #NAME? in @RISK 5.

- **Other new functions** specific to @RISK 6 and 7 such as RiskExtremeValMin, RiskF, RiskLevy, and others will return #NAME? in @RISK 5.

Using @RISK 7 Spreadsheets in @RISK 6

@RISK 7 spreadsheets can be used directly in @RISK 6 with the following exception:

- **The Distribution property function RiskCopula** specific to @RISK 7 will return #NAME? in @RISK 6.
Chapter 3: An Overview of Risk Analysis

Introduction

@RISK brings advanced modeling and risk analysis to Microsoft Excel. If you use data to solve problems, make forecasts, develop strategies, or make decisions, and if uncertainty is involved, then you should definitely consider performing risk analysis with @RISK.

Modeling usually indicates that you are trying to create a representation of a real-life situation so that you can analyze it, usually mathematically. Your representation, or model, can be used to examine the situation and hopefully provide insights into how the future might unfold. If you have ever performed “what-if” analysis in a spreadsheet, to see the effects of changing the values of various inputs, you are well on your way to understanding the importance of including uncertainty in a model.

What exactly is involved in incorporating risk in these analyses and models? This chapter will attempt to answer this question. Fortunately, you don't have to be an expert in statistics or decision theory to analyze situations under risk, and you certainly don't have to be an expert to use @RISK. Once you begin using @RISK, you will quickly begin to gain expertise in @RISK modeling.

Another purpose of this chapter is to provide an overview of how @RISK works in Excel to perform analyses. You don't have to understand everything about @RISK to use it successfully, but you will find some explanations useful and interesting. This chapter discusses:

- What risk is and how it can be assessed quantitatively.
- The nature of risk analysis and the techniques used in @RISK.
- How to run a simulation.
- How to interpret @RISK results.
- What risk analysis can and cannot do.
Everyone knows that risk affects the gambler about to roll the dice, the wildcatter about to drill an oil well, or the tightrope walker taking that first big step. But these simple illustrations aside, risk is a result of future uncertainty, that is, our inability to know how the future will unfold, given the decisions we make today. Risk implies that a given decision has more than one possible outcome.

In this simple sense, every decision is risky, from crossing the street to building a dam. However, the term “risk” is usually reserved for situations where the range of possible outcomes from a given decision is in some way significant. Common actions like crossing the street are usually not risky, whereas building a dam can involve significant risk. Somewhere in between, decisions pass from being nonrisky to risky. This distinction, although vague, is important. If you judge that a situation is risky, risk becomes one criterion for deciding what decision to make. At that point, some form of risk analysis becomes useful.

Characteristics of Risk

Risk derives from our inability to predict the future. It indicates a degree of uncertainty that is significant enough to make us notice it. This definition takes more shape by discussing several important characteristics of risk.

First, risk can be either objective or subjective. Flipping a coin is an objective risk because the chances of the possible outcomes are well known. Even though the outcome is uncertain, an objective risk can be described precisely based on theory, experiment, or common sense. Everyone agrees with the description of an objective risk. In contrast, describing the chances of rain next Thursday is not so clear, and this represents a subjective risk. Given the same information, weatherman A might say the chance of rain is 30%, whereas weatherman B might say the chance of rain is 45%. Neither is wrong. Describing a subjective risk is open-ended in the sense that anyone’s assessment could always refined with new information, further study, or by giving weight to the opinion of others. Most risks are subjective, and this has important implications for analyzing risk or making decisions based on a risk analysis.

Second, deciding that something is risky requires personal judgment, even for objective risks. For example, imagine flipping a coin where you win $1 for heads and lose $1 for tails. The range from $1 to -$1 would not be overly significant to most people. If the stakes were
$100,000 and -$100,000, however, most people would find the situation to be quite risky. Still, there might be a wealthy few who would not find this range of outcomes to be significant.

Third, risky actions, and therefore risk, are sometimes avoidable. Individuals differ in the amount of risk they are willing to accept. For example, two individuals of equal net worth might react quite differently to the $100,000 coin flip bet just described. One might accept it and the other might refuse it. Their personal preference for risk differs.

The Need for Risk Analysis

Here are a few common situations where risk analysis should be (and has been) applied. Maybe at least one of them applies to you.

- **Risks for New Product Development and Marketing.** Will the R&D department solve the technical problems involved? Will a competitor get to market first, or with a better product? Will government regulations and approvals delay product introduction? How much impact will the proposed advertising campaign have on sales levels? Will production costs be as forecast? Will the proposed sales price have to be changed to reflect unanticipated demand levels for the product?

- **Risks for Securities Analysis and Asset Management.** How will a tentative purchase affect portfolio value? Will a new management team affect market price? Will an acquired firm add earnings as forecast? How will a market correction impact a given industry sector?

- **Risks for Operations Management and Planning.** Will a given inventory level suffice for unpredictable demand levels? Will labor costs rise significantly with upcoming union contract negotiations? How will pending environmental legislation impact production costs? How will political and market events affect overseas suppliers in terms of exchange rates, trade barriers, and delivery schedules?

- **Risks for Design and Construction of a Structure (building, bridge, dam, and so on).** Will the cost of construction materials and labor be as forecast? Will a labor strike affect the construction schedule? Will the levels of stress placed on the structure by peak loads, crowds, and nature be as forecast? Will the structure ever be stressed to the point of failure?
• **Risks for Investment in Exploration for Oil and Minerals.**
  Will anything be found? If a deposit is found, will it be uneconomical, or a bonanza? Will the costs of developing the deposit be as forecast? Will some political event like an embargo, tax reform, or new environmental regulations drastically alter the economic viability of the project?

• **Risks for Policy Planning.** If the policy is subject to legislative approval, will it be approved? Will the level of compliance with any policy directives be complete or partial? Will the costs of implementation be as forecast? Will the level of benefits be what you projected?

**Assessing and Quantifying Risk**

Realizing that you have a need for risk analysis is only the first step. You must then *quantify* the risk(s) you have identified. Specifically, you must assess all possible values of a risky variable and the relative likelihoods of these values. Suppose your uncertain variable is the outcome from the flip of a coin. You could repeat the flip a large number of times. Assuming the coin is well-balanced, you would eventually find that it comes up heads about half the time and tails the other half. That is, you could estimate the relevant probabilities by repeating the “experiment” of flipping the coin.

In most real life situations, however, you can’t repeatedly perform an “experiment” to estimate probabilities. For example, when Apple or one of its rivals introduces a revolutionary new device into the market, it does this only once. It can’t repeat the “experiment” over and over. Therefore, when the company tries to assess probabilities of various levels of success, these probabilities are necessarily subjective, usually based partly of historical experience, expert judgment, and even gut feel.

If you can calculate the risks of your situation the way you would for a coin flip, the risk is objective. This means that everyone should agree with your risk assessment. Most risk quantification, however, involves subjective judgment, and different people, even experts in the area, might have quite different assessments.

In addition, your subjective assessments of risk are likely to change when you receive more information. If you have subjectively derived a risk assessment, you must always ask yourself whether additional information is available that would help you make a better assessment. If it is available, how hard and how expensive would it be to obtain? How much would it cause you to change your current assessment? How much would these changes affect the final results?
of the model you are analyzing? These are difficult questions to answer, but you need to keep them in mind.

**Describing Risk with a Probability Distribution**

If you have quantified risk, that is, you have determined possible outcomes and their probabilities of occurrence, you can summarize this information with a probability distribution. A probability distribution is a mathematical method for describing the uncertainty of a variable. @RISK uses probability distributions to describe uncertain values in your Excel models and to present results. There are many forms and types of probability distributions, each of which describes a range of possible values and their probabilities of occurrence. One popular type of distribution is the normal distribution, the traditional bell curve. But there is a wide variety of distribution types, including the uniform distribution, the triangular distribution, and many others.

All distribution “families,” such as the normal, use a set of arguments to specify a range of possible values and their probabilities. For example, the normal family uses a mean and standard deviation as its arguments. The mean defines the value around which the bell curve is centered, and the standard deviation defines the spread of values around the mean. Over thirty families of distributions, including the normal family, are available in @RISK for describing the uncertainty of variables in your Excel models.

The @RISK Define Distribution window allows you to graphically preview distributions and assign them to uncertain values. Using its graphs, you can quickly see the range of possible values your distribution describes.
What Is Risk Analysis?

In a broad sense, risk analysis is any method — quantitative and/or qualitative — for assessing the impacts of risk on the outcomes of decisions. Numerous techniques are used that blend both quantitative and qualitative techniques. The goal of these methods is to help a decision maker choose a course of action and to enable a better understanding of the possible outcomes that could occur.

Risk analysis in @RISK is a quantitative method that seeks to determine the probability distributions of the outcomes resulting from decisions. In general, the techniques in an @RISK risk analysis encompass four steps:

1. **Developing a Model.** Defining your problem or situation in an Excel model.

2. **Identifying Inputs and Outputs.** Determining uncertain inputs in your model and specifying their possible values with probability distributions, and identifying the important outputs you want to analyze.

3. **Analyzing the Model with Simulation.** Running many scenarios, each with sampled values for the uncertain inputs, to determine the probability distributions of your outputs.

4. **Making a Decision.** Using the simulated results and personal preferences to make informed choices.

@RISK helps with the first three steps, by providing a powerful and flexible set of tools in Excel to facilitate model building and risk analysis. The results generated by @RISK can then be used by a decision maker to help choose a course of action.

The techniques @RISK employs in a risk analysis are actually quite intuitive. As a result, you won't have to resort to calling @RISK a "black box" when your colleagues and superiors question you about the nature of your risk analysis. The following discussion will give you a firm understanding of what @RISK needs from you in terms of a model, and how an @RISK risk analysis proceeds.
Developing an @RISK Model

You are the expert at understanding the problems and situations you need to analyze. If you have a problem that is subject to risk, then @RISK and Excel can help you construct a complete and logical model.

A major strength of @RISK is that it allows you to work in a familiar model building environment: Microsoft Excel. @RISK works with your Excel model, allowing you to conduct a risk analysis, but it still preserves familiar spreadsheet capabilities. You presumably know how to build spreadsheet models in Excel. Then @RISK gives you the ability to modify these models easily for risk analysis.

Input Variables

Input variables (or simply “inputs”) are the basic “drivers” of your Excel model. If you are modeling a financial situation, your input variables might be quantities such as Unit Cost, Demand, and R&D Expenses, whereas if you are modeling a geologic situation, your input variables might be quantities such Depth to Deposit, Thickness of Coal Seam, and Porosity. Each situation has its own input variables, identified by you.

You might know the values of some of your input variables; these are certain (or “deterministic”) inputs. Conversely, you may not know the values of other input variables; these are uncertain inputs. If your input variables are uncertain, you need to describe the nature of their uncertainty. You do this with probability distributions, which specify both the range of possible values of the variable and the likelihood of occurrence of each value within the range. In @RISK, uncertain variables are entered as probability distribution functions in cell formulas. The following are typical examples:

- RiskNormal(100,10)
- RiskUniform(20,30)
- RiskExpon(A1+A2)
- RiskTriang(A3/2.01,A4,A5)

These distribution functions can be placed in your worksheet cell formulas just like any other Excel functions.
In addition to being certain or uncertain, input variables in a risk analysis model can be uncorrelated or correlated with one another. For example, if you have a financial model that evaluates the profitability of an agricultural crop, you might include an uncertain variable called Amount of Rainfall. It is reasonable to assume that other variables in your model such as Crop Price and Fertilizer Cost have no effect on the amount of rain, so that Amount of Rainfall would be uncorrelated with (that is, independent of) these other variables.

Correlated variables, in contrast, depend on one another. For example, you would expect the variable Crop Yield in the agricultural model to depend on the variable Amount of Rainfall. If there is too little or too much rain, then the crop yield will be low. If the amount of rain is about normal, then the crop yield will be somewhere in the normal range. There might also be other variables that are correlated with Crop Yield, such as Temperature and Loss to Insects.

When identifying the uncertain values in your Excel worksheet, you have to decide whether your input variables are uncorrelated or correlated. If you decide that a set of variables is correlated, you can use special @RISK functions to model their dependent behavior. It is extremely important to correctly recognize correlations between variables; otherwise, your model might generate nonsensical results. For example, if you ignored the relationship between Amount of Rainfall and Crop Yield, @RISK might simultaneously choose a low value for Rainfall and a high value for the Crop Yield, something that would almost never occur.

**Output Variables**

In addition to input variables that “drive” your model, you also need one or more “bottom-line” output variables (or simply “outputs”). These variables, such as Profit, Project Completion Time, and Ending Market Share, are the variables of primary interest in your model. Any @RISK model can have one or more output variables, and once you specify them as output variables, @RISK can use simulation to estimate their probability distributions. It then provides a number of tools for analyzing these probability distributions.
Analyzing a Model with Simulation

Once you have specified uncertain input variables and output variables of interest, you can run an @RISK simulation.

Simulation

@RISK uses simulation, often called Monte Carlo simulation, to perform a risk analysis. Here, simulation refers to a method where the distribution of possible outcomes is generated by letting a computer recalculate your worksheet repeatedly, each time using different randomly selected sets of values for the probability distributions in your input cells. In effect, the computer tries a wide range of valid combinations of the values of input variables to simulate a wide range of possible values of your output variables. This is just as if you ran hundreds or thousands of “what-if” analyses on your worksheet, all in one sitting.

As an example, suppose you have a model with only two input variables, Revenue and Cost. If there is no uncertainty in these variables, you will identify a single possible value for each variable. These two single values will then be combined by your Excel formulas to calculate a single value for an output variable such as Profit. For example, suppose the certain input values are:

Revenue = 100, Cost = 90

Then the single output value is:

Profit = 10

There is only one combination of the input variable values, because there is only one value possible for each variable.

Now suppose there is uncertainty in both input variables. For example,

Revenue = 100 or 120, Cost = 90 or 80

This gives two values for each input variable. In a simulation, @RISK would consider all possible combinations of these variable values to calculate possible values for Profit.

There are four combinations:

Profit = Revenue − Cost
10 = 100 − 90, 20 = 100 − 80, 30 = 120 − 90, 40 = 120 − 80

Now Profit is an uncertain variable because it is calculated from uncertain input variables.
How Simulation Works

In @RISK, simulation uses the following two distinct operations:

- It selects sets of values for the probability distribution functions contained in the input cells of your model.
- It recalculates the Excel worksheet using the new values.

The selection of values from probability distributions is called sampling, and each recalculation of the worksheet is called an iteration. Each iteration uses a set of single set of values sampled from input distribution functions to calculate values in the output cells. @RISK generates output distributions by consolidating the output values from all of the iterations.

The Alternative to Simulation

There are two basic approaches to quantitative risk analysis, and both generate valid results. Each of these approaches has the same goal: to derive probability distributions that describe the possible outcomes of output variables. The first approach is the one just described for @RISK: simulation. This approach relies on the ability of a computer to do a great deal of work very quickly.

The second approach to risk analysis is an analytic approach. Analytic methods require that the distributions for all uncertain variables in a model be described mathematically. Then the equations for these distributions are combined mathematically to derive formulas for the distributions of output variables. Unfortunately, this approach is usually not practical. It requires a strong mathematical background in probability theory, and even then, the mathematical details can be intimidating, if not impossible.

In short, the analytic approach is well beyond the realm of possibility for most users, whereas the simulation approach is possible for almost everyone, including you!
Making a Decision: Interpreting the Results

@RISK simulation results are presented in the form of probability distributions. You must interpret these probability distributions and then make decisions based on your interpretation.

**Interpreting a Traditional Analysis**

Let's start by looking at how you would interpret a single-valued result from a traditional analysis — an “expected” value. You would probably compare the expected value to some standard or minimum acceptable value. If it is at least as good as the standard, you would probably judge the result as acceptable. However, you should also recognize that the expected result doesn't include the impact of uncertainty. You must somehow manipulate the expected result to make some allowance for risk. You might arbitrarily raise the minimum acceptable result, or you might (non-rigorously) weigh the chances that the actual result could exceed, or fall short of, the expected result. At best, you might extend the analysis to include several other results, such as “worst case” and “best case,” in addition to the expected value. You could then decide whether the “expected” and “best case” values are good enough to outweigh the “worst case” value.

**Interpreting an @RISK Analysis**

In an @RISK risk analysis, the probability distributions of outputs provide a complete picture of the range of the possible outcomes. This is a tremendous elaboration on the “worst-expected-best” case approach just described. Beyond filling in the gaps between the three values, a probability distribution also does the following:

- **Determines a “Correct” Range.** Because you have more rigorously defined the uncertainty associated with the input variables, the possible range of outcomes is usually quite different, and more correct, than a “worst case” to “best case” range.

- **Shows Probability of Occurrence.** A probability distribution shows the probabilities of the possible outcomes.

As a result, you no longer just compare desirable outcomes with undesirable outcomes. Instead, you can recognize that some outcomes are more likely to occur than others and should be given more weight in your evaluation. This process is also a lot easier to understand than the traditional analysis because you see graphs of
the relevant output distributions. These graphs let you see the probabilities and understand the risks involved more clearly.

**Individual Preference**

The results provided by an @RISK analysis must be interpreted by *you*. The same results given to other individuals might be interpreted differently, and this could lead to a different courses of action. This is not a weakness of the technique. It is simply a result of the fact that individuals have varying preferences with regard to possible choices, time, and risk. You might feel that the shape of the output distribution favors a “no-go” decision. A colleague who is less risk averse might come to the opposite conclusion.

**The Distribution “Spread”**

Range and likelihoods of occurrence are directly related to the level of risk associated with a particular event. By looking at the spread and likelihoods of possible results, you can make an informed decision based on the level of risk you are willing to take. Risk averse decision makers prefer a small spread in possible results, with most of the probability associated with desirable results. But if you are a risk taker, you might accept a greater spread or possible variation in your outcome distribution. Furthermore, a risk taker might be influenced by “bonanza” outcomes, even if their probability of occurrence is small.

**Skewness**

A simulation output distribution can also show skewness, that is, an asymmetrical distribution of possible results. Suppose your distribution has a large positive “tail.” If you saw only a single number for the expected result, you might not realize the possibility of a highly positive outcome that could occur in the tail. Skewness such as this can be very important to a decision maker. By presenting a probability distribution for an outcome, including possible skewness, @RISK leads to a more enlightened decision.
What Risk Analysis Can (and Cannot) Do

Quantitative analysis techniques have gained a great deal of popularity with decision makers and analysts in recent years. Unfortunately, many people have mistakenly assumed that these techniques are magic “black boxes” that unequivocally specify the “correct” decision. No technique, including simulation with @RISK, can make this claim. All of these techniques are tools that can be used to help make good and enlightened decisions. Like any tools, they can be used to good advantage by skilled practitioners, or they can be used to create havoc in the hands of the unskilled. In the context of risk analysis, quantitative tools should used to complement personal good personal judgment, not replace it.

Finally, you should recognize that risk analysis cannot guarantee that the action you choose to follow, even if skillfully chosen to match your personal preferences, is the best action viewed from the perspective of hindsight. A “perfect” decision implies perfect information, which you almost never have at the time you have to make the decision. However, by using @RISK to conduct a logical risk analysis, you can be guaranteed that you have chosen the best strategy, given the available information when you have to make the decision. This is not a bad guarantee!
Chapter 4: Getting to Know @RISK

A Quick Overview of @RISK

This chapter provides a quick overview of using @RISK with Microsoft Excel. It guides you through the process of setting up an Excel model to be used with @RISK, simulating that model, and interpreting the results of your simulation.

How Does Risk Analysis Work?

@RISK extends the analytical capabilities of Microsoft Excel to include risk analysis and simulation. These techniques allow you to analyze your spreadsheet models for risk. Risk analysis identifies the range of possible outcomes you can expect for output variables and their probabilities of occurrence.

@RISK uses the technique of Monte Carlo simulation for risk analysis. With this technique, uncertain input values in your spreadsheet are specified as probability distributions. An input value is a value in a spreadsheet cell, or a formula, that “drives” the results in your spreadsheet model. In @RISK, a probability distribution that describes the range of possible values for an input is substituted for its original single fixed value. To learn more about inputs and probability distributions, see An Overview of Risk Analysis in this manual.
How Does @Risk Link to Excel?

To add risk analysis capabilities to your spreadsheet model, @RISK uses ribbon commands and custom distribution functions to Excel.

When @RISK is loaded, you will see the following @RISK ribbon. The icons and commands on this ribbon provide quick access to most @RISK options. (It is shown here in two parts so that you can see the details better.)

In @RISK, probability distributions are entered directly into your cell formulas using custom distribution functions. These functions, each of which represents a type of probability distribution (such as normal or binomial), are added to cell formulas, just like native Excel functions. To enter a distribution function, you enter both the function name, such as RiskTriang (a triangular distribution) and the arguments that describe the shape and range of the distribution, such as RiskTriang(10,20,30). Here, 10, 20, and 30 are the minimum, most likely, and maximum value, respectively.

@RISK distribution functions can be used in any uncertain input cell. These functions can be used just like you would use any native Excel functions, such as including them in mathematical expressions or having cell references or formulas for their arguments.

Entering Distributions in Cell Formulas

@RISK includes a Define Distribution window that allows you to easily add probability distribution functions to cell formulas. You display this window by clicking the Define Distributions icon on the @RISK ribbon.

The Define Distribution window graphically displays probability distributions which can be substituted for values in a cell formula. By changing the displayed distribution, you can see how various distributions describe the range of possible values for an uncertain input in your model.
The graphical display of an uncertain input is useful in showing your definition of the input to others. It displays the range of possible values for an input, their relative probabilities of occurring, and a number of descriptive statistics for the distribution. By working with distribution graphs, you can easily incorporate assessments of uncertainty from experts into your risk analysis models.

Simulation Outputs
Once distribution functions have been entered into your spreadsheet model, you need to designate the cells (or ranges of cells) for which you are interested in seeing simulation results. Typically, these output cells contain the “bottom-line” results of your spreadsheet model, such as Profit, but they can be any cells, anywhere in your spreadsheet model. To designate output cells, you simply select a cell or a range of cells and click the Add Output icon on the @RISK ribbon.
Model Window

@RISK’s Model window provides a table of all input probability distributions and simulation outputs in your model. From this window, you can:

- Edit any input distribution or output directly in the table.
- Drag and drop any thumbnail graph to expand it to a full-sized window.
- Quickly view thumbnail graphs of all defined inputs.
- Double-click any entry in the table to use the Graph Navigator to move through cells in your workbook with input distributions.

The Model window columns can be customized to show exactly the statistics you want to display for your inputs and outputs.
Using Data to Define Probability Distributions

@RISK’s Distribution Fitting menu (Professional and Industrial versions only) allows you to fit probability distributions to your data. You typically do this when you have a set of collected data that you want to use as the basis for an input distribution in your spreadsheet model. For example, you might have historical data on weekly demand for a product. Then you could fit a distribution based on this data to model possible future demands.

If desired, the fitted distributions can be assigned to an uncertain input in your spreadsheet model. In addition, if data in Excel are used in a fit, the fitted distribution can be linked to the data, so that the fit will automatically update whenever data change.
Running a Simulation

You run an @RISK simulation by clicking the Start Simulation icon on the @RISK ribbon.

When a simulation runs, your spreadsheet model recalculates repeatedly. Each recalculation is called an iteration. On each iteration, the spreadsheet model is recalculated with a new set of sampled values, thus generating new values for the output cells.

@RISK keeps track of these output values and displays them graphically, along with a number of relevant statistics, in a pop-up window.
Simulation Results

@RISK simulation results include distributions (graphs and statistics) of your outputs. In addition, @RISK generates sensitivity analysis and scenario analysis reports. These identify the inputs most critical to your results. These results are best presented graphically. Available graphs include frequency distributions of outputs, cumulative probability curves of outputs, tornado graphs for showing the sensitivities of an output to different inputs, and summary graphs for summarizing risk changes across a range of output cells.

Reports on an @RISK Simulation in Excel

@RISK makes it easy to copy a graph window from an @RISK simulation and paste it into a document, such as an Excel, Word, or PowerPoint document.

In addition, you can “export” any report window to an Excel worksheet, where you can access its values with formulas.

@RISK offers a set of standard reports that summarize your simulation results. In addition, @RISK reports generated in Excel can use pre-built “template” sheets that contain custom formatting, titles, and logos.

Advanced Analytical Capabilities

Advanced capabilities are available in @RISK to enable sophisticated analysis of simulation data. @RISK collects simulation data from all iterations for both input distributions and output variables. It analyzes this data set to determine:

- **Sensitivity Analysis.** This type of analysis identifies the input distributions with the most effect on outputs.
- **Scenario Analysis.** This type of analysis finds the combinations of input values most responsible for outputs achieving specified “target” values.
**Sensitivity Analysis**

@RISK’s sensitivity analysis, which identifies significant inputs, can be performed with three different analytical techniques: change in output statistic, regression analysis, and rank correlation analysis. The results of a sensitivity analysis can be displayed as a *tornado* graph, with longer bars at the top representing the most significant input variables.

**Scenario Analysis**

@RISK’s scenario analysis identifies combinations of input values that lead to output target values. This allows simulation results to be characterized by statements such as “when Profits are high, significant inputs are: low Operating Costs, very high Sales Prices, high Sales Volumes, and so on.”
Setting Up and Simulating an @RISK Model

This section steps you through the process of setting up an @RISK model and running a simulation on it. It briefly describes:

- Probability distributions in your worksheet model
- Correlations between distributions
- Running a simulation
- Simulation results
- Graphs of simulation results

Remember that you can also see a video of these basic operations in the @RISK Quick Start tutorial, available from the @RISK Welcome screen.

Probability Distributions in Your Worksheet

Recall that uncertainty in an @RISK model is entered with probability distribution functions. You can choose from over forty different @RISK distribution functions. Each function describes a different type of probability distribution. The simplest functions are those such as RiskTriang(min,most likely,max) or RiskUniform(min,max), where the arguments (the minimum, most likely, and maximum values) for the uncertain input are easily understood. More complex functions take arguments specific to the distribution, such as RiskBeta(alpha,beta).

For more sophisticated models, @RISK allows you to set up distribution functions that use cell references and spreadsheet formulas for function arguments. For example, you can set up a group of distribution functions across a spreadsheet row, with the mean of each function determined by the value sampled for the prior function. Mathematical expressions can also be used as arguments for distribution functions.
All distribution functions can be defined and edited using the pop-up Define Distribution window. The Define Distribution window can, among other things, also be used to enter multiple distribution functions in a cell’s formula, enter names that will be used to identify an input distribution, and truncate a distribution.
Argument values can be entered in the Distribution Argument panel or typed directly in the formula. This panel is displayed to the left of the graph.

By changing the Parameter type, you can enter Alternate Parameters or Truncate the distribution.
@RISK distribution functions have both required and optional arguments. The only required arguments are the numeric values that define the range and shape of the distribution. All other arguments, such as name, truncation, correlation, and others are optional, and are entered only when needed. These optional arguments are entered using property functions in a pop-up Input Properties window. This window is accessed by clicking the fx button at the top right of the Define Distribution window.

All entries made in the Define Distribution Window are converted to distribution functions that are placed in your spreadsheet model. For example, the distribution function created by the entries in the window displayed here would be:

=RiskNormal(20000,3000)

All of the distribution arguments that are assigned through the Define Distribution window can also be entered directly in the distribution itself. In addition, all arguments can be entered as cell references or as formulas, just like standard Excel functions.

As a beginner, you will probably want to use the Define Distribution window to enter your distribution functions. This helps you learn how to assign values to function arguments. Then once you better understand the syntax of distribution function arguments, you can enter the arguments yourself directly in Excel, bypassing the Define Distribution window.
Correlating Input Variables

In many situations, it is important to account for correlation between input variables. Correlation occurs when two or more input distributions are related. For example, it might be known that when one input is relatively high, a second input also tends to be relatively low. A good example is **Interest Rate** versus **Housing Starts**. The sampling from their distributions should be performed in a specific way to avoid nonsensical results. For example, when a high Interest Rate is sampled, the sampled value of Housing Starts should be relatively low, or vice versa.

There are two correlation methods available in @RISK. To add basic correlations between two or more input distributions, you can use a correlation matrix. Alternatively, for more advanced control over correlation patterns, you can consider using a copula.

To add correlations with a correlation matrix, you select the cells in Excel that contain the input distributions you want to correlate, and then you click the **Define Correlation Matrix** command from the **Define Correlations** menu. Once a matrix is displayed, you can enter correlation coefficients between inputs in the matrix cells, you can copy values in from a matrix in Excel, or you can use **scatter plots** to assess and enter correlations.
You can display a matrix of scatter plots by clicking the Scatter Plots icon at the bottom left of the Define Correlations window. The scatter plots in matrix cells show how values between any two input distributions are correlated. By dragging a scatter plot cell off of the matrix, you can expand the thumbnail scatter plot into a full-sized graph window.

When you use the Define Correlations window to correlate inputs, @RISK automatically adds RiskCorrmat functions to your individual distribution functions. Once you see the RiskCorrmat entries that are entered and you are comfortable with their syntax, you can enter these functions directly in your spreadsheet model, bypassing the Define Correlations window.
Fitting Distributions to Data

@RISK allows you to fit probability distributions to your data (Professional and Industrial versions only). Fitting is done when you have a data set that you want to use as the basis for an input distribution in your spreadsheet model. For example, you might have collected historical data on customer demand. Then you could fit a distribution to this data for use in modeling future demands.
**Fitting Options**

Several options are available for controlling the fitting process. Specific distributions can be selected to fit. In addition, input data can be in the form of sample, density, or cumulative data. You can also filter your data prior to fitting.
A number of graphs, including Comparison, P-P, and Q-Q plots, are available to help you examine the results of your fits. Delimiters on graphs allow you to quickly calculate probabilities associated with values in the fitted distributions.

Clicking **Write to Cell** places a fitted result in your spreadsheet model as a new distribution function. In addition, if you select the Linked or Live options (in the middle of the dialog), the fitted distribution will be dynamic, that is, it will change when the data changes.
The Fit Manager allows you to navigate between fitted data sets in your workbook or delete previously run fits.
@RISK Model Window

To help you view your model, @RISK detects all distribution functions, outputs, and correlations entered in your spreadsheet model and lists them in the Model window. From this window, you can:

- Edit any input distribution or output directly in the table.
- Drag and drop any thumbnail graph to expand it to a full-sized window.
- Quickly view thumbnail graphs of all inputs and outputs.
- Double-click any entry in the table to use the Graph Navigator to move through cells in your workbook with input distributions.
- Edit and preview correlation matrices.
The Model window can be customized to select the statistics you want to display. The **Columns** icon at the bottom of the window displays the **Columns for Table** dialog.

Inputs in the Model window can be grouped by category. By default, a category is created when a group of inputs share the same row (or column) name. In addition, you can define categories of inputs.
Simulation Settings

A variety of settings can be used to control the type of simulation @RISK performs. A simulation in @RISK supports nearly unlimited iterations and multiple simulations. Multiple simulations allow you to run one simulation after another on the same model. In each of these simulations, you can change values in your spreadsheet model so that you can compare simulation results across different assumptions.
A few simulation settings can be changed directly from the @RISK ribbon. Many others can be changed from the Simulation Settings dialog (shown above).

The items on the @RISK ribbon include:

- **Simulation Settings.** Opens the Simulation Settings dialog.
- **Iterations.** Lets you choose the number of iterations to run.
- **Simulations.** Lets you choose the number of simulations to run.
- **Random/Static Standard (F9) Recalc.** Toggles @RISK between returning static (usually expected) values from distributions and returning Monte Carlo samples in a standard Excel recalculation.
- **Automatically Show Output Graph.** Toggles between showing the distribution of a selected input or output as the simulation runs and not showing it.
- **Demo Mode.** Another toggle, useful for “stepping through” a simulation.

## Running a Simulation

A simulation in @RISK involves repetitive recalculation of your worksheet. Each recalculation is called an iteration. With each iteration:

- All distribution functions are sampled.
- Sampled values are returned to the cells and formulas of the spreadsheet model.
- The model is recalculated.
- Values calculated for output cells are stored.
- Any open @RISK graphs and reports are updated as necessary.

This repetitive recalculation process can run hundreds or even thousands of iterations.
Clicking the **Start Simulation** icon starts a simulation. When a simulation is running, you can watch Excel recalculate numerous times, you can monitor the convergence of your output distributions, and you can watch graphs of distributions of simulation results update in real time.

A Progress window is displayed during simulations. The icons in this window allow you to run, pause, or stop a simulation, as well as turn real-time updating of graphs and Excel recalculations on or off. The icons at the bottom (from left to right) are: Show Results Summary Window, Show Excel Recalculations, Update Graphs/Reports Real Time, Run, Pause, Stop, and Show Multi CPU Monitor.
Assuming that either Automatically Show Output Graph or Demo Mode is toggled “on,” @RISK shows you graphically how distributions of possible outcomes change during a simulation. Graph windows update to show calculated distributions of outcomes and their statistics.

This graph of the distribution of possible outcomes is created by combining all output values generated so far, analyzing them, and calculating statistics on how they are distributed across their minimum-to-maximum range.

The outcomes displayed depend on the selected cell when you run the simulation:

- If an output cell is selected, its distribution is shown.
- If an input cell is selected, its distribution is shown.
- If no input or output cell is selected, the distribution of the first output cell is shown.
@RISK includes a convergence monitoring capability to help evaluate the stability of the output distributions during a simulation. As more iterations are run, output distributions become more stable. It is important to run enough iterations so that the statistics generated for your outputs are reliable. However, at some point, the time spent for additional iterations is essentially wasted because the output statistics do not change significantly.

The Convergence settings control how the convergence of simulation outputs are monitored by @RISK as a simulation runs. Convergence testing can be controlled for individual outputs by using the RiskConvergence property function or with a global setting for all outputs in the Simulation Settings dialog.

@RISK monitors a set of convergence statistics on each output distribution during a simulation. During monitoring, @RISK calculates these statistics for each output at selected intervals (such as every 100 iterations) throughout the simulation. As more iterations are run, the amount of change in the statistics decreases until the specified Convergence Tolerance and Confidence Level are achieved.

You can also run @RISK in Auto-Stop mode, where @RISK will continue to run iterations until all outputs have converged. The number of iterations required for convergence is dependent on the model being simulated and the distribution functions included in the model. For example, more complex models with highly skewed distributions might require more iterations than simpler models.
Browse Results Mode

Browse Results mode is turned on by clicking the Browse Results icon on the @RISK ribbon. Also, Browse Results mode is automatically turned on at the end of a simulation run if Automatically Show Output Graph or Demo Mode is toggled “on.” In this case you don’t need to click the Browse Results icon.

In Browse Results mode, @RISK pops up graphs of simulation results as you select cells in your spreadsheet model, as follows:

- If the selected cell is a simulation output (or contains an input distribution function), @RISK displays its simulated distribution in a window pointing at the cell.
- If the selected cell is part of a correlation matrix, a matrix of the simulated scatter plots between the inputs in the matrix is displayed.

As you select different cells in your workbook, their results graphs are displayed. Alternatively, you can press <Tab> to move the Graph window through the various output cells in your model.

To exit the Browse Results mode, you can close the pop-up graph or click the Browse Results icon on the ribbon.
Results Summary Window

The **Results Summary** window summarizes the results of your model and displays thumbnail graphs and summary statistics for your simulated output cells and input distributions. Columns in the Results Summary window table can be customized to show exactly the statistics you want to display.

From the Results Summary Window, you can:

- Drag and drop any thumbnail graph to expand it to a full-sized window.
- Double-click any entry in the table to use the Graph Navigator to move through cells with input distributions in your model.
Detailed Statistics Window

Detailed statistics are available on simulated outputs and inputs, and target values can be entered for one or more outputs (or inputs).

Targets

Target values can be calculated on simulation results. A target shows the probability of achieving a specific outcome or the value associated with any probability level. Using targets you can answer questions such as: “What is the probability of a cash flow greater than one million?” or “What is the chance of a negative NPV?” Targets can be entered in the Detailed Statistics window, in the Results Summary window, and set directly with delimiters on graphs of simulation results.

By entering a desired target such as 1% for an output in the Results Summary window and copying it across several outputs, you can quickly see the same target calculated for these outputs.
Graphing Results

Simulation results can be displayed graphically in a variety of ways, and @RISK is extremely rich in graphical options. For example, the Results Summary window shows thumbnail graphs of the simulation results for all of your outputs and inputs. By dragging a thumbnail graph off this window, you can expand it to a full-sized graph window.

A graph of the results for an output shows the range of possible outcomes and their relative likelihood of occurrence. The graph can be displayed in standard histogram or frequency distribution form, and it can also be displayed in cumulative form. Each graph created by @RISK is accompanied by statistics for the output or input being graphed.

You can change the type of graph displayed with the icons at the bottom of the graph window. In addition, if you right-click a graph window, a pop-up menu allows you to change a graph’s format, scaling, colors, titles, and displayed statistics. Each graph can be copied to the clipboard and pasted into your spreadsheet. Because graphs are transferred as Windows metafiles, the pasted version can be resized and annotated.

You can use the Chart in Excel command to create graphs in Excel's native graph format. These graphs can be changed or customized just as with any Excel graph.
It is often useful to compare several simulated distributions graphically. This can be done with overlay graphs.

You can add overlays by clicking the Add Overlay icon at the bottom of a graph window, by dragging one graph onto another, or by dragging a thumbnail graph from the Results Summary window onto an open graph. Once overlays are added, delimiter statistics display probabilities for all distributions included in the overlay graph.

By dragging the delimiters displayed on a histogram or cumulative graph, you can calculate target probabilities or percentiles. When you move delimiters, calculated probabilities are shown in the delimiter bar above the graph. This is useful for graphically answering such questions as “What is the probability of a cash flow between 1 million and 2 million occurring?” or “What is the probability of a negative NPV occurring?”

Delimiters can be displayed for any number of overlays. The Graph Options dialog allows you to set the number of delimiter bars displayed.
Each distribution in an overlay graph can be formatted independently. By using the **Curves** tab options in the **Graph Options** dialog, the color, style, and pattern of each curve in the overlay graph can be set.
A Summary graph displays how risk changes across a range of output or input cells, such as a row of time series output values. You can create a summary graph for an output range, or you can select individual inputs or outputs to compare in a summary graph. Summary graphs have two forms: Summary Trend graphs and Summary Box Plot graphs. Either of these graphs can be displayed by:

- Clicking the Summary Graph icon at the bottom of a graph window and then selecting the cell(s) in Excel with results you want to include in the graph.

- Selecting the rows in the Results Summary window for the outputs (or inputs) you want to include in the summary graph, then clicking the Summary Graph icon at the bottom of the window (or right-clicking in the table) and selecting Summary Trend or Summary Box Plot.

A Summary Trend graph is especially useful in displaying trends such as how risk changes across time. For example, if a range of 10 output cells contain Cash Flow in years 1 through 10 of a project, a Summary Trend graph for this range shows how your risk changes across the 10-year period. The narrower the band, the less the uncertainty about your cash flow estimates. Conversely, the wider the band the greater the possible variance in cash flows and the greater the risk.
The center line of the Summary Trend graph represents the trend in mean value across the range. The two outer bands above the mean are one standard deviation above the mean and the 95th percentile, respectively. The two outer bands below the mean are one standard deviation below the mean and the 5th percentile, respectively. The definition of these bands can be changed using the Trend tab in the Graph Options dialog.

A Summary Box Plot displays a box plot for each distribution selected for the summary graph. A box plot (also called a box-whisker graph) shows a box for a defined inner range of a distribution, with lines showing the outer limits of the distribution. An inner line in the box marks the location of the mean, median, or mode of the distribution.
**Scatter Plots**

A scatter plot is an x-y graph showing the relationship between two variables. This type of graph is especially useful for examining the relationship between an input and an output from a simulation. @RISK displays an optional confidence ellipse to identify the region where, at a given confidence level, the x-y values will fall. Scatter plots can also be standardized so that multiple variables with different magnitudes can be more easily compared on a single scatter plot.

Scatter plot windows can be created by:

- Clicking the Scatter Plot icon on a displayed graph and then selecting the cell(s) in Excel with results you want to include in the plot.
- Selecting one or more outputs or inputs in the Results Summary window and clicking the Scatter Plot icon.
- Dragging a bar (representing the input you want to show in the scatter plot) from an output’s tornado graph.
- Displaying a scatter plot matrix in the Sensitivity Analysis report window.
- Selecting a correlation matrix in Browse mode. This displays a scatter plot matrix showing the simulated correlations between the inputs correlated in the matrix.
Sensitivity Analysis Results

Sensitivity analysis results are displayed by clicking the **Sensitivity Window** icon. These results show the sensitivity of each output variable to the input distributions in your model. This identifies the most “critical” inputs, the inputs you should concentrate on most when basing decisions on your model.

The data displayed in the Sensitivity window are ranked for the output selected in the **Rank Inputs for Output** entry in the **Sensitivity** window. The sensitivity of all other outputs to the ranked inputs is also shown.

Sensitivity analyses performed on the output variables and their associated inputs can be based on a change in output statistic analysis, a regression analysis, or a rank order correlation analysis. The type of analysis desired is set using the **Display Significant Inputs Using** entry in the **Sensitivity** window.

In the change in output statistic analysis, samples for an input are grouped in a set of equal-sized bins (or “scenarios”), ranging from the input’s lowest value to its highest. A value for a statistic of the output, such as its mean, is calculated for the output values in the iterations associated with each bin. Inputs are ranked by the amount of +/- swing they caused for the output statistic.

In the regression analysis, the coefficients calculated for each input variable measure the sensitivity of the output to that particular input distribution. The overall fit of the regression analysis is measured by the reported fit or R-squared value of the model. The lower the fit, the less stable the reported sensitivity statistics are. If the fit is too low — beneath 0.5 — a similar simulation with the same model could give a different ordering of input sensitivities.
The sensitivity analysis using rank correlations is based on the Spearman rank correlation coefficient calculations. With this analysis, the rank correlation coefficient is calculated between the selected output variable and the samples for each of the input distributions. The higher the correlation between the input and the output, the more significant the input is in determining the output's value.
You can also request scatter plots showing the sampled input values versus the output values calculated for all iterations of the simulation. In the Scatter Plot Matrix, ranked sensitivity analysis results are displayed with scatter plots. To show the scatter plot matrix, click the Scatter Plot icon in the lower left of the Sensitivity window.
You can also display sensitivity results with tornado graphs or spider graphs. These graph types can be generated by clicking the Tornado Graph icon on a graph window, or by right-clicking any output in the Results Summary window and selecting from the resulting pop-up menu.
Scenario Analysis Results

The Scenarios icon displays the Scenario analysis results for your outputs. Up to three scenario targets can be entered for each output variable.

The Scenario analysis performed on output variable is based on a conditional median analysis. In performing the Scenario analysis, @RISK first finds the subset of iterations of the simulation in which the output variable achieves the specified target. It then finds the median of this subset of sampled values for each input and compares it to the median of the input values for all iterations.

The objective of this process is to find the inputs with a subset median that differs significantly from the overall median. If the subset median for the input variable is close to the overall median, the input variable is marked as insignificant. This is because the values sampled for the input in the iterations where the target is achieved do not differ markedly from those for the entire simulation. However, if the subset median for the input variable deviates significantly from the overall median (at least half a standard deviation), the input variable is marked as significant. The reported scenarios show all inputs which are significant in achieving the specified target.
You can also request scatter plots in the Scenarios window with an overlay. Each such scatter plot shows:

1) The sampled input values versus the output values from all iterations of the simulation.

2) An overlay of the sampled input value versus the output values for iterations where the output value achieves the specified scenario.

Scenario analysis results can also be displayed graphically in tornado graphs. A tornado graph can be generated by clicking the Tornado graph icon in the Scenarios window or by clicking the Scenarios icon on a graph window. This tornado graph shows the key inputs affecting the output when the output achieves the specified scenario, such as when the output is below its 25th percentile.
Excel Reports

When you generate simulation reports and graphs in Excel, you have access to all of Excel’s formatting capabilities.

In addition, you can use pre-built @RISK template sheets to create your own custom simulation report. Simulation statistics and graphs are placed in a template using a set of @RISK functions added to Excel. When a statistics function or graphing function is included in a template sheet, the desired statistics and graphs are then generated at the end of a simulation in a copy of the template sheet to create your report. The original template sheet with the @RISK functions remains intact for use in generating reports for later simulations.

Template sheets are standard Excel spreadsheets. They are identified to @RISK by having a name that starts with RiskTemplate_. These files can also contain any standard Excel formulas, so that custom calculations can be performed using simulation results.
Chapter 5: @RISK Graphs

Simulation inputs and results are often displayed graphically, and @RISK takes full advantage of every graphical opportunity. For example, the Results Summary window shows thumbnail graphs of the simulation results for all your inputs and outputs. Dragging a thumbnail graph off the Results Summary window allows you to display the graph in a full-sized window. Graphs are also displayed when you select an output or input cell in Browse Results mode.

Overview

Graphs in @RISK appear in two types of windows:

- **Floating windows**, sitting by themselves on top of Excel. These windows stay open until you close them.

- **Callout windows**, attached to a cell. This is the type of window used in Browse Results mode. Only one of these windows can be open at a time, and the graph changes when a new cell is selected in Excel.

By using the icons on a graph, you can detach a callout window and turn it into a floating window, or you can re-attach a floating window to the cell it represents.

You can change the type of graph displayed by using the icons at the bottom of the graph window. In addition, you can right-click a graph window to see a pop-up menu. The items in this menu allow you to change a graph’s format, scaling, colors, titles, and other characteristics. They also enable you to copy a graph, print a graph, or save the image to a file. Finally, the **Chart in Excel** command allows you to create graphs in Excel's native graph format. These graphs can be changed, or customized, just as with any Excel graph.

A pane is displayed to the right of a graph where you can see a legend or statistics. By default, you see detailed statistics for the result graphed, but by changing the selector at the top, you can see a legend, a legend with a few statistics, or simulation data.
**Icons on Graphs**  
All @RISK graph windows have a set of icons in the lower left. These allow you to control the type, format, and placement of the displayed graphs. You can also use the Zoom icon to quickly zoom in on a region displayed in a graph.

**Formatting Graphs**  
@RISK graphics use a graphing engine designed specifically for processing simulation data. Graphs can be customized and enhanced as needed, often by simply clicking (or right-clicking) the appropriate element in the graph. For example, to change the title of a graph, simply click the title and type the new entry:
A displayed graph can also be customized through the Graph Options dialog. Customizations include colors, scaling, fonts, displayed statistics, and many others. You can display the Graph Options dialog by right-clicking a graph and selecting the Graph Options command, or by clicking the Graph Options icon in the bottom left of the graph window.

The Graph Options dialog changes depending on the type of graph being customized. Graph options specific to a certain graph type are discussed in the reference section for that graph type.
When you run multiple simulations, graphs for result distributions are available for each simulation. When you display an output graph through Browse Results, you will see the graph for the first simulation (the one with index 1). However, you can then click the Select Simulation icon (the one with the # sign) to select any of the simulations.

If you select All Simulations, you get an overlay graph for that output. This can be useful for comparing distributions across simulations.

The Select Simulation dialog is also available in report windows when you want to filter the report to show only results from a specific simulation.
Histogram and Cumulative Graphs

A histogram or a cumulative graph shows the range of possible outcomes and their relative likelihood of occurrence. These graphs can be displayed in several forms, as shown in the dropdown list in the following screenshot. It is also possible to display a histogram and a cumulative curve for a given output or input on the same graph.

Two vertical delimiters are available in these graphs. By dragging the delimiters, you can display target probabilities or percentiles. This is useful for graphically displaying answers to questions such as “What is the probability of a result between 1 million and 2 million occurring?” or “What is the probability of a negative result occurring?”

By default, the delimiters are placed at the 5th and 95th percentiles. However, you can move these by dragging the delimiters or by typing desired values (either a percentage or a value) above the graph.
It is often useful to compare several distributions graphically. You can do this with overlay graphs.
To add an overlay, you can either:

- Click the Add Overlay icon on a displayed graph and then select the cell(s) in Excel with results you want to include in the graph. Note that if you have run multiple simulations and you want to overlay the graphs of the same output for different simulations (as in the above screenshot), you can select the same output cell for the overlay. Then you will see the Select Simulations dialog, where you can choose the simulation(s) you want to overlay.

- Drag one graph onto another, or drag a thumbnail graph from the Model window or Results Summary window onto an open full-sized graph. Once overlays are added, delimiter statistics display probabilities for all distributions included in the overlay graph. The Graph Options dialog allows you to set the number of delimiter bars displayed for overlays.

Note: To remove an overlay quickly, you can right-click the colored legend for the curve you want to remove and select Remove Curve.

Overlaying Histogram and Cumulative Curves on a Single Graph

Sometimes it is useful to display a histogram and a cumulative curve for a given output or input on the same graph. This graph type has two Y-axes, one on the left for the histogram and a secondary Y-axis on the right for the cumulative curve.
To include a cumulative overlay on a Probability Density or Relative Frequency graph, click the Graph Type icon and select the Cumulative Overlay option.

The Graph Options dialog is displayed by right-clicking a graph and selecting the Graph Options command, or by clicking the Graph Options icon at the bottom left of a graph window. For histogram and cumulative graphs, the Graph Options — Distribution Tab sets the type of curve displayed along with binning options.

Options on the Graph Options — Distribution Tab include:

- **Distribution Format.** Changes the format of the displayed distribution. Settings include:
  - **Automatic.** Selects Probability Density graphs.
  - **Probability Density** and **Relative Frequency.** These are both histograms. The difference between them is the unit of measure reported on the y-axis. Relative Frequency shows the probability of a value in a bin range (observations in a bin/total observations). Probability Density shows the
relative frequency value divided by the width of the bin, insuring that y-axis values stay constant as the number of bins is changed.

- **Discrete Probability.** This option creates a “spike” chart, where the height of the spike above any value is the probability of that value. This setting should be used only for discrete distributions.

- **Cumulative Ascending** and **Cumulative Descending.** These options display either cumulative ascending probabilities (y-axis shows the probability of a value less than or equal to any x-axis value) or cumulative descending probabilities (y-axis shows the probability of a value greater than any x-axis value).

- **Histogram Binning.** This setting specifies how @RISK will bin the data in a histogram. The settings include:
  1. **Minimum.** Sets the minimum value where histogram bins start. **Automatic** specifies that @RISK will start the histogram bins based on the minimum of the data graphed.
  2. **Maximum.** Sets the maximum value where histogram bins end. **Automatic** specifies that @RISK will end the histogram bins based on the maximum of the data graphed.
  3. **Number of Bins.** Sets the number of histogram intervals calculated across the range of the graph. The value entered must be in the range 2 to 200. The setting **Automatic** calculates the best number of bins to use for your data, which is based on an internal heuristic.
  4. **Overlays.** Specifies how @RISK will align bins between distributions when overlay graphs are added. The options include:
     1) **Single Histogram.** The entire min-max range of data in all curves (including overlays) is binned, and each curve in the graph uses these same bins, which allows easy comparisons.
     2) **Single Histogram with Adjusted Limits.** This is the same as the Single Histogram option except at the endpoints of each curve. Smaller or larger bins are used at the endpoints to ensure that each curve does not extend below its minimum data value or above its maximum.
3) **Independent Histograms.** Each curve uses independent binning based on its own data range.

4) **Automatic.** Selects either Single Histogram with Adjusted Limits or Independent Histograms, depending on the overlap of the data between curves. Curves with sufficient data overlap will use Single Histogram with Adjusted Limits.

**Graph Options – Delimiters Tab**

For histogram and cumulative graphs, the **Graph Options – Delimiters Tab** specifies how delimiters are displayed with the graph.

When delimiters are moved, calculated probabilities and percentiles are shown in the delimiter bar above the graph. Delimiters can be shown for any or all of the curves in a graph.
For histogram and cumulative graphs, the **Graph Options — Markers Tab** specifies how markers are displayed with the graph. Markers annotate key values on a graph.

When markers are displayed, they are included in graphs when you copy them into a report.
Fitting a Distribution to a Simulated Result

You can click the **Fit Distributions to Data** icon in the lower left of a graph window to fit probability distributions to the data for a simulated output. All the options that can be used when fitting distributions to data in an Excel worksheet are available when fitting probability distributions to a simulated output. For more information on these options, see **Appendix A: Distribution Fitting** in this manual.

Tornado Graphs

Tornado graphs from a sensitivity analysis display a ranking of the input distributions that impact an output. Inputs with the largest impact on the distribution of the output have the longest (and topmost) bars in the graph.

Tornado graphs for an output can be displayed by selecting a row, or rows, in the Results Summary window, or by clicking **Tornado Graph** icon at the bottom of the window and selecting one of the tornado graph options. Alternatively, the graph for a simulated output can be changed to a tornado graph by clicking the **Tornado Graph** icon and selecting a tornado graph.
@RISK has four types of tornado graphs — Change in Output Statistic, Regression Coefficients, Regression — Mapped Values, and Correlation Coefficients.

For tornado graphs showing a Change in Output Statistic (such as Change in Output Mean), samples for an input are grouped into a set of equal-sized bins, ranging from the input’s lowest value to its highest. A value for a statistic of the output (such as its mean) is calculated for the output values in the iterations associated with each bin. The length of the bar shown for each input distribution is based on the range of the highest statistic value for any bin to the lowest statistic value. The highest statistic value is plotted on one end of the bar and the lowest statistic value for any bin is plotted on the other end. These are “double-sided” tornado graphs, where each bar ranges above and below the baseline, or center, of the graph.

For tornado graphs showing Regression Coefficients or Correlation Coefficients, the length of the bar shown for each input distribution is based on the coefficient value (the regression coefficient or the correlation coefficient) calculated for the input. These coefficient values are shown on each bar of the tornado graph.
For tornado graphs showing Regression — Mapped Values, the length of the bar shown for each input distribution is the amount of change in the output due to a 1 standard deviation change in the input. The values shown on each bar of the tornado graph are the output values associated with 1 standard deviation change in the input. (These “mapped” values are the “beta” coefficients from a regression that uses standardizes variables.)

The maximum number of bars that can be displayed in a tornado graph is 16 (or 10 for the Change in Output Statistic option). If you want to display tornado graphs with fewer bars, you can change the Maximum Number of Bars setting in the Graph Options dialog. To set a default maximum number of bars, you can change the Tornado Maximum # Bars setting in the Application Settings dialog.

If you right-click any bar in a tornado graph, you will see a number of options, including the Hide Bar option. If any bars are hidden and you right-click in the graph, you will see a Restore Hidden Bars option.

Note: If your tornado graph has many bars, there might not be enough room to display labels for each bar. In this case, you can drag a corner of the graph to increase its size, which will allow more bar labels to be shown.
Sensitivity analysis results can also be displayed in spider graphs. These graphs are created from the results of the **Change in Output Statistic** sensitivity analysis, explained for tornado graphs in the previous section.

A spider graph shows how the value of the output statistic changes as the sampled input value changes. The steeper the line, the greater the impact of the input on the output. This shows more information than a tornado graph. A tornado graph shows only the overall change in the value of the output statistic. In contrast, a spider provides information on the rate of change in output value as the input changes through its range.

The number of lines displayed on a spider graph can be set from the **Max # of Spider Lines** option in the **Settings** dialog. You can access this dialog by clicking the Tornado icon below any graph and selecting **Settings**.

Scenario analysis results can be displayed graphically in tornado graphs. You can generate a tornado graph by clicking the Tornado graph icon in the Output Scenarios window or by clicking the Scenarios icon in a graph window. This tornado graph shows the key inputs affecting the output for iterations where the output achieves the target scenario, such as when the output is above its 90th percentile. See the **Sensitivities Command** in this chapter for more details.
Scatter Plots

@RISK provides scatter plots to show the relationship between simulated values of two variables, often an output versus an input. You can create scatter plots by:

- Clicking the Scatter Plot icon on a displayed graph and then selecting the cell(s) in Excel with results you want to include in the plot.
- Selecting outputs or inputs in the Results Summary window and clicking the Scatter Plot icon.
- Dragging a bar (representing the input you want to show in the scatter) from an output’s tornado graph.
- Displaying a scatter plot matrix in the Sensitivity Analysis window (see the Sensitivities Command in this chapter for details).
- Clicking a correlation matrix in Browse mode. This displays a scatter plot matrix for the inputs correlated in the matrix.

When any scatter plot is displayed, you can click the Graph Options icon to bring up a Graph Options dialog specific to scatter plots. Among other things, you can then choose to display a confidence ellipse (a pale pink band) in the plot. This identifies the region where, at a given confidence level, the x-y values will fall.
The Graph Options dialog also allows you to request standardized values for the plot, so that values from inputs of different magnitudes can be more easily compared.

**Note:** Scatter plots are always displayed in floating, not callout windows.

*Scatter Plot Overlays*

Overlays can be added to a scatter plot by clicking the Add icon (with a plus sign) shown at the bottom of the graph window. This is useful for showing how the values for two (or more) inputs are related to an output.

Multiple outputs can also be included in a scatter plot overlay. This is useful for showing how the same input affects different outputs.
The Scatter tab in the Graph Options dialog specifies whether values displayed in a scatter plot will be standardized, and it provides settings for confidence ellipses.

Options on the Scatter include:

- **Standardization.** Selects whether X and/or Y values displayed in a scatter plot will be standardized. When values are standardized, values are displayed in terms of standard deviation change from the mean instead of actual values. Standardization is especially useful when overlaying scatter plots from different input distributions. This allows a common scale between the inputs, making comparisons of impacts on outputs easier.

- **Confidence Ellipses (Assuming Underlying Bivariate Normal).** A confidence ellipse is generated by fitting the best bivariate normal distribution to the data displayed in the scatter plot. The region shown by the ellipse is where a sample from that bivariate normal will fall for a specified confidence level. For example, if the confidence level is 99%, the probability is 99% that a sample from the best-fitting bivariate normal distribution will fall within the displayed ellipse.
Scatter plots have both X-axis and Y-axis delimiters that can be used to show the percentage of the points that fall in each of the delimited quadrants of the graph. If you have overlays in your scatter plot, the percentage values for each plot are color coded. (The number of plots in an overlay graph for which percentages are reported can be set in the Delimiters tab of the Graph Options dialog.)

If you zoom in on a region of the scatter plot, the percentage value shown in each quadrant represents the percentage of the total graph points that are in the visible quadrant (where total graph points = the total # of points in the original “non-zoomed” graph).

Note: You can click and drag the crossing point of the X and Y axis delimiters to adjust both delimiters at the same time.
Summary Graphs

@RISK has two types of graphs that summarize trends across a group of simulated outputs (or inputs). These are the Summary Trend graph and Box Plot. Each of these graphs can be displayed by:

- Clicking the Summary Graph icon at the bottom of a graph window and then selecting the range in Excel with results you want to include in the graph.
- Selecting the rows in the Results Summary window for the outputs (or inputs) you want to include in the summary graph, then clicking the Summary Graph icon at the bottom of the window (or right-clicking in the table), and selecting Summary Trend or Summary Box Plot.
- For an output range, you can also click the Range Name header and select Summary Graph.

Summary Trend and Summary Box Plot graphs can be interchanged once either type is displayed. To change the type of graph shown, simply click the appropriate icon in the bottom left of the graph window and select the new graph type.

Note: Elements can be added to a summary graph by clicking the Add icon (with a plus sign) shown at the bottom of the graph window.
A Summary Trend graph summarizes changes in multiple probability distributions or an output range. The Summary graph takes five parameters from each selected distribution (the mean, plus two upper and two lower band values) and graphs the changes in these five parameters across the output range. The upper band values default to +1 standard deviation and the 95th percentile of each distribution, while the two lower band values default to -1 standard deviation and the 5th percentile of each distribution. These can be changed in the Trend tab options of the Graph Options dialog.

The Summary Trend graph is especially useful for displaying changes in risk over time. For example, the output range might be a time series of yearly profits. The Summary Trend graph would then show any trends in the distributions of profit from year to year. The wider the band around the mean, the larger the variability in possible results.

When generating a Summary graph, @RISK calculates the mean and the four band values (such as 5th and 95th percentile) for each cell in the output range graphed. These points are graphed with hi-lo lines. Patterns in between the points for each cell are then added. The mean and band values for these added points are calculated by interpolation.
The Trend tab in the Graph Options dialog specifies the values displayed in each band of the Summary Trend graph, and the colors for those bands.

Options in the Trend tab include:

- **Statistics.** Selects the values displayed for the Center Line, the Inner Band, and the Outer Band of the Summary Trend graph. Settings include:
  - **Center Line.** Selects Mean, Median or Mode.
  - **Inner Band, Outer Band.** Selects the range for each band. The inner band must always be “narrower” than the outer band.

- **Formatting.** Selects the color and shading used for each of the three bands.
A Summary Box Plot displays a box plot for each distribution included in the summary graph. A box plot (also called a box-whisker plot) shows a box for a defined inner range of a distribution, with “whisker” lines showing the outer limits of the distribution. An inner line in the box marks the location of the mean, median, or mode of the distribution.
The **Box-Whisker** tab in the Graph Options dialog specifies the values used for the Center Line, Box, and Whiskers in each box of the Summary Box Plot graph, as well as the colors for the boxes.

Options in the **Box-Whisker** tab include:

- **Statistics.** Selects the values displayed for the Center Line, the Box, and the Whiskers of each box plot. Settings include:
  - **Center Line.** Selects Mean, Median or Mode.
  - **Box.** Selects the range each box will describe. The range for the box must always be “narrower” than the whiskers.
  - **Whiskers.** Selects the endpoints for the whiskers.

- **Formatting.** Selects the color and shading for each box.
When you run multiple simulations, it is often useful to compare the summary graphs, created for the same distributions, in different simulations. This comparison shows how the trend in expected value and risk changes for the different simulations.

To create a summary graph that compares the results for a range of cells in multiple simulations:

1) Set the number of simulations to a value greater than 1. Use the `RiskSimtable` function to change worksheet values across simulations.

2) Click the **Summary Graph** icon at the bottom of the displayed Browse window.

3) Select the cells in Excel with results you want to add to the graph.

4) Select **All Simulations** from the Select Simulation dialog.
To create a summary graph that compares the results for a single cell across multiple simulations, follow the previous steps but in step 3, select only a single cell to include in the summary graph. The displayed graph then shows the five parameters from the cell’s distribution in each simulation. This summarizes how the distribution for the cell changed by simulation. (Note that the x-axis now represents the simulation index.)

Summary graphs of multiple simulations can also be displayed by selecting the rows in the Results Summary window for the outputs or inputs (by simulation) you want to include in the summary graph. Then click the Summary Graph icon at the bottom of the window (or right-click in the table), and select Summary Trend or Summary Box Plot.
Formatting Graphs

@RISK graphics use a graphing engine designed specifically for processing simulation data. Graphs can be customized and enhanced as needed; titles, legends, colors, scaling, and other settings can all be controlled through the selections in the Graph Options dialog. You can display the Graph Options dialog by right clicking a graph and selecting Graph Options, or by clicking the Graph Options icon in the bottom left of the graph window.

The available options on tabs in the Graph Options dialog are described here. Note that not all options are available for all graph types.

**Graph Options – Title Tab**

The options on the Title tab specify the titles that will be displayed on the graph. If you do not enter a title, @RISK will automatically choose one, based on the name(s) of the output or input cells being graphed.
The options on the **X and Y Axis** tabs specify the scaling and axis titles that will be used in the graph. A **Scale Factor** (such as thousands or millions) can be applied to minimum and maximum axis values, and the number of axis ticks can be changed. Log scaling can also be selected for X and Y axes. Alternatively, you can change axis scaling directly on the graph by dragging the limits of an axis to a new minimum or maximum position.

**Note:** Different scaling options are available for different types of graphs (summary, distribution, scatter, etc.).
The options on the Curves tab specify the color, style, and value interpolation for each curve in the graph. The definition of a “curve” changes depending on the type of graph. For example, in a histogram or cumulative graph, a curve is associated with the primary graph and each overlay. In a scatter plot, a curve is associated with each X-Y data set shown in the graph. Clicking on a curve in the Curves list displays the available options for that curve.
The options on the Legend tab specify which statistics will be displayed with the graph and how they will be displayed.

Either summary statistics or simulation data can be displayed for each curve in a graph. The available statistics change depending on the type of graph displayed. These statistics can be shown in a grid next to the graph (with a column for each curve) or in a table in the graph legend.

Legend statistics are copied with the graph when it is pasted into a report. They also update as a simulation runs. To change the statistics displayed with a graph legend:

1) Uncheck Automatic to allow customization of the displayed statistics.

2) Check the Statistics desired.

3) Click Redefine to change the percentile values that will be reported, if desired.

To remove the statistics from a graph:

- Change the Style option to Simple Legend.

To remove legend and statistics from a graph:

- Change Show option to Never.
The options on the Other tab specify other available settings for a displayed graph. These include the color scheme used and the formatting of numbers and dates displayed in the graph.

Numbers displayed on a graph can be formatted to show the level of precision desired using the Number Formats options. The available numbers for formatting change depending on the type of graph displayed.

Dates displayed on a graph can be formatted to show the level of precision desired using the Date Formats options. The available dates for formatting change depending on the type of graph displayed.

For distribution graphs, Statistics (Unitless) refers to statistics such as skewness and kurtosis that are not in the units of the values for the graph. Statistics (with Units) refers to statistics such as mean and standard deviation that use the units of the graph.
Graphs can often be formatted by simply clicking the appropriated element in the graph. For example, to change the title of a graph, you can simply click the title and type the new entry.

Items that can be formatted directly on a graph include:

- **Titles.** Click the title in the graph and type in the new entry.
- **X-Axis Scaling.** Select and drag the end line of the axis to rescale the graph.
- **Delete an Overlay.** Right-click the colored legend of the curve you want to delete and select **Remove Curve.**
- **Delimiter Values.** Click the delimiter bar at the top of the graph or in the value above a delimiter and type a new entry.

In addition, the menu displayed when you right-click any part of a graph allows quick access to formatting items.
Chapter 6: @RISK Command Reference

@RISK Icons

The following table provides brief descriptions of the individual icons on the RISK ribbon. The rest of this chapter discusses the corresponding commands in detail.

<table>
<thead>
<tr>
<th>Icon</th>
<th>Function Performed</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Define Distributions" /></td>
<td>Add or edit probability distributions in the formula in the current cell</td>
</tr>
<tr>
<td><img src="image" alt="Add Output" /></td>
<td>Add the current selected cell (or range of cells) as a simulation output</td>
</tr>
<tr>
<td><img src="image" alt="Insert Function" /></td>
<td>Inserts an @RISK function in the formula of the active cell</td>
</tr>
<tr>
<td><img src="image" alt="Define Correlations" /></td>
<td>Defines correlations (or copulas) for probability distributions or related input variables</td>
</tr>
<tr>
<td><img src="image" alt="Distribution Fitting" /></td>
<td>Fit distributions to data</td>
</tr>
<tr>
<td><img src="image" alt="Data Viewer" /></td>
<td>Provides standard @RISK graphs and reports for any set of data in an Excel spreadsheet</td>
</tr>
<tr>
<td><img src="image" alt="Model Window" /></td>
<td>Displays current output cell(s) along with all distribution functions entered in the worksheet in the @RISK Model window</td>
</tr>
<tr>
<td>Icon</td>
<td>Description</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td><img src="iter.png" alt="" /></td>
<td>Sets the number of iterations to be run</td>
</tr>
<tr>
<td><img src="sim.png" alt="" /></td>
<td>Sets the number of simulations to be run</td>
</tr>
<tr>
<td><img src="settings.png" alt="" /></td>
<td>Displays simulation settings for changing # of iterations, # of simulations, sampling type, standard recalc method, executed macros, and other settings</td>
</tr>
<tr>
<td><img src="random.png" alt="" /></td>
<td>Sets the type of values (random or static) returned by @RISK distribution functions in a standard Excel recalculation</td>
</tr>
<tr>
<td><img src="graph.png" alt="" /></td>
<td>Selects whether to automatically show and output graph during (and after) a simulation</td>
</tr>
<tr>
<td><img src="demo.png" alt="" /></td>
<td>Turns Demo mode on or off</td>
</tr>
<tr>
<td><img src="start.png" alt="" /></td>
<td>Simulate the current worksheet(s)</td>
</tr>
<tr>
<td><img src="reports.png" alt="" /></td>
<td>Selects Excel reports to run</td>
</tr>
<tr>
<td><img src="browse.png" alt="" /></td>
<td>Opens the Browse Results window for examining an output distribution</td>
</tr>
<tr>
<td><img src="summary.png" alt="" /></td>
<td>Displays the Results Summary window for examining simulation results</td>
</tr>
<tr>
<td><img src="define.png" alt="" /></td>
<td>Opens the Filter window for creating input or output filters</td>
</tr>
<tr>
<td><img src="stats.png" alt="" /></td>
<td>Displays Detailed Statistics window</td>
</tr>
<tr>
<td><img src="data.png" alt="" /></td>
<td>Displays Data window for showing all of the simulation data</td>
</tr>
<tr>
<td><img src="sensitivity.png" alt="" /></td>
<td>Displays Sensitivity Analysis window</td>
</tr>
<tr>
<td><img src="scenario.png" alt="" /></td>
<td>Displays Scenario Analysis window</td>
</tr>
<tr>
<td>Command</td>
<td>Description</td>
</tr>
<tr>
<td>------------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Advanced Analyses</td>
<td>Performs an advanced analysis (Goal Seek, Stress Analysis, or Advanced Sensitivity Analysis)</td>
</tr>
<tr>
<td>RISK Optimizer</td>
<td>Performs an optimization of a simulation model</td>
</tr>
<tr>
<td>Time Series</td>
<td>Provides Time Series functions</td>
</tr>
<tr>
<td>Project</td>
<td>Develops simulation models for project schedules</td>
</tr>
<tr>
<td>Library</td>
<td>Adds results to, or displays, @RISK Library</td>
</tr>
<tr>
<td>Swap Out @RISK</td>
<td>Swaps @RISK functions in and out of cell formulas and places @RISK graphs in a workbook where @RISK is swapped out</td>
</tr>
<tr>
<td>Utilities</td>
<td>Opens Application Settings, Show Window Panel, Open Simulation File, Clear @RISK Data, Unload @RISK Add-In, and other utilities</td>
</tr>
<tr>
<td>Color Cells</td>
<td>Color-codes cells with @RISK functions</td>
</tr>
<tr>
<td>Thumbnails</td>
<td>Turns thumbnail graphs for @RISK function cells on and off</td>
</tr>
<tr>
<td>Help</td>
<td>Displays @RISK Help menu</td>
</tr>
</tbody>
</table>
Graph Window Icons

The following icons are shown at the bottom of @RISK graph windows. Depending on the type of displayed graph, some icons may not be shown.

<table>
<thead>
<tr>
<th>Icon</th>
<th>Function Performed</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Icon 1]</td>
<td>Displays the Graph Options dialog</td>
</tr>
<tr>
<td>![Icon 2]</td>
<td>Copies or reports the displayed result</td>
</tr>
<tr>
<td>![Icon 3]</td>
<td>Shows and sets the type of distribution graph shown</td>
</tr>
<tr>
<td>![Icon 4]</td>
<td>Shows and sets the type of tornado graph shown</td>
</tr>
<tr>
<td>![Icon 5]</td>
<td>Adds an overlay to the displayed graph</td>
</tr>
<tr>
<td>![Icon 6]</td>
<td>Creates a scatter plot using the data from the displayed graph</td>
</tr>
<tr>
<td>![Icon 7]</td>
<td>Shows a scenario tornado graph or edits scenarios</td>
</tr>
<tr>
<td>![Icon 8]</td>
<td>Creates a summary trend or box plot graph</td>
</tr>
<tr>
<td>![Icon 9]</td>
<td>Adds a new variable to a scatter plot or summary graph</td>
</tr>
<tr>
<td>![Icon 10]</td>
<td>Selects a graph from a simulation # in a multi-simulation run</td>
</tr>
<tr>
<td>![Icon 11]</td>
<td>Defines a filter for the displayed result</td>
</tr>
<tr>
<td>![Icon 12]</td>
<td>Fits distributions to a simulated result</td>
</tr>
<tr>
<td>![Icon 13]</td>
<td>Zooms in on a region of a graph</td>
</tr>
<tr>
<td>![Icon 14]</td>
<td>Resets zoom to default scaling</td>
</tr>
<tr>
<td>![Icon 15]</td>
<td>Changes a floating graph to a graph attached to the cell it references</td>
</tr>
</tbody>
</table>
Model Group Commands

This group lets you define inputs and outputs for your model, including correlations between inputs. This is also where you can fit distributions to your data, and where you can view graphs and statistics of data.
Define Distributions Command

Defines or edits probability distributions entered in the current cell formula

The Define Distributions command (in the Model group) displays the Define Distribution window. Using this window, probability distributions can be assigned to values contained in the formula of the selected cell. This window also allows you to edit distributions already present in a cell’s formula.

The @RISK Define Distribution window graphically displays probability distributions that can be substituted for values in the formula in the current cell. By changing the displayed distribution, you can see how various distributions would describe the range of possible values for an uncertain input in your model. The displayed statistics provide more information about a selected distribution.

The graphical display of an uncertain input is useful in showing your definition of risk to others. It clearly displays the range of possible values for an input, as well as the relative probabilities of value in the range. By working with distribution graphs, you can easily incorporate other individual assessments of uncertainty into your risk analysis models.
You can click the **Define Distributions** icon to display the Define Distribution window. If the selected cell doesn’t contain an @RISK distribution function, you see a gallery of distributions you can choose from.

From this window, you can choose a distribution from any of the tabs and then click Select Distribution. You can also select any distribution from any tab and then click Make Favorite to add this distribution to the Favorites tab, which contains the distributions you use most often.

After you click Select Distribution, or if you click the Define Distributions icon when the selected cell already contains a probability distribution, the Define Distribution window shows that distribution. If you then select other cells in your spreadsheet, the Define Distribution window updates to show the formula for each cell you select. (Alternatively, you can press `<Tab>` to move the window through all cells in open workbooks with distribution functions.)
When you click OK to close the window (or you select another input cell), all changes and edits made are added directly to the cell’s formula. In fact, you see this formula in the Define Distribution window, the formula in red near the top.

The Define Distribution window has a **Primary** curve, the one for the function entered in the cell formula, and up to ten **Overlay** curves, representing other distributions that you might want to graphically display on top of the Primary curve. You can add overlays by clicking the **Add Overlay** icon at the bottom of the window.
The Define Distribution window contains the following elements:

- **Name.** Displays the default name that @RISK identifies for the cell. By clicking the Reference Entry icon (the icon after the name), you can select an alternate cell in Excel that contains the name to use. Alternatively, you can simply type a name of your choice.

- **Cell Formula.** Displays the current cell formula, including any @RISK distribution functions. This formula can be edited here, just as in Excel. The text shown in red and underlined is the distribution that is graphed.

- **Select Distribution.** (Shown only when the Distribution Palette is visible, first screenshot above) Adds the currently selected distribution in the Distribution Palette. You get the same effect by double-clicking the distribution you want to use from the Distribution Palette.

- **Make Favorite.** (Shown only when the Distribution Palette is visible, first screenshot above) Adds the currently selected distribution in the Distribution Palette to the Favorites tab in the Palette.

- **Splitter Bars.** To make the Cell Formula box larger or smaller, you can move a splitter bar between the Cell Formula box and the graph up or down. Similarly, there are splitter bars you can move left or right to resize the Distribution Argument panel or the Statistics panel.

Delimiters and Statistics are used to display underlying statistics on displayed distribution graphs:

- **Delimiters.** Delimiters allow you to set target probabilities and x-axis scaling using the mouse. Dragging probability delimiters changes left and right x and p values, shown in the probability bar above the graph. Alternatively, you can enter x or p values directly above the graph. Dragging the delimiters at either end of the x-axis (at the bottom corners of the plot area) rescales the x-axis.

- **Statistics.** The statistics displayed for the graphed distributions, including any overlays, can be selected in the Legends tab of the Graph Options dialog. To display this dialog, click the **Graph Options** icon (second from left) at the bottom left of the window.
In the following screenshot, the Define Distributions icon was clicked when a cell with a formula not containing an @RISK distribution function was selected. In this case, you can assign a distribution to a specific part in the Cell Formula box by selecting that part and then double-clicking the distribution you want to use from the Distribution Palette.

Another situation is when you select a cell with a formula containing an @RISK distribution function, and you then want to change the distribution used in the formula. To do this, you can click the Replace Distribution in Formula button (the small button the cursor is pointing to in the following screenshot). This opens a small palette in another window, and you can double-click the distribution you want to change to from this palette.
Adding Overlays Using the Palette

To add overlays to a displayed distribution graph, you can click the Add Overlay icon at the bottom of the small palette window.
You can enter argument values in the **Distribution Argument** panel. (You can also type them directly in the formula shown in red.) This panel is displayed to the left of the graph (see the previous screenshot). Spinner buttons allow you to quickly change a parameter value. If you have Overlays, the Distribution Argument panel allows you to switch between entering arguments for the primary curve and any of the overlays.

Options in the Distribution Argument panel include:

- **Function.** This selects the distribution type displayed in the graph.

- **Parameters.** This selects the types of arguments used in the distribution function. They can include **Truncation Limits**, **Shift Factor**, **Date Formatting** and in many cases, **Alternate Parameters**. You can also select the **Static Value** to be used for the distribution.

  - Selecting **Truncation Limits** creates an entry for **Trunc. Min and Trunc. Max** in the Distribution Argument panel, allowing the distribution to be truncated at the value(s) specified.

  - Selecting **Shift Factor** creates an entry for **Shift** in the Distribution Argument panel. A shift factor shifts the range of the distribution by the selected shift amount.

  - Selecting **Alternate Parameters** allows the entry of alternate parameters, usually percentiles, for the distribution.

  - Selecting **Static Value** allows the entry of the static value for the distribution. This is the value shown in the cell when the Static/Random “dice” icon is toggled to Static.
Selecting **Date Formatting** instructs @RISK to display dates in the Distribution Argument panel and to display graphs and statistics using dates. This selection results in a **RiskIsDate** property function being added to the distribution function.

**Note: In the Application Settings dialog, you can specify that Truncation Limits, Shift Factor, and Static Value always be displayed in the Distribution Argument panel.**

**Alternate Parameters**

Alternate parameters allow you to specify values for specific percentiles of an input distribution instead of the traditional arguments used by the distribution. The percentiles to be entered are specified by the Alternate Distribution Parameters options, displayed when **Alternate Parameters** is selected.

With Alternate Parameters, you have the option to **Specify Using Cumulative Descending Percentiles**. This indicates that the percentiles used for alternate parameters will be in terms of cumulative descending probabilities. Percentiles entered in this case specify the probability of a value greater than the specified x-value.

When making Parameter Selections, **Percentile** parameters can be mixed with standard parameters by checking the appropriate options.
In the Application Settings dialog, opened through the @RISK Utilities menu, you can select the default parameters you want to use for Alternate Parameter Distributions. All such distributions have names that end in ALT (such as RiskNormalAlt). Your default parameters will be used each time you select an Alternate Parameter distribution from the Distribution Palette.
There are three small icons (besides the question mark Help icon) in the Distribution Argument panel. They are as follows:

- Deletes the curve whose arguments are shown in the selected region of the Distribution Argument panel.
- Displays the Distribution Palette for selecting a new distribution type for the selected curve.
- Displays the Distribution Argument panel so that Excel cell references can be selected for argument values. When in this mode, you can simply click the cells in Excel that contain the argument values you want to use. Then you can click the icon at the top right of the window when you are finished.

You can hide (or unhide) the Distribution Argument panel by clicking the second button from the right at the bottom of the Define Distribution window:
In the Define Distribution window (and in other graph windows), you can change the type of the displayed graph by clicking the Graph Type icon in the lower left of the window. This leads to the choices shown in the following screenshot.

@RISK distribution functions have both required and optional arguments. The only required arguments are the numeric values that define the range and shape of the distribution. All other arguments (such as name, truncation, correlation, and others) are optional and can be entered when needed. These optional arguments are entered in property functions and are specified in the Input Properties dialog. Clicking the fx icon at the end of the Cell Formula text box displays the Input Properties dialog.
Many properties can use references to Excel cells. You can simply click the Reference Entry icon next to the property to add a cell reference.
Distribution properties available in the Options tab of the Input Properties dialog include:

- **Name.** The name @RISK will use for the input distribution in reports and graphs. Initially, a default name determined by @RISK from row and column headings is shown. If you change this name, a **RiskName** property function will be added to the distribution function to hold the specified name.

- **Units.** The units @RISK will use for the input distribution to label the x-axis in graphs. If units are entered, a **RiskUnits** property function will be added to the distribution function to hold the defined units.

- **Use Static Value.** The value the distribution will return in static (not random) Excel recalculations. (This is also the value that will be substituted for the input distribution when @RISK functions are swapped out.) If no static value is entered, @RISK will use either the expected value, median, mode, or a percentile for the distribution. If a static value is entered, a **RiskStatic** property function will be added to the distribution function to hold the defined value.
• **Date Formatting.** Specifies whether the data for the input will be treated as dates in reports and graphs. The setting **Automatic** specifies that @RISK will automatically detect the date data using the format of the cell where the input is located. Selecting **Enabled** will force @RISK to always display graphs and statistics for the input using dates, regardless of the cell format. Likewise, **Disabled** will force @RISK to always generate graphs and statistics for the input in numeric format, regardless of the cell format. If Enabled or Disabled is selected, a **RiskIsDate** property function will be entered to hold the date setting.

Distribution properties available in the Sampling tab of Input Properties dialog include:

• **Separate Seed.** Sets the seed value for this input that will be used during the simulation. Setting a seed value for a specific input insures that any model that uses the input distribution will have the identical stream of sampled values for the input during a simulation. For example, this is useful when sharing input distributions between models using the @RISK Library.
- **Lock Input from Sampling**. Keeps the input from being sampled during a simulation. A locked input returns its static value (if specified) or alternatively, its expected value, or the value specified through the options under When a Simulation is Not Running, Distributions Return of the Simulation Settings dialog.

- **Collect Distribution Samples**. Instructs @RISK to collect samples for the input when the option Inputs Marked with Collect is selected in the Sampling tab of Simulation Settings dialog. If this option is chosen, only inputs marked to collect will be included in sensitivity analyses, statistics, and graphs available after a simulation.
Add Output Command

Adds a cell or range of cells as a simulation output or output range

The Add Output command (in the Model group) adds the currently selected range of worksheet cells as a simulation output. A distribution of possible outcomes is generated for every output cell selected. These probability distributions are created by combining the output values from all iterations of a simulation.

A Summary graph can be generated when a selected output range has more than one cell in it. For example, in one output range, you could select all the cells in a row in your worksheet. The output distributions from these cells would then be summarized in a Summary graph. You can also see an individual probability distribution for any cell in the range.

Sensitivity and Scenario analysis results are also generated for each output cell. For more information on these analyses, see the descriptions of these analyses in the Results Summary Window section of this chapter.
When a cell is designated as a simulation output, a RiskOutput function is added to the cell formula. These functions allow the easy copying, pasting, and moving of output cells. (You can also type RiskOutput functions in formulas, the same way you would type in any standard Excel function, bypassing the Add Output command.) RiskOutput functions optionally allow you to name your simulation outputs, and they allow you to add individual output cells to output ranges. A typical RiskOutput function might be:

\[ =\text{RiskOutput(“Profit”) + NPV(0.1,H1:H10)} \]

In this case, prior to its selection as a simulation output, the cell formula was:

\[ = \text{NPV(0.1,H1:H10)} \]

The added RiskOutput function designates the cell as a simulation output and supplies the output name Profit. For more information on RiskOutput functions, see the section: Reference: @RISK Functions.

Naming an Output

When you add an output, you are given the opportunity to name it, or you can use the default name @RISK has identified. You can also reference an Excel cell containing the name by clicking the desired cell. The name (if not the @RISK default name) is added as an argument to the RiskOutput function through a RiskName property function.

You can change an output name at any time by (1) editing the name argument of the RiskOutput function, (2) re-selecting the output cell and clicking the Add Output icon again, or (3) changing the name shown for the output in the Model window.
You can also designate a range, such as a time series of cash flows, as an output range. To add a new output range:

1) Select the range of cells in your model that you want to add as an output range.

2) Click the Add Output icon on the @RISK ribbon.

3) Add the name for the output range in the Add Output Range dialog or accept the default name. Each cell in the range will share the output name, along with a numeric index. Properties for individual output cells in the range can be added by selecting the output in the table and clicking the \(fx\) icon.
@RISK outputs (those designated with the RiskOutput function) have optional arguments that specify properties (such as name and units) that can be entered when needed. These optional arguments selected in the Output Properties dialog and are entered through property functions in the RiskOutput function.

Clicking the fx icon at the end of the Name text box displays the Output Properties dialog.

Many properties can use cell references to Excel cells. You can simply click the Reference Entry icon next to the property to add a cell reference.
Output properties available in the Options tab of the Output Properties dialog include:

- **Name.** The name @RISK will use for the output in reports and graphs. Initially, a default name determined by @RISK from row and column headings is shown, but you can change it.

- **Units.** The units @RISK will use for the output to label the x-axis in graphs. If units are entered, a `RiskUnits` property function is added to the RiskOutput function to hold the defined units.

- **Data Type.** Specifies the type of data that will be collected for the output during a simulation. The Automatic setting specifies that @RISK will automatically detect the type of data described by the simulated data and generate graphs and statistics for that type. Selecting Discrete forces @RISK to generate graphs and statistics for the output in discrete format. Similarly, Continuous forces @RISK to generate graphs and statistics for the output in continuous format. If Discrete or Continuous is selected, a `RiskIsDiscrete` property function is entered for the output in its RiskOutput function.
- **Date Formatting.** Specifies if the data for the output will be treated as dates in reports and graphs. The setting **Automatic** specifies that @RISK will automatically detect the date data using the format of the cell where the output is located. Selecting **Enabled** forces @RISK to display graphs and statistics for the output using dates, regardless of the cell format. Similarly, **Disabled** forces @RISK to generate graphs and statistics for the output in numeric format, regardless of the cell format.
The settings used in monitoring convergence of an output are set on the Convergence tab. All convergence monitoring settings are entered through the `RiskConvergence` property function. These settings include:

- **Convergence Tolerance**. Specifies the tolerance allowed for the statistic being tested. For example, the above settings specify that you want to estimate the mean of the output simulated within 3% of its actual value.

- **Confidence Level**. Specifies the confidence level for your estimate. For example, the above settings specify that you want your estimate of the mean of the output simulated (within the specified tolerance) to be accurate 95% of the time.

- **Perform Tests on Simulated**. Specifies the statistics of each output that will be tested.
The default settings for an output to be used in Six Sigma calculations are set on the Six Sigma tab. These properties include:

- **Calculate Capability Metrics for This Output**. Specifies that capability metrics will be displayed in reports and graphs for the output. These metrics will use the entered LSL, USL, and Target values.

- **LSL, USL, and Target**. Sets the LSL (lower specification limit), USL (upper specification limit) and Target values for the output.

- **Use Long Term Shift and Shift**. Specifies an optional shift for calculation of long-term capability metrics.

- **Upper/Lower X Bound**. The number of standard deviations to the right or the left of the mean for calculating the upper or lower X-axis values.
Any Six Sigma settings are entered through a RiskSixSigma property function of the output’s RiskOutput function. Only outputs that contain a RiskSixSigma property function will display six sigma markers and statistics in graphs and reports. @RISK six sigma statistics functions in Excel worksheets can reference any output cell that contains a RiskSixSigma property function.

**Note:** All graphs and reports in @RISK use the LSL, USL, and Target values from RiskSixSigma property functions that existed at the start of a simulation. If you change the Specification Limits for an output (and its associated RiskSixSigma property function), you need to re-run the simulation to view changed graphs and reports.
Insert Function Command

Inserts an @RISK function in the active cell

@RISK provides a variety of custom functions that can be used in Excel formulas for defining probability distributions, returning simulation statistics to Excel, or performing other modeling tasks. The Insert Function command (in the Model group) allows you to quickly insert an @RISK function into your spreadsheet model. You can also create a list of your favorite @RISK functions for quick access. When you use the @RISK Insert Function command, the Excel Insert Function Arguments dialog is displayed, where you can enter arguments of the functions.

If you use the @RISK Insert Function command to enter a distribution function, a graph of the distribution function can also be displayed. As with the Define Distribution window, you can add overlays to this graph, add input property functions, or even change the type of distribution function being entered.
Three categories of @RISK functions can be entered with the @RISK Insert Function command. These include:

- **Distribution Functions.** These include functions such as RiskNormal, RiskLognorm, and RiskTriang for specifying input distributions.

- **Statistics Functions.** These include functions such as RiskMean, RiskTheoMode, and RiskPNC, for calculating statistics of distribution functions.

- **Other Functions.** These include functions such as RiskOutput, RiskResultsGraph, and RiskConvergenceLevel for a variety of purposes.

To obtain more information on any of the @RISK functions listed with the Insert Function command, see the Reference: @RISK Functions section of this manual.

The Manage Favorites command displays the Manage Favorites dialog. You can check any number of functions in this dialog. These functions are then designated as Favorites so that you can quickly access them on the Insert Function menu or on the Favorites tab of the Distribution Palette.
If the @RISK Insert Function command is used to enter a distribution function, a graph of the distribution function can also be displayed. (Note: This functionality might not be available in Excel 2013 or in 64-bit versions of Excel.) This graph can also be displayed any time you edit an @RISK distribution using Excel’s Function Arguments dialog, for example, by clicking the small fx symbol next to the Excel formula bar or by using the Excel Insert function command.

A graph of a distribution function can be displayed (or hidden) by clicking the Graph button in Excel’s Function Arguments dialog.

If you do not want to display @RISK distribution functions graphically beside Excel’s Function Arguments dialog, you can select @RISK’s Application Settings command from the Utilities menu and set the ‘Insert Function’ Graph Window option to Disabled.

Note: Graphs of RiskCompound functions cannot be displayed in the Insert Function graph window. You can use the Define Distribution window to preview these functions.
Several buttons at the bottom of the Insert Function graph window allow you to:

- Access the Graph Options dialog to change the scaling, titles, colors, markers, and other settings for the graph.
- Create an Excel chart of the graph.
- Change the type of displayed graph (cumulative, relative frequency, and so on).
- Add overlays to the graph.
- Add properties (that is, distribution property functions such as RiskTruncate) to the distribution function.
- Change the type of distribution function graphed.
To add an overlay to an Insert Function Graph, click the **Add Overlay** button at the bottom of the window and select the distribution you wish to overlay from the Distribution Palette. Once an overlay is added, you can change the function argument values in the **Distribution Argument** panel. This panel is displayed to the left of the graph. Spinner buttons allow you to quickly change a parameter value. For more information on using the Distribution Argument panel, see the **Define Distribution Command** section in this chapter.

![Adding an Overlay in the Insert Function Graph Window](image)

To change the distribution used in the formula from the Insert Function Graph window, click the **Distribution Palette** button at the bottom of the window and select or double-click the distribution you want to change to from the Palette. Once selected, the new distribution and its arguments will be entered in the Excel formula bar and a graph of the new function will be shown.

![Changing the Distribution in the Insert Function Graph Window](image)
To add input properties in the Insert Function Graph window, click the Input Properties button at the bottom of the window and select the properties you want to include. If desired, you can edit the setting for the property in the Input Properties window.

Once you click OK and a distribution property function is entered, you can click the distribution property function in the Excel formula bar and the Excel Function Argument window will be displayed for the property function itself. The arguments can then be edited using Excel’s Function Argument window.
Define Correlation Matrix Command

Defines correlations between probability distributions in a correlation matrix

The Define Correlation Matrix command (from the Define Correlations menu in the Model group) can be used to correlate two or more input distributions according to a correlation matrix. When you click the Define Correlation Matrix command, a matrix is displayed with a row and column for each probability distribution in the currently selected cells. Correlation coefficients between the probability distributions can then be entered directly in this matrix.
Two input distributions are correlated when their samples should be related, that is, the value sampled for one distribution should depend on the value sampled for the other. This correlation is necessary when, in reality, two input variables move to some degree in tandem, either in the same direction or in opposite directions. For example, imagine a model with input distributions for Interest Rate and Housing Starts. These two inputs are related because the value sampled for Housing Starts depends on the value sampled for Interest Rate. A high Interest Rate tends to cause a low value for Housing Starts, and a low Interest Rate tends to cause a high value for Housing Starts. If this correlation were not accounted for in sampling, some iterations of the simulation would contain unrealistic sampled values, such as a high value for Interest Rate and a high value for Housing Starts.

Correlations between input distributions can be entered directly into the displayed matrix. The rows and columns of this matrix are labeled with each of the input distributions in the currently selected cells. Any specific cell in the matrix specifies the correlation coefficient between the two input distributions identified by the row and column of that cell.
Correlation coefficients range in value between -1 and 1. A value of 0 indicates there is no relationship (or more strictly, no linear relationship) between the two variables. A value of +1 indicates that the two variables are perfectly linearly related and vary in the same direction. A value of -1 is the same except that the variables now vary in opposite directions. Coefficient values in between, such as -0.5 or 0.5, specify varying degrees of linear relationship between the two variables. For example, a coefficient of 0.5 specifies that when the value sampled for one input is high, the value sampled for the second value will tend to, but not always, be high.

Correlations can be entered between any input distributions. One input distribution can be correlated with many other input distributions. You can often use historical data on the variables involved to estimate the correlation coefficients to be used in an @RISK model.

Note that any correlation matrix is symmetric about the diagonal, that is, the correlations above the diagonal are mirror images of those
below the diagonal. Because of this, @RISK allows you to enter correlation coefficients above the diagonal or below the diagonal, but it doesn’t require you to do both.

The Define Correlations window allows you to edit existing correlation matrices and create new instances of existing matrices. If you select a cell in Excel that includes a distribution that was previously correlated, or if you select a cell in an existing correlation matrix, and then you click the Define Correlations Matrix command, the existing matrix will be displayed. Then you can change coefficients, add new inputs, add instances, relocate the matrix, or edit the matrix.

Clicking the Add Inputs button in the Define Correlations window allows you to select Excel cells with @RISK distributions to add to the displayed matrix and instance. If any cells in a selected range do not include distribution functions, these cells are ignored.

**Note:** If the @RISK Model window is displayed, input distributions can be added to a matrix by dragging them from the Model window onto the matrix.

The Delete Matrix button deletes the displayed correlation matrix. All RiskCorrmat functions will be removed from distribution functions used in the matrix and the correlation matrix displayed in Excel will be deleted.
The options in the Define Correlation dialog include:

- **Matrix Name.** Specifies the name of the matrix. This name is used to name the range where the matrix is located in Excel and to identify the matrix in the RiskCorrmat functions created. This name must be a valid Excel range name.

- **Description.** Optional, describes the correlations.

- **Location.** Specifies the range in Excel that the matrix will occupy.

- **Add Heading Row/Column and Format.** Displays a heading row and column that includes the names and cell reference for the correlated inputs, and formats the matrix with colors and borders, as shown:
An instance is a new copy of an existing correlation matrix that can be used to correlate a new set of inputs. Each instance contains the same set of correlation coefficients. However, the inputs that are correlated with each instance are different. This allows you to easily set up groups of similarly correlated variables, without repeating the entry of the same correlation matrix. In addition, when a correlation coefficient is changed in any instance of a matrix, it is automatically changed in all instances.

Each instance of a correlation matrix has a name. Instances can be deleted or renamed at any time.

The instance argument is an optional third argument to the RiskCorrmat function. This allows you to easily specify instances when entering correlation matrices and RiskCorrmat functions directly in Excel. For more information on the RiskCorrmat function and the Instance argument, see RiskCorrmat in the Reference: @RISK Functions section of this chapter.

**Note:** When a correlation matrix with multiple instances is created in the Define Correlations window, and entered in Excel, only the inputs for the first instance are shown in the headings of the matrix. In addition, when you display a scatter plot matrix of simulated correlations for the matrix after a simulation run, only the scatter plots for the correlations in the first instance are shown.

Options for instances include:

- **Instance.** Selects the instance that will be shown in the displayed matrix. Inputs can be added to a displayed instance by clicking the Add Inputs button.

Icons next to the Instance name allow:

- **Rename Instance.** Renames the current instance of the displayed correlation matrix.
- **Delete Instance.** Deletes the current instance of the displayed correlation matrix.
- **Add New Instance.** Adds a new instance for the displayed correlation matrix.
You can create a **Correlated Time Series** from an Excel range that contains a set of similar distributions in each row (or column) of the range. In many cases, each row (or column) represents a time period, such as a month or a year. Often you would like to correlate each period’s distributions using the same correlation matrix but with a different instance of the matrix for each time period. For example, you might want to correlate sales of related products in any given month, but you might want sales to be uncorrelated across months.

When you click the Create Correlated Time Series icon, you are prompted to select the block of cells in Excel that contains the distributions of the time series. You can select to have each time period represented by the distributions in a column or row in the range.

When you create a correlated time series in this way, @RISK automatically sets up a correlation matrix instance for each set of similar distributions, in each row or column, in the selected range.

Note: the **Correlated Time Series** discussed here is not the same as correlated time series functions available with @RISK’s **Time Series** tool. @RISK’s Time Series tool uses array functions to model a variety of time series processes. These can also be correlated, as discussed in the Time Series chapter of this manual.
Columns in a correlation matrix can be reordered by dragging the column heading to the new desired position in the matrix.

When you right-click a correlation matrix, additional options allow you to delete rows or columns from a matrix, or remove an input from the matrix:

- **Insert Row/Column.** Inserts a new row and column in the active correlation matrix. A new column is placed in the matrix at the cursor location, shifting existing columns to the right. A new row is also added, at the same position as the added column, shifting existing rows down.

- **Delete Selected Row/Column(s).** Deletes the selected rows and columns from the active correlation matrix.

- **Delete Inputs in Selected Row/Column(s) from Matrix.** Removes the selected input(s) from the active correlation matrix. When inputs are deleted, only the inputs are removed; the coefficients specified in the matrix remain.
Displaying Scatter Plots

The **Show Scatter Plots** icon (in the bottom left of the Define Correlation dialog) displays a matrix of scatter plots of typical sampled values the correlated inputs. These scatter plots indicate graphically how the sampled values of any two inputs will be related during a simulation.

Moving the **Correlation Coefficient slider**, displayed with the scatter plot matrix, dynamically changes the correlation coefficient and scatter plot for any pair of inputs. If you have expanded or dragged the thumbnail scatter plot into a full-sized graph window, that window will also update dynamically.
After a simulation, you can check the correlations for the simulated inputs. You can do this by selecting a cell in the matrix when “browsing” simulation results in your model. The resulting scatter plot matrix shows the actual scatters of simulation input data, the corresponding correlations calculated from the simulation data, and the original correlations (in parentheses). If a correlation matrix has multiple instances, only the scatter plots for the correlations in the first instance are shown.
The Check Matrix Consistency command, displayed when you click the Check Matrix Consistency icon, verifies that the matrix specified in the correlation window is valid. @RISK can correct any invalid matrix and generate the closest valid matrix to the invalid one.

It is actually quite easy to create an invalid correlation matrix. A simple example is: correlate input $A$ and $B$ with a coefficient of $+1$, $B$ and $C$ with a coefficient of $+1$, and $C$ and $A$ with a coefficient of $-1$. This example is clearly illegal, but invalid matrices are not always this obvious. In general, a correlation matrix is valid only if it is positive semi-definite. A positive semi-definite matrix has all nonnegative eigenvalues and at least one positive eigenvalue. More simply, a correlation matrix is invalid if no set of actual data could have this set of correlations.

If @RISK detects an invalid matrix when you run the Check Matrix Consistency command, it gives you the option of having @RISK generate the closest valid matrix to the invalid one. The technical steps are the following:

1) It finds the smallest eigenvalue, $E_0$.
2) It shifts the eigenvalues so that the smallest eigenvalue equals zero by adding the product of $-E_0$ and the identity matrix ($I$) to the correlation matrix ($C$): $C' = C - E_0I$.
3) It divides the new matrix by $1 - E_0$ so that the diagonal terms equal: $C'' = (1/1-E_0)C'$

This new matrix is positive semi-definite, and therefore valid. It is important to check the new valid matrix to ensure that its correlation coefficients are realistic for your model. Optionally, you can control which coefficients are adjusted during the correction of a matrix by entering adjustment weights for individual coefficients.

**Note:** A correlation matrix entered in the Define Correlations dialog is automatically checked for consistency when the OK button is clicked, prior to entering the matrix in Excel and adding RiskCorrmat functions for each input in the matrix.
When an invalid correlation matrix is detected, you can specify adjustment weights for individual correlation coefficients. These weights control how coefficients are adjusted when the invalid matrix is corrected by @RISK. An adjustment weight ranges from 0 (any change is allowed) to 100 (no change is allowed). You might want to use adjustment weights when you are fairly confident in correlations between certain inputs, and you do not want @RISK to modify them (at least not very much) during the adjustment process.

To enter adjustment weights in a Define Correlation window, select the matrix cell(s) that you wish to enter weights for and select the Enter Adjustment Weight command displayed when you right-click the matrix or click the Check Matrix Consistency icon.

As adjustment weights are entered, cells with an adjustment weight are colored to indicate the degree to which their coefficients are fixed.
When you place a correlation matrix in Excel (or use the Check Matrix Consistency command), @RISK checks whether the entered correlation matrix is valid. If it is not, @RISK will use the adjustment weights to correct the matrix.

**Note:** If you enter an adjustment weight of 100, @RISK will try to keep the coefficient associated with that weight fixed. However, if no valid matrix can be generated with the fixed coefficient, it will have to be adjusted to create a valid matrix.

When you place a correlation matrix in Excel, its adjustment weights can also be placed in an **adjustment weight matrix** in Excel. This matrix has the same number of elements as the corresponding correlation matrix. Cells in this matrix contain the entered adjustment weights. Any matrix cells for which no weight was entered (shown as blanks in the matrix) have a weight of 0, indicating that they can be adjusted as necessary during matrix correction. An adjustment weight matrix in Excel is given an Excel range name using the name of the corresponding correlation matrix plus the extension _Weights. For example, a matrix named **Matrix1** would have an associated adjustment weight matrix with the name **Matrix1_Weights**.

<table>
<thead>
<tr>
<th>@RISK Corr</th>
<th>Stock 1 in $H$</th>
<th>Stock 2 in $I$</th>
<th>Commodity in $J$</th>
<th>Adjustment</th>
<th>Stock 1 in $H$</th>
<th>Stock 2 in $I$</th>
<th>Commodity in $J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock 1 in $H$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock 2 in $I$</td>
<td>0.1667313</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commodity in $J$</td>
<td>-0.58746</td>
<td>0.7</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** You do not have to request an adjustment weight matrix in Excel when exiting the Define Correlations window. You can just place the corrected correlation matrix in Excel and discard any entered weights if you are happy with the corrections made and do not need to access the weights at a later time.
You can view @RISK’s corrected matrix in Excel while simulating. If @RISK detects an inconsistent correlation matrix in your model, it will correct it, using any related adjustment weight matrix. However, it leaves your original invalid matrix as you entered it in Excel. To view the corrected matrix in your spreadsheet:

1. Select a range with the same number of rows and columns as the original correlation matrix
2. Enter the function
   =RiskCorrectCorrmat(CorrelationMatrixRange,AdjustmentMatrixRange)
3. Press Ctrl+Shift+Enter (all three at once) to enter your formula as an array formula. Note: The AdjustmentMatrixRange argument is optional and is used only when you are applying adjustment weights.

For example, if the correlation matrix is in the range A1:C3 and the adjustment weight matrix is in the range E1:G3, you would enter:

=RiskCorrectCorrmat(A1:C3,E1:G3)

The corrected coefficients for the matrix will be returned to the range.

The RiskCorrectCorrmat function will update the corrected matrix any time you change a coefficient in the matrix or a weight in the adjustment weight matrix.

When you specify a correlation matrix in the Define Correlations dialog and click OK, the following events take place:

1. The matrix is added to the specified location in Excel.
2. Optionally, any specified adjustment weights are placed in an adjustment weight matrix in Excel.
3. RiskCorrmat functions are added to each of the input distribution functions that are included in the matrix. The RiskCorrmat function is added as an argument to the distribution function itself, such as:

   =RiskNormal(200000, 30000,RiskCorrmat(NewMatrix,2))

Here, NewMatrix is the range name for the correlation matrix and 2 is the index of the distribution function in the matrix, that is, this is for the second input.
After the matrix and RiskCorrmat functions are added to Excel, you can change the coefficient values in your matrix (and weights in the adjustment weight matrix) without editing the matrix in the Define Correlations dialog. New input distributions, however, cannot be added to the matrix displayed in Excel unless you individually add the necessary RiskCorrmat functions in Excel. To add new inputs to a matrix, it is easier to edit the matrix in the Define Correlations dialog.

Correlations between input distributions can also be entered directly in your worksheet using the RiskCorrmat function. The correlations specified, using this function, are identical to those entered from the Define Correlations dialog. You can also enter an adjustment weight matrix directly in your worksheet. If you do this, remember to specify a range name for the correlation matrix, and then use the same range name with the extension _Weights for the adjustment weight matrix. If it is necessary for @RISK to correct the correlation matrix at the start of a simulation, it will use the specified adjustment weight matrix while correcting.

For more information on using these functions to enter correlations, see the description of these functions in the Reference: @RISK Functions section of this chapter.

The correlation of input distributions in @RISK is based on rank order correlations. The rank order correlation coefficient was developed by Spearman in the early 1900s, so often called the Spearman correlation coefficient. It is calculated from ranks of values, not from the actual values themselves (as is the linear, or Pearson, correlation coefficient). A value’s rank is determined by its position within the range of values for the variable. For example, its rank is 3 if it is the third-smallest value.

@RISK generates rank-correlated pairs of sampled values in a two-step process. First, a set of randomly distributed rank scores is generated for each variable. For example, if 100 iterations are to be run, 100 scores are generated for each variable. (Rank scores are simply values of varying magnitude between a minimum and maximum. @RISK uses van der Waerden scores based on the inverse function of the normal distribution.) These rank scores are then rearranged to give pairs of scores with the desired rank correlation coefficient.

In the second step, a set of random numbers between 0 and 1 is generated for each variable. Again, if 100 iterations are to be run, 100 random numbers are generated for each variable. These random numbers are then ranked smallest to largest.
For each variable, the smallest random number is then used in the iteration with the smallest rank score; the second smallest random number is used in the iteration with the second smallest rank score; and so on.

This method of generating correlated samples is known as a “distribution-free” approach because any distribution types can be correlated. For example, a normal distribution can be correlated with a triangular distribution. Although the samples drawn from the two distributions are correlated, the integrity of the individual distributions is maintained so that the resulting samples for each distribution reflect the input distributions from which they are drawn.
Define Copula Command

Defines a copula for special types of correlations structures

The Define Copula command (from the Define Correlations menu in the Model group) allows you to create, edit, and delete copulas. Like an @RISK correlation matrix, an @RISK copula can be used to correlate two or more input distributions. There are similarities between using a copula and using a correlation matrix in @RISK, but there are also some differences you should be aware of.

Why Use a Copula?

The correlation pattern generated from an @RISK correlation matrix always follows an “elliptical Gaussian” pattern. To see what this means, here are the two-dimensional patterns of correlations between two uniform distributions correlated with a correlation matrix with various coefficients.

Notice that not every possible pattern is available here. For example, suppose you have analyzed your data carefully, and you have found that the pattern should look something like the following:

Here, the scatter is quite tight in the lower-left section, while it is much more spread out in the upper-right section. A correlation
matrix doesn’t provide the flexibility to model this situation, but a copula does.

For many models, this level of detail either isn’t important or there isn’t enough data to determine an internal correlation structure with any degree of accuracy. However, in some cases (most commonly in the financial industry), copulas are used to mirror reality more closely.

A complete exposition on how copulas work is beyond the scope of this manual, but it is important to understand the basic theory. To do so, a little terminology is required. The univariate input distributions you are correlating are generally called the marginal distributions, or sometimes just the margins. The multivariate distribution formed when the distributions are correlated is called the joint distribution.

There is a key mathematical result for copulas, called Sklar’s Theorem. It states that any possible joint distribution can be written as a combination of the known marginal distributions and an object called a copula. You can think of a copula as being a special type of joint distribution with Uniform(0, 1) marginal distributions. This means that a copula pattern is a distilled version of the desired correlation pattern with the marginal distributions removed.

For example, here is a two-dimensional joint distribution pattern:
The x- and y- marginal distributions are a Normal and a Weibull distribution:

The underlying “demarginalized” copula pattern looks like this:
Just as there are many different types of univariate distributions, there are many different copula patterns. There are three broad classifications of copulas available in @RISK.

**Archimedean** – These simple copulas are used to correlate a potentially large number of similar variables, such as the returns for all the stocks in a particular industry. They are simple in that they require only a single parameter. This parameter controls the degree of correlation among *all* the variables. @RISK supports three types of Archimedean copulas: **Clayton**, **Gumbel**, and **Frank**.

In their standard form, the Archimedean copulas can model only *positive* correlations. However, there are also several “reflected” versions of the Archimedean copulas. For two-dimensional copulas, you can reflect about the x-axis, the y-axis, or both axes. (You can’t reflect the Frank copula about both axes due to its symmetry. This would yield an identical copula.) These reflected versions are identified with a suffix of “RX” (reflection about the x-axis), “RY” (reflection about the y-axis), or “R” (reflection about both axes.) For example, the possible reflections of a two-dimensional **Clayton** copula look as follows:
For three-dimensional or higher-dimensional copulas, only the “R” reflections are allowed, where all the axes are reflected. (Other reflections are mathematically impossible.) Again, the Frank does not have an “R” version because of its symmetry.
**Elliptical** – There are two elliptical copulas: **Gaussian** and **t**. A Gaussian copula is, in fact, identical to a standard @RISK correlation matrix. Therefore, it requires the specification of a full matrix of correlation values. A t-Copula is similar, but it has an additional parameter that controls the degree of dependence for extreme values.

**Gaussian**  
![Gaussian copula](image)

**t**  
![t-copula](image)

**Empirical** – You can base a copula on a set of existing data. This is similar to using a RiskGeneral function to create an arbitrary univariate probability distribution. Given a set of data, an empirical copula first removes the margins from your data. Then it creates a copula that allows you to correlate any distributions with that same pattern. When you specify an empirical copula, you can choose whether you want it to interpolate values. If you choose not to interpolate, you will get only the same (demarginalized) values present in your data set. More commonly, you will want to allow interpolation. Then @RISK uses Bayesian statistics to compute values between the values of your data set. For example, here are the results for a 10,000-iteration simulation of the same empirical copula of 40 data values. For the non-interpolated case, you get only 40 unique values, whereas in the interpolated case, the values are distributed around those 40 values, but are not restricted to them:

**Non-Interpolated**  
![Non-Interpolated copula](image)

**Interpolated**  
![Interpolated copula](image)
Defining a New Copula

To correlate inputs with a copula, you select two or more input distributions and click the Define Copula command on the Define Correlations menu. This brings up the Define Copula dialog.

At the top of this dialog, there are several fields you can specify:

- **Name.** Specifies the name of the copula. This name will be associated with the Excel range where the copula is defined in your model. It will also be used in the RiskCopula property functions inserted in your @RISK input distributions to identify the copula they are attached to. This name must be a valid Excel range name.

- **Location.** Specifies the range in Excel that the copula will occupy.

- **Copula Type.** Specifies the copula type. Several of the copula types are available only for two-dimensional copulas.

- **Copula Parameters.** Depending on the copula type specified, various parameters will be available to allow you to adjust the copula.

- **Empirical Source Range.** If you choose an empirical copula, you need to specify the range containing the source data.
The Define Copula dialog also contains an image grid. The diagonal images in this window show the marginal distributions being correlated, and the off-diagonal images are scatter plots showing the correlation pattern between any pair of them. Note that the items below the diagonal are always reflections of the items above the diagonal. The Elliptical copulas (Gaussian and t) require the specification of matrix values. These can be edited directly in the image grid.

After you specify the name, location, and copula properties, clicking the OK button writes the new copula into your spreadsheet model. It also adds RiskCopula property functions to your input distribution functions to “attach” them to that copula.

For example, the three uniform distribution in cells B2:B4 below are correlated using a Clayton copula called “MyCopula”:

```
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V1</td>
<td>0.5</td>
<td><strong>@RISK Copula: MyCopula</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>V2</td>
<td>0.5</td>
<td>Type</td>
<td>Clayton</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>V3</td>
<td>0.5</td>
<td>Dimension</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Parameter</td>
<td>8.000</td>
<td></td>
</tr>
</tbody>
</table>
```

“MyCopula” is an Excel range name that refers to the cells in yellow (E3:E5). In each input cell, a RiskCopula property function has been also added to attach the input to the copula. The input functions is cells B2:B4 are:

- `=RiskUniform(0,1,RiskCopula(MyCopula,1))`
- `=RiskUniform(0,1,RiskCopula(MyCopula,2))`
- `=RiskUniform(0,1,RiskCopula(MyCopula,3))`

Notice how the RiskCopula function is almost identical in operation to the RiskCorrmat function for adding a correlation matrix. The first argument specifies the copula to use, and the second indicates the index of the input in the copula.
You can make basic edits to copulas, such as changing parameters or matrix values, directly in your spreadsheet model. However, major changes, such as changing a copula’s type, attachments, or dimensionality, should be done through the Define Copula dialog.

To redisplay the Define Copula dialog for an existing copula, you can select a cell within the existing copula or one of the inputs attached to it and then choose the Define Copula menu item. You can then make changes to your copula.

Sometimes you might want to remove, modify, or attach new inputs to an existing copula. Although it is possible to do this directly by modifying the RiskCopula property functions in your spreadsheet functions, it is much easier to use the Attachment Editor of the Define Copula dialog. This can be accessed by clicking the Attachment Editor icon at the bottom left.

This changes the Define Copula window to display a list of every input currently attached to the copula, along with a set of buttons to make changes to this list.

- **Attach More Inputs.** Allows you to attach additional inputs to the copula. After clicking this button, you will be asked to choose the Excel cells containing the inputs, and then you will be given several options for controlling how they will be added to the list.

- **Detach Inputs.** Removes the currently selected inputs from the attachment list.

- **Resize.** Allows you to change the dimensionality of the copula.
• **Instances.** Just as with correlation matrices, you can have multiple independent instances of the same copula. Use this button to create, delete, or name new instances. See the **Define Correlations** section of this manual for more information about instances.

**Deleting a Copula**

The easiest way to delete a copula from your model is to bring it up in the Define Copula dialog and click the Delete button at the bottom. This removes the copula definition, the associated defined name, and any RiskCopula property functions from the attached input distributions.

**Checking Matrix Consistency**

The **Elliptical Copulas** (*Gaussian* and *t*) both require the specification of a matrix of coefficients. Just as with standard correlation matrices, these coefficients must be valid (positive semi-definite.) See the **Define Correlations** section of this manual for more information.

This consistency check is performed automatically when you close the Define Copula window, but you can also perform it by clicking **Check Matrix Consistency** button.

Unlike correlation matrices, you cannot specify an adjustment weight matrix for copula coefficients.
Fit Copula Command

Fits a copula to an existing data set

The Fit Copula command (from the Define Correlations menu in the Model group) allows you to create a copula based on an existing data set. You can specify a copula from scratch, but more commonly you will want to create a copula based on an existing set of data. One way to do this is to use an empirical copula from within the Define Copula dialog. A second, more mathematical, way is to fit a copula to your data. The basic steps @RISK performs to fit a copula to your data are described here.

It is important to remember that, because copulas have all marginal distribution information removed from them, the first step in fitting a copula is to remove the margins. Several methods for doing this have been proposed in the statistics literature. In @RISK, this demarginalization process is performed by transforming the raw data into so-called pseudo-samples. The pseudo-samples are generated by replacing all the samples with their corresponding ranks and then dividing by the sample size plus one.

There are two main fitting methodologies used to determine copula parameters from the pseudo-samples. This manual will not go into full detail about the fitting process (which you can find in the statistics literature), but briefly, they are:

- **Maximum Likelihood Estimation.** This method (called MLE for short) is very similar to the MLE method used in univariate distribution fitting. The parameters of each fitted copula are chosen to maximize the joint likelihood function of observing the given pseudo-samples. In the statistics literature, the combination of demarginalization just described and MLE methodology is often called the canonical maximum likelihood method (sometimes abbreviated CMLE or CML).

- **Kendall’s Tau Inversion.** This method, sometimes called the itau method, is a type of moment matching. It first calculates the Kendall’s tau statistic for each pair of variables. This statistic measures how often two variables both move in the same direction (up or down). It then matches the parameters of the copula to give the same theoretical statistic.
The MLE method is generally better in most circumstances, and you should use it if possible. It has better convergence and stability properties, but it is quite time consuming to calculate, especially for the elliptical copulas, and it takes longer as the number of variables (D) increases. After about D = 20, the process starts to take a very long time to calculate. @RISK provides an approximate MLE option that is much quicker for large D, but it sacrifices some accuracy. However, even with the approximate method, there is an additional problem. At about D = 50, most of the MLE fitting calculations break down because of numerical problems. Therefore, @RISK will not allow you to fit D > 50 with the MLE method.

@RISK provides the Kendall’s Tau method as an alternative to MLE. It is much faster, and it allows you to have very high dimensionality (up to D = 1000). Nevertheless, it has its own drawbacks. First, its convergence is slower. This means that you need to have more samples than with the MLE method to achieve the same level of accuracy. A second issue, which affects only elliptical copulas, is that the method can find a solution that isn’t strictly legal because it fails the “consistency” (positive semi-definite) requirement of the copula’s matrix. There are ways to correct this problem and @RISK implements them, but it inherently involves some arbitrariness in the solution you will get.

To fit a copula to your data, you first select the range of data and then choose the Fit Copula command to see the Fit Copulas to Data dialog.

**Specifying Copula Fitting Options**

![Image of the Fit Copulas to Data dialog]
This dialog has the following options:

- **Range.** Specifies the range containing the data to be fit. This will already be filled in when you bring up the dialog, but you can change it if necessary. The fitting process requires your data to be arranged in columns.

- **Variable Names in First Row.** If the first row of the data range contains names of each variable being fitted, you should check this option. Again, @RISK determines this for you automatically, but you can change it if necessary.

- **Data Already Demarginalized.** You will almost always want @RISK to “demarginalize” your data before fitting. However, in the unlikely case where your data has already been demarginalized (either because you did it yourself manually, or your data was generated directly from a raw copula), you should check this option so that @RISK doesn’t incorrectly demarginalize it a second time.

- **Fitting Method.** Specifies which fitting method to use. (See the Copula Fitting Methods section for more information about how to choose the appropriate method.) The choice are:
  - Maximum Likelihood Estimation (High Accuracy)
  - Maximum Likelihood Estimation (Approximate)
  - Kendall’s Tau Inversion

- **Selection of Copula Types.** Each copula type that is compatible with your data is listed here. By default, you will fit every possibility, but you can deselect one or more items if you want to make the fitting process faster, or if you know that a particular copula type should not be considered.

- **Load Fit From File.** If you have saved the results of a previous fit to a file, you can load those results with this command.
Once you have fit one or more copulas to your data, you can view the fitted results in the Copula Fit Results window.

The left panel of this window contains a list of every copula that was fit, sorted by a model selection statistic. You can control this sorting by choosing the **Fit Ranking** dropdown list. The options are:

- Akaike Information Criteria (AIC)
- Bayesian Information Criteria (BIC)
- Average Log-Likelihood (Av. LogL)
- Name

The AIC and BIC statistics are described in detail in the Distribution Fitting appendix of this manual. It is important to note that these statistics are all good methods for performing model selection. That is, they are all good for identifying that one copula is a better fit than the others. However, they do not provide any absolute measure of goodness-of-fit.

Selecting one of the fits in the list changes the graph(s) shown in the window. For two-dimensional fits, there will be a single large graph. For more dimensions, there will be one graph for each pair of variables. Any of these graphs can be dragged off to create a full-sized graph, which you can then investigate in more detail.
Each graph has two sets of points, one showing the actual (demarginalized) data and one showing a simulated set of data from the fitted copula. By comparing the overlap between these two sets of points, you can determine the quality of the fit visually.

**Save Fit to File.** This button allows you to save the fitted result to a fit for later viewing.

**Write To Spreadsheet.** Use this command to add a fitted copula to your spreadsheet model. Choosing this command brings up a dialog where you can name the new copula, specify its location in your model, and optionally attach it to any input distributions.
Fit Command

Fits probability distributions to Excel data and displays the results

The Fit command (from the Distribution Fitting menu in the Model group) fits probability distributions to the data in a selected Excel range. This command is available only in @RISK Professional and Industrial versions.

You typically use the Fit command to fit probability distributions to a set of data. The resulting Fit Distributions to Data multi-tab dialog has all the commands necessary for fitting distributions to data. After fitting, the distribution can be placed in your model as an @RISK distribution function for use during simulations.

Note: Output data from a simulation can also be used as the source of the data to be fit. To fit distributions to simulated data, you click the Fit Distributions to Data icon in the lower left of the graph window that displays the simulated distribution of the data you want to use in the fit.

Fit Dialog – Data Tab

The Data tab specifies the source and type of input data entered, whether it represents a continuous or discrete distribution, and whether it should be filtered in any way.
The **Data Set** options specify the source and type of the data to be fitted. The options include:

- **Name.** Specifies a name for the fitted data set. This will be the name shown in the Fit Manager and in any RiskFit functions that link a distribution function to the results from a fit.

- **Range.** Specifies a range in Excel that contains the data to be fitted.

The **Data Set Type Options** specify the type of data to be fitted. Six different types of data can be entered:

- **Continuous Sample Data.** Specifies that the data are a sample from a continuous population. The sample data are used to estimate the properties of that population. The data can be in a column, a row, or a block of cells in Excel.

- **Discrete Sample Data.** Specifies that the data are a sample from a discrete population, where only integer values are possible. The data can be in a column, a row, or a block of cells in Excel.

- **Discrete Sample Data (Counted Format).** Specifies that the data are from a discrete population in Counted format. In this case, the input data will be in the form of \((X, \text{Count})\) pairs, where \(\text{Count}\) specifies the number of data values equal to \(X\). The data must be in two columns in Excel, with the \(X\) values in the first column, and the corresponding \(\text{Count}\) values in the second column.

- **Density (X-Y) Points (Unnormalized).** Specifies that the data are from a density curve in the form of \((X, Y)\) pairs. The \(Y\) value specifies the relative height of the density curve at each \(X\) value. The data must be in two columns in Excel, with the \(X\) values in the first column, and the corresponding \(Y\) values in the second column.

- **Density (X-Y) Points (Normalized).** Specifies that the data are from a density curve in the form of \((X, Y)\) pairs. Typically, this option is used if the \(Y\) data are taken from a curve that has already been normalized (the area under the curve is 1). The \(Y\) value specifies the height of the density curve at each \(X\) value. It is recommended that you select this option to improve fitting of density curve data. The data must be in two columns in Excel, with the \(X\) values in the first column, and the corresponding \(Y\) values in the second column.
- **Cumulative (X-P) Points.** Specifies that the data are from a cumulative curve in the form of \((X, p)\) pairs, where each pair has a \(X\) value, and a cumulative probability \(p\) that specifies the height of the cumulative probability curve at the \(X\) value. A probability \(p\) represents the probability of a value occurring that is less than or equal to the corresponding \(X\) value. The data must be in two columns in Excel, with the \(X\) values in the first column, and the corresponding \(p\) values in the second column.

- **Values are Dates.** Specifies that you will be fitting date data, and that graphs and statistics will be displayed using dates. If @RISK detects dates in the referenced data set, this option will be checked by default.

**Filter Options**

Filtering allows you to exclude unwanted values, those outside an entered range, from your input data set. For example, filtering allows you to ignore outliers, negative values, or other possibilities. The filtering options include:

- **None.** Specifies that the data will be fitted as entered.
- **Absolute.** Specifies a minimum \(X\)-value, a maximum \(X\)-value, or both. Values outside the entered range will be ignored. If just a minimum, or just a maximum, is entered for the range, the data will be filtered only below the minimum or above the maximum.
- **Relative.** Specifies that data outside the entered number of standard deviations from the mean will be ignored.
The options on the **Distributions to Fit** tab let you select the probability distributions to include in a fit. These options can also be used to specify predefined distributions, with preset parameter values to fit. Probability distributions to be included in a fit can also be selected by entering information on the lower and upper limits of the allowable distributions.

The **Fitting Method** options control whether (1) a group of distribution types will be fitted or (2) a set of predefined distributions will be used. The selection for Fitting Method determines the other options displayed in the Distributions to Fit tab. The Fitting Method options include:

- **Parameter Estimation**. The procedure will find the parameters for the selected distribution types that best fit your data set.
- **Predefined Distributions**. The procedure will find the best fitted distribution from a set of distributions you predefine.
When **Parameter Estimation** is selected as the fitting method, the following options are available in the Distributions to Fit tab:

- **List of Distribution Types.** Checking (or unchecking) a specific distribution type includes (or removes) that type from the fit to be performed. The list of distribution types that are displayed changes depending on the options selected for **Lower Limit** and **Upper Limit.** By default, some distribution types in the list are unchecked. This is because: (1) these types are specialized forms of an already checked distribution type (for example, an Erlang distribution is a Gamma distribution with an integer shape parameter) and fitting them would be redundant, or (2) they are a distribution type not normally used in fitting (such as a Student or ChiSq). You can override @RISK’s choices if you like.

Each distribution type has different limits on the data it can describe. Using the **Lower Limit** and **Upper Limit** options, you can select the types of distributions to include, based on your knowledge of the range of possible values for your variable.

**Lower Limit** and **Upper Limit** options include:

- **Fixed Bound of.** Specifies a lower and/or upper limit. Only specific distribution types, such as Triangular, have fixed lower and upper limits, so an entry for Fixed Bound will restrict the fit to certain types of distributions.

- **Bounded, but Unknown.** Specifies that the fitted distribution has a finite lower and/or upper limit, but you don’t know its value.

- **Open (Extends to +/- infinity).** Specifies that the the fitted distribution can possibly extend to any possible positive, or negative, value.

- **Unsure.** Specifies that you are not sure about the possible values that could occur.
When *Predefined Distributions* is selected as the fitting method, you can enter a set of predefined distributions. Then only these predefined distributions will be tested during the fitting process.

Predefined distributions are specified using the following options:

- **Name.** Specifies the name for a predefined distribution.
- **Function.** Specifies the predefined distribution in distribution function format.

Predefined distributions can be included or excluded from a fit by checking or unchecking their entries in the table.

The *Suppress Questionable Fits* option indicates that fits that are mathematically valid but fail various common sense heuristics should not be considered as possible fits. For example, it is often possible to fit normal-like data to a BetaGeneral distribution with very large shape parameters and artificially wide minimum and maximum parameters. Although this can lead to a very good mathematical fit, the fit might be objectionable from a practical standpoint.
For certain distributions, @RISK allows you to fix parameters during fitting. For example, you can fix the mean of any fitted normal distribution to 10, only allowing the standard deviation to vary during the fitting process. Any fixed parameters are applied in addition to boundary limits that have been set using Lower Limit and Upper Limit options.

The **Bootstrap** tab in the **Fit Distributions to Data** dialog sets up a parametric bootstrap for a fit. A parametric bootstrap is used to generate confidence intervals for parameters of fitted distributions, to create distributions for test statistics, and to calculate critical values.
A bootstrap is performed by sampling values from a fitted distribution and refitting those values. The number of samples drawn equals the number of values in the original data set. For example, if your best fit was a Normal(10,1.25) distribution, and there were 100 values in your data set, the bootstrap would sample 100 values from this normal distribution and fit those samples. This process is done repeatedly, with the number of sampling/refitting cycles set by the Number of Resamples entry.

The bootstrap generates a distribution of parameter values for the fitted distribution, along with a confidence interval. For example, if your best fit is a normal distribution, the bootstrap provides a confidence interval for both the fitted mean and the fitted standard deviation. In addition, the bootstrap generates a distribution of test statistic values. This information provides insights into the quality and stability of the estimated parameters and statistics for a fitted distribution.

For more information on a bootstrap and distribution fitting, see Appendix A: Distribution Fitting.
The Chi-Sq Binning tab in the *Fit Distributions to Data* dialog defines the number of bins, type of bins, and custom binning to be used for chi-square goodness-of-fit tests. Here, each bin a particular group of data, such as all values from 100 to 150. Binning can affect the results of chi-square tests and the fit results that are generated. By using the Chi-Sq Binning options, you can ensure that the chi-square test is using the bins that you deem appropriate. For more information on how the number of bins is used in a chi-square test, see Appendix A: Distribution Fitting.

*Note: If you are unsure about the number or type of bins to use for a chi-square test, set “Number of Bins” to “Automatic” and set “Bin Arrangement” to “Equal Probabilities”.*
The following Bin Arrangement options specify the type of binning that will be performed:

- **Equal Probabilities.** Specifies that bins will be made at equal-probability intervals across the fitted distribution. This usually results in unequal length bins. For example, if ten bins are used, the first bin will extend from the minimum to the 10th percentile, the second from the 10th percentile to the 20th, and so on. In this mode, @RISK will adjust the bin sizes, based on the fitted distribution, trying to make all bin have the same probability. This is straightforward for continuous distributions. For discrete distributions, however, @RISK will only be able to make the bin probabilities approximately equal.

- **Equal Intervals.** Specifies that bins will be of equal length across the input data set. Several options are available for entering equal-interval bins across an input data set. Any, or all, of these options can be selected:
  - **Automatic Minimum and Maximum Based on Input Data.** Specifies that the minimum and maximum of your data set will be used to calculate the minimum and maximum of equal-interval bins. However, the first and last bins will be defined according to the settings for the Extend First Bin and Extend Last Bin options. If the Automatic Minimum and Maximum Based on Input Data option is not checked, you can enter a specific Minimum and Maximum value where your bins will start and end. This allows you to enter a specific range for binning, without regard to the minimum and maximum values in your data set.
  - **Extend First Bin from Minimum to -Infinity.** Specifies that the first bin used will extend from minus infinity to the specified minimum. All other bins will be of equal length. In certain situations, this improves fitting for data sets with unknown lower bounds.
  - **Extend Last Bin from Maximum to +Infinity.** Specifies that the last bin used will extend from the specified maximum to plus infinity. All other bins will be of equal length. In certain situations, this improves fitting for data sets with unknown upper bounds.
• **Custom Bins.** You might want to have complete control over the bins used for chi-square testing. For example, you might want to use custom bins when there is a natural grouping of sample data. Then you could design the bins to reflect that grouping. Entering custom bins allows you to enter a specific range for each bin that is defined.

To enter custom bins:

1) Select **Custom** in Bin Arrangement.

2) Enter a value for the Bin Limit for each of your bins. As you enter subsequent values, the range for each bin will be filled in automatically.

**Number of Bins**

The **Number of Bins** options lets you specify a fixed number of bins. Alternatively, you can use the Automatic setting to let @RISK specify the number of bins automatically (based on the sample size of your data set).
The Fit Results window displays a list of fitted distributions and graphs that illustrate how the selected distribution fits your data. It also includes statistics on both the fitted distribution and the input data, as well as the results of the goodness-of-fit (GOF) tests on the fit.

**Note:** No goodness-of-fit test information is generated if the input data type is Density Points or Cumulative Points. In addition, only Comparison and Difference graphs are available for these data types.

**Fit Ranking**

The Fit Ranking list displays all distributions for which valid fits were generated. These distributions are ranked, according to the goodness-of-fit test selected from the **Fit Ranking** selector at the top of the Fit Ranking table. Only distribution types checked in the Distributions to Fit tab of the Fit Distributions to Data dialog are tested when fitting.

A goodness-of-fit statistic provides a quantitative measure of how closely the fitted distribution matches the distribution of the data. In general, a lower statistic indicates a better fit. The goodness-of-fit statistic can be used for comparing the fits from various distributions. Note that goodness-of-Fit information is available only when the Input Data type is Sampled Values.

Checking a distribution listed in the Fit Ranking list displays the fit for that distribution, including graphs and statistics on the selected fit.

The **Fit Ranking** selector specifies the goodness-of-fit statistic to use for ranking distributions. Each goodness-of-fit statistic indicates how well a potential fitted distribution matches the distribution of the sample data. Five types of statistics are available:
• **Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC).** The AIC and BIC statistics are calculated from the log-likelihood function and take into account the number of parameters of the fitted distribution. (To understand why the number of parameters is important, consider the case where a normal distribution and a beta-general distribution are both good fits to a data set. All else being equal, the normal distribution is preferable because it has only two adjustable parameters and the beta-general has four.) We generally recommend that you use the AIC statistic or the BIC statistic for ranking the fits.

• **Chi-Sq.** The chi-square statistic corresponds to the most common goodness-of-fit test. It can be used with any type of sample data and any type of distribution, discrete or continuous. Its weakness is it requires bins, and there are no clear guidelines for selecting these bins. It is sometimes possible to reach different conclusions from the same data, depending on how the bins are specified. The bins used in the chi-square statistic can be defined in the Define Chi-Sq Binning tab of the Fit Distributions to Data dialog.

• **K-S.** The Kolmogorov-Smirnov statistic is generally preferred over the chi-square statistic because it doesn’t rely on bins.

• **A-D.** The Anderson-Darling statistic is similar to the Kolmogorov-Smirnov statistic, but it places more emphasis on tail values. It doesn’t rely bins.

• **RMS Error, or root mean squared error.** If the data type is a Density Curve or Cumulative Curve (as specified in the Data tab of the Fit Distributions to Data dialog), only the RMS Error statistic can be used for ranking fits.

For more information on the available goodness-of-fit statistics, see Appendix A: Distribution Fitting.
To display fit results for multiple distributions in a single graph, you can check all distributions of interest in the Fit Ranking list.

When the input data type is **Sampled Values**, three graphs — **Comparison**, **P-P**, and **Q-Q** — are available for any fit, selected by clicking in the Fitted Distribution list. If the input data type is **Density Curve** or **Cumulative Curve**, only the **Comparison** and **Difference** graphs are available.

For all graph types, **delimiters** can be used to graphically set specific X-P values on the graph.
A Comparison Graph displays two curves: the best fitting distribution and the distribution of the sample data.

Two delimiters are available for a Comparison graph. These delimiters set the **Left X** and **Left P** values, along with the **Right X** and **Right P** values. Values returned by the delimiters are displayed in the probability bar above the graph.

**P-P Graph**

The P-P (Probability-Probability) graph plots the p-value of the fitted distribution versus the p-value of the input data. If the fit is good, the plot will be nearly linear.
A Q-Q (or Quantile-Quantile graph) plots the percentile values of the fitted distribution versus the percentile values of the input data. If the fit is good, the plot will be nearly linear.

A bootstrap analysis provides distributions and statistics for fitted parameters and goodness-of-fit statistics.
When a bootstrap analysis is run, a distribution is generated for the values for each parameter in the fitted distribution. A bootstrap is performed by sampling a set of values from a fitted distribution and refitting those values. Each fit in the bootstrap generates a new value for each fitted parameter of the distribution type. For example, for a Gamma distribution type, a distribution is generated for the alpha parameter and the beta parameter. A distribution of these values is displayed, along with values for the selected confidence interval. The confidence interval allows you to make statements such as, “You are 95% confident that the alpha parameter is in the range 2.09 to 3.25, with a fitted value of 2.57.”

When a bootstrap analysis is run, a distribution is generated for the goodness-of-fit statistics for the chi-square, Kolmogorov-Smirnov, and Anderson-Darling tests. Each fit in the bootstrap generates a new value of the goodness-of-fit statistic and, after the bootstrap runs, a distribution of these values is displayed, along with with a P-value. The P-value, which can be read from the delimiter on the graph, ranges from 0 to 1, with larger values indicating a better fit.

For more information on bootstrap and distribution fitting, see Appendix A: Distribution Fitting.
The **Write to Cell** button in the Fit Results window writes a fit result to a cell in your spreadsheet as an @RISK distribution function.

The options in the Write to Cell dialog include:

- **Select Distribution.** The distribution function to be written to Excel can be either **Best Fit Based on** (the best fitting distribution based on the selected statistic) or **By Name** (a specific fitted distribution in the list).

- **@RISK Function Format.** The distribution function to be written to Excel can be automatically updated (1) when the input data in the referenced data range in Excel changes and a new simulation is run, or (2) whenever the input data changes. If **Link - Updates Each New Simulation** is selected, a new fit will be run when @RISK starts a simulation and detects that the data has changed. Linking is done with a RiskFit property function, such as:

  \[ =\text{RiskNormal}(2.5, 1, \text{RiskFit}(\text{"Price Data"}, \text{"Best A-D"})) \]

This specifies that the distribution is linked to the best-fitting distribution according to the Anderson-Darling statistic, for the data associated with the fit named “Price Data”. Currently, this distribution is a Normal distribution with mean 2.5 and standard deviation 1.
The RiskFit property function is automatically added to the function written to Excel when **Link - Updates Each New Simulation** is selected. If no RiskFit function is used in the distribution function for a fit result, the distribution will not be linked to the data used for fitting. If the data is later changed, the distribution will remain as is.

The option **Live – Updates Whenever Data Changes Option** writes a **RiskFitDistribution** function to Excel. This function automatically updates the fitted distribution when the input data changes. Using this capability, you can have fitted distributions update automatically as new data is received or data changes during a simulation.

The RiskFitDistribution function fits data interactively and returns samples from the best-fitting distribution during a simulation. It operates the same as an @RISK distribution function for the best fit that is entered in a cell. It can be correlated, named, and include property functions, just as with standard @RISK distribution functions.

- **Function to Add.** This displays the @RISK distribution function that will be added to Excel.
The Fit Summary window displays a summary of calculated statistics and other information for all distributions fit to the current data set.

The following entries are shown in the Fit Summary window:

- **Function.** Lists the distribution and arguments for the fitted distribution. When a fit is used as an input to an @RISK model, this formula matches the distribution function that will be placed in your spreadsheet model.

- **Parameter Statistics (Bootstrap only).** These entries display the confidence intervals for the fitted parameters for each fit.

- **Distribution Statistics (Minimum, Maximum, Mean, etc.).** These entries display the statistics calculated for all fitted distributions and the distribution of the input data.

- **Percentiles.** Lists the probabilities of achieving specific outcomes at various probability levels.

- **AIC and BIC.** Lists Information Criteria values.
For Chi-Sq, A-D and K-S options, the Fit Summary window also displays:

- **Test Value.** The test statistic for the fitted probability distribution for each of the three tests.

- **P-Value.** The observed level of the significance of the fit. For more information on P-values, see Appendix A: Distribution Fitting.

- **Rank.** The the rank of the fitted distribution among all distributions fitted for each of the three tests. Depending on the test, the returned rank can change.

- **Cr. Value (Bootstrap only).** The critical values calculated at various significance levels for chi-square, Kolmogorov-Smirnov, and Anderson-Darling tests.

- **Bin statistics.** Lists statistics for each bin, for both the input and the fitted distribution (chi-square test only). These statistics include the minimum and maximum of each bin and the probability value for the bin.
Batch Fit Command

Fits a group of data sets at one time and generates reports for fit results

The Batch Fit command (from the Distribution Fitting menu in the Model group) is relevant when you have data on multiple variables (often related variables) in a data set, typically in adjacent columns. It fits the data for each variable in a single run and generates Excel reports on the fit results. The report includes information on the best fit for each variable.

Batch Fit can also generate a correlation matrix showing correlations between the fitted variables. When correlations are generated, the appropriate RiskCorrmat entries for the fitted distributions are also added.

The Batch Fit dialog is similar to the Fit dialog. It allows you to select the parameters to use when fitting the data for each variable. An additional setting, **Best Fit Selection**, specifies the statistic (AIC, BIC, Chi-Sq, A-D, or K-S) to use for ranking the fits.
The Report tab specifies the type and location of the reports that will be generated from a batch fit.

**Batch Fit – Report Tab**
Two types of reports can be generated: a **Standard Report** or a **Live Report**. A standard report has a worksheet for each fitted variable. Each of these worksheets lists an @RISK distribution function for the best fit and statistics on fit results. A standard report does not update if your data changes; you need to re-run the batch fit.
A live report has a single worksheet with a **RiskFitDistribution** function for each fitted variable. In addition, @RISK fit statistics functions are used to return information on each fit to the report.

The **RiskFitDistribution** function fits data interactively and returns samples from the best-fitting distribution during a simulation. It can be correlated, named, or include property functions, just as with standard @RISK distribution functions.

@RISK fit statistics functions return information on the results from a fit performed by **RiskFitDistribution**. For example, **RiskFitDescription** returns the name and arguments for the best-fitting distribution, and **RiskFitStatistic** returns a specified statistic for the fit.

The RiskFitDistribution function automatically updates the fitted distribution when the input data changes in Excel. Correlations are also updated. Using this capability, you can have fitted distributions update automatically as new data is received or data changes during a simulation.
Fit Manager Command

Displays a list of fitted data sets in the current workbook for editing and deleting

The Fit Manager command (from the Distribution Fitting menu in the Model group) displays a list of the fitted data sets in all open workbooks.

Fitted data sets and their settings are saved when you save your workbook. From the Fit Manager dialog, you can easily select a fitted data set, or you can delete ones you no longer need.
Artist Command

Displays the Distribution Artist window where a curve to be used as a probability distribution can be drawn

The Artist command (from the Distribution Fitting menu in the Model group) is used to draw free-form curves for creating probability distributions. This is useful for graphically assessing probabilities and then creating probability distributions in the graph. Distributions may be drawn as density functions, histograms, cumulative curves, or discrete distributions.

After an Artist window has been displayed with the Artist command, you can draw a curve simply by dragging the mouse through the window.

A curve in the Distribution Artist window can be fitted to a probability distribution by clicking the Fit Distribution to Data icon. A curve in the Distribution Artist window can also be written to a cell in Excel as a RiskGeneral, RiskHistogram, RiskCumul, RiskCumulD, or RiskDiscrete distribution, where the points on the curve will be entered as arguments to the distribution.

If you select the Artist command and the active cell in Excel contains a distribution function, the Artist window will display a probability density graph of that function with points you can adjust. You can also use this capability to review previously drawn curves you wrote to Excel.
The scaling and type of graph drawn in the Artist window are set using the Distribution Artist Options dialog. This is displayed by clicking the Draw New Curve icon, or by right-clicking the graph and selecting the Draw New Curve command.

The Distribution Artist options include:

- **Name.** The default name given to the selected cell by @RISK, or the name of the distribution used to create the displayed curve as given in its RiskName property function.

- **Distribution Format.** Specifies the type of curve that will be created, where Probability Density (General) is a probability density curve with x-y points, Probability Density (Histogram) is a density curve with histogram bars, Cumulative Ascending is an ascending cumulative curve, Cumulative Descending is a descending cumulative curve, and Discrete Probability is a “spike chart” with discrete probabilities.

- **Date Formatting.** Specifies that dates will be used for X-axis values.

- **Minimum and Maximum.** Specifies the X-axis scaling for the graph.

- **Number of Points or Bars.** Sets the number of points or bars that will be drawn as you drag across the range of the graph. You can drag the points on the curve or move the bars on a histogram up or down to change a curve’s shape.

If you are drawing an ascending cumulative distribution, you must draw a curve with ascending Y values (or descending Y values for a descending cumulative curve).
When you have completed a curve, the end-points on your curve will be automatically plotted.

Some items to note about drawing curves with the Distribution Artist:

- After drawing a curve, you can drag one of the points to a new location. This automatically redraws the curve.
- You can move data points along the X-axis or Y-axis (except with a histogram).
- You can drag endpoints outside the axes by dragging an endpoint.
- You can move a dashed vertical endline to reposition the entire curve.
- By right-clicking the curve, you can add new points or bars as necessary.

Icons at the bottom right of the Distribution Artist window include:

- **Copy.** The Copy commands copy the selected data or the graph from the Artist window to the clipboard. **Copy Data** copies X and Y data points for markers only. **Copy Graph** places a copy of the drawn graph in the clipboard.
- **Distribution Format.** Displays the current curve in one of the other available distribution formats.
- **Draw New Curve.** Erases the active curve in the Artist window and begins a new curve.
- **Fit Distributions to Data.** Fits a probability distribution to the curve drawn. When a curve is fitted, the X and Y values associated with the curve are fitted. The results of the fit are displayed in a standard Fit Results window, where each of the fitted distributions can be reviewed. All options that can be used when fitting distributions to data are available when fitting probability distributions to a curve drawn in the Artist window. For more information on these options, see **Appendix A: Distribution Fitting** in this manual.
Clicking the **Write to Cell** button enters a RiskGeneral, RiskHistogrm, RiskCumul, RiskCumulD, or RiskDiscrete distribution function from the drawn curve into a selected cell. These functions correspond to the type of graph drawn. You can find details about these functions in the Function Reference section of this manual.
Model Window Command

Displays all input distributions and output cells in the @RISK Model window

The Model Window command (in the Model group) displays the @RISK Model window. This window provides a table of all input probability distributions and simulation outputs described in your model. (Actually, if multiple workbooks are open, it shows all inputs and outputs from all of these workbooks.) From this window, you can:

- Edit any input distribution, or output, by making appropriate changes in the table.
- Drag and drop any thumbnail graph to expand it to a full-sized window.
- Quickly view thumbnail graphs of all defined inputs.
- Double-click any entry in the table to use the Graph Navigator to move through input or output cells in your workbook.
- View and edit any correlation matrices in your model(s).
The items in the Model window are linked to your spreadsheet model. When you click an input or output in the table, the corresponding cell is highlighted in Excel. If you double-click an input or output in the table, the corresponding graph will be displayed, with a pointer to the cell where the input or output is located.
Options for the Model window can be accessed by clicking the icons displayed at the bottom of the window or by right-clicking and selecting from the pop-up menu. Selected commands will be performed on the current selected rows in the table.

The tables of inputs and outputs displayed in the Model window are set up automatically when you display the window. When the window is displayed, your worksheets are scanned or re-scanned for @RISK functions.

If a name is not entered explicitly in a RiskOutput function or in a distribution function, @RISK will automatically try to create a name. These names are created by scanning the worksheet around the cell where the input or output is located. To identify names, @RISK moves from the input or output cell, to the left and up, until it finds a label cell, or a cell without a formula in it. It then combines these row and column labels to create a name for the input or output. The resulting names can always be changed.

**How Are Variable Names Generated?**
Model Window – Inputs Tab

The **Inputs** tab in the Model window lists all distribution functions in your model. By default, the table shows for each input:

- **Name.** The name of the input. To change the name of the input, simply type a new name in the table, or click the Reference Entry icon to select a cell in Excel containing the name you want to use.

- **Cell.** The cell where the distribution is located.

- **Graph.** A thumbnail graph of the distribution. To expand a graph into a full-sized window, you can drag the thumbnail off the table.

- **Function.** The distribution function entered in the Excel formula. You can edit this function directly in the table.

- **Min, Mean, Max.** Summary measures of the distribution.
The Model window columns can be customized to select the statistics you want to display. The Select Columns for Table icon at the bottom of the window displays the following dialog.

![Select Columns for Table: Model dialog]

Note that the only percentiles listed are for 5% and 95%. However, you can change these by using the ellipsis buttons. Editable p1,x1 and p2,x2 columns are also available, where you can enter specific target values and/or target probabilities in the table.
Inputs in the Model window can be grouped by category. By default, a category is created when a group of inputs share the same row (or column) name. In addition, you can place inputs in any category you wish. Each category of inputs can be expanded or collapsed by clicking on the — or + buttons in the left margin.

The **Arrange** icon at the bottom of the Model window allows you to turn category grouping on and off, change the type of default categories used, create new categories, and move inputs between categories. The property function **RiskCategory** is used to specify the category for an input (when it is not located in the default category identified by @RISK).
The commands on the Arrange menu include:

- **Group Inputs By Category.** Specifies whether the table of inputs is arranged by category. When Group Inputs By Category is checked, categories entered using a RiskCategory function will always be shown.

- **Default Categories.** Specifies how @RISK automatically generates category names from input names. Default category names are easily created from the default input names created by @RISK. (For more information on how default names are generated for an input using a Row Heading and a Column heading in your spreadsheet, see the section How Are Default Names Created?) A typical default name might be Profit / Year 1, where Profit is the row label and Year 1 is the column label.
  - **Row Heading.** Specifies that names that share a common row heading will be grouped together in a category.
  - **Column Heading.** Specifies that names that share a common column heading will be grouped together in a category.

Default Categories can also be created from input names you specify with a RiskName function, as long as a “/” separator is included to separate row or column headings in the name. For example, the input

```
=RiskNormal(100,10,RiskName("R&D Costs / 2015"))
```

would be included in a default category named “R&D Costs” (assuming the Default Categories Row Heading option is checked), and it would be included in a default category named “2010” (assuming the Default Categories Column Heading option is checked).

- **Assign Input to Category Command.** This command places an input, or set of inputs, into a category. The Input Categories dialog allows you to create a new category, or select a previously created category, for the selected inputs. (In the following, the selected inputs will be placed into a new category called Revenue inputs.)
When you assign an input to a category, the input category is defined in an @RISK function using the RiskCategory property function. For more information on this function, see the Listing of Property Functions in the Function Reference of this manual.

**Edit Menu**

The Model window can be copied to the clipboard or exported to Excel using the commands on the Edit menu. In addition, where appropriate, values in the table can be filled down or copied and pasted. This allows you to quickly copy an @RISK distribution function across multiple inputs, or copy editable P1 and X1 values.

Commands on the Edit menu include:

- **Copy Selection.** Copies the current selection in the table to the clipboard.

- **Paste, Fill Down.** Pastes or fills values into the current selection in the table.

The Graph menu can be accessed by clicking the **Graph** icon at the bottom of the Model window or by right-clicking in the table. The commands shown will be performed on the selected rows in the table. This allows you to quickly display graphs of multiple input distributions in your model by selecting the type of graph you wish to display. The **Automatic** command creates the graph using the default type (probability density) for input distributions.
The Outputs tab in the Model window lists all outputs in your model. These are the cells where RiskOutput functions are located.

For each output, the table shows:

- **Name.** The name of the output used in @RISK reports. To change the name of the output, simply type a new name in the table. Alternatively, you can click the Reference Entry icon to select a cell in Excel that contains the name you want to use.

- **Cell.** The cell where the output is located.

- **Function.** The RiskOutput function entered in the Excel formula. You can edit this function directly in the table.

The properties of each output can be entered by clicking the fx icon (which appears at the right when you select a function). For more on properties for outputs, see the Add Output command in this chapter.
The **Correlations** tab in the Model window lists all correlations in open workbooks, along with any defined correlation instances. Each input distribution contained in each correlation matrix (or copula) and each instance is listed.

Inputs can be edited in the Correlations tab, just as in the Inputs tab. A correlation matrix corresponding to any input can be edited by:

- Clicking the **Correlation Matrix** icon (which appears when you selected a function).
- Right-clicking the input, in the Correlations tab or the Inputs tab, and selecting **Correlate Edit Matrix**.
- Selecting the cell in Excel where the input distribution is located (or a cell in the correlation matrix or copula), and selecting **Define Correlations** or **Define Copula**.

For more information on correlation, see the **Define Correlations** command in this manual.
Data Viewer Command

Provides standard @RISK graphs and reports for an Excel data set

The Data Viewer command (in the Model group) provides a useful tool for examining any Excel data set. This can be applied to data from any source, including data generated from an @RISK simulation. All Data Viewer results are displayed in standard @RISK graphs and tables.

To use the Data Viewer, select the data in your spreadsheet and click the Data Viewer icon on the @RISK ribbon. (As a shortcut, if you select a single cell within a data set, @RISK will automatically expand the selection to include the full data set.) This displays the Data Viewer Options dialog, where you can specify properties of the data set.

- **Range.** The range where your data is located. @RISK will guess this range based on the current selection, but you can change it if necessary.

- **Variable Arrangement.** The arrangement of variables can be in rows or columns, or it can be a block of data that should be treated as a single variable.

- **Variable Names in First Row/Column.** If the first row or column contains the names of the variables in the data set, check this option. @RISK will try to detect this automatically, but you can change it if necessary.
• **Ignore Entire Row/Column if Any Cell Has a Missing or Non-Numeric Value.** Checking this option will discard an entire row or column if any of the values in that row or column are missing or non-numeric.

• **Correlation Calculations.** The Data Viewer automatically displays correlation coefficients between your variables. This option controls how these coefficients are calculated.

**Multivariate Data Viewer Window**

After you specify the Data Viewer options and click OK, you will be presented with the Multivariate Data Viewer window (if your data set contains more than one variable). This window has several different views, each giving a different perspective of your data. You can change the view by clicking the view buttons at the bottom of the window.

**Summary View**

The summary view gives a quick overview of your data set in tabular format. Each row of the table corresponds to one variable and shows the variables name, a thumbnail graph, and several statistical summary measures. You can control which columns are displayed in the table by clicking the **Configure Columns** button at the bottom of the window.

You can drag an individual thumbnail graph off the window to get a full-sized graph window.
You can export this table to Excel by clicking the **Export** button at the bottom on the window and choosing **Report in Excel**. Alternatively, you can select one or more rows in the table, click the **Export** button, and choose **Copy** to copy and paste the selected data to Excel.

**Correlation View**

The correlation view displays graphical and numeric information about the correlation patterns between variables in the data set. It displays histograms of individual variables on the diagonal and scatter plots for pairs of variables off the diagonal. The correlation coefficients associated with each pair are shown at the top of each scatter plot.

You can click and drag an individual graph off the window to get a full-sized graph window.

You can export the correlation coefficient matrix to Excel by clicking the **Export** button at the bottom on the window and choosing **Matrix in Excel**. Alternatively, you can click an individual graph, click the **Export** button, and choose **Copy** to copy and paste the selected graph.
The overlay view shows all the variables simultaneously in a single graph. You can control which inputs are displayed (up to a maximum of 10) in the overlay graph by clicking the **Graph Options** button at the bottom of the window and choosing **Select Variables to Graph**. By default, @RISK will only display variables with the same formatting as the first variable in the data set. For example, if you have data with some numeric values, some date values, and some percentage values, @RISK will display only the numeric values together on the overlay graph.

You can copy or export this graph to Excel by clicking the **Export** button at the bottom of the window.

You can change the formatting of the graph by clicking the **Graph Options** button at the bottom of the window.
The Trend and Boxplot views show summary statistics of all the variables in a compact graphical format.
You can control which inputs are displayed in the graphs (up to a maximum of 500) by clicking the Graph Options button at the bottom of the window and choosing Select Variables to Graph. By default, @RISK will only show variables with the same formatting as the first variable in the data set. For example, if you have data with some numeric values, some date values, and some percentage values, @RISK will show only the numeric values together on the graph.

You can copy or export the graphs to Excel by clicking the Export button at the bottom of the window.

You can change the formatting of the graphs by clicking the Graph Options button at the bottom of the window.
Simulation Group Commands

This group is where you specify simulation settings for the current simulation and eventually run the simulation.
Simulation Settings Command

Changes the settings that control the @RISK simulations

The **Simulation Settings** command (in the **Simulation** group) affects the operations performed during an @RISK simulation. All settings have default values, but these can be modified. The simulation settings affect the type of sampling @RISK performs, the updating of the worksheet display during simulation, the values returned by Excel in a standard recalculation, the seeding of the random number generator used for sampling, the status of convergence monitoring, and macro execution during simulation. All simulation settings are saved with the workbook when you save your workbook in Excel.

Note that a few of the simulation settings, particularly the number of iterations and the number of simulations, can be set directly from the @RISK ribbon. This saves you from having to get into the Simulation Settings dialog each time you want to change these common settings.

To save simulation settings so they will be used as the default settings each time you start @RISK, you can use the **Application Settings** command on the **Utilities** menu. However, be aware that the simulation settings saved in any workbook override the default application settings.
The General tab lets you specify the number of iterations and simulations to be executed. It also lets you specify the type of values returned by @RISK distributions in normal Excel recalculations.

The Simulation Runtime options include:

- **Number of Iterations.** Specifies the number of iterations to be executed during a simulation. Any positive integer value (up to 2,147,483,647) can be entered. The default value is 100. In each iteration:
  1) All distribution functions are sampled.
  2) Sampled values are returned to the cells and formulas of the worksheet.
  3) The worksheet is recalculated.
  4) The new calculated values in all output cells are saved for use in creating output distributions.
The number of iterations performed affects both your simulation execution time and the accuracy of your results. To get a quick look at results, you can run 100 iterations or fewer. For more accurate results, you will probably need to run 1000 or more iterations, although this depends on the model. You can use the Convergence Monitoring options to run the number of iterations required for accurate and stable results. The Automatic setting allows @RISK to determine the number of iterations to run. It is used with convergence monitoring to stop the simulation when all output distributions have converged. See the Convergence Tab later in this section for more information on convergence monitoring.

Note that the Calculation command in Excel’s Options dialog has its own Maximum Iterations setting. This is used for resolving worksheet calculations that contain circular references. You can simulate worksheets that use this option; @RISK will not interfere with the resolution of circular references. @RISK allows Excel to “iterate” to resolve circular references during each iteration of a simulation.

**Important! A single recalc when @RISK’s “dice” button is toggled to Random might fail to resolve circular references. If an @RISK distribution function is located in a cell that is recalculated during an Excel iteration, it will be resampled on each iteration of the single recalc. Because of this, the “dice” button should be toggled to Static for worksheets that use Excel Iterations capabilities to resolve circular references.**

- **Number of Simulations.** Specifies the number of simulations to be executed when you click the Start Simulation icon. You can enter any positive integer value. The default value is 1. Each simulation will be run for the number of iterations selected. In each iteration of each simulation:
  1) All distribution functions are sampled.
  2) RiskSimtable functions return the argument corresponding to the index of the simulation being executed.
  3) The worksheet is recalculated.
  4) The new calculated values in all output cells are saved for use in creating output distributions.
The number of simulations requested should be no greater than the number of arguments entered in the RiskSimtable functions. Otherwise, errors will occur. For example, if the number of simulations is specified as 5 and a RiskSimtable function has only 3 arguments, the RiskSimtable function will return an error value during simulations 4 and 5.

**Important!** Each simulation executed, when the number of simulations is greater than 1, uses the same random number generator seed value. This isolates differences between simulations to the changes in the values returned by RiskSimtable functions. If you want to override this behavior, you can select Multiple Simulations Use Different Seed Values in the Random Number Generator section of the Sampling tab prior to running the simulations.

- **Multiple CPU Support.** Instructs @RISK to use all CPUs in your computer to speed simulations. Additionally, a simulation setting that is not visible in the @RISK user interface allows control over the maximum number of cores used with the multiple CPU option. To use this setting, you need to create a defined name in your simulation workbook called `_AtRisk_SimSetting_MaxCores` and set its value to the maximum number of cores you want to allow. Note that if more than one workbook is open with different values for this name, the minimum of the values will be used.
If you run multiple simulations, you can enter a name for each simulation to be run. This name will be used to label results in reports and graphs. To do this, set the Number of Simulations to a value greater than 1 and then, from the Simulation Settings dialog, click the **Simulation Names** button and enter a name for each simulation.
When a Simulation is Not Running, Distributions Return Options

The **When a Simulation is Not Running, Distributions Return** options control the display when a standard Excel recalculation is performed. The options include:

- **Random Values (Monte Carlo).** In this mode, distribution functions return a random Monte Carlo sample during a recalculation. This setting allows your worksheet values to appear as they would during execution of a simulation, with new samples drawn for distribution functions on each recalculation.

- **Static Values.** In this mode, distribution functions return static values entered in a RiskStatic property function during a recalculation. If a static value is not defined for a distribution function, it will return:
  - **Expected Value.** Uses the expected value of the distribution (the mean). For discrete distributions, this setting uses the discrete value in the distribution closest to the mean as the static value. If a distribution doesn’t have a mean (the Pareto distribution, for example), the 50th percentile (the median) is returned instead.
  - **‘True’ Expected Value.** Uses the same static value as in the previous option, except in the case of discrete distributions, such as Binomial and Poisson distributions. For discrete distributions, the true mean is used, even if it is not one of the discrete possible values of the distribution.
  - **Mode.** Uses the distribution’s mode value.
  - **Percentile.** Uses a specified percentile value.

- **By Default, Functions Use Simulation Number.** This setting allows you to control which simulation index is used (in cases where one is not explicitly specified) in @RISK functions. This setting is relevant (and is shown in the Simulation Settings dialog) when the number of simulations is greater than 1.

The Random (Monte Carlo) versus Static Values setting can be quickly changed using the Random/Static (dice) icon on the @RISK ribbon.
The View tab settings control what is displayed by @RISK while a simulation is running and when a simulation ends.

The **Automatic Results Display** options include:

- **Show Output Graph.** In this mode, a graph of simulation results for the selected cell in Excel automatically pops up:
  - When a simulation run starts (if real-time results are enabled with **Update Windows During Simulation Every XXX Seconds**), or
  - When a simulation run is complete.

In addition, **Browse Results** mode is enabled at the end of a simulation run. If the selected cell is not an @RISK output or input, a graph of the first output cell in your model will be displayed.

- **Show Results Summary Window.** This option displays the Results Summary window when a run starts (if real-time results are enabled with **Update Windows During Simulation Every XXX Seconds**), or when a simulation run is complete.
• **Demo mode.** This is a preset view, where @RISK updates the workbook each iteration to show how values are changing, and displays an updated graph of the first output in your model. This mode is useful for illustrating a simulation in @RISK.

• **None.** No @RISK results windows are displayed at the start, or end, of a simulation.

Settings under **Options** on the **View** tab of the Simulation Settings dialog include:

• **Minimize Excel at Start of Simulation.** Minimizes the Excel window, and all @RISK windows, at the start of a simulation. Any window can be viewed during the run by clicking it in the Task bar.

• **Show Simulation Progress Window.** Allows you to show or hide the progress window that normally appears during a simulation.

• **Update Windows During Simulation Every XXX Seconds.** Turns real-time updating of open @RISK windows on or off, and sets the frequency with which windows are updated. When **Automatic** is selected, @RISK selects an update frequency, based on the number of iterations performed and the runtime per iteration.

• **Show Excel Recalculations.** Toggles the updating of the worksheet display during a simulation on or off. For each iteration of a simulation, all distribution functions are sampled and the spreadsheet is recalculated. This setting allows you to display the results of each recalculation on the screen or suppress the display. Because updating the display for new values every iteration slows down the simulation, the default is off.
- **Pause on Output Errors.** Toggles the Pause on Error capability on or off. Pause on Error causes a simulation to pause when an error value is generated in any output. When an error is generated, the Pause on Error in Outputs dialog provides a detailed listing of the outputs for which errors occurred and the cells in your spreadsheet that caused the errors.

![@RISK - Pause On Error In Outputs - Iteration # 3](image)

The Pause on Error in Outputs dialog shows, on the left, an explorer-type list containing each output for which an error was generated. A cell whose formula caused an error is shown on the right when you select an output with an error in the list. @RISK identifies this cell, by searching through the list of precedent cells for the output with the error, until values switch from error to a non-error value. The last precedent cell(s) returning error prior to precedent cells returning non-error values, is identified as the “error-causing” cell.

You can also review the formulas and values for cells that are precedents to the “error-causing” cell by expanding the error-causing cell in the list on the right. This allows you to examine values which feed into the error-causing formula. For example, a formula might return #VALUE because of a combination of values referenced by the formula. Looking at
precedents to the error-causing formula allows you to examine these referenced values.

- **Automatically Generate Reports at End of Simulation.** Selects Excel reports that will be generated automatically at the end of a simulation.

For more information on these available Excel reports, see the Excel Reports command.
The Sampling tab specifies how samples are drawn and saved during a simulation.

The Random Numbers settings include:

**Sampling Type**

- **Sampling Type.** Sets the type of sampling used during an @RISK simulation. Sampling types differ in how they draw samples from a distribution. Latin Hypercube sampling, the recommended type, more accurately recreates the probability distributions specified by distribution functions (or takes fewer iterations to achieve the same accuracy) than Monte Carlo sampling. The technical details on each sampling type are presented in the Technical Appendices.
  - **Latin Hypercube.** Selects stratified sampling.
  - **Monte Carlo.** Selects standard Monte Carlo sampling.
Generator

- **Generator.** Selects any of eight different random number generators for use when simulating:
  - MersenneTwister
  - MRG32k3a
  - MWC
  - KISS
  - LFIB4
  - SWB
  - KISS_SWB
  - RAN3I

Each of the available random number generators is described here:

1. **Mersenne Twister.** This is the default generator in @RISK. For more information on its characteristics, visit http://www.math.sci.hiroshima-u.ac.jp/~m-mat/MT/emt.html.

2. **MRG32k3a.** This is a robust generator from Pierre L’Ecuyer. For more information on its characteristics, visit http://www.iro.umontreal.ca/~lecuyer/myftp/papers/streams00s.pdf.

3. **MWC.** The MWC generator concatenates two 16-bit multiply-with-carry generators, \(x(n) = 36969x(n-1) + \text{carry}\), \(y(n) = 18000y(n-1) + \text{carry} \mod 2^{16}\), has period about \(2^{60}\) and seems to pass all tests of randomness. A favorite stand-alone generator, it is faster than KISS.

4. **KISS.** The KISS generator (Keep It Simple Stupid) is designed to combine the two multiply-with-carry generators in MWC with the 3-shift register SHR3 and the congruential generator CONG, using addition and exclusive-or. Period about \(2^{123}\).

5. **LFIB4.** LFIB4 is defined as a lagged Fibonacci generator: \(x(n) = x(n-r) \oplus x(n-s)\), with the \(x\) values in a finite set over which there is a binary operation \(\oplus\), such as +/- on integers mod \(2^{32}\), * on odd such integers, exclusive-or(xor) on binary vectors.
6. **SWB.** SWB is a subtract-with-borrow generator developed to give a simple method for producing extremely long periods:

\[ x(n) = x(n-222) - x(n-237) - \text{borrow mod } 2^{32} \]

The 'borrow' is 0, or set to 1 if computing \(x(n-1)\) caused overflow in 32-bit integer arithmetic. This generator has a very long period, \(2^{7098}(2^{480}-1)\), about \(2^{7578}\). It seems to pass all tests of randomness, except for the Birthday Spacings test, which it fails badly, as do all lagged Fibonacci generators using +=, -= or xor.

7. **KISS_SWB.** KISS+SWB has period \(>2^{7700}\) and is highly recommended. Subtract-with-borrow (SWB) has the same local behaviour as lagged Fibonacci using +=, -=, xor- -- the borrow merely provides a much longer period. SWB fails the birthday spacings test, as do all lagged Fibonacci and other generators that merely combine two previous values by means of +=, -= or xor. Those failures are for a particular case: m=512 birthdays in a year of n=2^24 days. There are choices of m and n for which lags >1000 will also fail the test. A reasonable precaution is to always combine a 2-lag Fibonacci or SWB generator with another kind of generator, unless the generator uses *, for which a very satisfactory sequence of odd 32-bit integers results.

8. **RAN3I.** This is the RNG used in @RISK 3 & 4. It is from Numerical Recipes, and is based on a portable “subtractive” random number generator of Knuth.

The MWC, KISS, LFIB4, SWB, and KISS+SWB generators are all from George Marsaglia at Florida State University. Visit http://www.lns.cornell.edu/spr/1999-01/msg0014148.html for more information.
Seed

- **Initial Seed.** The initial seed for the random number generator can be set to either:
  - **Choose Randomly.** @RISK will randomly pick a new seed for each simulation.
  - **Fixed.** @RISK will use the same seed each simulation. When you enter a fixed nonzero seed value for the random number generator, exactly the same sequence of random numbers will be repeated each time the simulation runs. The seed value must be an integer in the range 1 to 2147483647.
  - Setting a fixed seed value is useful when you want to control the simulation sampling environment. For example, you might want to simulate the same model twice, changing only the argument values for one distribution function. By setting a fixed seed, the same values will be sampled from all distribution functions, except the one you changed, during each iteration. Therefore, any differences in the results between the two runs will be due to the changed argument values of the single distribution function.

- **Multiple Simulations.** Specifies the seed used when @RISK performs multiple simulations. Options include:
  - **All Use Same Seed.** The same seed will be used for each simulation when @RISK performs multiple simulations in a single run. This allows you to isolate differences across simulations to changes introduced by RiskSimtable functions.
  - **Use Different Seed Values.** Different seeds will be used for each simulation in a multiple-simulation run.

If a **Fixed** seed is used and the **Multiple Simulations — Different Seed Values** option is selected, each simulation will use a different seed, but the same sequence of seed values will be used each time the entire multiple-simulation run is executed. This implies that the results will be reproducible from run to run.

**Note:** The Initial Seed set in the Sampling tab affects only the random numbers generated for input distributions that do not have an independent seed specified with the RiskSeed property function. Input distributions that use RiskSeed always have their own reproducible stream of random numbers.
Other settings on the Sampling tab include:

- **Collect Distribution Samples.** Specifies how @RISK collects random samples from input distribution functions during a simulation. The options include:
  
  - **All.** Specifies that samples will be collected for all input distribution functions.
  
  - **Inputs Marked with Collect.** Specifies that samples will be collected only for input distributions that include a RiskCollect property function as part of their distribution function. Then Sensitivity and Scenario analyses will include only these distributions and ignore the others.
  
  - **None.** Specifies that no samples will be collected during a simulation. If no samples are collected, Sensitivity and Scenario analyses will not be available as simulation results. In addition, statistics will not be provided on samples drawn for input distribution functions. Deselecting sample collection, however, does allow simulations to run faster, and will sometimes allow large simulations with many outputs to be run on memory-constrained systems.

  **Note.** When simulating project schedules using Microsoft Project, Distribution Samples must be collected.

- **Smart Sensitivity Analysis.** Enables or disables Smart Sensitivity Analysis. For more information on Smart Sensitivity Analysis and situations where you might want to disable it, see the Sensitivities Command in this manual.

- **Update Statistic Functions.** Specifies when @RISK’s statistics functions (such as RiskMean, RiskStdDev, etc.) will be updated during a simulation. In most cases, statistics do not need to be updated until the end of a simulation, when you wish to view final simulation statistics in Excel. However, if the calculations in your model require a new statistic to be returned each iteration (for example, when a custom convergence calculation has been entered using Excel formulas), the Each Iteration setting should be used.
The Macros tab allows you to specify an Excel macro to be executed before, during, or after a simulation.

The Run an Excel Macro options allows Excel macros to be executed during an @RISK simulation. The options include:

- **Before Each Simulation.** The macro runs before each simulation begins.

- **Before Each Iteration’s Recalc.** The macro runs before @RISK has placed new sampled values into the spreadsheet model, and before Excel is recalculated with these values.

- **After Each Iteration’s Recalc.** The macro runs after @RISK performs its sampling and worksheet recalculation, but before @RISK stores values for outputs. This means that the macro can update values in @RISK output cells, and @RISK reports and calculations will use these values and not the results of the Excel recalculation.

- **After Each Simulation.** The macro runs after each simulation ends.
The If Excel Recalculations Occur During Macros, Distributions Return options allows you to control how @RISK distribution functions handle Excel recalculations that occur inside macros. The options are:

You can specify that functions return Different Samples Each Recalculation that occurs during your macro, or you can have functions return Fixed Samples, that is, the sample that was drawn by @RISK in the iteration when the macro is running.

Note that when Fixed Samples is selected, @RISK will return a different sample each macro recalculation in the case where the macro changes the argument values for an @RISK distribution function. For example, if your worksheet has a distribution function RiskNormal(A1,10) and the macro changes the value of cell A1, @RISK will return a new sample for RiskNormal(A1,10).

Macros can be run at any, or all, of the possible times during a simulation. This feature allows calculations that can be performed only through the use of a macro to be performed during a simulation. Examples of such macro-performed calculations are optimizations, iterative “looping” calculations, and calculations that require new data from external sources. In addition, a macro can include @RISK distribution functions that are sampled during the execution of the macro.

The Macro Name specified should be fully qualified, that is, it should contain the full address (including filename) of the macro to be run.

There are no restrictions on the operations performed by the macro each iteration. However, you should avoid macros that perform operations such as closing the worksheet being simulated, quitting Excel, and so on.

@RISK includes an object oriented programming interface (API) that allows custom applications to be built using @RISK. This programming interface is described in the @RISK XDK Reference manual, accessed from the @RISK Help menu.
The Convergence tab lets you define settings for monitoring convergence of simulation results.

![Simulation Settings Dialog - Convergence Tab](image)

The Convergence tab settings specify how @RISK monitors convergence during a simulation. Convergence monitoring shows how statistics on output distributions change as additional iterations are run during the simulation.

As more iterations are executed, the output distributions generated become more stable in the sense that their summary statistics change less. The number of iterations required to generate stable output distributions varies, depending on the model being simulated and the distribution functions in the model.

By monitoring convergence, you can ensure that you have run a sufficient, but not excessive, number of iterations. This is especially important with complex models that take a long time to recalculate.

Convergence monitoring does add to simulation runtime. If the fastest simulation is desired for a preset number of iterations, you should turn convergence monitoring off to maximize speed.

Convergence testing in @RISK can also be controlled for individual outputs using the `RiskConvergence` property function. However, convergence testing performed with any RiskConvergence functions in your worksheet is independent of the convergence testing specified on the Convergence tab. The `RiskConvergenceLevel` function returns
the convergence level of the output cell it references. In addition, a simulation will be halted when any RiskStopRun function has passed an argument value of TRUE, regardless of the status of the convergence testing specified on the Convergence tab.

Default Convergence Options include:

- **Convergence Tolerance.** Specifies the tolerance allowed for the statistic you are testing. For example, the settings in the previous screenshot specify that you want to estimate the mean of each output simulated to within 3% of its actual value.

- **Confidence Level.** Specifies the confidence level for your estimate. For example, the settings in the previous screenshot specify that you want your estimate of the mean of each output simulated (within the specified tolerance) to be accurate 95% of the time.

- **Perform Tests on Simulated.** Specifies the statistic of each output that will be tested: mean, standard deviation, or selected percentile.

If the Number of Iterations is set to Auto, @RISK will automatically stop a simulation when convergence is achieved for all simulation outputs.

The Results Summary window reports convergence status when a simulation is running and convergence monitoring is enabled. The first column in the window displays the status for each output (as a value from 1 to 99) and displays OK when an output has converged.
Start Simulation Command

Starts a simulation

The **Start Simulation** command (in the **Simulation** group) runs a simulation using the current settings.

A Progress window is displayed during the simulation. The icons at the bottom (from left to right) are: Show Results Summary Window, Show Excel Recalculations, Update Graphs/Reports Real Time, Run, Pause, Stop, and Show Multi CPU Monitor.

![Start Simulation Icon](image1.png)

The Update Display option can also be toggled on or off by pressing the `<Num Lock>` key during the simulation.

Clicking the arrow button at the bottom right of the Progress window displays the **Performance Monitor**. This monitor shows additional information on the status of each CPU used during a run.

![Performance Monitor](image2.png)

Simulation messages are also available. These messages provide recommendations for increasing the speed of lengthy simulations.
All open @RISK windows will update during a simulation if the Simulation setting `Update Windows During Simulation Every XXX Seconds` is on. This is especially useful for updating the Results Summary window. The small thumbnail graphs in this window will update to show a “dashboard” summary of simulation progress.
Results Group Commands

This group provides a variety of reports for the results of a simulation. Note that most of the icons in this group are disabled until a simulation has been run.
Excel Reports Command

Selects the reports on simulation results to generate in Excel

The Excel Reports command (in the Results group) selects reports to be generated on the simulation results or the model definition.

A variety of different pre-built simulation reports are available directly in Excel at the end of a simulation. The Quick Reports option provides a report on simulation results designed for printing. This report usually contains a single worksheet for each output in a simulation. The Custom Reports option is similar, but it enables the Custom Report Settings tab, where you can select exactly the reports you want. The other available reports, starting with Input Results Summary, contain the same information as the equivalent report in the Results Summary window or other @RISK report windows.
The location of your reports is set using the Place Report In option. Two selections are available for locating reports in Excel:

- **New Workbook.** Places simulation reports in a new workbook each time reports are generated.
- **Active Workbook.** Places simulation reports in new sheets in the active workbook each time reports are generated.

**Custom Reports**

You can check Custom Reports and then click the Custom Report Settings tab to request exactly the reports you want.

The **Custom Items** section lists the @RISK items (such as graphs and statistics) you want to include on each simulation output in your report. You can use Add, Delete, Move Up, and Move Down to change or rearrange these items. You can use Edit to change the definition of an item. For example, you might want to include a Tornado Graph by changing the Report Item to Sensitivity Graph and selecting the desired Sensitivity calculation method to be used for the Tornado graph.
The **Preview** button provides a quick look at your custom report.

Custom reports can span multiple pages. In this case, the Preview will display a preview of each page in the report.
**Custom Report Outputs**

You have the option to include only specific outputs when generating a custom report. This is useful when you have many simulation outputs in your model, but you want to report only on a few key outputs.

In addition, you can use the **Multiple Simulation Reports** option to generate separate reports for each simulation for a given output, instead of combining the results from all simulations into a single report.

**Template Sheets**

You can use template sheets to create your own custom reports. Simulation statistics and graphs are placed in a template by using @RISK statistics functions, such as `RiskMean`, or the graph function `RiskResultsGraph`. When a statistics function or graph function is located in a template sheet, the desired statistics and graphs are then generated at the end of a simulation in a copy of the template sheet when you choose the **Template Sheets** option in the Excel Reports dialog. The original template sheet with the @RISK functions remains intact for use in generating reports from your next simulation.

Template sheets are standard Excel sheets. They are identified to @RISK by having a name that starts with `RiskTemplate_`. These sheets can also contain any standard Excel formulas so that custom calculations can be performed using simulation results. The example file `RiskTemplate.xlsx` shown here contains a template sheet. You can open this file in Excel and review this sheet to learn how to set up your own custom reports with template sheets.
**Browse Results Command**

**Turns on Browse Results mode so that a graph of simulation results is displayed when a cell is selected in Excel**

The *Browse Results* command (in the *Results* group) turns on Browse Results mode. While in this mode, you can see a graph of simulation results in Excel simply by selecting the cell of interest in your spreadsheet model. Alternatively, you can press `<Tab>` to move the graph through all output cells in your model.

In Browse Results mode, @RISK displays graphs of simulation results as you click (or tab) to cells in your spreadsheet:

- If the selected cell is a simulation output cell, or it is an input cell with a distribution function, @RISK will display a graph of its simulated distribution.

- If the selected cell is part of a correlation matrix, @RISK will display a scatter plot matrix of the simulated correlations between the inputs in the matrix.

If the Simulation Setting, *Automatic Results Display — Show Output Graph* is selected, Browse mode will be active at the end of a run.

To exit Browse Results mode, simply close the graph window or click the Browse Results icon on the @RISK ribbon.
Summary Command

Displays all simulation results, including statistics and small thumbnail graphs

Results group) displays the Results Summary window. This window provides a concise summary of the results of your model. It displays thumbnail graphs and summary statistics for your simulated output cells and input distributions. As with the Model window, you can:

- Drag off any thumbnail graph to expand it to a full-sized window.
- Double-click any entry in the table to use the Graph Navigator to move through input or output cells in your model.
- Customize the table by selecting the statistics (columns) you want to display.

Note: If a name of an input or an output is shown in red in the Results Summary window, the referenced cell for the simulated result cannot be found. This can happen if you open simulation results and do not have a workbook that was used in the simulation open, or if you have deleted the cell in your workbook after the simulation was run. In this case you will still be able to drag a graph of the result off the Results Summary window, but you will not be able to browse to the cell and pop up a graph.
The Results Summary window is linked to your Excel model. When you select a simulated output or input in the table, the cells where the result and its name are located are highlighted in Excel. If you double-click a thumbnail graph in the table, the graph of the simulated input or output is displayed in Excel, with a pointer to the cell where it is located.
The commands for the Results Summary window can be accessed by clicking the icons displayed at the bottom of the table or by right-clicking and selecting from a pop-up menu. Commands will then be performed on the current selected rows in the table.
Drag and Drop Graphs

Many graphs can be created in @RISK simply by dragging thumbnails off the Results Summary window. In addition, overlays can be added to a graph by dragging one graph (or thumbnail) onto another.
Multiple graphs can be created in one step by selecting multiple rows in the Results Summary window and clicking Graph icon at the bottom of the window.

As you make edits to a graph in one of these full-sized windows, the thumbnail graph in the Results Summary window will update to store your edits. This allows you to close an open graph window without losing your edits. It is also possible to open multiple graph windows from the thumbnail of single output or input and then edit any of these graphs. In this case, only the edits for the most recently changed graph are stored.
The columns displayed in the Results Summary window can be customized. Clicking the Columns icon at the bottom of the window displays the following dialog.

![Select Columns for Table: Results Summary](image)

Note that the only two percentile options are 5% and 95%. However, you can change either of these by clicking the corresponding ellipsis button. The list also contains editable $p_1,x_1$ and $p_2,x_2$ columns. You can use these columns to enter specific target values and/or target probabilities directly in the table. You can also select **Fill Down** from the **Edit** menu to quickly copy $p$ or $x$ values across multiple outputs or inputs.

**Note:** Separate column selections can be made for the Model and Results Summary windows.

When convergence monitoring is turned on through the Simulation Settings dialog, a **Status** column is automatically added as the first column in the Results Summary window. It displays the convergence level for each output.
The Graph menu is accessed by clicking the **Graph** icon at the bottom of the Results Summary window or by right-clicking in the table. Selected commands will be performed on the selected rows in the table. This allows you to quickly make graphs of multiple simulation results from your model. The **Automatic** option creates graphs using the default type (relative frequency) for distributions of simulation results.
The Results Summary window can be copied to the clipboard, or exported to Excel, using the commands on the Edit and Export menu. In addition, where appropriate, values in the table can be filled down or copied and pasted. For example, this allows you to quickly copy P1 and X1 values.

Commands on the Edit and Export menu include:

- **Report in Excel.** Exports the table to a new worksheet in Excel.
- **Copy Selection.**Copies the current selection in the table to the clipboard.
- **Copy Grid.**Copies the entire grid (text only, no thumbnail graphs) to the clipboard.
- **Paste, Fill Down.** Pastes or fills values into the current selection in the table.
Define Filters Command

Filters values from simulation statistics calculations and graphs

The Define Filters command (in the Results group) lets you enter filters for each input or output cell in your model. Filters allow you to remove unwanted values from the statistics calculations and graphs generated by @RISK. You enter filters by clicking the Define Filters icon on the ribbon, or by clicking the Filter icon on the graph of a simulation result or in the Data window.

A filter can be defined for input or output listed in the Name column of the Filter Settings table. To define a filter, you must enter a Type, a Values Are type (Percentiles or Values), and Minimum and Maximum allowed values. If the Minimum or Maximum entry is left blank, the Filter range will be unbounded on one end.
Icons and options in the Filters dialog include:

- **Show Only Outputs or Inputs With Filters** (middle icon at the bottom). Displays only those inputs or outputs for which filters have been entered.

- **Same Filter For All Simulations** (right icon at the bottom, visible only if the number of simulations is greater than 1). Copies the first filter entered, for an input or output, to the results for the same input or output in all other simulations.

- **Apply**. Filters are applied as soon as you click this button.

- **Clear Filters**. Click this button and then the Apply button to remove the filters from the currently selected rows in the table. To disable a filter but keep the entered filter range, set the Filter Type to Off.

The available Filter Types are:

- **Standard Filter**. This type of filter is applied only to the input or output for which the filter was entered. Values below the specified minimum or above the specified maximum are removed from the statistics, sensitivity, and scenario calculations for the result, and they are not included in graphs for the simulation result.

- **Iteration Filter**. This type of filter affects all simulation results. @RISK first applies the filter to the input or output for which the filter was entered. Values below the specified minimum or above the specified maximum are removed from the statistics, sensitivity, and scenario calculations for the result, and they are not included in graphs for the simulation result. The iterations where the conditions of this filter for the input or output are satisfied are then “marked” and all other inputs and outputs are filtered to include only values generated in these iterations. This type of filter is especially useful when you want to review simulation results (for all outputs and inputs) for only those iterations that satisfy a specific filter condition, such as where NPV is positive.
When you click the Filter icon at the bottom of a graph window, a quick filter dialog is displayed that allows you to set a filter for just the input or output displayed in the graph.

When filtering from a graph window, you set the type of filter, the type of values to be entered, and the minimum-maximum range, and click **Apply**. The graph is redisplayed (with new statistics) and the number of values used (not filtered out) is shown at the bottom of the graph. As with any filter, values outside the minimum-maximum range are removed from the statistics, sensitivity, and scenario calculations for the result, and they are not included in graphs for the simulation result.

Then if you want to see the full Filter dialog that lists all active Filters, you can click the **Show All** button.
Simulation Detailed Statistics Command

Displays the Detailed Statistics window

The **Simulation Detailed Statistics** command (in the **Results** group) displays detailed statistics on simulation results for all inputs and outputs. In addition, percentile values (in increments of 5%) are listed, along with filter information and up to 10 target values and/or probabilities.

The Detailed Statistics window can be pivoted so that it displays statistics in columns, and outputs and inputs in rows. To pivot the table, click the **Pivot Table of Statistics** icon (third from left) at the bottom of the window.
Targets in @RISK may be calculated for any input or output distribution. These targets identify the probability of achieving a value, or the value associated with any probability level. Either values or probabilities can be entered into the target entry area toward the bottom of the Detailed Statistics window.

If a value is entered, @RISK calculates the probability of a value occurring that is less than or equal to the entered value. (If the @RISK Defaults menu Display Cumulative Descending Percentiles option is selected, the reported target probability will be the probability of exceeding the entered target value.)

If a probability is entered, @RISK calculates the value in the distribution whose cumulative probability equals the entered probability.

Default target percentiles shown in the Detailed Statistics window can be set using the Application Settings command from the Utilities menu.

Once a target value or probability has been entered, you can be copy it across a range of simulation results by dragging across. An example of this is shown in the previous screenshot, with the 600000 target entered for each of three output cells. To copy targets:

1) Enter the desired target value or probability in a single cell in a target row.

2) Highlight a range of cells across the row, adjacent to the entered value, by dragging the mouse across the range.

3) Right-click, and select Fill Right from the Edit menu.
The Detailed Statistics window, like other @RISK reports windows, can be exported to an Excel worksheet. Click the Edit and Export icon at the bottom of the window and select Report in Excel to export the window.
Simulation Data Command
Displays the Data window

The Simulation Data command (in the Results group) displays all of the input and output data values generated during the simulation. This allows you to analyze the data in any way you like.

Each row contains all of the data from a single iteration. By moving across any row, you can see the exact combination of input values that led to the output values in that iteration.

You can use the middle Data Sort button at the bottom of the window to sort the data in a number of ways. For example, you could sort to show the values from an important output in decreasing order, or you could sort to show those iterations where an error occurred. Optionally, you can hide filtered values or errors.
The Data Sort dialog controls how the Data window will be sorted.

**Data Sort Dialog**

- **Iteration Number**. Lets you show All Iterations, Iterations Where an Error Occurred, or Iterations Remaining After Iteration Filters are Applied. For more information on Iteration Filters, see the Filters command in this chapter. The option Iterations Where an Error Occurred is useful for debugging a model. In this case, you would first filter to show the iterations with errors. Then you could use the Iteration Step command (discussed next) to set Excel to the values calculated for those iterations. This process, plus a careful examination of your model, should help you determine the conditions that led to the error.

- **Specific Result**. Each column in the Data window can be sorted individually. Use this option to show the highest, or lowest, values for a result. Selecting Hide Filtered Values For this Result or Hide Error Values for this Result hides iterations where the selected result has a filtered value or an error.
You can “step through” the iterations displayed in the Data window. This updates Excel with the values that were sampled and calculated during the simulation. This is useful for investigating iterations with errors or iterations that led to certain output scenarios.

To step through iterations:

1) Click the **Iteration Step** icon (the shoe icon) at the bottom of the Data window.

2) Select the row of interest in the Data window. The sampled values of all inputs in that iteration are placed in Excel and the workbook is recalculated.

3) Select a particular value in that row, and the corresponding output or input cell in Excel will be selected.

*Note: If your workbook in Excel has been changed since the simulation was run, the Iteration values that were calculated in the simulation might no longer match those calculated during the Iteration Step. When this happens, the error is reported in the Title Bar of the Data window.*
Simulation Sensitivities Command

Displays the Sensitivity Analysis window

The **Simulation Sensitivities** command (in the **Results** group) displays sensitivity analysis results for output cells. These results show the sensitivity of each output variable to the input variables.

The sensitivity analysis uses either a change in output statistic analysis, a regression analysis, or a rank-order correlation analysis. The input distributions in the model are ranked by their impact on the output selected from the dropdown list at the upper left of the window. The method for ranking is selected from the dropdown list to its right. Then you can click the **Tornado Graph** icon to display a tornado graph for the values in any selected column.

**Note:** Clicking a column header ranks the inputs for the output in the selected column.

By default, @RISK uses a **Smart Sensitivity Analysis**. To do this, it pre-screens inputs based on their precedence in formulas for outputs. Inputs that have no link (via the model’s formulas) to an output cell are removed from the sensitivity analysis, thus avoiding spurious results. In the Sensitivity Analysis window, these unrelated inputs are indicated by `n/a`.

Smart Sensitivity Analysis is performed because it is possible for simulation data to show a correlation between an input and an output when, in reality, the input has no effect on the output in the model. Without Smart Sensitivity Analysis, tornado graph bars could be displayed for those unrelated inputs.
There are some situations where you should disable Smart Sensitivity Analysis (on the **Sampling tab** of the **Simulation Settings** dialog) to improve performance and sensitivity analysis results. Here are two such situations:

1) The Smart Sensitivity Analysis setup time for scanning precedents adds significant runtime for a very large model. This increased runtime might outweigh your desire to avoid spurious correlations.

2) You use a macro or DLL that performs calculations involving @RISK input values in cells that have no relationship (via workbook formulas) with the output. This macro or DLL then returns a result to a cell that is used in calculating the output values. In this case, Smart Sensitivity Analysis should be disabled. To avoid situations like this, we recommend that you create macro functions (UDFs) that explicitly reference all used input cells in their argument lists.

**Change in Output Statistic Analysis**

In the change in output statistic analysis, samples for an input are grouped in a set of equal-sized bins (or “scenarios”), ranging from the input’s lowest value to its highest. A value for a statistic of the output (such as its mean) is calculated for the output values in the iterations associated with each bin. Inputs are ranked by the amount of +/- swing they caused for the output statistic.

This analysis is similar to @RISK’s **Scenario Analysis**, except that scenarios for each input are used instead of scenarios for an output. In the change in output statistic analysis:

1) @RISK sorts all iterations in ascending order of that input.

2) It divides the sorted iterations into 10 bins. For example, with 2500 iterations, the first bin would contain the 250 iterations with the 250 lowest values of this input; the second bin would contain the 250 iterations with the next lowest values of this input; and so on.

3) It computes a selected statistic (usually the mean, but not necessarily) of the output values in each bin.

4) In the tornado chart, the lowest of the ten output statistics is the number at the left edge of this input’s bar, and the highest of the ten output statistics is the number at the right edge of the bar.
The key here is that the inputs are first sorted in ascending order and binned in that order, and then an output statistics is computed for just the iterations in each bin.

In the Sensitivity Analysis window, inputs are ranked by the range between their highest statistic value for any bin and their lowest statistic value, that is, by tornado graph bar length.

You can control the output statistic for this analysis and the number of bins used for each input. You do this through the **Change in Output Statistic** settings dialog.

The statistic used for an output can be its mean, mode, or a percentile value. The number of iterations in each bin is determined by the total number of iterations divided by the number of bins. For example, if you run 1000 iterations and have 10 bins, the output statistic for each bin will be calculated from 100 values. If you run too few iterations, you will have a limited number of values in each bin and your results will not be stable.

Two other methods, **Regression** and **Rank Order Correlation**, can also used as the basis for a sensitivity analysis. They are explained in the **Tornado Graphs** section of this manual. However, the **Change in Output Statistic** method is arguably the preferred method, primarily for ease of interpretation.
Sensitivity Analysis results can also be displayed in a scatter plot matrix. In this matrix, ranked sensitivity analysis results are displayed with scatter plots. To show the scatter plot matrix, click the Scatter Plot icon in the lower left of the Sensitivity Analysis window.

You can drag any thumbnail scatter plot to a full-sized graph window. In addition, you can create overlays of scatter plots by dragging additional thumbnail scatter plots from the matrix onto a full-sized scatter plot window.
Simulation Scenarios Command

Displays the Output Scenarios Window

The Simulation Scenarios command (in the Results group) displays scenario analysis results for output cells. Up to three scenarios can be entered for each output variable. Scenarios are shown in the top row of the Output Scenario window (and in the Scenarios section of the Detailed Statistics window). Targets are preceded by an inequality operator and can be specified in terms of percentiles or actual values.

Scenario analysis allows you to determine which input variables contribute significantly toward reaching a goal. For example, it can show which variables contribute to exceptionally high sales values, or which variables contribute to negative NPV values.

@RISK allows you to define target scenarios for each output. You might be interested in the highest quartile of values in the Total Sales output, or the value less than 1 million in the Net Profit output. You enter these target conditions directly in the top row of the Output Scenarios window.

When you display the Output Scenarios window, @RISK examines the data created by your simulation. For each output, it performs the following steps:

1) For each input distribution, it calculates the median and standard deviation of the samples.

2) It creates a subset containing only the iterations in which the output achieves a defined target.

3) The median of each input is calculated for this subset.

What is Scenario Analysis?
4) For each input, the difference between the simulation median (found in step 1) and the subset median (found in step 3) is calculated and compared to the standard deviation of the input data (found in step 1). If the absolute value of the difference in medians is greater than half a standard deviation, the input is termed significant; otherwise the input is ignored in the scenario analysis.

5) Each significant input found in step 4 is listed in the scenario report.

Interpreting the Results

For example, @RISK might report that the Retail Price input is significant when the target is the highest quartile of Total Sales. Then you know that when Total Sales are high, the median Retail Price is significantly different from the median Retail Price for the entire simulation.

@RISK calculates three statistics for each significant input in a scenario:

- **Actual Median of Samples in Iterations Meeting Target.** This is the median of the subset for the selected input (the one calculated in step 3 above). You can compare this with the median of the selected output for the entire simulation (the 50% percentile reported in the statistics report).

- **Percentile Median of Samples in Iterations Meeting Target.** This is the percentile value for the subset median in the distribution generated for the entire simulation. If this value is less than 50%, the subset median is smaller than the median for the entire simulation; if it is greater than 50%, the subset median is greater than the median for the entire simulation.

- For example, when the target is high Total Sales, you might find that the subset median for Retail Price is lower than the median Retail Price for the entire simulation. This would indicate that a lower Retail Price can help you reach the goal of high Total Sales.
• **Ratio Shown Median to Original Standard Deviation.** This is the difference between the subset median and the median for the entire simulation, divided by the standard deviation of the input for the entire simulation. The larger the magnitude of this ratio, the more significant the variable is in reaching the defined target. For example, suppose Number of Salespeople is significant toward reaching the target of high Total Sales, but its ratio of median to standard deviation is only half the magnitude of the ratio for the Retail Price input. Then you could conclude that, although Number of Salespeople affects your goal of high Total Sales, Retail Price is more significant and might require more attention.

**Caution:** The greatest danger in using scenario analysis is that the results can be deceiving if the subset contains a small number of data points. For example, in a simulation of 100 iterations and a scenario target of “>90%”, the subset will contain only 10 data points.
The default scenarios can be changed by clicking the Edit Scenarios icon (in the Output Scenarios window or in a graph window) or by double-clicking a scenario displayed in the first row of the Output Scenarios window.

Up to three scenarios can be entered for each simulation output. Each scenario can have one or two bounds. If you enter two bounds, you will be specifying a scenario that has a min-max range for the output, such as from 90% to 99%. If you don’t want to specify a second bound, you can leave it blank. Each bound can be specified as a percentile or an actual value, such as >1000000.

Note: Default scenarios settings can be entered using the Application Settings command on the Utilities menu.
You can click the **Scatter Plot** icon in the Output Scenarios window to generate matrix of scatter plots. Each scatter plot shows:

1. The output values versus the input values for the entire simulation (blue).

2. An overlay of the output values versus the input values for the subset of iterations where the output target is achieved (red).

*Note: You can overlay the same input and output, under different scenarios, only in a scatter plot that displays scenario analysis results.*
You can also click the **Tornado** icon at the bottom of the Output Scenarios window to display a tornado graph. This graph shows the key inputs affecting the output when the output achieves a particular target scenario.
Advanced Analyses Commands

@RISK Professional and Industrial versions allow you to perform advanced analyses on your model. The advanced analyses include Goal Seek, Stress Analysis, and Advanced Sensitivity Analysis. These advanced analyses can be used to design your model, verify your model, or obtain many “what-if” types of results.

Each of the advanced analyses generates its own sets of reports in Excel to show the results of the analysis being run. However, each of the analyses uses standard @RISK multiple simulations to generate its results. Because of this, the Results Summary window can also be used to review advanced analysis results. This is useful when you want to generate a graph of results not included in the Excel reports, or when you want to review advanced analysis data in more detail.

The simulation settings specified in the Simulation Settings dialog (except for # of Simulations) are those used in each of the advanced analyses. Because advanced analyses can involve large numbers of simulations, you should review your simulation settings to insure that advanced analysis runtimes are reasonable. For example, when testing the setup of an advanced analysis, you should set the number of iterations to a relatively low value. Then you can set the number of iterations back to a value necessary for stable simulation results and run the full advanced analysis.
**Goal Seek Command**

**Sets up and runs a Goal Seek analysis**

The Goal Seek command (from the Advanced Analyses menu in the Tools group) allows you to achieve the value for a simulated statistic of an output by adjusting the value of another cell. This statistic could be the mean, the standard deviation, a percentile, or others. The setup of an @RISK Goal Seek is very similar to Excel’s Goal Seek; they share the same basic goal. Unlike Excel’s Goal Seek, however, @RISK’s Goal Seek utilizes multiple simulations to find the adjustable cell value that achieves your specified target.

When you know the desired statistic value for an output, but not the input value needed to obtain that value, you can use the Goal Seek feature. An input can be any cell in your Excel workbook. An output is any cell that contains a RiskOutput function. The input should be a precedent of the output cell being targeted. Then @RISK varies the value in the input cell and runs a full simulation. This process is repeated until the target statistic for the output equals (or actually, is very close to) the value you want.

When you click the Goal Seek icon, the Goal Seek dialog opens, where you can specify the Goal Seek problem you are trying to solve.

![Goal Seek Dialog](image)
The following options are available in the Goal Seek dialog:

- **Cell.** A cell reference to the output cell. If the cell does not contain a RiskOutput function, you will be prompted to add one. You can click the ellipsis button next to the Cell Entry icon to display a list of current @RISK output cells and then choose one from this list:

![@RISK Outputs](image)

- **Statistic.** The output statistic to monitor for convergence to the target value. The list includes: Minimum, Maximum, Kurtosis, Mean, Mode, Median, 5th Percentile, 95th Percentile, Skewness, Standard Deviation, and Variance.

- **Value.** The value you want the output statistic to achieve.

- **By Changing Cell.** Identifies the cell Goal Seek should change to achieve the target. The output cell must be dependent on this changing cell (through precedence in Excel formulas). If it is not, Goal Seek will not be able to find a solution.
If you click the Options button, you see the Goal Seek Options dialog. This allows you to set parameters that can affect the success and quality of the Goal Seek solution.

The options include:

- **Minimum, Maximum**. Allows you to set the minimum and maximum values to use for the changing cell. Goal Seek will then try to find a solution within these limits.

- **Comparison Accuracy**. Determines how close the actual solution must be to the target. This entry can be viewed as a range around the target value that is acceptable for the output statistic. Any result within this range is accepted as achieving the goal.
  - **Percent of Target Value**. Specifies the accuracy as a percentage of the target value.
  - **+/− Actual Value**. Specifies the accuracy as the maximum difference between the goal and the output statistic found by Goal Seek.

- **Maximum Number of Simulations**. Specifies the number of simulations Goal Seek will attempt while trying to meet the goal. If a solution is found before all simulations are completed, the procedure will stop and the Goal Seek Status dialog will be displayed.
• **Generate Complete Simulation Results for Solution.** When this option is selected, Goal Seek will perform an additional simulation, using the found value for the changing cell, after a solution has been found. The statistics for that simulation are displayed in the Results Summary window. This option does *not* replace the original value of the changing cell with the found value in the spreadsheet. Instead, it enables you to see the effect that such a replacement would have without performing it.

When you click the **Analyze** button on the Goal Seek dialog, Goal Seek cycles through the following process until the target statistic value is achieved, the maximum number of simulations are run, or:

1) A new value is placed in the changing cell.
2) A full simulation of all open workbooks is run using the current settings, as specified in the @RISK Simulation Settings dialog.

If a solution is found within the requested accuracy, Goal Seek will display a Status dialog. This allows you to replace the contents of the changing cell with the solution value. If you choose to do this, the entire cell contents will be replaced by the solution value, and any formula or values that were previously in that cell will be lost.

![@RISK Goal Seek Status](image)

It is possible that Goal Seek will not be able to converge to within the requested accuracy. In this case Goal Seek will prompt you with its best solution.
The @RISK Goal Seek procedure uses a two-tiered approach:

1) If no minimum and maximum brackets are set, Goal Seek will attempt to bracket the target value using a geometric expansion around the original value.

2) Once a solution is bracketed, Goal Seek uses Ridders' method of root finding. Goal Seek first simulates the model with the input value set to the midpoint of the bracketed range. It then factors out that unique exponential function, which turns the residual function, into a straight line. This has some important benefits for the Goal Seek procedure. It insures that the tested input values never jump outside the brackets, and it helps insure that Goal Seek moves toward a solution in as few cycles as possible (an important benefit when each “cycle” is a full simulation of your model).

It is possible that Goal Seek will not converge to a solution. Some desired solutions may just be impossible to achieve, or the model may behave so unpredictably that the root finding algorithm cannot converge to a solution. You can help Goal Seek converge by:

- **Starting Goal Seek with a Different Value in the Changing Cell.** Because the iteration process begins with guesses around the original changing cell value, starting Goal Seek with a different value in this cell might help.

- **Changing Your Brackets.** Trying different Minimum and Maximum in the Options dialog might help point Goal Seek toward a solution.

**Note:** Goal Seek is not designed to work with multiple simulation models. If a RiskSimtable function is present, its first argument will be used for all simulations.
Stress Analysis Command

Sets up and runs a Stress Analysis

The Stress Analysis command (from the Advanced Analysis menu in the Tools group) allows you to analyze the effects of stressing @RISK distributions. “Stressing” a distribution restricts samples drawn from the distribution to values between a specified pair of percentiles. Alternatively, stressing can be done by specifying a new “stress” distribution that will be sampled instead of the original distribution in your model. With Stress Analysis you can select a number of @RISK distributions. Then you can run simulations while stressing these distributions jointly in one simulation, or separately in multiple simulations. By stressing the selected distributions, you can analyze scenarios without changing your model.

After completing a simulation, Stress Analysis provides a collection of reports and graphs useful for analyzing the effects of stressing certain distributions on a selected model output.

When you click the Stress Analysis icon, you see the Stress Analysis dialog. This dialog lets you specify the output and input(s) for a stress analysis.
The options in the Stress Analysis dialog are as follows:

- **Cell To Monitor.** This is a single output cell you want to monitor as the specified input distributions are stressed. This output cell can be specified by entering a cell reference, clicking the desired output cell, or clicking the ellipsis button. The latter displays a list of all @RISK outputs in your model, and you can choose an output from the list.

The **Inputs** section of the Stress Analysis dialog allows you to add, edit, or delete the input distributions to be stressed. The specified distributions are maintained in a list that you can edit.

- **Add, Edit.** These display the **Input Definition** dialog. This allows you to specify an input distribution, or a range of input distributions, to be stressed. You can then select from **Low, High, or Custom** sampling ranges, or specify an alternate stress distribution or formula.

- **Delete.** This completely removes the highlighted @RISK distribution(s) in the list from the Stress Analysis. To temporarily exclude a distribution, or a range of distributions, from the analysis without deleting them, you can remove the corresponding checkmark(s).
The Input Definition dialog is used to indicate how a specific input will be stressed.

The options in the Input Definition dialog are as follows:

- **Type.** For Stress Analysis, only @RISK distributions can be selected as inputs, so the only option for Type is **Distributions**.

- **Reference.** Selects the distributions to stress. Distributions can be specified by typing appropriate cell references, selecting a range of cells in the worksheet, or by clicking the ellipsis button. The latter opens the Distribution Functions dialog, listing all the distributions in the model.

The Variation Method options allow you to enter a range to sample from, or enter an alternate distribution, or formula, to substitute for the selected probability distribution(s) during the analysis.

- **Stress Low Values.** Specifies a low range to sample from, bounded at the low end by the distribution minimum. The default is from 0% to 5%, that is, only values below the 5th percentile of the distribution. Any upper percentile can be entered instead of 5%.

- **Stress High Values.** Specifies a high range to sample from, bounded at the high end by the distribution maximum. The default is from 95% to 100%, that is, only values above the 95th percentile. Any desired lower percentile can be entered instead of 95%.
- **Stress Custom Range of Values.** Allows you to specify any percentile range to sample from.

- **Substitute Function or Distribution.** Allows you to enter an alternate @RISK distribution function (or any valid Excel formula) that will be substituted for the selected distribution during a Stress Analysis. By clicking the icon to the right of the Distribution/Formula box (which appears when you choose this option), you can use Excel’s Function Wizard to help enter an alternate distribution,
If you click the Options button in the Stress Analysis dialog, you see the Stress Options dialog. This allows you to specify how stressing will be performed and which reports or graphs to generate.

The **Multiple Inputs** section allows you to stress all of your specified @RISK distributions during one simulation, or to run a separate simulation for each @RISK distribution.

- **Stress Each Input in its Own Simulation.** Specifies that a full simulation will be run for each stress range entered. The only change made to the model during each simulation will be the stressing of a single input. The number of simulations run will equal the number of stress ranges entered.

- **Stress All Inputs in a Single Simulation.** Specifies that a single simulation will be run using all stress ranges entered. The simulation results will combine the effects of all stress ranges.

The **Reports** section allows you to choose which reports and graphs you want to generate at the end of the stress simulations. The options include a Summary report, Box-Whisker Plots, Comparison Graphs, Histograms, Cumulative Distribution Functions and Quick Reports. For more information on the reports generated by a Stress Analysis, see **Reports** in this section.
The **Place Reports in** options indicate where your reports will be placed.

- **New Workbook.** All reports will be placed in a new workbook.
- **Active Workbook.** All reports will be placed in the active workbook with your model.

**Running Stress Analysis**

Once you have selected the cell to monitor and at least one input distribution to be stressed, you can click the **Analyze** button in the Stress Analysis dialog to run the analysis. The analysis runs one or more simulation that restrict sampling of the selected input distributions to the specified stress range(s), or it substitutes in any alternate stress distributions or formulas you have entered. The results of the Stress Analysis simulations are reported in a summary sheet and several graphs.

The results of the Stress Analysis are also available in the Results Summary window. This allows you to further analyze the results of stressing the specified inputs.

**Stress Analysis Reports**

The reports generated by a Stress Analysis include a summary report, a quick report, and several types of graphs.

A **Summary** report describes the stressed inputs, and it lists the corresponding statistics of the monitored output: Mean, Minimum, Maximum, Mode, Standard Deviation, Variance, Kurtosis, Skewness, 5\textsuperscript{th} Percentile, and 95\textsuperscript{th} Percentile.

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<th>Stress Analysis Summary</th>
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<td>(none)</td>
</tr>
<tr>
<td>Damage</td>
<td>Stress Analytic Model</td>
</tr>
</tbody>
</table>

A Quick report (not shown here) provides a page-sized summary of the Stress Analysis as a whole. This report is designed to fit on standard-sized pages. It includes the graphs that you can also request on separate worksheets.
The purpose of Stress Analysis graphs is to let you compare the results from stressing to the “base” case where there is no stressing. The possible graphs are shown in order below:

**Comparison Graph**

![Comparison Graph Image]

**Box-Whisker Plot**

![Box-Whisker Plot Image]
Histogram

Baseline

Damage $3 \ 90.00\% \ to \ 100.00\%$
CDFs (Cumulative Distribution Functions)

Baseline

Damage C3 90.00% to 100.00%
Advanced Sensitivity Analysis

Sets up and runs an Advanced Sensitivity Analysis

The Advanced Sensitivity Analysis command (from the Advanced Analyses menu in the Tools group) allows you to determine the effects of inputs on @RISK outputs. Unlike typical @RISK sensitivity analysis, an input can be either an @RISK distribution or a cell in your Excel workbook. That is, an input cell does not have to contain an @RISK distribution function. Advanced Sensitivity Analysis allows you to select a number of @RISK distributions, or worksheet cells, and run trial simulations while varying these inputs across a range. Advanced Sensitivity Analysis runs a full simulation at each of a set of possible values for an input, tracking the simulation results at each value. The results then show how simulation results changed as the input value was changed.

Advanced Sensitivity Analysis can be used to test the sensitivity of an @RISK output to the input distributions in a model. When testing an @RISK distribution, @RISK runs a set of simulations for the input. In each simulation, the input distribution is fixed at a different value across its range. Typically these “step” values are different percentile values for the input distribution.

When you click the Advanced Sensitivity Analysis icon, you see the Advanced Sensitivity Analysis dialog, where you can select an output and define inputs for the analysis.
The options in the Advanced Sensitivity Analysis dialog are as follows:

- **Cell To Monitor.** This is a single @RISK output that you want to monitor during simulations, while stepping through possible input values. This cell can be specified by entering a cell reference, clicking the desired cell, or clicking the ellipsis button. The latter displays a dialog that contains a list of all @RISK outputs in your model.

The **Inputs** options allow you to add, edit, or delete the inputs to be included in the analysis. The specified inputs are maintained in a list that you can edit.

- **Add, Edit.** These options display the Input Definition dialog, where you can specify either a single @RISK distribution or worksheet cell, or a range of @RISK distributions or worksheet cells, to be analyzed.

- **Delete.** This option completely removes inputs from the Advanced Sensitivity Analysis. To temporarily exclude an input, or a group of inputs, from the analysis without deleting them, you can remove the appropriate check marks.
The Input Definition dialog allows you to enter the type of input, its name, a base value, and the possible values for the input to be tested. A full simulation will be run at each value you enter for an input.

![Input Definition Dialog]

The options in the Input Definition dialog include:

- **Type.** Type specifies the type of input you are entering: either a distribution or a worksheet cell. (Remember that input cells for an Advanced Sensitivity Analysis do not have to contain @RISK distribution functions.)

- **Reference.** Specifies the location of your input(s). If you are selecting distribution input(s) you can click the ellipsis button. This opens the @RISK Distributions Functions dialog, listing all the distributions in all open worksheets.
• **Name.** Specifies names for your input(s). Cell names are defined in the Input Definition dialog only for the purpose of Advanced Sensitivity Analysis. These names are used in the Results Summary window, and in the reports generated by Advanced Sensitivity Analysis. However, these names do not become part of your Excel model.

  - If you are selecting distribution inputs, the existing @RISK name for each input is shown. If you want to use a different name for a distribution, you can change the @RISK name by adding a **RiskName** function to the distribution in Excel or by editing the name in @RISK’s Model window.

  - If you are selecting worksheet cells as inputs, the name of a single input can be typed directly in the Name entry. When you have selected a range of inputs, the Name entry shows the names of each cell, separated by commas. These names can be edited by typing in the box (keeping the comma-separated format) or by clicking the ellipsis button, which opens the **Sensitivity Analysis Cell Names** dialog.
• **Base Value.** This value is used to determine the sequence of values to step through for an input, and as a reference point in the Percent Change report graph. The base value is especially important when you wish to apply a Variation Method that is a change from base, such as *+/- Percent Change from Base*. By default, the base value is the value a distribution or cell evaluates to when Excel recalculates the worksheet, but you can change it to a different value. Note: If your distribution or cell evaluates to 0, and Base Value is set to **Auto**, you need to enter a nonzero base value if you use the *+/- Percent Change from Base* option.
Variation

The Variation options in the Input Definition dialog describe the type of variation @RISK will use to select the test values for your input(s). During an analysis, inputs are “stepped” across a range of possible values, and a full simulation is run at each step value. Variation defines the nature of this range: % Change from Base, Change from Base Value, Values Across Range, Percentiles of Distribution, Table of Values, or Table from Excel Range. These different Variation options provide a great deal of flexibility in describing the input values to be tested. Depending on the Variation method you select, the entry information for defining the actual range and step values will change.

Each Variation method, and its associated range and value entries, is described here.

- **% Change from Base Value.** With this Variation method, the first and the last value in the sequence to step through are obtained by incrementing, or decrementing, the input's Base Value by the percentages specified in Min Change (%) and Max Change (%) entries. The intermediate values are at equal intervals, with the number of values to test set by # of Steps.

- **Change from Base Value.** With this Variation method, the first and the last value in the sequence to step through are obtained by adding the values specified in Min Change and Max Change entries to the Base Value. The intermediate values are at equal intervals, with the number of values to test set by # of Steps.
• **Values Across Range.** With this Variation method, the sequence of values to step through starts at the **Minimum** and ends at the **Maximum**. The intermediate values are at equal intervals, with the number of values to test is set by **# of Steps**.

![Variation Method](image)

• **Percentiles of Distribution.** This Variation method is used only when the selected input type is a distribution. Then you can specify steps as percentiles of the selected @RISK distribution, with up to 20 steps.

![Variation Method](image)

• **Table of Values.** With this Variation method, you can enter a sequence of values to step through directly in a table in the Input Definition dialog. The Base Value is irrelevant because only the specific values you enter are tested.

![Variation Method](image)
• **Table from Excel Range.** With this Variation method, the sequence of values to step through is found in the range of worksheet cells specified in **Excel Range** entry. This range can contain any number of values; however, it is important to remember that a full simulation will be run for each value in the referenced range.

![Add Analysis Names]

By clicking the **Add Analysis Names** button in the Input Definition dialog, a descriptive name can be added to each input value that will be tested in an Advanced Sensitivity Analysis. This name will be used to identify the simulation run when an input is fixed at a particular value. The purpose of these names is to make your reports more readable and to help identify individual simulations.

![Sensitivity Analysis Names]

The **Sensitivity Analysis Names** dialog allows you to enter a name for the simulation to be run at each stepped input value. The default name @RISK has created is initially shown, but you can change this as desired.
When you click the Options button in the Advanced Sensitivity Analysis dialog, you see the Sensitivity Options dialog. This allows you to select the output statistic you want to evaluate during the sensitivity analysis, identify the reports you want to generate, and specify the behavior of RiskSimtable functions in the analysis.

The Sensitivity Options dialog is invoked by clicking the Options button from the main Advanced Sensitivity Analysis dialog. The selections in this dialog include:

- **Tracking Statistic.** The particular statistic you want to monitor for the output during each simulation. The comparison graphs and reports from the analysis will show the change in the value of this statistic from simulation to simulation.

- **Reports.** Allows you to choose which analysis reports are generated at the end of the Advanced Sensitivity Analysis. They include **Summary, Box-Whisker Plot, Input Graphs, Quick Reports, Percentile Graphs, Percent Change Graphs,** and **Tornado Graphs.** For more information on each of these reports, see **Reports** in this section.
The **Place Reports** setting indicates where your reports will be placed.

- **New Workbook.** All reports are placed in a new workbook.
- **Active Workbook.** All reports are placed in the active workbook with your model.

If an Advanced Sensitivity Analysis is run on worksheets that include **RiskSimtable** functions, the **Include Simtable Functions as Inputs to Analyze** option causes the values specified by these functions to be included in the analysis. If this option is checked, open workbooks will be scanned for RiskSimtable functions. The Advanced Sensitivity Analysis will then step through the values specified in the RiskSimtable function arguments, running a full simulation at each value. The reports generated after the run will show the sensitivity of the output statistic to the variation of the inputs set up in the Advanced Sensitivity Analysis dialog, and to the variation of the values from RiskSimtable functions.

This option is especially useful when an Advanced Sensitivity Analysis is run on an @RISK model that was set up for multiple simulations. @RISK's multiple simulation capability are often used together with RiskSimtable functions to analyze how simulation results change when an input value is changed. This analysis is similar to that performed by an Advanced Sensitivity Analysis. By checking the **Include Simtable Functions as Inputs to Analyze** option and running an Advanced Sensitivity Analysis, multiple simulation models can get the benefit of all Advanced Sensitivity Analysis reports and graphs with no additional setup.

For more information on the RiskSimtable function, see the section **Reference @RISK: Functions** in this manual.

When you click the Analyze button on the Advanced Sensitivity Analysis dialog, you see the number of simulations, iterations per simulation, and total number of iterations. If these appear excessive, you can stop the analysis right away. This lets you decrease the number of iterations, for example, and then retry the analysis.
When an Advanced Sensitivity Analysis is run, the following actions occur for each input in the analysis:

1) A single step value for the input is substituted for the existing cell value, or @RISK distribution, in the model.

2) A full simulation of the model is run.

3) The simulation results for the tracked output cell are collected and stored.

4) This process is repeated until a simulation has been run for each possible step value for the input.

The results of the Sensitivity Analysis are also available in the Results Summary window. You can analyze these results further with the tools available in this window.

The Advanced Sensitivity Analysis reports include a summary report, a quick report, and several graphs.

A Summary report describes the values assigned to the analyzed inputs, and the corresponding statistics of the monitored output: Mean, Minimum, Maximum, Mode, Median, Standard Deviation, Variance, Kurtosis, Skewness, 5th Percentile, and 95th Percentile.
A Quick report (not shown here) provides a summary of the Advanced Sensitivity Analysis as a whole. This report is designed to fit on a single page. It includes the graphs that you can also request on separate worksheets.
You can request Input graphs (not shown here). Each of these indicates how an input for the analysis varied over its range. You are more likely to request any or all of the following graphs, each of which shows how the output varies as the various inputs vary.

The Percent Change Graph plots the output statistic against each of the selected inputs as a Percent Change from Base. The input value, listed on the X-axis, is calculated by comparing each input value tested to the base value for the input.

![Mean of NPV of profit C36 vs Percentage Change of Inputs](chart.png)
The **Percentile Graph** plots the output statistic against percentiles of each of the @RISK distributions that were selected for analysis with step type **Percentiles of Distribution**. Note: Only inputs that are @RISK distributions are displayed on this graph.

![Percentile Graph](image1)

The **Tornado Graph** shows a bar for each of the inputs defined for analysis, showing the minimum and maximum values of the specified output statistic as the input varies over its range.

![Tornado Graph](image2)
RISKOptimizer

Introduction to RISKOptimizer

@RISK includes RISKOptimizer, a powerful tool that combines simulation and optimization to find optimal solutions to models that contain uncertainty. (RISKOptimizer used to be a separate Palisade add-in, but it is now totally integrated into @RISK.) By using optimization techniques and Monte Carlo simulation, RISKOptimizer can find optimal solutions to problems that cannot be solved by standard linear and nonlinear optimizers. RISKOptimizer combines the simulation technology of @RISK and the optimization engines of Evolver. The latter employs Palisade’s genetic algorithms (GA) and OptQuest, a widely used optimizer, to solve deterministic optimization models—those with no uncertainty. RISKOptimizer takes the process one giant step farther by optimizing models with explicit uncertainty.

What Is Optimization?

Optimization is the process of trying to find the “best” solution to a problem. For example, a company might have three manufacturing plants, each manufacturing different quantities of different goods. Given the cost for each plant to produce each good, the costs for each plant to ship to each store, and the limitations of each plant, the goal is to find the optimal way to adequately meet the demand of local retail stores while minimizing the transportation costs.

This is the type of problem that optimization is designed to answer, but it is certainly not the only type. Optimization models of various types appear in the fields of operations management, finance, marketing, economics, engineering, and others. They all share a common goal: to select values of inputs to maximize or minimize an objective value, possibly subject to constraints.

When a problem includes uncertainty, traditional optimization procedures fail because they are not capable of dealing with uncertainty. In the logistics example, suppose the demand of local retail stores is uncertain, that is, the company doesn’t know exactly what quantities of products will be demanded by each store. With a traditional optimization method, the company would assume a single estimate of demand at each store, and the model would be optimized for these assumed demands. However, this approach completely ignores uncertainty, and it can easily lead to an “optimal” solution
that isn’t really optimal, especially if there is a wide range of possible future demands.

**Why Use RISKOptimizer?**

RISKOptimizer opens up a whole new spectrum of problems that can be optimized, specifically those that involve uncertainty. Current optimizers such as Evolver (a Palisade add-in based on GA and OptQuest optimization methods) can find optimal solutions only for deterministic models, those involving no uncertainty. RISKOptimizer overcomes this limitation.

Traditional Excel-based optimization models analyzed by Evolver or other optimizers are comprised of:

- A target cell (also called an objective cell) that you want to minimize or maximize.
- A set of adjustable cells (also called decision variable cells) with values you can change to improve the target value.
- A set of constraints that must be satisfied, usually specified as inequalities such as TotalCost<100 or A11>=0.

During an optimization, the adjustable cells are changed across specified ranges. For each possible set of adjustable cell values, the model is recalculated, which leads to a new value for the target cell. An optimization is complete when an optimal solution has been found (or, in some cases, when the procedure determines that no optimal solution exists). This solution is the combination of adjustable cell values that yields the optimal value for the target cell, while satisfying all of the constraints.

However, when a model includes uncertainty, most optimizers cannot generate optimal solutions. In the past, many optimization models simply ignored uncertainty, making models unrealistic but optimizable. If an attempt was made to find optimal values by using simulation, a “brute force” approach was employed to search possible adjustable cell values iteratively. This involved running an initial simulation, changing one or more values, rerunning the simulation, and repeating this process until a solution was found that appeared to be optimal. This is a lengthy process, and it is usually not clear how to change adjustable cell values from one simulation to the next.
With RISKOptimizer, the uncertainty in a model can be included explicitly, and reliable optimal solutions that take the uncertainty into account can be found. RISKOptimizer uses simulation to deal with the uncertainty present in the model, and it uses advanced optimization techniques to generate new values for the adjustable cells. The result of this “simulation/optimization” approach is the set of adjustable cell values that optimizes a statistic of the distribution of the target output cell, such as its mean.

To model uncertainty, RISKOptimizer lets you use any of the probability distribution functions available in @RISK. For example, you could replace the value in an input cell with the function \texttt{RiskNormal(10,2)}. This function behaves exactly as in any (non-optimization) @RISK model.

When optimizing, RISKOptimizer runs a full simulation for each possible trial solution that is generated by Optquest or the GA-based optimizer. In each iteration of a trial solution’s simulation, probability distribution functions in the model are sampled and a new value for the target cell is generated. At the end of a simulation, the result for the trial solution is the statistic of the target cell’s distribution that you want to optimize. This value is then used by the optimizer to generate new and better trial solutions. For each new trial solution, another simulation is run and another value for the target statistic is generated.

To avoid confusion, it is helpful to distinguish between the terms “trials” and “iterations” in the RISKOptimizer process. The overall process involves a sequence of “trials,” where a given set of adjustable cell values is tested on each trial. To evaluate the target cell (and the constraint cells) for a trial, a standard @RISK simulation is run for a specified number of “iterations.” This distinction helps you understand why RISKOptimizer can take some time. For example, it might require 1000 trials to converge to optimality, and each of these trials might require a 500-iteration simulation. This is a lot of computer calculation, especially for complex models.

As in traditional optimizers, constraints can be entered in RISKOptimizer. When the value of the constrained cell does not change during a simulation, the constraint is called non-probabilistic. Otherwise, it is called probabilistic. A probabilistic constraint is based on a statistic of the distribution of the cell’s values, such as “the mean of A1 <= 100.” RISKOptimizer calculates the statistic at the end of each simulation to decide whether the constraint is satisfied.
Each constraint can be hard (it must be satisfied) or soft (it can be violated to some extent). Trial solutions that violate any hard constraints are discarded.

As large numbers of simulations are being run by RISKOptimizer, two important techniques are used to minimize runtimes and generate optimal solutions as quickly as possible. First, RISKOptimizer uses convergence monitoring to determine when a sufficient number of iterations have been run. This insures that the resulting statistic from the target cell’s probability distribution is stable, and that any statistics from output distributions referenced in constraints are stable. Second, RISKOptimizer optimization engines generate trial solutions that move toward an optimal solution as quickly as possible.

All @RISK graphs and reports are available for viewing the results of solutions generated by RISKOptimizer. @RISK statistical functions such as RiskMean can also be used to return simulation results directly to your spreadsheet model via cell formulas.

The availability of optimization for models with uncertainty allows you to solve many previously “unoptimizable” problems. As a rule, any model that has uncertain elements can be optimized through the combination of simulation and optimization, including:

- Selection of optimal production and capacity levels for new products with uncertain market conditions
- Identification of optimal inventory levels with uncertain demand
- Portfolio allocation for risk minimization
- Identification of an optimal product mix when product markets are geographically distributed and demand levels are uncertain
- Determining optimal levels for options purchases when hedging
- Yield management when the same product is sold at different prices under different restrictions
- Scheduling with uncertain task times
How does RISKOptimizer work?

RISKOptimizer uses two optimization engines (OptQuest and genetic algorithms, or GAs) to search for optimal solutions, along with probability distributions and simulation to deal with the uncertainty in your model.

**OptQuest**

OptQuest Engine uses metaheuristics, mathematical optimization, and neural network components to guide the search for best solutions to decision and planning problems of all types. OptQuest’s methods integrate state-of-the-art metaheuristic procedures, including Tabu Search, Neural Networks, Scatter Search, and Linear/Integer Programming, into a single composite method. To learn more about OptQuest, see Appendix B - Optimization.

**Genetic Algorithms**

Genetic algorithms used in RISKOptimizer mimic Darwinian principles of natural selection by creating an environment where hundreds of possible solutions to a problem compete with one another, and only the “fittest” survive. As in biological evolution, each solution can pass along its good “genes” through “offspring” solutions so that the entire population of solutions will continue to evolve better solutions.

The terminology used when working with GAs is often similar to that of its inspiration. They involve “crossover” functions to help focus the search for solutions, “mutation” rates to help diversify the “gene pool”, and they evaluate the entire “population” of solutions or “organisms”. To learn more about how RISKOptimizer’s GA works, see Appendix B - Optimization.

**Probability Distributions and Simulation**

Probability distributions and simulation are used in RISKOptimizer to describe the uncertainty in your model. They are entered with the usual @RISK distribution functions, such as `RiskTriang(10,20,30)`. Simulation is then used to generate a distribution of possible outcomes for each possible trial solution that is generated by the optimizer.
What Are RISKOptimizer’s Advantages?

When you are dealing with large numbers of interacting variables, and you are trying to find the optimal mix, the optimal ordering, or the optimal grouping of these variables, you might be tempted to just make an “educated guess.” Now that powerful computers are affordable, and the Excel and RISKOptimizer software are readily available, there is little reason to guess at solutions or waste time trying to find “good” solutions in an ad hoc manner.

**More Accurate, More Meaningful**

With RISKOptimizer, you can use the entire range of Excel formulas and probability distributions to build more realistic models of any system. When you use RISKOptimizer, you do not have to compromise the accuracy of your model because the algorithm you are using cannot handle real-world complexities. Traditional optimizers force you to make assumptions about the way the variables in your model interact, often resulting in over-simplified and unrealistic models of your problem. They also force you to ignore uncertainty. By the time you have simplified a system enough that these optimizers can be used, the resulting solution is often of little use for your real problem.

**More Flexible**

There are many solving algorithms that work well for solving simple linear and nonlinear types of problems. For larger and more complex problems, you might be able to write customized algorithms to achieve good results, but this can require a lot of research and development. Even then, the resulting program will probably require modification each time your model changes.

Among its other claims, RISKOptimizer is the only commercial program that can solve most combinatorial problems involving uncertainty. These are problems where the variables must be permuted or combined in some way. For example, RISKOptimizer can find the optimal ordering of tasks to be performed as part of a project, evaluating only solutions that meet specified precedence constraints, that is, constraints that require certain tasks to be performed before others. Complex scheduling problems are also combinatorial. RISKOptimizer can solve these types of problems and many more that other optimizers cannot solve. RISKOptimizer’s unique optimization and simulation technology allows you to optimize virtually any type of model, of any size and any complexity.
In spite of its power and flexibility advantages, RISKOptimizer remains easy to use. You first set up the model, just as you set up any @RISK simulation model. Then all you need to do is specify the target to optimize, the adjustable cells, and the constraints. RISKOptimizer hides the difficult technical details and automates the optimization process behind the scenes.

Although many commercial programs have been developed for mathematical programming and model-building, spreadsheets are by far the most popular. With their intuitive row and column format, spreadsheets are easier to set up and maintain than other dedicated packages. Because RISKOptimizer is an add-in to Microsoft Excel, you have access to the entire range of functions and development tools to build realistic models and then optimize them.

**Traditional Optimization versus Simulation Optimization**

This section describes in more detail how RISKOptimizer works.

The traditional process for optimizing a spreadsheet model is as follows:

1) You identify a **target** (or **objective**) cell to minimize or maximize.

2) You identify a set of **adjustable** (or **decision variable**) cells. These are the cells you can change to improve the target value. Their values are often limited to specified ranges.

3) You identify a set of **constraints** that must be satisfied. They are usually in the form of inequalities, such as TotalCost<=100 or A11>=0.

4) You run the optimization, where the spreadsheet is recalculated repeatedly. Each time, a different set of possible values for the adjustable cells is used. These sets of adjustable cell values are often called **trial** solutions.

5) During this process:
   a) Each recalculation generates a new value for the target cell.
   b) The optimizer uses this new target cell value and other information about the model to choose the next trial solution.

This last step is repeated until some **stopping** condition is reached, that is, until the process recognizes that the optimal solution has been obtained or that no further progress is possible.
Simulation optimization using RISKOptimizer follows many of these same steps. However, changes are made to allow uncertainty in the model and to use simulation, instead of a simple spreadsheet recalculation, to evaluate the target cell and choose the next trial solution.

RISKOptimizer uses the following simulation/optimization process, with differences from traditional spreadsheet optimization shown in bold:

1) **You enter probability distribution functions to describe the uncertain inputs in your model.**

2) **You identify a target (or objective) cell and the simulation statistic (mean, standard deviation, etc.) of this cell to minimize or maximize.**

3) **You identify a set of adjustable (or decision variable) cells. These are the cells you can change to improve the target value. Their values are often limited to specified ranges.**

4) **You identify a set of constraints that must be satisfied. They are usually in the form of inequalities, such as TotalCost<=100 or A11>=0. They can also be based on simulation statistics, such as “the 95th percentile of the value in cell A11 must be greater than 1000.”**

5) **You run the optimization, where the spreadsheet is simulated repeatedly, with each simulation using different values for the adjustable cells. During this process:**
   a) **Each simulation generates a new distribution of values for the target cell. The statistic you want to minimize or maximize is calculated from this distribution.**
   b) **The optimizer uses the new value of this statistic for the target cell to select the next trial solution.**

This last step is repeated until some stopping condition is reached, that is, the process recognizes that the optimal solution has been obtained or that no further progress is possible.

This explanation should be enough to give you the big picture. The RISKOptimizer Commands section in the @RISK Command Reference chapter provides details for implementing the procedure.
Chapter 6: @RISK Command Reference
Efficient Frontier Analysis

Efficient frontier analysis is a special kind of optimization. It is used when there are two competing goals. You choose one of them as the target for an optimization and constrain the other to be no “worse” than a specified limit. Then you perform a sequence of optimizations, each time changing the limit value for the constraint.

Although efficient frontier analysis is generally used for financial portfolio optimization, it can be applied to a variety of problems where there are two competing goals. As one example, you might be trying to determine a waste management system that minimizes cost and minimizes the pollution level. Because these goals are pulling in opposite directions, you might minimize cost while putting various upper limits on the allowable pollution level. Alternatively, you could minimize the pollution level by putting various upper limits on the allowable cost.

For illustration, however, this discussion will be in usual context of portfolio optimization. In this context, you want to find a portfolio of securities that maximizes expected portfolio return and minimizes risk, usually measured by the standard deviation of portfolio return. As in the pollution example, these two goals pull in opposite directions, so efficient frontier analysis typically minimizes the risk (standard deviation), while putting a lower bound on the expected return. (It could be performed in the opposite way, by maximizing expected return with an upper bound on risk.) By performing several optimizations, each with a different lower bound on the expected return, the efficient frontier is swept out. It tells you, for any given required expected return, the portfolio that minimizes your risk. Then from all the portfolios on the efficient frontier, you can choose the portfolio that you prefer, given your risk preferences.

The following graph illustrates an efficient frontier, the yellow curve. Optimal portfolios can be found for any point on this curve, and these are the only portfolios an investor should consider. Points in the darker green area below and to the right of the efficient frontier correspond to portfolios that have more risk than necessary for any specified expected return, so they are suboptimal and shouldn’t be considered. Points in the lighter green area above and to the left of the efficient frontier are impossible to achieve.
Starting with version 7, RISKOptimizer can perform efficient portfolio analysis for any model with two competing goals. You choose one of the goals as the target to optimize, you add an “efficient frontier” type of constraint on the other goal, where you list the bounds to try, and you specify that you are performing an efficient frontier analysis. Then RISKOptimizer solves a sequence of optimizations, one for each bound you specify, and it presents the results in graphical and tabular form.

This procedure takes longer because RISKOptimizer is solving several optimizations, but it doesn’t take as much longer as you might expect. RISKOptimizer uses a very efficient method of simultaneously solving all of the optimizations in “one pass.”
RISKOptimizer Commands

The RISKOptimizer commands are available from the RISKOptimizer icon on the @RISK ribbon.
Model Definition Command

Defines the goal, adjustable cells, and constraints for a model

The Model Definition command (from the RISKOptimizer menu in the Tools group) opens the RISKOptimizer Model dialog. This is where you define most of the RISKOptimizer settings.

The RISKOptimizer Model dialog is used to specify or describe an optimization problem to RISKOptimizer. It starts as shown in the above screenshot with each new Excel workbook, but your settings are saved when you save the workbook. That means that when you open this workbook again, it will show your settings. This section describes each component of the dialog.
Options in the Model dialog include:

- **Optimization Goal.** This determines the type of optimization to perform. Typically, it will be *Maximum* or *Minimum*, indicating that you want to maximize or minimize a target. However, you can also choose *Target Value*. In this case, RISKOptimizer will try to make the selected statistic of the target cell as close as possible to the value you specify.

- **Cell.** This is the target cell you want to optimize. The target cell should contain a formula that depends (either directly or through a series of calculations) on the adjustable cells. This formula can include standard Excel functions or user-defined VBA macro functions. By using VBA macro functions, you can have RISKOptimizer evaluate very complex models.

  As RISKOptimizer searches for a solution, it uses the statistic for the simulation results of the target cell to evaluate how good each trial solution is, and to determine which variable values should continue to be adjusted. When you build your model, your target cell must reflect the “goodness” of any given scenario, so as RISKOptimizer calculates the possibilities, it can accurately measure its progress.

- **Optimize.** This is the statistic of the target cell you want to minimize, maximize, or set to a specific value. You can select from the dropdown list shown below.
If you want to select a Percentile or Target for the target cell's distribution (items that appear at the bottom of the dropdown list), select **Percentile (X for given P)** or **Target (P for given X)**.

1) For Percentile (X for given P), enter the desired "P" value between 0 and 100 in the % field. The value that will be minimized or maximized will be the value associated with the entered percentile. For example, you could enter 99% to maximize the 99th percentile.

2) For Target (P for given X), enter the desired "X" value. This optimizes the cumulative probability for X. For example, you could enter 1000 to minimize the probability of the target value being less than or equal to 1000.

**Optimize Cell Value**

Although not as common, you can also optimize the **Value** of a cell, not one of its simulation statistics. For example, when engineering a complex system, money can be spent on backup systems, thereby reducing the probability of failure. The adjustable cells would specify the number of different backup systems. The cell formula calculating the cost of the backup systems depends on the values of the adjustable cells, but it contains no uncertainty. In this case, you would
minimize the cost cell using the **Value** option in the dropdown list.

Another situation where you would choose the **Value** option is when the target cell involves @RISK statistics functions for your goal. This method is especially useful when we want to optimize a statistic not included in the dropdown list. For example, an investor might want to maximize the Sharpe ratio of a portfolio of securities, defined in terms of the mean and standard deviation of the portfolio returns. If the portfolio return (the uncertain quantity) is in cell F10 and the riskfree rate of return is in cell F3, an appropriate formula for the Sharpe ratio is:

\[
=(\text{RiskMean}(F10) - F3) / \text{RiskStdDev}(F10)
\]

If this formula is in cell F12, say, then the **Value** option for cell F12 should be selected in the Optimize dropdown list as the goal of the optimization.

- **Analysis Type.** You should choose the Standard type unless you are calculating an efficient frontier. Then you should choose the Efficient Frontier type.

The Adjustable Cell Ranges table displays each range of adjustable cells in a row. One or more adjustable cell ranges can be included in an **Adjustable Cell Group**. All cell ranges in an Adjustable Cell Group share a common solving method; if the Genetic Algorithm is used, they also share the crossover rate, mutation rate, and operators. (All of this is explained in detail below.)

Because the adjustable cells contain the decision variables of the problem, you must define at least one group of adjustable cells to use RISKOptimizer. Most problems will contain only one group of adjustable cells, but more complex problems may require different blocks of variables to be solved with different solving methods simultaneously. This unique architecture allows for highly complex problems to be built from multiple groups of adjustable cells.
The following options are available for entering Adjustable Cell Ranges:

- **Check boxes.** Each adjustable cell range has a check box to its left. You can activate or deactivate any (or all) of the current adjustable cell ranges by checking or unchecking its box.

- **Add.** You can add new adjustable cells by clicking the Add button. You are immediately asked for the adjustable cell range, and then a new row is added to the table with the following fields.

- **Range.** The range you already specified is in the Range field. You can change this if necessary by typing or pointing to a new range.

- **Minimum** and **Maximum.** The Minimum and Maximum entries set the range of acceptable values for each adjustable cell in the range. These values are required. RISKOptimizer will not allow any values outside the specified ranges (unless you specify soft constraints). In cases where natural limits don’t exist (or aren’t known), you should specify limits wide enough to allow all reasonable solutions, but the tighter you can keep these limits, the quicker the solution process will be, because tighter limits restrict the scope of the search.

- **Values.** The Values entry (which is relevant only for the Recipe and Budget solving methods) specifies the type of adjustable cell values RISKOptimizer will search through. In each of these settings, only values between the specified minimum and the specified maximum are included in the search.

![Adjustable Cell Values](image)

There are three possible settings:

- **Any.** All real numbers are included. This is the default.
• **Integer.** Only integer values are included. This is usually chosen when non-integer values make no sense in your model.

• **Discrete.** With this setting, you choose a step size. Then only multiples of this step size are included. This setting is useful when “rounded” values of the adjustable cells, such as multiples of $10, are fine for all practical purposes.

The default is to include all real numbers in the specified range. The use of Discrete (or Integer) can dramatically the number of possible solutions and will often result in much faster optimizations.

**Adjustable Cell Groups**

RISKOptimizer uses groups of adjustable cell ranges to provide more flexibility in the solution process. Each group can contain multiple cell ranges. This allows you to build a hierarchy of groups of related cell ranges. Within each group, each cell range can have its own minimum-to-maximum range.

All cell ranges in an Adjustable Cell Group share a common solving method. This is specified in the Adjust Cell Group Settings dialog. This dialog is accessed by clicking the Group button. You can create a new group to which you can add adjustable cell ranges, or you can edit the settings for an existing group.

By default, each adjustable cell range you add (by clicking the Add button) is added to a single group using the Recipe solving method. However, you have other options.

• To add a new group that uses another solving method, click the Group button and select New. You will be asked for an adjustable cell range, and then you will see the Adjustable Cell Group Settings dialog, where you can choose the solving method for the new group.
Once you have two more groups, the adjustable cell table will look something as follows. You can select either group bar (here, the Budget group is selected). Then when you click Add to add another adjustable cell range, it will be added to the selected group. Alternatively, you can click the Group button and select Edit to edit the selected group (change its solving method or description).

![Adjustable cell table](image)

**Solving Methods**

Again, remember that each group of adjustable cell ranges shares a common solving method. RISKOptimizer can use any of six solving methods, depending on the type of optimization model. Three of the solving methods (Recipe, Order, and Grouping) use entirely different algorithms. The other three are descendants of these three, adding additional constraints.

The following section describes each solving method. To get a better understanding of how each solving method is used, you are also encouraged to explore the example files included with the software.

**Recipe Method**

The **Recipe** solving method is the simplest and most popular type of solving method. It is used when the values in an adjustable cell group can be varied independently of one another. You can think of each variable as the amount of an ingredient in a cake; when you use the Recipe solving method, you are asking RISKOptimizer to find values that produce the best mix. The only constraint you place on the Recipe values themselves is their minimum-to-maximum range.

**Order Method**

The **Order** solving method is the second most popular type. This method is used in a very specific context: when you want to find the best ordering, or permutation, of items. For example, if you have 10 jobs to schedule on a machine, one after the other, and you want the best ordering in terms of total job completion time, you are searching for the best permutation of the integers 1 to 10 and you should use the Order solving method.
The **Grouping** solving method is used when your problem involves multiple items to be grouped into sets. For example, suppose you want to group a large number of securities into 6 sets so that the variability within groups (on some measure such as 6-month return) is minimized. Simply stated, you want the securities in each group to be similar. Then you could initially assign each security to any group index from 1 to 6 and then ask RISKOptimizer to find the optimal group indexes for the securities. These indexes, one for each security, will be the adjustable cell values.

By default, the number of different groups that RISKOptimizer creates will be equal to the number of unique values present in the adjustable cells at the start of an optimization, in this case 6. If any of the group indexes are not included in the initial set of adjustable cell values (maybe you didn’t enter any 5 indexes, for example), you can edit the group in the following dialog:

![RISKOptimizer - Adjustable Cell Group Settings](image)

The Group IDs field allows you to specify a cell range containing all the group indexes. If the bottom option is checked, RISKOptimizer will not allow any solutions where there is a group without any items in it.

The **Budget** solving method is the same as Recipe except that sum of the adjustable cell values is fixed at the initial value of the sum. For example, the adjustable cell values might be percentages of a total investment in various stocks. If the initial values of these percentages add to 100% and the Budget method is used, then the percentages will continue to sum to 100% in all trial solutions.

An alternative in this situation is to use the Recipe method and add a constraint on the sum of the adjustable cell values. This approach work well if the OptQuest engine is used. However, if the Genetic Algorithm is used, the Budget method is more efficient.
**Project Method**

The **Project** solving method is similar to the Order solving method except that certain items (tasks) must precede others. This solving method is especially useful in project management to order tasks with the precedence constraints. In this case, the adjustable cells containing the task order should be in a single column, rather than in a row. This is because the solving method expects the preceding tasks cells to be arranged vertically rather than horizontally.

After you have specified the location of the adjustable cells, you must edit the adjustable cell group and specify the location of the preceding tasks cells. An example is shown here, where the ordering (of towns to visit) is shown in yellow and the precedence constraints are indicated in columns I to K. For example, towns 2 and 7 must be visited before town 3. The precedence tasks range should have $n$ rows and $m$ columns, where $n$ is the total number of tasks (number of adjustable cells), and $m$ is the largest number of preceding tasks that any one task has.

![Example of Project Method](image)

**Schedule Method**

The **Schedule** solving method is used to schedule tasks to times. Each task is assumed to take the same amount of time, much as classes at a school are all of the same length. In this case, the Adjustable Cell Group Settings dialog lets you directly specify the number of time blocks to be used.

Here is typical example where college classes are being assigned to time slots. The example below illustrates this. Each yellow cell specifies a time slot from 1 to 6 for the class. These are the adjustable cells. The Constraint Cells consist of the bordered range. This range uses “codes” for the spelled-out constraints to their left.

![Example of Schedule Method](image)
Eight kinds of constraints are possible:

1. (with) The tasks in columns 1 and 3 must occur in the same time block.
2. (not with) The tasks in columns 1 and 3 must not occur in the same time block.
3. (before) The task in column 1 must occur before the task in column 3.
4. (at) The task in column 1 must occur in the time block in column 3.
5. (not after) The task in column 1 must occur at the same time or before the task in column 3.
6. (not before) The task in column 1 must occur at the same time or after the task in column 3.
7. (not at) The task in column 1 must not occur in the time block in column 3.
8. (after) The task in column 1 must occur after the task in column 3.

You can enter either a numeric code (1 through 8) or the description (after, not at, etc.) for a constraint. (All language versions of RISKOptimizer will recognize the English description entered for a constraint as well as the translated form). Note: When you select the schedule solving method, integers starting from 1 are always used regardless of the original values in the adjustable cells.
**Constraints**

RISKOptimizer allows you to enter constraints, or conditions that must be satisfied for a solution to be valid. Constraints you have entered are shown in the **Constraints table** in the Model Definition dialog. As with adjustable cell ranges, there is a check box to the left of each defined constraint. This allows you to activate or deactivate any constraint in the list.

![Constraints Table](image)

**Add Constraints**

You click the Add button to add a new constraint. This displays the **Constraint Settings** dialog, where you specify the constraint.

![Constraint Settings Dialog](image)

The main entries in this dialog are:

- **Range to Constrain.** The cell(s) in Excel for which values will be constrained during the optimization.

- **Constraining Values.** The values you want to limit the cell to.

The **operator entry** in the middle controls whether the Range to Constrain will be >, >=, <, <= or = the Constraining Values.

You can constrain either the **Value** or a **Simulation Statistic** calculated for the Range to Constrain.
If you select to constrain the **Value** of the **Range to Constrain**, RISKOptimizer checks the value in the range during an optimization to see if the constraint is satisfied. In this case, the Range to Constrain usually contains a formula that calculates a value you want to constrain. The formula typically references adjustable cell values in your model and/or @RISK statistics functions that are updated during the optimization process. For example, the Range to Constrain might contain a formula that sums a set of adjustable cell values. Then your constraint could be that the sum must not exceed a certain value.

If you select to constrain a **Simulation Statistic** of the **Range to Constrain**, RISKOptimizer checks the simulation statistic calculated for the range during each simulation of an optimization and compares it with the entered **Constraining Value**.

During this process, a distribution of possible values for the cell (or cells) specified as the **Range to Constrain** is generated during each trial solution's simulation. At the end of each simulation, the constraint is checked to see if it has been satisfied. If the simulation constraint is a hard constraint and the constraint is not satisfied, the trial solution is discarded. If the constraint is a soft constraint and the constraint is not satisfied, the target cell's statistic that is being minimized or maximized is penalized according to the entered penalty function. (See the section **Soft Constraints**).

If the statistic you want to constrain is not in the **Simulation Statistic** list, you can still define the constraint in RISKOptimizer. For example, you might to constrain the Sharpe ratio of a portfolio of securities, which is defined in terms of the mean and the standard deviation of the portfolio return. To accomplish this in RISKOptimizer, you can place an appropriate formula for the Sharpe ratio in a cell using the RiskMean and RiskStdDev functions. Then you can specify this cell in the **Range to Constrain**, and select the **Value** option for **Constrain**.

Three styles – **One-Sided**, **Two-Sided**, and **Formula** – can be used for entering constraints. The **Entry Style** button in the Constraints dialog lets you select the style you want to use.

The **One-Sided** and **Two-Sided** styles allow constraints to be entered using $<, <=, >, >=$ or $=$ relations. A typical **Two-Sided** constraint would be $0<=Value \ of \ A1<=10$, where $A1$ is entered in the Range to Constrain, 0 is entered as one constraining value, and 10 is entered as the second.
A Formula constraint, on the other hand, allows you to enter any valid Excel formula as a constraint. For example, you could enter the formula \(=\text{IF}(A1>100, B1>0, B1<0)\). This constraint requires the value in cell B1 to be positive or negative depending on the value in cell A1. Alternatively, the formula can be entered in a cell; if that cell is C1, for example, then you can enter \(=C1\) in the Formula field of the Constraint Settings dialog.

In general, entering constraints in the One-Sided or Two-Sided style helps Evolver find the optimal solution faster. The formula just discussed could be entered in cell D1 as \(=\text{IF}(A1>100, B1, -B1)\). Then a simple One-Sided constraint can be added, requiring that D1>0.

Clicking the More button expands the Constraint Settings dialog to provide additional options for setting up constraints.
Other constraint options include:

**Constraint Type**

Two types of constraints can be specified in RISKOptimizer:

- **Hard**, or conditions that must be satisfied for a solution to be valid. Trial solutions that do not satisfy hard constraints are discarded.

- **Soft**, or conditions that you would like to satisfy, but can be violated at a certain “penalty” cost. This penalty cost is subtracted from or added to the target value to discourage invalid solutions.

**Precision**

**Precision** refers to violations of constraints that are so small that RISKOptimizer disregards them and will treat a solution as valid despite these small violations. This small imprecision in the handling of constraints relates to the fact that computers can only handle mathematical operations with finite precision.

Leaving Precision as **Automatic** is recommended. In this case the Optimization Summary will report the actual precision number, which is the maximum violation of the constraint that is disregarded. Alternatively, a specific number can be entered in the Precision field.

**Evaluation Time**

When you select to constrain the **Value** of a range (not a **Simulation Statistic**), you can control when the constraint is evaluated. Options are:

- **Automatic**. RISKOptimizer decides when to evaluate the constraint.

- **Every Iteration of Each Simulation (Iteration Constraint)**. RISKOptimizer evaluates the constraint every iteration of a simulation.

- **At the End of Each Simulation (Simulation Constraint)**. RISKOptimizer evaluates the constraint at the end of each simulation.

When the evaluation takes place, a constraint is checked to see if it has been met. If the constraint is a hard constraint and the constraint is not met, the trial solution is discarded. If the constraint is a soft constraint and the constraint is not met, the target cell’s statistic that is being minimized or maximized is penalized according to the specified penalty function.
To perform the Efficient Frontier Analysis, you need to specify a special type of constraint with multiple alternative “constraining values.” To do this, you first must select Efficient Frontier as the Analysis Type in the Model dialog.

Then you select Use for Efficient Frontier in the Constraint Settings dialog to display the options for defining the list of constraining values.

The List of Constraining Values option specifies how you will enter the list of values. For example, suppose you want the following list of alternative constraining values: 0.08, 0.085, 0.09, 0.095, 0.10. You can define the list by specifying that it contains 5 values between 0.08 and 0.10 (the Between Minimum and Maximum option). Alternatively, you can type the 5 values in the dialog (Table option), or type them in an Excel range (In Excel Range option).

To identify the efficient frontier, RISKOptimizer performs multiple optimizations to find the optimal value for each possible value of the constraint. In the example here, RISKOptimizer would perform an optimization at each of the constraining values and then graph the optimal solution across the possible constraining values.
Soft Constraints are conditions you would like to satisfy as much as possible, but you are willing to violate as a compromise (to improve the target value). When a soft constraint is violated, a penalty is attached to the target. The amount of penalty is determined by a penalty function that you specify when you specify the soft constraint.

More information about penalty functions is as follows:

- **Entering a Penalty Function.** RISKOptimizer has a default penalty function that is displayed when you first enter a soft constraint. However, you can replace this with any valid Excel formula. Any penalty function you enter should include the keyword `deviation`. This “placeholder” represents the absolute amount by which the constraint value is beyond its limit. For each trial solution, RISKOptimizer checks whether the soft constraint has been satisfied; if not, it places the amount of deviation in the penalty formula and then calculates the amount of penalty to apply to the target cell value.

  The penalty amount is either added to (for minimization) or subtracted from (for maximization) the target cell value to penalize it.
• **Viewing the Effects of an Entered Penalty Function.**
  RISKOptimizer includes an Excel worksheet *Penalty Functions and Soft Constraints.xlsx* that can be used to evaluate the effects of different penalty functions on specific soft constraints and target cell results.

  ![Graph showing penalty effects](image)

• **Viewing the Penalties Applied.** When a penalty is applied to the target cell due to an unsatisfied soft constraint, you can see the amount of penalty applied in the RISKOptimizer Watcher. In addition, penalty values are shown in Optimization Log worksheets, created optionally after optimization.

  **NOTE:** If you place a solution in your worksheet at the end of an optimization, the calculated target cell result shown in the spreadsheet will not include any penalties applied due to unsatisfied soft constraints. You can check the Optimization Log worksheet to see the penalized target cell value and the amount of penalty imposed due to each unsatisfied soft constraint.
Optimization Settings Command

The Optimization Settings command (from the RISKOptimizer menu in the Tools group) opens the RISKOptimizer Optimization Settings dialog. This dialog has four tabs.

The **Runtime** tab on the Optimization Settings dialog displays RISKOptimizer settings that determine the runtime of the optimization. These stopping conditions specify how and when RISKOptimizer will stop during an optimization. Once you select the Start Optimization command, RISKOptimizer will continue to run, searching for better solutions and running simulations until the selected stopping criteria are met. You can turn on any number of these conditions, or none at all; if you do not select any stopping conditions, RISKOptimizer will run until all possible solutions have been tried, or until you stop it manually by pressing the Stop button. When multiple conditions are checked, RISKOptimizer will stop as soon as one of the chosen conditions is met. You can also override these selections and stop RISKOptimizer at any time manually by using the stop button in the RISKOptimizer Watcher or Progress windows.
The options in the Runtime tab include:

- **Trials** - This option, when set, stops RISKOptimizer when the given number of simulations have been executed. A simulation is run for each trial solution generated by RISKOptimizer.

- The Trials setting is particularly useful when comparing RISKOptimizer’s efficiency across different modeling methods. By changing the way you model a problem, or by choosing a different solving method, you might be able to increase RISKOptimizer’s efficiency. Having a model run for a specified number of simulations will indicate how efficiently RISKOptimizer is converging to a solution, regardless of any differences in the number of variables chosen, the speed of the computer hardware being used, or the screen re-drawing time. The RISKOptimizer optimization summary worksheet is also useful for comparing results across runs. For more information on Optimization Summary worksheets, see the RISKOptimizer Watcher – Stopping Options section in this chapter.

- **Time** - This option, when set, stops RISKOptimizer from simulating scenarios after the given number of hours, minutes, or seconds has elapsed. This entry can be any positive number (600, 5.2, etc.).

- **Progress** - This option, when set, stops RISKOptimizer from simulating scenarios when the improvement in the target cell is less than the specified amount (change criterion). You can specify, as an integer, the number of simulations over which to check the improvement. A percentage value such as 1% can be entered as the maximum change value in the Maximum Change field.

For example, suppose you are trying to maximize the mean of the target cell with Maximum Change set to 0.1, and after 500 simulations, the best value of the target found so far is 354.8. If the Progress option is the only stopping condition selected, RISKOptimizer will pause at simulation #600 and will continue only if it is able to find a target value of at least 354.9 during those last 100 simulations. In other words, the target value has not improved at least 0.1 over the last 100 simulations, so RISKOptimizer assumes there is little more improvement to be found, and stops the search. For more complex problems, you might want to boost the number of
simulations that RISKOptimizer runs before deciding whether there is still sufficient improvement to continue.

This is the most popular stopping condition, because it provides an effective way to stop RISKOptimizer after the improvement rate is slowing down and better solutions are not being found. If you are viewing the graphs of the best results on the Progress tab of the RISKOptimizer Watcher, you will see the graphs flatten out for a while before this condition is met and RISKOptimizer stops. The Progress option is really just an automatic way to do what you could do yourself with the graph: let it run until the improvement levels off.

- **Formula is True.** This stopping condition causes the optimization to stop whenever the entered (or referenced) Excel formula evaluates to TRUE during the optimization.

- **Stop on Error.** This stopping condition causes the optimization to stop whenever an Error value is calculated for the target cell.

**Note:** You can also select no stopping conditions, in which case RISKOptimizer will run until all the possible solutions have been tried, or until you stop it manually by pressing the “stop” button.

RISKOptimizer runs a full simulation of your model for each trial solution it generates, stopping the simulation based on @RISK simulation settings. Those settings can specify a fixed number of iterations or they can instruct @RISK to stop automatically based on convergence of the values of statistics.
The Efficient Frontier Runtime tab on the Optimization Settings dialog displays RISKOptimizer settings that determine the runtime of an Efficient Frontier analysis. If you do not select any stopping conditions, RISKOptimizer will run until all the possible solutions have been tried, or until you stop it manually by pressing the Stop button. When multiple conditions are checked, RISKOptimizer will stop as soon as one of the chosen conditions is met. You can also override these selections and stop RISKOptimizer manually at any time by clicking the Stop button in the RISKOptimizer Watcher or Progress windows.

The stopping conditions on the Efficient Frontier Runtime tab include:

- **Trials per Constraining Value.** If this field is set to 250, say, and you defined the Efficient Frontier analysis by specifying 5 alternative values in a constraint, the Efficient Frontier analysis will stop after 1250 simulations (5 x 250 simulations).

- **Time.** This option, when set, stops RISKOptimizer from simulating scenarios after the given number of hours, minutes, or seconds has elapsed. This entry can be any positive real number (600, 5.2, etc.).
Chapter 6: @RISK Command Reference

- **Progress.** This option, when set, stops RISKOptimizer from simulating scenarios when the improvement in the target cell is less than the specified amount (change criterion). You can specify, as an integer, the number of simulations after which to check the improvement. A percentage value such as 1% can be entered as the maximum change value in the **Maximum Change** field. The improvement in the target cell needs to be understood in the context of the fact that in the Efficient Frontier analysis, you are running multiple parallel optimizations. The analysis will stop when the progress is insufficient in each of the parallel optimizations.

**Optimization Settings Dialog – Engine Tab**

The Engine tab on the Optimization Settings dialog selects the optimization engine and settings that will be used during an optimization. RISKOptimizer uses two optimization engines, OptQuest and Genetic Algorithm, to search for optimum solutions to a problem.

**Optimization Mode**

In most cases, RISKOptimizer can automatically detect which engine will give the best and fastest solutions to your model. The **Automatic** setting specifies this. However, there might be times when you wish to specify an engine to use. You might, for example, have a model that requires the Project or Schedule solving methods that are available only with the Genetic Algorithm engine.
Two optimization engines, **Genetic Algorithm** and **OptQuest**, are available to search for optimum solutions to a problem.

The **Genetic Algorithm** engine originated in Evolver, Palisade’s optimizer for deterministic models, and was used in pre-6.0 versions of RISKOptimizer. The **Genetic Algorithm** engine mimics Darwinian principles of natural selection by creating an environment where hundreds of possible solutions to a problem can compete with one another, and only the fittest survive. Just as in biological evolution, each solution can pass along its good “genes” through “offspring” solutions so that the entire population of solutions will continue to evolve better solutions.

The **OptQuest Engine** uses metaheuristic, mathematical optimization, and neural network components to guide the search for best solutions to decision and planning problems of all types. OptQuest’s methods integrate state-of-the-art metaheuristic procedures, including Tabu Search, Neural Networks, Scatter Search, and Linear/Integer Programming, into a single composite method.

**Genetic Algorithm Settings** on the Engine tab include:

**Population Size.** The population size tells RISKOptimizer how many organisms (or complete sets of variables) to store in memory at any given time. Although there is still much debate and research regarding the optimal population size to use on different problems, we generally recommend using 30-100 organisms in your population, depending on the size of your problem (bigger population for larger problems). The common view is that a larger population takes longer to settle on a solution but is more likely to find a global solution because of its more diverse gene pool.

**Crossover and Mutation.** One of the most difficult problems with searching for optimal solutions, when your problem has seemingly endless possibilities, is in determining where to focus your energy. In other words, how much computational time should be devoted to looking in new areas of the “solution space” and how much time should be devoted to fine-tuning the solutions in the population that have already proven to be fairly good?

A big part of the genetic algorithm success has been attributed to its ability to achieve this balance. The structure of the GA allows good solutions to “breed,” but it also keeps “less fit” organisms in memory to maintain diversity, reasoning that a latent “gene” will prove important to the ultimate solution.

Crossover and Mutation are two parameters that affect the scope of the search, and RISKOptimizer allows you to change these parameters.
before, and also during, the evolutionary process. This way, you can help the GA by deciding where it should focus its energy. For most purposes, you should not need to adjust the default crossover and mutation settings (0.5 and 0.1 respectively). However, if you want to fine-tune the algorithm to your problem, conduct comparative studies, or just to experiment, you can try adjusting these two parameters.

Crossover. The crossover rate can be set to between 0.01 and 1.0, and reflects the likelihood that future scenarios or “organisms” will contain a mix of information from the previous generation of parent organisms.

In other words, a rate of 0.5 means that an offspring organism will contain roughly 50% of its variable values from one parent and the remaining values from the other parent. A rate of 0.9 means that roughly 90% of an offspring organism’s values will come from the first parent and 10% will come from the second parent. A Crossover rate of 1 means that no crossover will occur, so only clones of the parents will be evaluated.

The default rate used by RISKOptimizer is 0.5. Once RISKOptimizer has started solving a problem, you can change the crossover rate by using the RISKOptimizer Watcher (see the RISKOptimizer Watcher section in this chapter).

Mutation Rate. The mutation rate can be set to between 0.0 and 1.0, and reflects the likelihood that future scenarios will contain some random values. A higher mutation rate means that more mutations, or random “gene” values, will be introduced into the population. Because mutation occurs after crossover, setting the mutation rate to 1 (100% random values) will effectively prevent the crossover from having any effect, and RISKOptimizer will generate totally random scenarios.

If all the data of the optimal solution is somewhere in the population, then the crossover operator alone is enough to eventually piece together the solution. Mutation has proven to be a powerful force in the biological world for many of the same reasons that it is needed in a genetic algorithm; it is vital to maintaining a diverse population of individual organisms, thereby preventing the population from becoming too rigid, and unable to adapt to a dynamic environment. As in a genetic algorithm, it is often the genetic mutations in animals that eventually lead to the development of critical new functions.

For most purposes, the default mutation setting does not need adjustment. However, you can change it to fine-tune
RISKOptimizer’s performance on complex problems. You might want to boost the mutation rate if RISKOptimizer’s population is fairly homogenous and no new solutions have been found in the last several hundred trials. Typical setting changes are from 0.06 to 0.2. Once RISKOptimizer has started solving a problem, you can change the mutation rate dynamically by using the RISKOptimizer Watcher (see the RISKOptimizer Watcher section later in this chapter).

By selecting Auto from for the Mutation rate, auto-mutation rate adjustment is selected. Auto-mutation rate adjustment allows RISKOptimizer to increase the mutation rate automatically when an organism "ages" significantly, that is, when it has not changed over an extended number of trials. For many models, especially where the “best” mutation rate is not known, the Auto setting is preferred.

RISKOptimizer includes selectable genetic operators when used with the Recipe solving method. Clicking the Operators button in Engine tab allows you to select a specific genetic operator (such as heuristic crossover or boundary mutation) to be used when generating possible values for the adjustable cells. In addition, you can have RISKOptimizer automatically test all available operators and identify the one that performs best for your problem.

Genetic algorithms use genetic operators to create new members of the population from current members. Two of the types of genetic operators RISKOptimizer employs are mutation and crossover. The mutation operator determines whether random changes in “genes” (variables) will occur and how they occur. The crossover operator determines how pairs of members in a population swap genes to produce “offspring” that may be better solutions than either of their “parents.”
RISKOptimizer includes the following specialized genetic operators:

- **Linear Operators.** These are designed to solve problems where the optimal solution lies on the boundary of the search space defined by the constraints. This mutation and crossover operator pair is well suited for solving linear optimization problems.

- **Boundary Mutation.** This is designed to quickly optimize variables that affect the result in a monotonic fashion and can be set to the extremes of their range without violating constraints.

- **Cauchy Mutation.** This is designed to produce small changes in variables most of the time, but can occasionally generate large changes.

- **Non-uniform Mutation.** This produces smaller and smaller mutations as more trials are calculated. This allows RISKOptimizer to fine-tune solutions.

- **Arithmetic Crossover.** This creates new offspring by arithmetically combining the two parents (as opposed to swapping genes).

- **Heuristic Crossover.** This uses values produced by the parents to determine how the offspring is produced. It searches in the most promising direction and provides local fine-tuning.

Depending on the type of optimization problem, different combinations of mutation and crossover operators can produce better results than others. In the Operators tab of the Adjustable Cell Group Settings dialog, when using the Recipe solving method, you can select any number of operators. When multiple selections are made, RISKOptimizer will test valid combinations of the selected operators to identify the best performing ones for your model. After a run, the Optimization Summary worksheet ranks each of the selected operators by their performance during the run. For subsequent runs of the same model, selecting just the top performing operators can lead to faster, better performing optimizations.
VBA macros can be run at different times during an optimization and during the simulation run for each trial solution. This allows you to develop custom calculations that will be invoked during an optimization.

Macros can be executed at the following times during an optimization:

- **At the Start of the Optimization.** The macro runs after the Run icon is clicked, prior to the first trial solution being generated.

- **After Storing Output.** The macro runs after each simulation that is executed and after the statistic that is being optimized for the target cell's distribution is stored.

- **At the End of the Optimization.** The macro runs when the optimization is completed.

This feature allows calculations that can be performed only through the use of a macro to be made during an optimization. Examples of such macro-performed calculations are iterative looping calculations and calculations that require new data from external sources.

The **Macro Name** defines the macro to be run. Macros that you want to run each simulation, or each iteration of each simulation, can be specified in the Macro tab of the @RISK Simulation Settings dialog.
Start Command

Starts an optimization

The Start command (from the RISKOptimizer menu in the Tools group) starts an optimization of the active model and workbook. While RISKOptimizer is running, you will see the following RISKOptimizer Progress window.

The Progress window displays:

- **Iteration.** The number of iterations run so far in the current simulation.
- **Simulation.** The total number of simulations that have been run so far and (if there are constraints) the number of these simulations for which all constraints were met.
- **Runtime.** The elapsed time in the run so far.
- **Original.** The original value of the target cell statistic. This is calculated from an initial simulation run using the existing values in the worksheet for the adjustable cells.
- **Best.** The best value found so far for the target cell statistic being optimized.

During an optimization the status bar in Excel also displays the current progress of the analysis.

Best=$20,130 (Trial #151) Original=$20,048 Trials=1000 (489 Valid) Time=00:00:44
The toolbar at the bottom of the Progress window includes the following options (from left to right):

- **Display Excel Updating Options.** This turns Excel Updating Options on or off. With this option on, the screen will update every iteration.

- **Display RISKOptimizer Watcher.** This displays the RISKOptimizer Watcher window.

- **Run.** This resumes the RISKOptimizer process after it has been paused.

- **Pause.** This allows you to pause the RISKOptimizer process. While it is paused, you can open and explore the RISKOptimizer Watcher and change parameters, look at the whole population, view a status report, or copy a graph.

- **Stop.** This stops the optimization process.

**Graph of Best Solution**

If the **Automatically Display Output Graph** option in the @RISK Simulation Settings dialog is on, then during an optimization @RISK also displays the distribution for the optimization goal cell for each new best solution found.

**RISKOptimizer Watcher**

The magnifying glass icon on the RISKOptimizer Progress window displays the RISKOptimizer Watcher, a multi-tab dialog. RISKOptimizer Watcher is responsible for regulating and reporting all RISKOptimizer activity.

From RISKOptimizer Watcher, you can change parameters and analyze the progress of the optimization. You can also see real-time information about the problem and information on RISKOptimizer’s progress in the status bar across the bottom of RISKOptimizer Watcher.
The Progress tab shows how results are changing, by simulation, for the selected target cell.

Progress graphs show the simulation count on the X-axis and target cell value on the Y-axis. You can right-click either graph to display the Graph Options dialog, where you can customize the graph as you like.
The **Graph Options** dialog displays settings that control the titles, legends, scaling and fonts used on the displayed graph.
When an Efficient Frontier analysis is underway, the Progress tab in the RISKOptimizer Watcher graphically shows how results are changing by optimization (for each possible efficient frontier constraint value), along with the current Efficient Frontier graph itself.

To identify the efficient frontier, RISKOptimizer performs multiple optimizations to find the optimal value for each value of the constraint. Instead of performing these optimizations separately in a sequential manner, RISKOptimizer employs a more efficient algorithm in which the multiple optimizations are performed as part of a single analytical process. Each line in the Progress for Constraining Values graph shows the progress for the optimal value at each possible constraint value.

The Efficient Frontier graph shows the current optimal value across all possible constraint values.
The Summary tab displays a summary table of adjustable cell values tested during the optimization. If the Genetic Algorithm engine is in use, tools for adjusting the crossover and mutation rate for each Adjustable Cell Group in the model are also displayed.

Genetic Algorithm Parameters allow you to change the Crossover and Mutation rates of the genetic algorithm as the optimization is in progress. Any changes made here will override the original setting of these parameters and will take place immediately, affecting the population (or group of adjustable cells) that was selected in the Group Shown field.

We almost always recommend using the default crossover of 0.5. You might try turning it up as high as about 0.4 if you want to find the best solution and are willing to wait longer. You can find more information about these settings earlier in this chapter.
The Log tab displays a summary table of each simulation run during the optimization. The log includes the results for the target cell, each adjustable cell, and all constraints.

From the Show dropdown, you can request a log of All Trials or only those simulations where there was a Progress Step (that is, where the target cell value improved). The log shows a row for each simulation, including:

1. **Elapsed Time.** The start time of the simulation.
2. **Iters.** The number of iterations run.
3. **Result.** The value of the target cell’s statistic being optimized (including penalties for soft constraints)
4. **Output Mean, Output StdDev, Output Min, Output Max.** Statistics of the target cell distribution.
5. **Input columns.** The values in the adjustable cells.
6. **Constraint columns.** Indicating whether the constraints are satisfied.
If the Genetic Algorithm engine is being used, the Population tab is shown. This displays a grid that lists all the variables of each organism (each possible solution) in the current population. These organisms are ranked in order from worst to best. Because this table lists all organisms in the population, the Population Size setting in the RISKOptimizer Settings dialog determines the number of organisms listed here. (The default is 50.) In addition, the first column shows the value of the target statistic for each organism.
If the Genetic Algorithm engine is being used, the Diversity tab is shown. The plot on the Diversity tab assigns colors to adjustable cell values, based on how much the value of a given cell differs across the population of organisms (solutions) that are stored in memory at a given point. (Using the genetic optimization terminology, this is an indication of the diversity that exists in the gene pool.) Each vertical bar in the plot corresponds to one adjustable cell. Horizontal stripes within each bar represent the values of that adjustable cell in different organisms (solutions). The colors of the stripes are assigned by dividing the range between the minimum and maximum value for a given adjustable cell into 16 equal-length intervals; each of the intervals is represented by a different color. For example, if a vertical bar is single-color, the corresponding adjustable cell has the same value for each solution in memory.
When you click the Stop button, the Stopping Options tab is displayed. This dialog also appears as soon as one of the specified stopping conditions has been met. This tab includes the options available for updating your worksheet with the best calculated values for adjustable cells, restoring original values, and generating an optimization summary report.
The **Reports to Generate** options include the following reports. (The same options are also available from the RISKOptimizer menu on the @RISK ribbon.)

- **Optimization Summary.** This summary report contains information such as date and time of the run, the optimization settings used, the value calculated for the target cell, and the value for each of the adjustable cells.
- **Log of All Trials.** This report logs the results of all trial simulations performed. The values shown in magenta indicate that a constraint was not met.
- **Log of Progress Steps.** This report logs the results of all trial simulations that improved the result for the target cell.
Constraint Solver Command

Runs the Constraint Solver

The Constraint Solver command (from the RISKOptimizer menu in the Tools group) enhances RISKOptimizer's ability to handle model constraints. When RISKOptimizer runs an optimization, it is assumed that the initial adjustable cell values satisfy all of the hard constraints, that is, the initial solution is valid. If this is not the case, the algorithm can run many simulations before finding a first valid solution. The problem is that if a model contains multiple constraints, it can be difficult to find an initial valid solution.

Note: The Constraint Solver is useful when optimizing with the Genetic Algorithm, whereas OptQuest optimizations do not generally require the use of the Constraint Solver. Therefore, the description in this section is relevant only for the Genetic Algorithm.

If a RISKOptimizer model contains multiple hard constraints, and the optimization process fails with all solutions invalid, you will be notified so that the Constraint Solver can be run. The Constraint Solver runs an optimization in a special mode, in which the goal is to find a solution that satisfies all the hard constraints. The optimization progress is shown in the same way as in regular optimizations. The Progress Window shows the number of constraints that are met in the original and best solutions.
A button in the Progress window allows you to switch to the RISKOptimizer Watcher. In the Constraint Solver mode the details of optimization progress are available as in regular mode optimizations, in Progress, Summary, Log, Population and Diversity tabs. However, in Constraint Solver mode, the Watcher contains an additional Constraint Solver tab. This tab shows the status of each hard constraint (Met or Not Met) for the Best, Original, and Last solution.

A Constraint Solver optimization stops automatically when a solution meeting all the hard constraints is found; it can also be stopped by clicking a button in the progress window or in the RISKOptimizer Watcher. After a Constraint Solver run, you can choose to keep the Best, Original, or Last solution (from the RISKOptimizer Watcher Stopping Options tab) just like in regular-mode optimizations.

Note there is no need to set up the Constraint Solver before a run. It uses the settings specified in the model. The only difference is the optimization goal, to find a solution that satisfies all the hard constraints.
In the **Stopping Options** tab there is an additional recommended option to **Set Seed to Value Used in this Optimization**. This option is provided because if the random number generator seed is not fixed, then constraints that were met during a Constraint Solver run might not be met during a regular-mode run, even if adjustable cell values are the same (since simulation results depend on the seed). The option is disabled (as it is in the screenshot below) if the seed was fixed in the Simulation Settings dialog before a Constraint Solver run.
Time Series Commands

In statistics, economics, and mathematical finance, a time series is a sequence of observations, typically measured at regularly spaced times, such as every week, every month, or every quarter. Examples of time series are weekly currency exchange rates, the daily closing value of the NASDAQ Composite index, and monthly crude oil prices.

The Time Series section of @RISK provides two types of tools: (1) Fit and Batch Fit tools for fitting various time series processes to historical data, and then projecting these to the future, and (2) a Define tool for simulating data from a selected time series process for use in an @RISK model. The time series results from such a simulation can be viewed with the normal @RISK results options or by using the Time Series Results window.

The Fit and Batch Fit commands are analogous, in a time series context, to the Fit and Batch Fit commands in @RISK’s Distribution Fitting. The Define command is analogous, in a time series context, to @RISK’s Define Distributions command. Whereas Distribution Fitting and Define Distributions are relevant for individual probability distributions, the Time Series commands are relevant for time series processes. These Time Series commands add @RISK functions to your spreadsheet model, just like the Define Distributions command. However, the @RISK time series functions are array functions. They change the cells where your time series forecast is located as a group in each iteration of a simulation. For details on these time series functions, see the section Time Series Functions in this chapter.
Fit Command

Fits a Time Series process to data

@RISK allows you to fit time series processes to your data. This is appropriate when you have a column of historical data that you want to use as the basis for a time series process in your spreadsheet model. For example, you might want to create forecasts of future portfolio values that are based on historical portfolio values.

Fitting times series processes to data using @RISK involves three steps:

1. Define the input data.
2. Specify the time series processes to fit.
3. Run the fit and interpret the results.

**Step 1: Define Input Data**

Select any cell in the time series column you want to fit. Then select Fit from the Time Series menu to see the Data tab of the following dialog.
The range of data, including the variable name at the top, is shown. You can change this range if necessary.

Data requirements for sample data include:

- You must have at least six data values.
- All sample values should fall in the range \(-1\times10^{37} \leq x \leq +1\times10^{37}\).
- There must not be any missing data.

The fitting procedure assumes that the time series is stationary. For example, this implies that the mean and standard deviation are constant through time. The charts in the right section of the dialog (of the series, the autocorrelations, or ACF, and the partial autocorrelations, or PACF), help you check for stationarity visually. If you click the Auto Detect button, @RISK will automatically look for transformations of the data that produce stationarity. These include logarithms, first differences, and seasonal differences. Alternatively, you can ignore Auto Detect and check any transformations you want to apply. If the series is transformed, the transformed series will be fit by the various time series processes. Note that when you transform the data, the charts change automatically to reflect the transformed data.

The auto-detect feature uses a set of heuristics to determine good guesses for the transformations to apply to your data before fitting occurs. However, if you have knowledge about your data, you can override these guesses. For example, if you have stock price data, you should probably use a log transform with single differencing (which takes into account compounded returns) even if this is not the transformation set determined by the auto-detect feature.

The Function option allows you to select Logarithmic or Square Root transformations. Because logarithms require positive numbers and square roots require nonnegative numbers, you can enter a Shift (an additive constant) to avoid illegal values.

The Detrend option allows you to select First Order Differencing or Second Order Differencing. First order differences are differences between successive values. Second order differences are differences between successive first differences.
If you have seasonal data, you can check the Deseasonalize option. There are three options: First Order Differencing, Second Order Differencing, and Additive. You should also enter a Period, such as 4 for quarterly data. Note that Auto Detect checks for seasonality in a complex way (spectral analysis), so even if your data are, say, monthly, it might recommend a period other than 12. In any case, first order differences are differences between observations the period length apart, second order differences are differences between successive first differences, and additive implies that a seasonal correction is added to each observation.

When you fit data to a time series process, the fitted function requires a starting point. If the fitted time series function is used to forecast future values, it is usually appropriate to use the last value of the historical data as a starting value for the forecast. Then you should select Last Value of Dataset from the Starting Point dropdown list. If you want to start the forecasts from the first historical value, you should select First Value of Dataset.

You can select AIC or BIC from the Statistic dropdown list. These are both good criteria for measuring the goodness-of-fit to your data.
**Step 2: Specify Processes to Fit**

Select the **Series to Fit** tab of the Time Series Fitting dialog. It shows the same charts as before, along with a list of time series processes you can try fitting your data to. These include ARMA (autoregressive, moving average) processes, GBM (geometric Brownian motion) and its variations, and ARCH (autoregressive conditional heteroskedasticity) and its variations. Some of these processes might be disabled, due to context. You can check or uncheck as many candidate processes as you like.
Click the **Fit** button to perform the fitting. For each of the time series processes specified in the previous step, @RISK uses maximum likelihood estimates (MLE) of the parameters to achieve the closest match between the time series process and your data. As with any MLE procedure, @RISK can’t guarantee that any of these processes produced your data. It can only identify one or more processes that are most in line with your data. Always evaluate your @RISK results quantitatively and qualitatively, examining both the comparison graphs and statistics before using a result.

The ranking of the processes (with the lowest values best) and a live chart of the time series with future projections and confidence bands is then shown.
The **Toggle Synchronization** button (third from the left at the bottom of the window) switches between synchronizing to the **First Value of Dataset** and the **Last Value of Dataset**. This is the **Starting Point** option in the Fitting setup dialog. This toggle is useful because it is sometimes easier to compare the fit with the original data when it is overlaid, rather than graphed sequentially.

To obtain future forecasts, click the **Write to Cells** button. This leads to a dialog where you should enter a range with as many cells as you want forecasts for.

This enters an *array formula* in these cells with a function such as RiskAR1. These results are live, just like cells with a RiskNormal function, for example, but they change as a group because they are based on an array formula.

Note that if you fit transformed data, the future forecasts will automatically be “untransformed” back to the original units.
Batch Fit Command

Fits several time series simultaneously, and estimates the correlations between them

@RISK allows you to fit time series processes to several times series simultaneously with its Batch Fit tool. This has two advantages over using the Fit tool on each time series separately. First, it is faster. You go through the various steps only once, instead of once per time series. Second, it has the ability to estimate correlations between the time series and use these in its forecasts of future values. This is especially useful for time series that tend to move together, such as commodity prices for oil products.

Batch fitting time series processes using @RISK involves four steps:

1. Define the input data.
2. Specify the processes to fit.
3. Check the report settings.
4. Run the fit and interpret the results.
Select any cell in the time series data set you want to fit. Then select **Batch Fit** from the **Time Series** dropdown list to see the **Data** tab of the following dialog.

**Name, Range**

The range of data, including the variable names at the top, is shown. You can change this range if necessary and supply a meaningful name to the data set. Note that if there is a date variable in the data set, it will be included in the suggested range. You should change the range so that the date variable is not included. (Alternatively, you can separate the date variable from the rest of the data with a blank column.)

Data requirements for sample data include:

- You must have at least six data values for each time series.
- All sample values should fall in the range $-1E+37 \leq x \leq +1E+37$.
- There must not be any missing data.
The fitting procedure assumes that each time series is stationary. For example, this implies that the mean and standard deviation of each time series are constant through time. If you click the Auto Detect button, @RISK will automatically look for transformations of the data to produce stationarity. These include logarithms, first differences, and seasonal differences. Alternatively, you can ignore Auto Detect and check transformation options you would like. If the series is transformed, the transformed series will be fit by the various time series processes. Note that when you select a transformation, each series is transformed in the same way, through first differencing, for example.

The auto-detect feature uses a set of heuristics to determine good guesses for the transformations to apply to your data before fitting occurs. However, if you have knowledge about your data, you can override these guesses. For example, if you have data that are stock prices, you should probably use a log transform with single differencing (which takes into account compounded returns), even if this is not the transformation set determined by the auto-detect feature.

The Function option allows you to select Logarithmic or Square Root transformations. Because logarithms require positive numbers and square roots require nonnegative numbers, you can enter a Shift (an additive constant) to avoid illegal values.

The Detrend option allows you to select First Order Differencing or Second Order Differencing. First order differences are differences between successive values. Second order differences are differences between successive first differences.

If you have seasonal data, you can check the Deseasonalize option. There are three options: First Order Differencing, Second Order Differencing, and Additive. You should also enter a Period, such as 4 for quarterly data. Note that Auto Detect checks for seasonality in a complex way (spectral analysis), so even if your data are, say, monthly, it might recommend a period other than 12. In any case, first order differences are differences between observations the period length apart, second order differences are differences between successive first differences, and additive implies that a seasonal index is added to each observation.
Starting Point
When you fit data to a time series process, the fitted function requires a starting point. If the fitted time series function is used to forecast future values, it is usually appropriate to use the last value of the historical data as a starting value for the forecast. Then you should select Last Value of Dataset from the Starting Point dropdown list. If you want to start the forecasts from the first historical value, you should select First Value of Dataset.

Statistic
You can select AIC or BIC from the Statistic dropdown list. These are both good criteria for measuring the goodness-of-fit to your data.

Step 2: Specify Processes to Fit
Select the Series to Fit tab of the Time Series Fitting dialog. It shows a list of time series processes you can try fitting your data to. These include ARMA (autoregressive, moving average) processes, GBM (geometric Brownian motion) and its variations, and ARCH (autoregressive conditional heteroskedasticity) and its variations. Some of these processes might be disabled, due to context. You can check or uncheck as many candidate processes as you like.
Click the **Report** tab to bring up the following dialog. Here you choose one of the four report placements shown. Also, if you check the Include Correlations option, correlations across the series will be estimated and included in the future forecast formulas.
Step 4: Run the Fit and Interpret the Results

Click the **Fit** button to perform the fitting. For each of the time series processes specified in the previous step, @RISK uses maximum likelihood estimates (MLE) of the parameters to achieve the closest match between the time series process and your data. As with any MLE procedure, @RISK can’t guarantee that any of these processes produced your data. It can only identify one or more processes that are most in line with your data. Always evaluate your @RISK results quantitatively and qualitatively, examining both the comparison graphs and statistics before using a result.

A summary of the results appears on a Time Series Fit Summary sheet. This includes an array formula for the best-fitting process to each series and, if requested, a correlations table. Note that different processes can produce the best fits to different time series. For example, AR1 might provide the best fit to the first series, and MA1 might provide the best fit to the second series. The array formulas below each chart are live, and they include the RiskCorrmat functions for the correlations if you requested correlations. These array formulas (as a group) can be copied to any other place in the workbook if you like. Note that if you fit transformed data, the future forecasts will automatically be “untransformed” back to original units.
The results also include a sheet for each time series. This sheet includes the estimated parameters for each process, as well as the rankings of the processes on the AIC and BIC goodness-of-fit criteria.

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<th>AR1</th>
<th>AR2</th>
<th>ARCH</th>
<th>ARMA</th>
<th>GARCH</th>
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<td>Function</td>
<td>Data Transform</td>
<td>Function</td>
<td>Shift</td>
<td>Defend</td>
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</table>

Parameter #1: μ
Value: 7.812

Parameter #2: σ_μ
Value: 1.983935264

Parameter #3: Ω
Value: 0.939297889

Parameter #4: Σ
Value: 0.939297889

Parameter #5: Ω
Value: 0.939297889
Define Command

Defines a Time Series process to use in a simulation model

If you want random values for a time series in an @RISK simulation model, you can use the Time Series Define tool. Unlike the fitting tools, this requires no historical data. You simply choose one of the available time series processes and its parameters to include it in an @RISK simulation model.

Using the Define Tool requires three steps:

1. Choose a time series process.
2. Specify the parameters of the process.
3. Select the range for the simulated data.

Select Define from the Time Series dropdown list to see a gallery of time series processes, choose the process you want, and click Select Series.
Step 2: Select the Parameters of the Process

Enter the parameters for the selected process. For more information about these parameters, you can hover the mouse over any of the labels on the left, or you can consult the Time Series Functions section of @RISK Help.

Data Transformations

You can check the Data Transformations option to apply a number of possible transformations to your Time Series.
Data Sync

If you want to synchronize the simulated data to historical data on your worksheet, you can check the Data Sync option. Here you can choose one of the types from the Sync Type dropdown list, probably Last value of Dataset, and enter the range for the historical data in the Sync Data box. The effect is that the simulated data will essentially start where the historical data stopped. (The initialization parameters of the time series process will be modified appropriately.)

Step 2: Select the Range for the Simulated Data

Select the range for the simulated data:
When you click OK in the Define Time Series dialog, an array formula is entered in the range you selected. The random data behave just like random data from any other @RISK distribution function, but because of the array formula, all cells now change as a group. (And as usual, all values appear as the mean of the process unless you toggle the @RISK Random/Static Standard Recalc “dice” button on the ribbon to Random.)
Results Command

Views the results for a simulated Time Series function

When you use the Define tool to embed a Time Series function in an @RISK simulation, you can view the simulation results in the standard @RISK results windows, or you can view them in the Time Series Results window.

When you use the standard @RISK results windows, individual elements of the time series can be analyzed using tables or graphs. For example, you can click Browse Results with any time series cell selected.

As another example, you can click Summary to see results for each time series cell.
The **Time Series Results** window provides an overview of the entire time series process. To see it, select **Results** from the **Time Series** dropdown list (after running the simulation). By clicking the backward and forward buttons at the bottom, you can see only selected iterations. Also, you can click the button between the backward and forward buttons to see an animated chart of the iterations.
By clicking the **Overlay** button (third from left), you can overlay results from other simulated time series. The **Rescale Overlay to Current Iteration** button (to the right of the Overlay button) rescales an added overlay, normalizing its Y-scale so it can be compared on the same graph with the original time series.
The Graph Options dialog for Time Series graphs can be displayed by right-clicking the graph. The options available are the same as those for standard @RISK Summary Graphs. You can change the statistic displayed on the center-line and the range of the percentile bands around it. You can also adjust the colors and patterns used for the bands.
Time Series Functions

The Time Series tools add @RISK functions to your spreadsheet, just as does @RISK’s Define Distribution tool. There is a different function for each available Time Series process, such as RiskAR1(D, V, a1, Y0), RiskMA1(D, V, b1, e0) and RiskGBM(D, V). A full description of each available function is included in the @RISK Functions chapter of this manual.

Most @RISK property functions, such as RiskName, may be added to @RISK time series functions, just as with @RISK distribution functions. Some property functions, like RiskTruncate and RiskShift, are specific to distribution functions and are ignored if used in times series functions. In addition, there are several property functions that are specific to time series functions and are used to specify properties for a time series process. For example, the RiskTSSeasonality property function specifies that a time series function will have the specified seasonality applied to the process result.

The @RISK time series functions are array functions as they change the cells where your time series forecast is located as a group each iteration of a simulation. A single time series function is used for the entire range of a time series forecast. As with other Excel array functions, formulas for a cell in the range may not be edited individually.

To edit a time series function directly in your spreadsheet, you need to select the entire range of the forecast where the array function is located as a group each iteration of a simulation. Most of the time this will not be necessary as the @RISK Time Series Fit, Batch Fit and Define tools enter the array functions automatically for you in the range you select.

It is possible to correlate two or more time-series functions using the @RISK Define Correlations window (or manually using RiskCorrmat property functions) just as you would with regular @RISK distribution functions. However it is important to understand that correlation between time-series is fundamentally different from the correlation of standard distributions. A correlation between two time series functions indicates that each iteration the array of values returned by the two time series are subject to the specified correlation coefficient. In contrast, the correlation between two standard @RISK distribution functions requires the entire simulation for that correlation to be apparent.
To understand how @RISK’s correlation is implemented, it is important to realize that time series models generate the value at a particular time based on one or more known values from previous time periods plus a randomly distributed noise term. It is the distributions of noise that obey the correlations you specify.

Be aware that the correlations you specify always apply to the underlying stationary time series model before any transformations (such as exponentiation or integration) are applied. Most commonly you will generate sets of correlated time-series using the time series batch fit command, which as part of its output constructs a correlation matrix. The coefficients in that matrix will be the correlations between the data after all the specified data transformations have been applied to each of the series. For example, if you have two series of data that are stock prices, it is very common to use a log transform and first differencing to convert the raw values into period returns before fitting them. It is these returns, and not the raw series of data, for which the correlation coefficients are calculated.

Some time series functions, the so-called regressive models, have a state of equilibrium to which the series is strongly pulled if appreciable deviations from that equilibrium occur. If you correlate two time series, one or both of which are out-of-equilibrium at the start of the series, the correlation you specify between them will be overwhelmed at the start of the forecast by the need for the series to bring itself back to the equilibrium state. Often you will find the correlations you have specified are only actually realized after a “burn in” period where the series have settled back to equilibrium. (Incidentally, this also means that correlations for the BMMRJD time-series will only be approximate, since every time a jump occurs, its need to recover from that jump will dominate the correlations you have specified.)
Using @RISK with Microsoft Project

@RISK’s Project tools allow you to import a project into Microsoft Excel, introduce uncertainty to the project, and view the results using the graphs, reports and interface of @RISK. @RISK allows you to use the full power of @RISK and Excel, including all distribution functions and correlation capabilities, for projects from Microsoft Project. You simply define the uncertain elements in a project and select the outputs. @RISK does the rest!

Compatibility with Earlier Versions of @RISK for Project

The Project tools are now part of the @RISK add-in; when you install @RISK, you have access to all of the Project tools (assuming you have Microsoft Project installed on your PC). Note, however, that there used to be a separate add-in called @RISK for Project, used exclusively for simulations involving projects. It was separate from “@RISK for Excel,” the add-in now simply called @RISK for general simulations. If you have used @RISK for Project, keep in mind throughout this section that “@RISK” refers to the current add-in, the add-in that is relevant for all simulations, including those involving projects, and that there is no longer a separate @RISK for Project add-in.

Projects developed with @RISK for Project (version 4 and earlier) are supported with @RISK’s Project capabilities. When a project used with earlier versions of @RISK for Project is imported into @RISK, the @RISK elements in that project are converted to their equivalent form in @RISK. Distributions from the @RISK: Functions column in the project are changed to distribution functions in Excel. Global variables, correlations, probabilistic branching, and other @RISK for Project specific features are similarly converted.

Unlike earlier versions of @RISK for Project, @RISK makes no modifications to your project’s .MPP file when risk models are built and run. All information is stored in the Excel workbook that links to the .MPP file.

Why Introduce Uncertainty To A Project?

Suppose you are planning a big job that could span a year or more. Things can and will go wrong but, with @RISK, you can make informed decisions by accounting for high risk elements such as learning curves, inflation, weather, and so on. You can calculate not
only the most likely project end date, but also dates for the best and worst case scenarios. You can also view the uncertainty in cost, project duration, or critical indices. You simply choose any task or resource field in Project as an @RISK output for your simulation. The result is improved decision making backed up by a complete statistical analysis and presentation quality reports and graphics.

**Modeling Features**

@RISK allows you to simulate projects through a unique link between Microsoft Excel and Microsoft Project. @RISK “imports” a project .MPP file into Excel, where it can be enhanced with Excel formulas and @RISK distributions. An Excel workbook becomes a new “view” of your project. It even includes a Gantt chart similar to what you see in Microsoft Project.

Once in Excel, changes to project schedules can be made and relevant dates and costs in the schedule will be updated. This is done by linking the values shown for the project in Excel with relevant tasks and fields in Microsoft Project. Behind the scenes, @RISK passes changed Excel values to Microsoft Project for recalculation and then returns newly calculated values back to Excel. All scheduling calculations are performed in Microsoft Project, but the results of these calculations are shown in Excel.

@RISK allows a far wider scope of modeling capabilities for projects than those available in Microsoft Project alone. For example, Excel formulas can be used to perform calculations on values that will be passed to Microsoft Project. A formula in a cell representing a project task or resource field can contain an @RISK distribution function or an Excel function. This will calculate a value in Excel. The calculated value will then be passed to Microsoft Project for schedule recalculation. Alternatively, values returned from Microsoft Project, such as a Cost calculation, can be referenced in formulas in other cells in Excel.

The entire range of modeling and reporting capabilities of @RISK are available for use with projects. This includes all probability distribution functions, correlations, alternate parameters, sensitivity analyses, and more. You should understand how to use @RISK in standard Excel simulation models before you use it with projects.

Additional modeling capabilities specific to the use of @RISK with projects include:
• **Risk Categories.** These are where common risk definitions can be applied to categories or groups of tasks, such as the duration of all testing tasks.

• **Parameter Entry Tables.** These tables that allow easy entry of parameter values to be set up in Excel.

• **Probabilistic branching.** This allows branching between tasks to change during a simulation based on probabilities you enter.

• **Probabilistic calendars.** With these, working and non-working periods can change based on probabilities you enter.

Additional reporting capabilities specific to the use of @RISK with projects include:

• **Probabilistic Gantt Chart.** This chart displays task critical indices and probabilistic start and finish dates.

• **Timescaled Data Report.** This table displays probabilistic information for timescaled project data, such as cost per time period.

@RISK with projects often requires the use of dates in probability distribution functions. For example, you might want to use a function that describes an uncertain start date for a task or project. To learn more about using dates with @RISK functions, see the section **Dates in Functions** in the chapter **@RISK: Functions** in this manual.

**System Requirements**

To use the Project simulation capabilities of @RISK, a copy of Microsoft Project Version 2003 or higher needs to be installed on your system. This is in addition to the standard requirements for using @RISK. The 64-bit version of Microsoft Project is not supported by the Project simulation capabilities of @RISK.

**Simulation Speed**

@RISK offers two simulation engines to achieve the fastest possible simulation speed when simulating projects. The **Accelerated** engine supports most models and provides the fastest simulations. The **Standard** engine is slower but supports distributions and outputs for all fields in a project. @RISK automatically detects the engine to use for your project.

Recalculation speed has changed between versions of Microsoft Project and this impacts the runtimes of @RISK simulations run with the Standard engine. Recalculations are the fastest in Microsoft
Project 2003 and slowest in Microsoft Project 2007. Microsoft Project 2010 is an improvement over 2007, but it is still substantially slower than Microsoft Project 2003. If you have large projects in which simulation runtime is an issue, you should run the simulation using the Accelerated engine if possible. If the Standard engine must be used, Microsoft Project 2003 is recommended for the fastest possible hardware configuration.

**Using @RISK with Project Schedules**

@RISK allows you to simulate projects through a unique link between Microsoft Excel and Microsoft Project. @RISK “imports” a project contained in an .MPP file into Excel, where it can be enhanced with Excel formulas and @RISK distributions. An Excel workbook becomes a new “view” of your project. It even includes a Gantt chart similar to what you see in Microsoft Project.

When @RISK is used with projects, a Project menu is added to the @RISK ribbon. (This Project menu does not appear unless Microsoft Project is installed on your PC.) The commands on this menu allow you to import project .MPP files into Excel, use Project-specific modeling tools, generate reports, and more.

![Project menu in @RISK](image.png)

Except for the Project menu, using @RISK with project schedules is virtually the same as using @RISK with standard Excel spreadsheet models. If you know @RISK, you’ll know most of what you need to know for conducting a risk analysis of a project. If you are new to @RISK, you should become familiar with it Excel. What you learn will make it easy to model the risk in your projects.

If you don’t have a project open, you can use @RISK as it is normally used on Excel spreadsheet models. Only when you import a project, or open a workbook with a saved project, do the commands on the Project menu become available.
To see some of the project capabilities, you can examine the example files (under the @RISK Help menu) in the Project category. Each example has two files: the Excel file and the corresponding project .MPP file.

When you use @RISK with a project, a copy of Microsoft Project is also running at the same time. The project you are working on in Excel is also open in Microsoft Project. This is because @RISK uses Microsoft Project to perform scheduling recalculations.

In the Excel view of your project, changes to project schedules can be made, and relevant dates and costs in the schedule shown in Excel will be updated when you “sync” Excel with your project. @RISK links the values shown for the project in Excel with relevant tasks and fields in Microsoft Project. Behind the scenes, @RISK passes changed Excel values to Microsoft Project for recalculation and then returns newly calculated values back to Excel. All scheduling calculations are performed in Microsoft Project, but the results of the calculations are shown in Excel.

You can view and change your project in Microsoft Project while using @RISK. If you make changes that affect what is displayed in Excel, @RISK will “sync” up those changes when you select the Sync Now command on @RISK’s Project menu.

**Importing a Project into Excel**

To start building an @RISK simulation model for a project, you first need to open the project .MPP file and import it into Excel. To do this, select the Import .MPP File command from the @RISK Project menu and then select the project you want to display in Excel. You will see the following progress window.

![Reading .MPP File](image)

When importing an .MPP file into Excel, @RISK first opens the selected project in Microsoft Project and then reads task, resource, and field values from the project. It then builds one or more new Excel
worksheets, creating a view of your project in Excel. Separate worksheets are created for project tasks and resources.

When a project developed with earlier versions of @RISK for Project is imported into @RISK (recall that @RISK for Project was a separate add-in), the @RISK elements in that project are converted to their equivalent form in @RISK. Distributions from the @RISK: Functions column in the project are changed to distribution functions in cell formulas in Excel. Global variables, correlations, probabilistic branching and other @RISK specific features are similarly converted. You might see additional worksheets in Excel for other @RISK elements in the imported project, such as sheets for probabilistic calendars, global variables, and correlations.

You can save the Excel workbook with an imported project at any time. When the saved project workbook is reopened, @RISK will automatically open the associated project in Microsoft Project, set up the links between Excel and Microsoft Project, and update Excel with any changes that have been made to the project in the interim. This means that you have to import a project to Excel only once.

**Recalculating the Project by Changing Values in Excel**

Values for a project’s task and resource fields that are shown in Excel can be changed and the associated project values in Excel can be updated. This typically is done by selecting the Sync Now command on the @RISK Project menu. For example, if the cell value for duration of a task is changed, the cell with the finish date of the task (and the start and finish date of successor tasks) will update when Sync Now is
selected. The Gantt chart shown in Excel will also update to reflect the new values. For smaller projects, schedule recalculation can be done automatically, just as Excel does its recalculations. This is selected using the **Project Settings** command on the @RISK Project menu.

![Project Settings dialog box](image)

When a cell value associated a task or resource field is changed, behind the scenes, @RISK passes changed Excel values to Microsoft Project for recalculation and then returns newly calculated values back to Excel.

**Building a Risk Model**

Once a project is displayed in Excel, the standard @RISK modeling tools available for any spreadsheet model in Excel can be used to set up a risk model for the project. For example, you can assign a probability distribution to a cell representing a task or resource field by using @RISK’s Define Distribution window:

*Defining Probability Distributions*
As in other @RISK models, you can also enter the distribution function directly in the formula for the cell of a task or resource field. In either case, the cell formula for the task or resource field will include an @RISK distribution function, such as the formula:

\[=\text{RiskPert}(53.1,59,80,\text{RiskStatic(ProjectFieldVal)})\]

Similarly, a field whose value you want to track as an output from an @RISK simulation, such as the Finish Date or Total Cost of the project, can be designated as an @RISK Output with the @RISK Add Output command.

In this case, the @RISK function RiskOutput is added to the cell formula, such as:

\[=\text{RiskOutput()}+\text{DATE}(2011,5,9)\]

As Excel formulas can be used to calculate the value for a task or resource field, any allowable Excel notation can be used. Formulas can reference cells containing values for other task or resource fields in the project. In the following example, the value for the Duration of one task is given by the result of the formula

\[=\text{NETWORKDAYS(D8,E6)}\]
This formula calculates the difference, in workdays, between the start date of one task and the finish date of a second task.

Calculated project values can also be referenced in other supporting Excel worksheets or workbooks. For example, if you had a workbook with formulas for a cost calculation, it could reference the cell showing the value for the Total Cost of the project.

**Project-Specific Modeling Options**

Most risk modeling for projects in Excel uses standard @RISK modeling tools. However, there are additional @RISK tools available specifically for use with project schedules. These include:

- Risk Categories
- Parameter Entry Table
- Probabilistic Branching
- Probabilistic Calendars
- ProjectFieldVal
- RiskProject functions
**Risk Categories** allow distributions to be assigned quickly to a field for groups of tasks or resources in a project. You can quickly apply a min-max range for all estimates of a field in a set of tasks in your project and then run a simulation of corresponding project outcomes.

Risk Categories allow you to easily change assumptions and re-simulate. For example, you could run a simulation using a –10% to +10% possible swing in duration estimates, and then you could compare those results with a –20% to +20% possible change.

**Categories** are tasks or resources that you want to apply a common risk to. For example, you can vary the duration of a group of all Planning tasks by –10% to +10%, while varying a group of all Training tasks by –30% to +30%. The estimated variation in a field in each of the tasks in a group can be changed at any time by changing the category's definition in the Risk Categories dialog.
To ease data entry, you can create a table in Excel for entering the possible values for a field for tasks or resources. For example, you could have three columns where you enter the minimum, most likely and, maximum possible values for the duration of each task. The Parameter Entry Table dialog will create these columns for you and automatically generate the @RISK distribution functions that reference the values entered in these columns.
Probabilistic branching allows a project to branch from one task to any number of other tasks during a simulation. Each of the task groups that could be branched to has a probability value. For example, upon completion of a Startup task, there might be a 70% chance that the Market Research tasks will follow and a 30% chance that the standard rollout tasks will follow.

In a simulation, after the task for which the probabilistic branching is entered is finished, @RISK will sample a task group to branch to, based on the probabilities entered. It then makes the tasks in the selected group the successors of the finished task and recalculates the project using the new successors.
**Probabilistic Calendars**

Probabilistic calendars allow you to enter non-working probabilities in calendars to be used in a simulation. This accounts for events that might affect the outcome of your project, such as weather conditions during certain seasons. By default, @RISK uses the calendars created in Microsoft Project during a simulation. However, you can model potential circumstances that affect calendars by associating probabilities with specific dates and date ranges. You can apply these probabilities to individual workdays and ranges of workdays. You can also include any nonworkdays in the range as worktime.

![Probabilistic Calendars](image)

**ProjectFieldVal**

The Excel name **ProjectFieldVal** has a special meaning to @RISK when project schedules are open. When used in an Excel formula, this name returns the value of a field directly from Microsoft Project to the related cell in Excel. This is useful to allow @RISK distributions (when a simulation is not running) to return the same value for a field as is shown in Microsoft Project. Otherwise, you might see the mean of a distribution in Excel, which might not be the same as the value in Project. For example, suppose the following @RISK distribution is entered in the cell associated with the Duration field of a task:

=RiskPert(53.1,59,80,RiskStatic(ProjectFieldVal))

The value shown in Excel when a simulation is not running (the "static" value) will be the value entered in the matching Duration field in Microsoft Project.

ProjectFieldVal can also be used to allow a percentage variation around the deterministic estimate entered in the schedule in Microsoft Project. Then even if the value in Microsoft Project is later changed, the same distribution can be used to describe the uncertainty.
@RISK for Excel includes several new functions that can be included in formulas in Excel. (All of these functions begin with **RiskProject**.) These functions make changes to a project schedule during a simulation. They are especially useful when formulas calculated in Excel, such as those in a Risk Register, need to be linked to the logic of a schedule in Microsoft Project.

RiskProject functions include:

- **RiskProjectAddDelay** (*PrecedingTask*, *DelayLength*, *DelayCost*). This function adds a new task to a project after *PrecedingTask* completes. This task has the specified length and cost. You could use this to add an additional task to the project being simulated in iterations when a risk event occurs.

- **RiskProjectAddCost** (*CostToAdd*, *TimeToAdd*). This function adds a new cost to a project at the date given by *TimeToAdd*. You could use this to add an additional cost to the project being simulated in iterations when a risk event occurs.

- **RiskProjectRemoveTask** (*TaskToRemove*). This function removes a task from a project being simulated. You could use this to *not* execute certain tasks in the project being simulated when a risk event occurs.

- **RiskProjectResourceUse** (*Task*, *Resource*, *UsageValue*). This function changes the units of a material resource (or work for a work resource) that is assigned to a task. Costs calculated in Project will reflect the changed usage in each iteration of a simulation.

- **RiskProjectResourceAdd** (*Task*, *Resource*, *Units*). This function assigns a new resource to a task. Costs calculated in Project will reflect the new resource assignment in each iteration of a simulation.

- **RiskProjectResourceRemove** (*Task*, *Resource*). This function removes a resource assigned to a task. Costs calculated in Project will reflect the removed resource assignment in each iteration of a simulation.
Running a Project Simulation

@RISK simulations of projects run just like simulations of standard Excel spreadsheet models. The number of iterations and simulations to be run can be set on the @RISK ribbon. Clicking the Start Simulation button on the ribbon starts the simulation.

A graph of possible results, in this case the Finish Date of the project, is updated as the simulation runs. Once the simulation is completed, all @RISK reports and graphs are available for examining the results. For example, sensitivity analyses and scatter plots identify the key factors impacting a particular output.
Sensitivity Analysis

By default, @RISK uses a **Smart Sensitivity Analysis**, where it pre-screens inputs based on their precedence in the project schedule to outputs. Inputs located in tasks that have no link (via your model’s precedence/successor relationships) to the task of an output are removed from the sensitivity analysis, thus avoiding spurious results. In the Sensitivity Analysis window these unrelated inputs are shown with the entry *n/a*.

There are a couple of limitations to the use of **Smart Sensitivity Analysis** with projects. If you use probabilistic branching, Smart Sensitivity Analysis is disabled. This is done because successor/predecessor relationships change during the run, so @RISK can’t accurately determine precedent tasks before simulation. Smart Sensitivity Analysis is also disabled if you have formula references across tasks within a project, such as when an argument for a distribution in one task references the finish date of another task.

**Project-Specific Reports on Simulation Results**

Simulations of project schedules provide additional reports and statistics not available with non-project simulations. These can be generated from the Project menu: the **Probabilistic Gantt Chart** and the **Timescaled Data** report.
A Probabilistic Gantt Chart shows, by default, the earliest, 10\textsuperscript{th} percentile (P10), and expected start date, and the expected, 90\textsuperscript{th} percentile (P90) and latest finish date for project tasks.

In addition, the report lists the Critical Index for each task, that is, the percentage of time during the simulation that the task fell on the critical path of the project. The critical index lets managers rate the importance of tasks.

The information displayed in the Probabilistic Gantt Chart is customizable using the Probabilistic Gantt Chart settings dialog. You can select the probability values to be displayed in the chart and optionally include sensitivity information as well.
To generate a Probabilistic Gantt Chart, you need to select to collect the data necessary for the report in the Project Settings dialog. This is done by default but does slow down simulations slightly.
Timescaled (or timephased) data is available, by time period, throughout the duration of a project. Many types of timescaled data, including costs, cumulative costs, and work, are available in Microsoft Project. This data is available for both tasks and resources.

@RISK can collect timescaled data during a simulation. With this data, it can generate probability distributions that display a range of possible values for each time period in a project. For example, in addition to a single distribution for the possible Total Cost of a project, you might want to see the distribution for Total Cost in each month or year of a project. The Timescaled Data report provides such information after a simulation.

To report timescaled data, you first need to identify the data you want to collect. To do this, open the Project Settings dialog, and on the Simulation tab, check the Collect Timescaled Data option and click the Data to Collect button to get the following dialog:

Timescaled data can be collected for the entire project or for individual tasks or resources. The Field to Collect can be any Microsoft Project field that is available on a timescaled basis.
Once you have identified the data to collect, you run the simulation. For each iteration, the value of the selected field(s) in each time period of the project is collected. When the simulation is complete, the following report describes the probabilities for the values of the collected timescaled data:

<table>
<thead>
<tr>
<th></th>
<th>Dec-2009</th>
<th>Jan-2010</th>
<th>Feb-2010</th>
<th>Mar-2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>14706.19371</td>
<td>121945.8127</td>
<td>228631.2394</td>
<td>351322.8215</td>
</tr>
<tr>
<td>Cumulative Cost</td>
<td>Minimum</td>
<td>6769.190396</td>
<td>73244.44578</td>
<td>140035.0519</td>
</tr>
<tr>
<td></td>
<td>10 Perct</td>
<td>7864.153846</td>
<td>86370.08547</td>
<td>165517.9487</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>11913.92856</td>
<td>109574.1567</td>
<td>209464.0825</td>
</tr>
<tr>
<td></td>
<td>80 Perct</td>
<td>14703.54837</td>
<td>133515.3416</td>
<td>230207.3768</td>
</tr>
<tr>
<td>Maximum</td>
<td>15653.48384</td>
<td>188268.3460</td>
<td>316654.4115</td>
<td>444064.0412</td>
</tr>
</tbody>
</table>

Graphs are generated that describe the trends for statistics on the collected timescaled data:
Project Commands

When @RISK is used with projects, a Project menu is added to the @RISK ribbon. (This Project menu does not appear unless Microsoft Project is installed on your PC.) The commands on this menu allow you to import project .MPP files into Excel, use Project-specific modeling tools, generate reports, and more.

@RISK includes a set of new names and functions specifically for working with project schedules. These are used to return the current value in a Microsoft Project field to Excel and to make changes to a project schedule during a simulation. For details on these functions, see the Function Reference chapter.
Import .MPP File Command

Reads a project schedule in a Microsoft Project .MPP file and displays the project in Excel

The Project menu Import .MPP File command opens a project .MPP file and imports it into Excel.

When importing an .MPP file into Excel, @RISK first opens the selected project in Microsoft Project and then reads task, resource, and field values from the project. It then builds one or more new Excel worksheets, creating a view of your project in Excel. Separate worksheets are created for project tasks and resources.

The fields imported reflect those present in the active Gantt Chart and Resource table in Microsoft Project. This can be changed in the Import Settings dialog prior to the import. You can show additional fields in Excel by following the instructions displayed when the Insert or Hide Field command on the Project Link menu is selected.
After you select an .MPP file to import, you have the option of reviewing and changing the settings to be used during the import.

A project can be imported into a new workbook or into new worksheets in the active workbook. The **Project Tables** to import specifies the task and resource tables whose fields will be set up in new worksheets in Excel.

If the .MPP was used with @RISK for Project 4.x, @RISK information (such as probability distributions) is located in text fields in the project. You can select to import only @RISK text fields that are displayed in the tables you are importing, or alternatively, have @RISK import all @RISK information found in the project. Use the **Imported Table** option to import task and resource data only from an .MPP file, skipping @RISK 4.x data.

You can save the Excel workbook with an imported project at any time. When the saved project workbook is reopened, @RISK will automatically open the associated project in Microsoft Project, set up the links between Excel and Microsoft Project, and update Excel with any changes that have been made to the project in the interim. This means that you have to import a project to Excel only once.
A conversion log displays any errors or problems encountered during import.

Projects used with @RISK for Project Version 4 and earlier are supported with @RISK’s Project capabilities. When a project developed with earlier versions of @RISK for Project is imported into @RISK, the @RISK elements in that project are converted to their equivalent form in @RISK. Distributions from the @RISK: Functions column in the project are changed to distribution functions in Excel. Global variables, correlations, probabilistic branching, and other @RISK-specific features are similarly converted.
Risk Categories Command

Displays the Risk Categories dialog where risk can be applied to a given field for a set of tasks or resources in a project

The Risk Categories command on the Model Tools menu displays the Risk Categories dialog. This allows distributions to be assigned quickly to a field for groups of tasks in a project. You can apply a min-max range for all estimates for a field in a set of tasks in your project and then run a simulation of corresponding project outcomes.

Categories are tasks or resources that you want to apply a common risk to. For example, you can vary the duration of a group of all Planning tasks by -10% to +10%, while varying a group of all Training tasks by -30% to +30%. The estimated variation in a field in each of the tasks in a group can be changed at any time by changing the category's definition in the Risk Categories dialog.

When Risk Categories are used, RiskVary functions are entered in the cell associated with the selected field for each task or resource in a
category. The syntax of a Vary function entered from the Risk Categories dialog is as follows:

\[=\text{RiskVary(}\text{ProjectFieldVal, Minimum, Maximum, RangeType,\text{,, Distribution)}\]

The Excel name ProjectFieldVal is used in the position of the expected value of the RiskVary function. The value shown in Excel when a simulation is not running (the “static” value) will be the value returned by ProjectFieldVal or the value entered in the matching field in Microsoft Project. ProjectFieldVal allows a percentage variation around the deterministic value entered in the schedule in Microsoft Project. This means that if the value in Microsoft Project is later updated or changed, the same distribution can be used to describe the uncertainty.

The options in the Risk Categories dialog include:

- **Category List.** This section lets you specify the Name of a category that contains the tasks or resources you want to apply a common variation in value to. You can click New to enter a new category, or you can click the name of an existing category to update the dialog with the settings and tasks or resources that belong to that category.

- **Vary Values Using.** This section lets you specify the minimum and maximum possible values for the range to be applied, the type of units that the range is measured in, and the manner in which possible values are distributed across the min-max range.
  - **Distribution.** Selects the type of distribution that will be used to model any field to which the entered risk estimates are applied. The options are Normal, Triang, Trigen, Uniform, and Pert. If the selected distribution takes three arguments (such as Triang), the entered Minimum value is the minimum argument of the distribution, the existing value for the field from the project is the most likely value, and the entered Maximum value is the maximum argument of the distribution.
  - **Range Type.** Specifies the type of range that will be applied and the min and max values for the range. The options for Range Type are a +/- percentage change from the current value for the field or an actual +/- change from the current value for the field.
- **Minimum, Maximum.** The minimum and maximum values for the range.

- **Apply to.** The options in this section allow the selection of the field and the tasks or resources to which the entered risk estimates will be. Selected tasks or resources are added to the selected category.
  - **Field.** The field to which the entered range of possible values will be applied.
  - **Add.** Displays the selector so that tasks or resources can be selected directly from your project.
  - **Delete.** Deletes one or more selected tasks or resources from the list.
  - **Add Marked.** Allows you to select a cell associated with a field that has a value you want to use to identify the tasks or resources for a category. For example, a text field in an imported project could contain the label “Construction” for each task in a “Construction” category. Using **Add Marked**, you could then select a single cell in the text field with the label Construction, and then @RISK would place all tasks with the same label in the category.

An asterisk (*) next to a task or resource name in the list indicates that the current distribution for the selected field in that task or resource does not match the definition of the category. This can happen if you define a category and apply a common variation to all task or resources in the category and then, at a later time, edit a distribution for a member of the category. The next time the Risk Categories dialog is displayed, @RISK will indicate (with an asterisk) that the edited distribution does not match the category definition. If you click OK from the Risk Categories dialog, you will apply the defined +/- change to all members of the category, overwriting the edited distribution.

Clicking the OK button writes the Vary functions generated from the selections in the Risk Categories dialog directly into your project in Excel. Once applied, you can run a simulation that will use the specified risks.
Parameter Entry Table Command

Displays the Parameter Entry Table dialog where columns can be added to a project for entering the possible values for a field

To ease data entry, you can add columns in Excel for entering the possible values for a field for tasks or resources. For example, you could have three columns where you enter the minimum, most likely, and maximum possible values for the duration of each task. The Parameter Entry Table dialog will create these columns for you and automatically generate the @RISK distribution functions that reference the values entered in these columns.

A Parameter Entry Table is typically created when you start using @RISK with a project. Creating a table will overwrite any distributions you have already entered in your project for the selected tasks and field. The new distribution is added to each selected task in your project.
The options in the Parameter Entry Table dialog include:

- **Field.** Selects the field to which the entered distribution and columns of possible values will be applied.

- **Using Distribution.** Selects the type of distribution that will be used for the selected field.

- **Min, Max.** Enters the desired minimum and maximum changes that will be used to calculate a default value in the Parameter Entry table for each task or resource.

  **Note:** The Min and Max entries will be used by @RISK where possible for calculating default parameter values to enter in the table. For some distribution types, it is not possible to use the default minimum or maximum percentage changes when creating argument values in the table. In this case, you will receive default values for the arguments of the selected distribution. You should change these to enter the uncertainty you expect for the field for each task or resource.

- **Build Entry Table for.** Selects the tasks or resources to which the entered distribution and columns of possible values will be applied. If you select a task field, you will be selecting tasks, and if you select a resource field, you will be selecting resources. **All Tasks (or Resources) enters a new distribution for the selected field for every task or resource in your project.**

- **Selected Tasks (or Resources).** Specifies that a new distribution for the selected field will be added to each task or resource that you select with the Add button. The **Delete** button removes selections from the list. **Add Marked** allows you to select a cell associated with a field that has a value you want to use to identify the tasks or resources for the table. For example, a text field in an imported project could contain the label “Construction” for each task in a “Construction” category. Using Add Marked, you could select a single cell in the text field with the label Construction, and then @RISK would place all tasks with the same label in the table.
• Also Add Entry Table to .MPP in Microsoft Project. Selects to add columns in Microsoft Project where the values for the Parameter Entry Table will be displayed. These columns are for text fields starting at the Starting Text Field for Table entry. This option allows values for the Parameter Entry Table to be entered directly in the .MPP file. When the workbook linked to the .MPP file is later opened (or the Sync Now command is selected) the values for Parameter Entry Table located in the .MPP file will be copied to the table in Excel.

Creating a Parameter Entry Table results in new columns being added to your project in Excel. Possible values for the selected field can be entered in these columns.

A typical function in Excel that is used with a parameter entry table is the following:

=RiskTriang(K3,L3,M3,RiskStatic(ProjectFieldVal))

Note: Only one Parameter Entry Table can be used in a single project. Re-entering a Parameter Entry Table causes distributions for an existing table (and the table itself) to be removed.
Probabilistic Branching Command

Displays the Probabilistic Branching dialog that lists probabilistic branching information for the active project.

The Probabilistic Branching command on the Model Tools menu displays the Probabilistic Branching dialog. This dialog specifies probabilistic branches for the active project.

Probabilistic branching allows a project to branch from one task to any number of other tasks during a simulation. Each of the task groups that could be branched to has a probability value. For example, upon completion of a design task, there might be a 10% chance that the bad weather construction tasks will follow and a 90% chance that the standard construction tasks will follow.

**Note:** All tasks for which probabilistic branches are entered should have a default successor task in Project. Probabilistic branches can change this successor task and are applied only during simulation or single-step Monte Carlo recalculations. The default successor task is used in standard, deterministic, Project schedule calculations.

To speed entry of task names, the Add button displays a selection editor that allow you to select tasks in your project to include in a group of tasks to branch to. If tasks are added with the By Group option, the tasks selected are added to a single group, or row in the table. If tasks are added with the All Branches at Once option, each
selected task is placed in its own group, or row in the table. Multiple
tasks can be entered as a group of tasks to branch to. This is done
when you want to branch to a group of tasks, each of which will
become a successor.

- **Probabilistic Branching During a Simulation.** In a
  simulation, after the task for which the probabilistic
  branching is entered is complete, @RISK will sample a task
group to branch to, based on the probabilities entered. It then
makes the tasks in the selected group the successors of the
completed task and recalculates Project using the new
successors.

- **Zeroing Out Unselected Branches.** In any given iteration,
tasks that are not branched to, along with unique successor
tasks of these unused branches, are zeroed out. Values for
the fields for these tasks will be #VALUE as they not used in
the iteration. This keeps resources and costs from being
applied to unused tasks. For a task to be zeroed out, it must
meet one of the following conditions:
  - The task is in an unselected branch, and it has no other
    predecessor tasks except the task where the branch is
    located.
  - The only predecessors of the task are tasks that have been
    zeroed out. That is, this task is a successor to a task
    belonging to an unused branch.

**Distribution Functions for Probabilistic Branches**

When probabilistic branching is defined for a task, @RISK creates a
discrete distribution function in the cell associated with the Successors
field for the task with branching. A typical distribution function
associated with a probabilistic branch would be:

=CHOOSE(RiskDiscrete({1,2,3},{0.7,0.2,0.1},RiskStatic(1),
RiskCategory("Probabilistic Branch")),"2","8","4")

For this task, the possible successor tasks would be tasks 2, 8, or 4.
(Note that these are the Unique IDs for these tasks, not the task IDs).
When a simulation is not running, the value for the first task in the list
will be returned by the function.

**Note:** If you want to execute a task that is not shown in the schedule
during a simulation, you might want to use the
RiskProjectAddDelay function. In each iteration, this function can
optionally add a “new” task, with a cost and duration, following a
task you specify. Thus, when risks occur, new tasks can be executed.
Probabilistic Calendars Command

Displays the Probabilistic Calendars dialog which lists probabilistic calendar information

The Probabilistic Calendars command on the Model menu allows you to enter non-working probabilities in calendars to be used in a simulation. This accounts for events that can affect the outcome of your project, such as weather conditions during certain seasons. By default, @RISK uses the calendars created in Project during a simulation. However, you can model potential circumstances that affect calendars by associating probabilities with specific dates and date ranges. You can apply these probabilities to individual workdays and ranges of workdays. You can also include any non-workdays in the range as worktime.

Non-working probabilities are entered by date range. Any number of date ranges can be set up for a given calendar. A non-working probability can be applied to each day in a given date range, or the entire range as a whole. You can also repeat the use of non-working probabilities outside the date range, reapplying the probability every week or month.

The entries in the Probabilistic Calendar dialog include:

- **View Non-Working % for Calendar.** Selects the project calendar to enter probabilities for.
- **In .MPP.** Selects the open project for which available calendars will be listed. (This is relevant only when multiple projects are open.)
- **Range Name.** A descriptive name for the range of dates for which non-working probabilities will be entered.
• **Start Date, End Date.** The first and last dates in a range of dates for which non-working probabilities will be entered.

• **Non-Working Probability.** The probability that the range or a date in the range will be a non-working day.

• **1 Sample for.** Selects to apply a sampled working/non-working value to each day in the entered range or the entire range as a whole. For example, if you have a 5-day range and you select “1 sample for” each day in the entered range, 5 total samples will be drawn and the working/non-working of each day in the range will be set individually. If you select “1 sample for” an entire range, one sample will be drawn and the working/non-working of all days in the range will be set together based on that sample.

• **Outside Range, Repeat.** Allows the specified range to be automatically repeated each week, month or year until the entered **Repeat Until** date is reached. This is useful when you have an entered working/non-working probability for a range, such as a single Friday. The application of this probability can then be repeated each week, without entering additional ranges.

• **Outside Range, Repeat Until.** Sets the end date for repeating the entered range as specified in the **Outside Range, Repeat** setting.

• **Apply to Non-Work Periods.** Specifies that all dates in the range (including those originally set to non-work periods, such as weekends) will be work periods when the working/non-working state is set.
In each iteration of a simulation, @RISK first uses your entered probabilities to determine whether a given day in the entered date ranges for a given calendar will be a working day or non-working day. All subsequent calculations of Project in the iteration will be made using the new calendars.

Options available for entering Probabilistic Calendars include:

- **Disable Risk for this Calendar.** Turns off the use of non-working probabilities for the selected calendar but leaves the entered probabilities intact. This allows the testing of the impacts of entered non-working probabilities on simulation results.

Clicking the **Apply to All Calendars** button copies the date ranges entered in the current calendar to all calendars defined for the selected project.

Clicking the **Delete Range** button deletes the date range in the selected row. However, although a range is deleted from the dialog, it is not deleted from your project until the OK button is pressed to dismiss the Probabilistic Calendars dialog.
Calculations for probabilistic calendars use distributions on a new worksheet in Excel that is added to the workbook for your project. This sheet, named **Probabilistic Calendars**, contains all necessary distribution functions for calendar calculations.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Calendar Name</td>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Range Name</td>
<td>Entire Project</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Resource Calendar?</td>
<td>FALSE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Start Date</td>
<td>5/4/2010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>End Date</td>
<td>7/15/2010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Apply In Range</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Apply Out of Range</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Apply Out of Range Until</td>
<td>12/31/2010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Apply to Non Work Periods</td>
<td>FALSE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Disable?</td>
<td>FALSE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Non-Working Prob%</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Working Prob%</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Dist #1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Dist #2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Dist #3</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Dist #4</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Dist #5</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Dist #6</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Dist #7</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Dist #8</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Dist #9</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Dist #10</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Standard Gantt Command

Displays the options for the Gantt chart available in Excel for a project schedule

@RISK can display a Gantt chart for a schedule imported into Excel from Microsoft Project. The Standard Gantt command on the Charts and Reports menu displays the Standard Gantt Chart dialog for controlling how these Gantt charts are displayed.

- **Display Gantt Chart in Excel.** By default, Gantt charts are displayed for projects imported by @RISK. If you wish, you can select not to display the Gantt chart. You could do this if you wanted to use the cells in the worksheet that are used by the Gantt chart for formulas that support the calculations of the project in Excel.

- **Layout.** Options that control the dates and units used for the Gantt chart. **Start Date** sets the starting date for the left of the chart. **Timescale Units** control the units (days, weeks, months, years, etc.) used in the chart. **Date Format** sets the format for the dates displayed in the top header for the chart.

- **Display Links/Connectors Between Tasks.** Specifies whether to display the predecessor/successor links between the bars displayed in the Gantt chart. For large projects this setting is off by default.

- **# Columns Between Task Data and Chart.** Allows columns to be inserted between a Gantt Chart and the project task data. These columns can be used to hold formulas and text that you want to link to task data in the project.

**Note:** There is a 3000-task limit to any displayed standard Gantt chart. This limit affects only the graphical display, not the simulation of projects with more than 3000 tasks.
Probabilistic Gantt Command

Displays the options for the Probabilistic Gantt chart available after a simulation has been run on a project schedule

The **Probabilistic Gantt** command on the Charts and Reports menu displays the Probabilistic Gantt Chart dialog for controlling how Probabilistic Gantt charts are displayed and generating the chart.

A Probabilistic Gantt Chart shows, by default, the earliest, 10th percentile (P10), and expected start date, and the expected, 90th percentile (P90) and latest finish date for each project task. In addition, the report lists the Critical Index for each task, or the percentage of iterations during the simulation where the task fell on the critical path of the project. The critical index lets managers rate the importance of tasks.

In each iteration of a simulation, @RISK collects the start and finish dates of each task, plus whether the task fell on the critical path. It then uses this data to calculate the statistics to display in the Probabilistic Gantt Chart.
The items displayed in the Probabilistic Gantt Chart for each task are as follows:

- **Blue bars and connectors.** Show the deterministic schedule for the project. This is similar to what is displayed on a standard Gantt chart.

- **Small solid red bar.** Shows the range between the earliest possible start date and latest possible finish date for each task.

- **Larger lightly shaded red bar.** Shows the range between the specified start date Perc% value (P10 by default) and finish date Perc% value (P90 by default).

- **Red diamond markers.** Shows the mid Start and mid Finish dates.

The information displayed in the Probabilistic Gantt Chart is customizable. You can select the probability values to be displayed in the chart and optionally include sensitivity information as well.
The information displayed in the General tab specifies the information shown in the Probabilistic Gantt Chart. The **Display Start Dates** options specify the simulated start dates to be displayed. Available options include:

- **Perc%**. Selects to display, for each task, the start date associated with the specified percentile value. For example, the 10 Perc% date is the date for which there is only a 10% chance of an earlier start date occurring.

- **Mid**. Selects to display, for each task, the mean or median start date (as calculated from the distribution of possible start dates).

The **Display Finish Dates** options specify simulated finish dates to be displayed. Available options include:

- **Perc%**. Selects to display, for each task, the finish date associated with the entered percentile value. For example, the 90 Perc% date is the date for which there is only a 10% chance of a later finish date occurring.

- **Mid**. Selects to display, for each task, the mean or median finish date (as calculated from the distribution of possible finish dates).

The **Criticality** options specify simulated critical index information to be displayed. Available options include:

- **Display Critical Index**. Selects to label, above the bar for each task in the Probabilistic Gantt chart, the simulated critical index, or the percentage of iterations where a task is on the critical path.

- **Highlight Critical Tasks with Critical Index >**. Selects to highlight tasks in the Probabilistic Gantt Chart that have a critical index above the specified percentage. Bars for these tasks will be shown in yellow.

The **Report Location** option allows you to select whether to place the Probabilistic Gantt Chart in a worksheet in Excel or in a new table in Microsoft Project.
The Tracked Output Tab of the Probabilistic Gantt Chart dialog specifies the output whose sensitivity analysis results will be displayed in the Probabilistic Gantt chart. The type of sensitivity analysis results to be displayed can also be selected. Sensitivity results are reported only for tasks that have input distributions assigned to one or more fields for the task (that is, tasks that contain uncertainty).

The **Name** entry selects the simulation output for which sensitivities will be calculated. All user-selected outputs (identified with RiskOutput functions) are listed.
The options in the **Sensitivity Results** section specify the type of sensitivity analysis results to be displayed:

- **Show on Tasks With Input Risks.** Selects whether to display sensitivity analysis results in the chart. If this option is checked, columns with sensitivity information are added to the data table for the chart.

- **Type.** Selects the type of sensitivity analysis data to display for each task with an input distribution. Available options include **Change in Output Mean,** **Correlation,** **Regression,** or **Cruciality** values. Correlation and Regression are sensitivity analysis methods built into @RISK. For more information on these see the **@RISK Sensitivities Command** in this manual.

Cruciality is a calculated coefficient that combines the critical index and correlation sensitivity coefficient. By multiplying these two values, this index weights the reported sensitivity for a task by the percentage of the time the task is on the critical path.
Timescaled Data Command

Displays the options for the Timescaled Data report available after a simulation has been run on a project schedule

The Timescaled Data command on the Charts and Reports menu displays the Timescaled Data dialog for controlling the information shown in then Timescaled Data report and generating the report itself.

Timescaled (or timephased) data is data that is available, by time period, throughout the duration of a project. Many types of timescaled data, including costs, cumulative costs, and work, are available in Microsoft Project. This data is available for both tasks and resources

@RISK can collect timescaled data during a simulation. It can then use this data to generate probability distributions for each time period in a project. For example, in addition to a single distribution for the possible Total Cost of a project, you might want to see the distribution for Total Cost in each month or year of a project. The Timescaled Data report provides such information after a simulation.

To report timescaled data, you need to select the timescaled data you want to collect prior to running a simulation. The Collect Timescaled Data command from Project Settings allows you to do this. For more information on this command, see its entry later in this chapter.

Once you have selected the data to collect, you can run a simulation. In each iteration, the value of the selected field(s) is collected for each time period of the project. When the simulation is complete, the following report describes the probabilities for the values of the collected timescaled data:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Register and Advanced Modeling.mpp -</td>
<td>Deterministic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative Cost</td>
<td>Minimum</td>
<td>0769.190396</td>
<td>73244.44578</td>
<td>140223.0259</td>
</tr>
<tr>
<td></td>
<td>20 PerC</td>
<td>7846.153846</td>
<td>86370.68547</td>
<td>165517.9487</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>15913.52856</td>
<td>169374.1367</td>
<td>209464.0823</td>
</tr>
<tr>
<td></td>
<td>80 PerC</td>
<td>14792.54507</td>
<td>122515.9416</td>
<td>250297.2788</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>15692.48284</td>
<td>188268.8400</td>
<td>316694.4115</td>
</tr>
</tbody>
</table>
Graphs are generated that describe the trends for statistics on the collected timescaled data:

![Graph](image)

**Timescaled Data Report Dialog**

The Timescaled Data Report dialog provides options for data to be included in a timescaled data report.

![Dialog](image)

Up to six values can be included in the table shown in the report. The **Low Perc% and High Perc%** values specify the percentile values for the data collected for each time period.
• **Deterministic (non-simulated values).** The timescaled values for the project schedule when a simulation is not running. This is the same as if you reported on the same timescaled values in Microsoft Project without @RISK running.

• **Chart Selected Data.** Selects to create charts in Excel showing the trends for the timescaled data over the life of the project. Both **Line Graphs** and **Box-Whisker graphs** are created as standard Excel charts. With the Line Graph, individual line graphs are created, showing the change for each of the selected statistics over the time of the project.
With the Box-Whisker graph, individual box plots are created for each time period in the project. A box shows the range between the selected Low Perc% and High Perc% values. The whiskers extend to the minimum and maximum values for each time period.

**Include Row for Real-Time Timescaled Data.** You can optionally generate, for each time period for selected timescaled data, a complete @RISK distribution during a simulation. When this is selected, a new row is added to the table in the Timescaled Data report. In this row, a RiskOutput function is automatically placed in each cell for each time period. When the simulation is re-run, the distribution displayed for each cell in the row will be the distribution of possible values for the timescaled data in the time period.
All standard @RISK analyses can be used on the generated simulation outputs. Sensitivity analysis can be used to show the critical factors affecting the values in a time period, such as the key drivers of Year 1 Costs. Scatter plots can be displayed that compare the values in a time period with other model inputs and outputs.

**Note:** Once the **Include Row for Real-Time Timescaled Data** option is selected, you must re-run the simulation to obtain complete @RISK distributions on timescaled data. This is because simulations will run slightly slower when complete @RISK distributions are generated, and @RISK does not do this by default. However, once RiskOutput functions are added, each subsequent simulation will generate complete @RISK distributions for timescaled data.

- **Update Automatically After Simulation.** This option instructs @RISK to automatically update the generated report with new values each time a simulation is run. This overwrites the prior report. If you check the **Include Row for Real-Time Timescaled Data** option, this will always occur, so that statistics from @RISK distributions match the data shown in the report.
Schedule Audit Command

Audits a project schedule to check its readiness for risk analysis

The Schedule Audit command checks a project schedule to identify misspecified or incomplete entries that can affect the results of a risk analysis.

Many errors or omissions in a project schedule can impact the results of a simulation. For example, if a predecessor-successor link between two tasks is missing, changes in task durations during simulation might not propagate through a schedule. Constraints can also affect simulation results. For example, if a task has a Start No Earlier Than constraint, simulated schedule changes might not have an impact on the task, because it cannot start earlier than the entered date.

It is important to review and correct problems identified by a schedule audit prior to a risk analysis. Some identified issues might not require change, as they may be needed to accurately schedule the project. Other issues, however, might simply be errors that were made when the schedule was created. These should be corrected.

![Schedule Audit](image)

Schedule constraints may impact a risk analysis. During simulation, delays or early completion in preceding tasks will not change this task's schedule. Change constraint to As Soon As Possible to correct. Note: Constraints are valid when dates are truly fixed.
A Schedule Audit reports the following items found in individual tasks:

1. **Constraints: Start No Earlier Than.**
Description: During simulation, delays or early completion in preceding tasks may not change the schedule of a task with a Start No Earlier Than constraint. Change to a Finish-Start task dependency to correct. Note: Constraints are valid when dates are truly fixed.

2. **Constraints: Finish No Earlier Than**
Description: During simulation, delays or early completion in preceding tasks may not change the schedule of a task with a Finish No Earlier Than constraint. Change to a Finish-Start task dependency to correct. Note: Constraints are valid when dates are truly fixed.

3. **Negative Lags**
Description: No uncertainty can be specified for the length of a negative lag, but in many cases it is uncertain. You may wish to use a Start-Start dependency between the tasks, add a task for the lag itself, and then assign uncertainty to the new task.

4. **No Predecessors**
Description: No predecessor task was assigned. During simulation, simulated changes to schedules will not affect a task without predecessors. Check if task dependencies are missing.

5. **No Successors**
Description: No successor task was assigned. During simulation, simulated schedule changes for a task without successors will not delay other tasks. Check if task dependencies are missing.

6. **Positive Lags**
Description: No uncertainty can be specified for the length of a lag, but in many cases it is uncertain. You may wish to add a task for the lag itself and assign uncertainty to the new task. Click the Options button to automatically convert lags to tasks.

Note: Instead of a lag, you can use the **RiskProjectAddDelay** function to add a delay that follows a task due to a risk event. See the **RiskProject Functions** section of this chapter for more information on this.
7. **Start to Finish Links**

Description: A start to finish dependency is associated with the listed task. Thus, the successor occurs prior to the predecessor. Check whether the task dependency logic is correct.

8. **Task Out of Sequence**

Description: The listed task starts earlier than its predecessor, even though it has a Finish to Start dependency with that task. Check whether the task dependency logic is correct.

The Schedule Audit Options dialog allows you to configure the information reported in a Schedule Audit. You can select the type of errors that are checked and reported.

![Schedule Audit Options Dialog]

You can select to check for any of the issues described in the previous section. You can also fine-tune the checking of lags, using the **Positive Lags with Length >** and **Negative Lags with Length >** options, to identifying lags only over a specified length in days. You also can skip the checking of tasks with small durations using the **Filter Out Tasks with Duration < =** option.

The **Hide Issues Marked as OK** option keeps issues that have been checked as OK (using the checkbox in the first column of the Schedule Audit report) from appearing in a report.

The Navigate button displays the Microsoft Project window and navigates among the tasks with the issues displayed in the Schedule Audit report. This makes it easier to make adjustments to the problem tasks in Microsoft Project to correct the identified issues.
Project Settings Command

Specifies settings for simulating, recalculating and collecting data in Microsoft Project

The Project Settings command specifies settings for recalculating Microsoft Project in a standard Excel recalculation and in an @RISK simulation. In addition, this command identifies the data that will be collected in Microsoft Project when a simulation is run. All Project Settings are stored with your Excel workbook when it is saved.

Simulation Tab – Project Settings Command
Options in the Simulation Tab of the Project Settings dialog include:

- **During Simulation.** These options control the data that is collected in Microsoft Project during a simulation, along with other options that control Project’s calculations in a simulation.
  
  - **Collect Critical Indices.** Requests @RISK to track the critical index value for each task in open projects in each iteration of a simulation. If you have a large project, this option can slow down a simulation. However, you need to use this option if you want to report a critical index for each task after a simulation.
  
  - **Calculate Statistics for Probabilistic Gantt Chart.** Requests @RISK to track the start and finish date for each task in open projects in each iteration of a simulation. If you have a large project, this option can slow down a simulation. This option also allows a Probabilistic Chart to be generated from simulation results.
  
  - **Collect Timescaled Data.** Requests @RISK to collect timescaled data during a simulation. Timescaled (or timephased) data is data that is available, by time period, throughout the duration of a project. Many types of timescaled data, including costs, cumulative costs, and work, are available in Microsoft Project. This data is available for both tasks and resources.

**Note:** You might want to see changes in schedules updated during a simulation in the Microsoft Project window. If you want to see this, you can use the @RISK Simulation Settings command and select the Show Excel Recalculations option. This also cause Microsoft Project to update. You can activate the Project window during a simulation so you can see it change when the simulation is running.
The **Data to Collect** button selects the timescaled data to collect during a simulation.

The **For Project** option lets you select the project to report on.

The **By Time Units** option selects the time units for which data will be collected. With the **Automatic** option, the time units for data collection are the same as the time units shown in the Gantt chart displayed for the project.

The rest of the options specify the actual data to collect during a simulation. Data can be collected for individual tasks or resources, or for the project as a whole. The available **Fields to Collect** are any of the fields in Microsoft Project whose values are time-sequenced. Separate table and charts in the Timescaled Data report are generated for each task or resource field selected.
@RISK offers two simulation engines to achieve the fastest possible simulation speed when simulating projects. The Accelerated engine supports most models and provides the fastest simulations. The Standard engine is slower but supports distributions and outputs for all fields in a project. If the Automatic option is selected or the Check Engine command is selected, @RISK will automatically detect which engine should be used for your project.

The Accelerated Engine allows probability distributions to be assigned to the following fields in a project. Simulation outputs can be assigned to any of these task fields.

<table>
<thead>
<tr>
<th>Task</th>
<th>Resource</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>Standard Rate</td>
</tr>
<tr>
<td>Remaining Duration</td>
<td>Overtime Rate</td>
</tr>
<tr>
<td>Start</td>
<td>Cost Per Use</td>
</tr>
<tr>
<td>Finish</td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td></td>
</tr>
<tr>
<td>Fixed Cost</td>
<td></td>
</tr>
</tbody>
</table>

Probabilistic branching, probabilistic calendars, timescaled data collection, and RiskProject functions can also be used with the Accelerated Engine.

The speed increase obtained with the Accelerated Engine will vary depending on the structure of the project being simulated and the version of Microsoft Project in use. However, most projects will see a significant speed increase.

The Standard Engine supports distributions and outputs for all fields in a project. In addition, if resource constraints that affect activity duration or resource leveling are active during a simulation, the Standard Engine should be used. The Automatic option will detect any differences in simulation results caused by resource constraints or resource leveling and then automatically use the Standard Engine.

You can select either the Standard Engine or Automatic option as the default engine with @RISK’s Application Settings command.
@RISK can perform a check to insure that your project is compatible with the Accelerated Engine. This check happens automatically at the start of a simulation if the Automatic option is selected for Simulation Engine. It also is performed when the Check Engine command is selected. During the compatibility check, a short simulation is run using both the Standard and Accelerated engines, and results are compared. Differences are flagged and reported. Any significant differences will require that the Standard Engine be used. It might be necessary to use the Schedule Audit command to check a project that is incompatible with the Accelerated engine to see where any scheduling issues exist that could affect simulation results.

Once a project has been checked for compatibility, it is marked as compatible and retesting is not automatically performed at the start of a simulation. A project generally needs retesting only when structural changes (such as new tasks or links between tasks) are made to the project in Microsoft Project. @RISK distributions can be added and changed without retesting. However, if you add distributions or outputs to fields not supported by the Accelerated engine, simulations will automatically switch to the Standard Engine.

Because two different simulation engines are used, simulation data will be similar, but not identical, when the same simulation is run in the two engines.

Other differences might be seen in critical path calculations. The Accelerated Engine identifies the critical path as the longest path through the project schedule in each iteration. This is consistent with standard definitions of a schedule’s critical path. Microsoft Project alternatively calculates the critical path as the collection of tasks in a schedule with a nonnegative total float. In general, these methods generate similar results. However, with certain projects there can be differences. If you want to use Microsoft Project’s method of calculating the critical path, you should use the Standard Engine.
Larger projects (10,000 tasks or more) require a longer initialization time at a start of a simulation and longer simulation run times. Several methods can be used to speed up simulation of these projects:

- Use the **Accelerated Engine** if the project supports it.

- As described in the previous section, a project is checked for compatibility with the Accelerated Engine at the start of a simulation or after the **Check Engine** command is selected. Once this is done, you should save your workbook in Excel. @RISK will then be able to skip the compatibility check the next time the workbook and project are reopened in Excel and re-simulated.

- Enable **Smart Sensitivity Analysis** only when necessary. The initialization of Smart Sensitivity Analysis adds to the initialization time at a start of a simulation. You can use disable Smart Sensitivity Analysis in the Sampling tab of the @RISK Simulation Settings dialog.

- Avoid references to the project values on Task and Resource sheets from Excel formulas that are in other sheets and workbooks, or in formulas that are outside the range where project data is kept. These “external” references cause a longer initialization time and require that simulated project values be returned to Excel in each iteration of a simulation, slowing processing. If you need these links, try disabling them while you are setting up the simulation of your project so that your test simulations will run faster. (Note: If you see the message @RISK: Checking for Formulas Dependent on Project Calculations in the Excel status bar during simulation initialization, @RISK has found “external” formula references as described here.)

The **Date Range For Simulation** options control how @RISK simulates a project that is underway. You can select to simulate an **Entire Schedule** or only activities that take place **After the Current Project Date** or **After Project Status Date**. You would select to simulate only activities that take place after the Current Project or Status Date when you are simulating a project that is already underway. In this case, you do not want to have dates and durations for already completed tasks changed during simulation. Probability distributions may have been previously assigned to those tasks, but because the tasks are complete, their schedules should not be varied in the simulation.
If you select to simulate only activities that take place after the Current Project Date or Project Status Date, a task with uncertainty may be partially complete, based on the Current Project Date or Status Date. In this case, entered uncertainty in duration will be prorated across the remaining duration for the task.

By default, when a task’s % Complete is positive, @RISK will prorate any uncertainty in duration across the remaining duration for the task. If you do not want @RISK to prorate duration uncertainty, you can select Ignore % Complete Information. Note that any setting for Date Range For Simulation will override any specific % Complete information for a task. For example, if a task is marked as 50% complete, but its start and finish are before the Current Project Date or After Project Status Date, no duration changes will be applied to the task during simulation.

The General tab options control how Excel calculates Project during a standard Excel recalculation and the linkages between an Excel workbook and its associated .MPP file.
- **Excel Recalculation of Project.** Values for a project’s task and resource fields that are shown in Excel can be changed and the associated project values in Excel will be updated. This typically is done by selecting the **Sync Now** command on the @RISK Project menu. For example, if the cell value for duration of a task is changed, the cell with the finish date of the task (and the start and finish date of successor tasks) will update when Sync Now is selected. The Gantt chart shown in Excel will also update to reflect the new values. For smaller projects, schedule recalculation can be done automatically, just as Excel does its recalculations. Selecting the **Automatic** option turns this on.

- **Linked Project.** Each Excel workbook created from importing a Project .MPP file is “linked” to that .MPP file. This way, changes to the .MPP file will be updated in Excel when the linked workbook is opened or the **Sync Now** command is selected. For example, if a new task is added to a linked .MPP file, that task will be displayed in the Tasks worksheet in Excel after a sync or when the workbook is opened in Excel.

You can change the .MPP file to which an Excel workbook is linked by clicking the **Change...** button. You would do this if you had created a new version of a project and saved it under a different .MPP file name. By changing the link to the new .MPP, you could load changes from the new .MPP into Excel, while maintaining any @RISK functions and Excel formulas you had previously created.
Read Active Project Command

Reads the active project in Microsoft Project and displays it in Excel

The Read Active Project command reads task, resource, and field values from the active project in Microsoft Project. It then builds one or more new Excel worksheets, creating a view of your project in Excel. Separate worksheets are created for project tasks and resources.

The fields imported reflect those present in the active Gantt Chart and Resource view in Microsoft Project. You can show additional fields in Excel by following the instructions displayed when the Project Link Insert or Hide Field command is selected.
Insert or Hide Field Command

Details the steps for inserting a new field in a project displayed in Excel.

The Insert or Hide Field command indicates the steps to be followed to show or hide project fields in columns in the worksheet with the Excel “view” of a project.

New fields from Project can be inserted into the displayed Tasks or Resources worksheet in Excel. Optionally, fields shown in Excel can be hidden from view.
Update Project Filters Command

Updates displayed tasks and resources in Excel to reflect filters entered in Microsoft Project

The Update Project Filters command displays only tasks and resources in Excel that are shown due to filters set in Microsoft Project. To display filters in Excel, you first set the filter in Microsoft Project and then select Update Project Filters.
Sync Now Command

Synchronizes changes made between Microsoft Project and linked Excel workbooks

The Sync Now command transfers changes between Excel and Microsoft Project and updates values displayed in Excel.

It is important to note that:

1. @RISK does not overwrite Excel cells with formulas when syncing changes from Project.

2. @RISK automatically syncs changes from Project when an existing Excel workbook that links to a .MPP file is reopened. This ensures that any changes to the schedule made in Project when @RISK was not running are reflected in Excel.

3. Changes made to Excel cells for field values result in similar changes to those same fields and values in Project itself. When exiting, these changes will be preserved only if the .MPP file is saved. Otherwise, the old field values will remain in the .MPP file. These will be synced back to Excel when the workbook and .MPP file are later reopened.

If you want to review any Project changes prior to updating your spreadsheet, you can select the Display Changes during Sync of Project to Excel option in the Project Settings dialog.

From here, you can review any changes @RISK will make and check the option for the formula you want to use to update Excel.
Library Commands

@RISK Professional and Industrial versions include the @RISK Library. The @RISK Library is a separate database application for sharing @RISK’s input probability distributions and simulation results. It uses SQL Server to store @RISK data.

Different users in an organization can access a shared @RISK Library to access:

- Common input probability distributions that have been pre-defined for use in an organization’s risk models.
- Simulation results from different users.
- An archive of simulations run on different versions of a model.

The @RISK Library is accessed two ways:

- You can click the Library icon on the @RISK ribbon and select Show @RISK Library to display the @RISK Library window. This allows you to review current distributions and stored simulation results. Its Add Results to Library command adds a current simulation result to the Library.
- You can click the Add Distribution to Library icon in the Define Distribution window to add a probability distribution to the Library. Once a distribution is added, it is available to others who use the Library.

Multiple libraries can be accessed from different SQL servers. For example, you might want to keep a local library where you store simulations and distributions for your personal use. A different library could be used to share distributions and results among @RISK users in a workgroup or division. A corporate library could hold common distributions for company-wide assumptions on such variables as future interest rates, prices, and so on.
The @RISK Library includes two types of stored information for @RISK models: **Distributions** and **Results**. These are shown in the two tabs in the @RISK Library window.

**Distributions in the @RISK Library**

The @RISK Library allows the sharing of probability distributions among @RISK users. This is done to insure that all @RISK users in an organization use the same, and most current, definition for common risk inputs that are used in different models. By using the same definitions for key inputs, an organization can insure that all models are run using the same common assumptions. This allows results to be compared from model to model.

@RISK automatically updates all library distributions present in a model each time a simulation is run. This is done with the **RiskLibrary** property function that is present in any input distribution function that is added from the @RISK Library. The RiskLibrary property function includes a special identifier that allows @RISK to use the most recent definition of the distribution from the library, changing the function if necessary. For example, if the Corporate Planning Department has updated the distribution for next year’s Oil Price, your model will automatically use this distribution when you resimulate.
Two different methods can be used for adding probability distributions to the @RISK Library:

- **Adding from the Define Distribution Window.** Any distribution displayed in the Define Distribution window can be added to the @RISK Library. The Add Input to Library icon adds the displayed distribution to the @RISK Library.

- **Entering a Distribution Directly in the @RISK Library.** Clicking the Add button in the Distributions tab in the @RISK Library allows you to define a new distribution and make it available to users who access your library.

The @RISK Library allows you to enter additional information about a distribution you add. Properties of a Library distribution include:

- **Name.** The Name of the distribution
- **Description.** A custom description you can add.
- **Function.** The functional definition of the distribution. This can be edited at any time by those who have write access to the database.
- **Revisions.** Tracks the revisions made to any distribution while it is stored in the library.
Cell References in Library Distributions

Distribution functions that include Excel cell references can be added to the @RISK Library, but this should be done with caution. Typically this would be done only when the library distribution was going to be used locally in the same workbook where it was originally defined. Inserting a library distribution with cell references into a different model might not properly resolve the argument values because the model structure could be different and the specified cell references would not contain the values you expect.

Seeding Library Distributions

Often you will want to add a RiskSeed property function for seeding its random number stream. This insures that each model in which the distribution is used receives the same sequence of sampled values for the library distribution. This ensures that a valid comparison of results from different models with the library distribution can be made.
Graphing a library distribution is done much like the graphing of input distributions in @RISK’s Define Distributions and Model windows. Clicking the Graph icon at the bottom of the Distributions tab selects the type of graph to display for the selected distributions (that is, rows) in the list. Dragging an input off the list to the bottom of the @RISK Library window also generates a graph. Right-clicking a graph displays the Graph Options dialog where graph settings can be entered. The definition of a library distribution can be changed by clicking the Edit button and using the Argument Panel when a graph of distribution is displayed.
The Distribution Tab columns can be customized to select the statistics and information you want to display on the input distributions in the library. The **Columns** icon at the bottom of the window displays the Columns For Table dialog.

Library distributions are added to a model in Excel from either the **Define Distribution** window, @RISK’s **Insert Function** menu, or the **@RISK Library** itself.

The Distribution Palette has a **@RISK Library** tab that lists all distributions available in the library. Clicking one of these distributions selects it and adds it to the displayed cell formula.
To add a distribution to a model in Excel from the Distributions tab in the @RISK Library itself, select the distribution you want to add in the Distributions list and click the **Add to Cell** icon. Then select the cell in Excel where you want to place the function.

@RISK automatically updates all library distributions present in a model each time a simulation is run. This is done with the **RiskLibrary** property function that is present in any input that is added from the @RISK Library. For example, the formula

\[
=RiskNormal(50000,10000,\text{RiskName(”Product Development/2015”),RiskLibrary(5,”8RENDCKN”)})
\]

instructs @RISK to update the definition of this function from the library identified by ”8RENDCKN” at the start of the simulation. This identifier links to a unique library on your system. If the library is not available, @RISK will use the current definition in your model, in this case, RiskNormal(50000,10000).

The @RISK Library allows results from different models and simulations to be stored and compared. In the @RISK Library, results from multiple @RISK simulation runs can be active at any time.

Once results are stored in the Library, overlay graphs can be used to compare results from different runs. For example, you can run a simulation using an initial set of parameters and store that result in the @RISK Library. Then you could change your model in Excel and re-run the analysis, storing the second result in the library. Overlaying the graphs for outputs from the two runs will indicate how your results changed.
You can also sample from an output stored in the @RISK Library in a new simulation in Excel. The @RISK Library can place a RiskResample function in Excel that references the data that was collected for the output and stored in the @RISK Library. This is useful for combining the results of many separate models in a new single simulation or portfolio optimization.
Simulation results are stored in the @RISK Library by selecting Add Result to Library from the Library dropdown on the @RISK ribbon. You can select to store a new simulation in the library or overwrite a currently stored simulation.

When a simulation is placed in the Library, the simulation data and associated Excel workbooks are automatically placed in the @RISK Library. By using the Open Model icon (the yellow “folder” at the bottom of the Results tab), you can re-open any stored simulation (and the workbooks used in that simulation) in Excel. This allows you to quickly “roll back” to a prior simulation and model.

Note: A shortcut for reverting to a prior simulation and its workbooks in Excel is to right-click the list in the Results tab and select the Open Model command.
Graphing a simulation result in the library is done much like the graphing of results in @RISK’s **Results Summary** window. Clicking the Graph icon at the bottom of the Results tab selects the type of graph to display for the selected outputs (that is, rows) in the list. Dragging a result off the list to the bottom of the @RISK Library window also generates a graph. Right-clicking a graph displays the Graph Options dialog, where graph settings can be entered.

To overlay different results, you can drag a result from the list onto an existing graph.
You can sample from an output stored in the @RISK Library in a new simulation in Excel. There are times when you might want to use output distributions from many different simulations as inputs in a new simulation in Excel. For example, you might want to create a portfolio optimization model that uses the output distributions from a set of different models to select an optimal mix of projects or investments. In this case, each possible project or investment in the portfolio has an individual simulation associated with it that is stored in the @RISK Library. The portfolio optimization model will then reference these individual output distributions. It samples from them in each iteration while calculating the results for the portfolio as a whole.

The output distribution from each project or investment becomes an input that can be sampled via the `RiskResample` function. You can place an output in the library into a workbook in Excel using the `Add to Model as Resampled Input` command. When you do this, the data that was collected and stored for the output becomes the data set that is sampled from during the portfolio simulation. This data is stored in the workbook with the portfolio simulation.

The `RiskResample` function that uses a previous output as an input distribution has different options for sampling its referenced data set. You can sample the data in order, randomly sample with replacement, or randomly sample without replacement. However, you will usually use the `Order` option. This preserves the ordering of iteration data from the stored simulations during the combined simulation.

Preserving the ordering of iteration data from the stored simulations is important when the individual simulations share common input distributions. These common distributions often have a `RiskSeed` property function that causes them to return the same sample values in the same order each time they are used. In the portfolio example, each simulation for an individual project or investment will use the same sampled values for the common distributions in each iteration.

If the Order option is not used, inaccurate combinations of the output values from individual projects or investments could be introduced into the combined simulation. For example, suppose you are simulating a portfolio of individual oil and gas projects, and Random, not Order, resampling is used. A given iteration could then resample a value from the output distribution of one project where a high oil price was used and then randomly resample a value from the output distribution of a second project where a low oil price was used. This
would be a combination that could not occur and would lead to inaccurate simulation results for the portfolio.

To enter an output from the library as a resampled input:

1. Select the output distribution you want to resample from in the Results tab of the @RISK Library.

2. Click the Add to Model as Resampled Input icon, or right-click and select Add to Model as Resampled Input.

3. Select the Sampling Method you wish to use: In Order, Random with Replacement, or Randomly without Replacement.

4. Select Update at the Start of Each Simulation if you want to update the data for the output at the start of each simulation. With this option, @RISK will check the @RISK Library at the start of each simulation to see whether the stored simulation for the output has been updated with more current results. This would happen if you overwrote the original simulation stored in the library with a newer version.

Updating is done with the RiskLibrary property function. This function is present in a resampled output that is added from the @RISK Library when the Update at the Start of Each Simulation option is selected. For example the formula

=RiskResample(1,RiskLibraryExtractedData!B1:B100,RiskIsDiscrete(FALSE),RiskLibrary(407,"TB8GKF8C","RiskLibraryLocal"),RiskName("NPV (10%)"))

instructs @RISK to update the data for the output from the library identified by "TB8GKF8C" at the start of the simulation. This identifier links to a unique library on your system. If the library is not
available, @RISK will use the data for the output that was stored in the workbook the last time the data was updated and the workbook was saved.

5. Select **Graph as Continuous Distribution** if you want the resampled data to be graphed continuously (as opposed to a discrete distribution). This is done with a `RiskIsDiscrete(FALSE)` property function entry in the RiskResample function. By default, the RiskResample distribution is a discrete distribution because only values in the referenced data set can be sampled. However, graphing continuously shows graphs in a form that is easier to present to others. **Note:** Selecting Graph as Continuous Distribution has no effect on the values resampled or simulation results.

6. Select the cell in Excel where you want to place the resampled output.

![Excel Sheet](image)

**Technical Notes**
The @RISK Library uses Microsoft SQL Server to store saved simulations and workbooks. Accessing an @RISK Library file is the same as accessing any SQL database. Multiple @RISK Library databases can be open at a single time. By clicking the **Library** icon at the bottom of the @RISK Library window, connections to existing @RISK Library databases can be set up and new databases can be created.
Connecting to an Existing Library

Clicking the Connect button allows you to navigate to a server where SQL is installed and an @RISK Library database is available. Clicking a server name will check that server for available databases.
Clicking the **Create** button allows you to navigate to a server where SQL is installed. You enter a name for the new library in the **Library Name** field and click **Create**. Once created, the library will be available for storing @RISK distributions and simulation results.

The @RISK Library uses **SQL Server Express** as the platform for storage and retrieval of RiskLibrary functions and simulation results. It is Microsoft’s free database product that is based on SQL Server 20xx technology.

SQL Server Express uses the same database engine as the other versions of SQL Server 20xx, but it has several limitations, including a 1 CPU, 1 GB RAM, and a 4 GB database size limit.

Although SQL Server Express can be used as a server product, @RISK also uses it as a local client data store where the @RISK Library data access functionality does not depend on the network.

SQL Server Express can install and run on multiprocessor machines, but only a single CPU is used at any time. The 4 GB database size limit applies to all data files. However, there are no limits to the number of databases that can be attached to the server, and @RISK Library users can create or connect to several databases.

Multiple SQL Server 20xx Express installations can coexist on the same machine along with other installations of SQL Server 20xx.
By default, SQL Server Express installs as a named instance called SQLEXPRESS. We recommend that you use this instance unless other applications have special configuration needs.

You will notice when connecting to or creating databases, or editing RiskLibrary functions, that there are SQL Server Authentication options. For most users, and for all local instances of SQL Server Express, **Windows Authentication** should be adequate. Windows Authentication uses your network credentials to connect to SQL Server as a login. When you log onto your workstation, your password is authenticated by Windows, and these credentials allow you to access SQL Server, as well as other applications on your workstation or network. This does not automatically grant you access to an @Risk Library database, but you should be able to connect to the server.

With SQL Server Authentication, a login name and password are stored inside SQL Server Express. When you attempt to connect using SQL Server Authentication, the login name is checked for a matching one. If a match is found, the password is then checked against the stored value. If this also matches, you are granted access to the server.

SQL Server Authentication allows you to protect your database by granting or denying permissions to specific users or user groups. The details of setting and managing these permissions are normally handled by a database or network administrator, and are not discussed here. These permissions allow you to grant or deny specific permissions to specific users on a database server.

The SA or System Admin account is disabled by default if Windows Authentication is used. Normal users on the machine have almost no privileges on the SQL Server Express instance. A local administrator on the server must explicitly grant relevant permissions for normal users so that they can use SQL functionality.

**Library Capacity**

In SQL Server Express, a single library database can hold approximately 2000 representative simulations with 10 outputs, 100 inputs, and 1000 iterations. Different sized simulations will have different storage requirements. There are no limits to the number of databases that can be attached to the server, and @RISK Library users can create or connect to several databases.
Utilities Group Commands

The Utilities group on the @RISK ribbon has a number of handy tools, including a Swap Out @RISK command for sharing models with non-@RISK users; a Utilities dropdown list with several items, including access to @RISK Application Settings; tools for color-coding your spreadsheet model and providing thumbnail graphs as cell comments; and various Help items.
Swap Out @RISK Command

Swaps @RISK functions in and out of cell formulas and places @RISK graphs in a workbook where @RISK is swapped out.

The Swap Out @RISK command (in the Utilities group) lets you swap @RISK functions in and out of your workbooks. This makes it easy to give models to colleagues who do not have @RISK. If your model is changed when @RISK functions are swapped out, @RISK will update the locations and static values of @RISK functions when they are swapped back in.

@RISK uses the RiskStatic property function to help in its function swap. RiskStatic holds the value that will replace the function when it is swapped out. It also specifies the value that @RISK will return for the distribution in a standard Excel recalculation.

When you swap functions out, you can also include @RISK thumbnail graphs and a summary report that will be displayed in Excel when @RISK is not running. This makes it easy to show input distributions and simulation results to users who do not have @RISK installed on their computers. They can simply open the swapped-out workbook and see your @RISK graphs. For more on thumbnail graphs, see the Thumbnails command in this chapter.

When you click the Swap Out @RISK icon, you can immediately swap out functions using the current swap settings, or you can change the settings to be used.
When functions are swapped out, the @RISK ribbon is disabled, and any @RISK functions you enter will not be recognized. In addition, if you have asked to have @RISK place thumbnail graphs and a summary report in your swapped-out worksheet, you will see @RISK graphs pop up as you hover the mouse over @RISK output and input cells in the workbook (as shown below). The summary sheet displays statistics on simulation inputs and results. Clicking a link in the report will move you to the appropriate @RISK input or output cell in the workbook.

@RISK will automatically swap your functions back in when a swapped-out workbook is opened and @RISK is running. If your workbook was changed after @RISK functions were swapped out, @RISK can report how it will re-insert @RISK functions into your changed model. In most cases, @RISK will be able to automatically handle changes to a workbook when functions are swapped out.

Clicking the Options button in the Swap Out dialog displays the Swap Options dialog. This dialog allows you to specify how @RISK will operate when functions are swapped in and out.

Swap Options are available for:

- **Swap Out** (when @RISK functions are removed)
- **Swap In** (when @RISK functions are returned to your workbook)
Three optional items can be added to a swapped-out workbook. These items make it easy to show input distributions and simulation results to users who do not have @RISK installed on their computers.

- **Thumbnail graphs.** Images of the graphs of input distributions used in the workbook and simulation results for outputs. When a thumbnail graph is added, @RISK uses the most recent edited graph for the @RISK input or output cell. For example, if you last displayed a Tornado graph for an @RISK output cell, that graph will be included in the thumbnail during the swap. This allows you to control what type of graphs you include in a swapped-out workbook.

- **Cell Color.** Color codes @RISK inputs, outputs, and statistics functions in a workbook.

- **@RISK Summary Sheet** A new worksheet that contains a report on inputs and simulation results.
In addition, Swap Out options control the value that will be placed in your workbook where @RISK functions have been removed. When swapping out, the primary value used for replacing an @RISK function is its static value. Typically this is the value in a formula in your model that was replaced by an @RISK function. It is stored in an @RISK distribution with the RiskStatic property function.

If you enter a new distribution using the Define Distribution window, @RISK can automatically store the value you are replacing with a distribution in a RiskStatic property function. For example; if a cell C10 has the value 1000 in it, as shown in the formula =1000, Then by using the Define Distribution window, you can replace this value with a normal distribution with mean 990 and standard deviation 100. The formula in Excel will be as follows, where the original cell value of 1000 has been retained in the RiskStatic property function.

=RiskNormal(990,100,RiskStatic(1000))

If a Static value is not defined (that is, no RiskStatic function is present), a set of different values are available for replacing the @RISK functions value. These are selected in the Where RiskStatic is Not Defined, Use options, and include:

- **Expected Value.** The distribution’s expected or mean value, except for discrete distributions. For discrete distributions, the setting “Corrected” Expected Value will use the discrete value in the distribution closest to the true expected value as the swap value.

- **True Expected Value.** This setting causes the same values to be swapped as the Expected Value option, except for discrete distribution. For these distributions, the true expected value will be used as the swap value, even if the expected value is not one of the possible discrete values in the distribution.

- **Mode.** The distribution’s mode value.

- **Percentile.** A specified percentile value for the distribution.
The **Swap In** options control how @RISK will report changes it will make to your spreadsheet, prior to inserting distribution functions back into formulas. Spreadsheet formulas and values might be changed after @RISK functions are swapped out. When swapping in, @RISK identifies where it should re-insert @RISK functions and, if desired, it will show all the changes it is going to make to your formulas. You can check these changes to make sure @RISK functions are returned the way you want. In most cases, Swap In is automatic, as @RISK captures all changes to static values that were made when functions were swapped out. It also handles moved formulas and inserted rows and columns automatically. However, if formulas where @RISK functions were previously located were deleted after functions were swapped out, @RISK will notify you of the problem formulas prior to swapping functions back in.

Swap In options include:

- **All**. With this option, all changes to be made to the model are reported, even if a formula and swapped-out value were not changed after @RISK functions were swapped out.
• **Only Where Formulas and Static Values Were Modified.** With this option only changes to be made, including a changed static value or formula, are reported. For example, suppose the original @RISK distribution in cell C10 was

\[ =\text{RiskNormal}(990,100,\text{RiskStatic}(1000)) \]

After swap out, the formula would be:

\[ =1000 \]

Next, suppose the value in cell C10 was changed while functions were swapped out to

\[ =2000 \]

Then @RISK would swap the following function back in, updating the static value:

\[ =\text{RiskNormal}(990,100,\text{RiskStatic}(2000)) \]

If the Swap In option **Only Where Formulas and Static Values Were Modified** is selected, @RISK would report this change prior to swapping in.

• **Only Where Formulas Were Modified.** Only changes to be made, that include a changed formula, are reported with this option. For example, suppose the original @RISK distribution in cell C10 was in the formula:

\[ =1.12+\text{RiskNormal}(990,100,\text{RiskStatic}(1000)) \]

After swap out, the formula would be

\[ =1.12+1000 \]

Next, suppose the formula in cell C10 was changed when functions were swapped out to

\[ =1000 \]

Then @RISK would swap the following formula and function back in:

\[ =\text{RiskNormal}(990,100,\text{RiskStatic}(1000)) \]

If the options **Only Where Formulas and Static Values Were Modified** or **Only Where Formulas Were Modified** are selected, @RISK would report this change prior to swapping in.

• **None.** No changes to be made to the model are reported, and @RISK automatically swaps in its recommended change.
@RISK creates a report you can use to preview the changes that will be made to a workbook when swapping functions in. The report includes the **Original (Before Swap)**, the **Original (After Swap)**, the **Current**, and the **Recommended** formulas to be swapped back in.

You can edit the **Recommended** formula to be swapped back in. Alternatively, you can select one of the other displayed formulas to be used when swapping back in. By clicking the **Create Report to Excel** command from the Edit icon at the bottom of the window, you can choose to create an Excel report of the changes made to the model.

If @RISK is running, it will automatically offer to swap in functions when a swapped-out workbook is opened. However, this will not happen if the swapped-out workbook is opened while @RISK’s ribbon is disabled.
Application Settings Command

Displays the Application Settings dialog where program defaults can be set

The Application Settings command (from the Utilities menu) lets you view and change a wide variety of @RISK settings that will be used each time @RISK runs. These include report placement, graph color, displayed statistics, coloring of @RISK cells in Excel, and many others.

All @RISK windows and graphs will update when Application Settings are changed. Therefore, this is an easy way to apply desired changes, across all open windows and graphs during an @RISK session.
Many defaults are self-explanatory, and most reflect settings found in other @RISK dialogs and screens. Defaults that require more clarification include:

- **Percentiles — Ascending or Descending.** Selecting Descending as the Percentiles defaults switches all @RISK statistics reports, targets, and graph x and p values, to display cumulative descending percentiles. By default, @RISK reports percentile values in terms of cumulative ascending percentiles, or the probability that a value is less than or equal to a given x value. Selecting Descending Percentiles causes @RISK to report cumulative descending percentiles, or the probability that a value is greater than a given x value.

Selecting Descending Percentiles also causes @RISK to default to the entry of cumulative descending percentiles when alternate parameter distributions are entered in the Define Distribution window. In this case, the probability of a value greater than the entered value is specified.

- **Insert Static Values.** If set to True, a RiskStatic function will automatically be inserted in @RISK distributions entered from the Define Distribution window. In this case, when an existing value in a cell formula is replaced by an @RISK distribution, the value that was replaced will be included in a RiskStatic property function.

- **Smart Sensitivity Analysis.** Enables or disables Smart Sensitivity Analysis. For more information on Smart Sensitivity Analysis and situations where you might want to disable it, see the Sensitivities Command section of this manual.

- **Show Window List.** By default, the @RISK Window List (displayed when Windows command on the Utilities menu is selected) is displayed automatically when more than five @RISK windows are shown on screen. This option either suppresses the window list, always displays it, or allows it to come up automatically. Note that “windows” in this context refers only to @RISK results windows, not the Excel windows that contain your @RISK models.
• **Color @RISK Function Cells.** If desired, you can apply formatting to cells in your workbook where @RISK inputs, outputs, statistics functions, and optimization variables are located. You can select a color for cell font, border, and/or background. You can also use the **Cell Color** command in the Utilities group to access these options.

• **Preferred Distribution Format.** Specifies the format to be used for @RISK distribution graphs, model inputs, and simulation results. If a specific graph cannot be displayed in the preferred format, this setting is ignored.

• **Number of Delimited Curves.** Sets the maximum number of delimiter bars, shown at the top of the graph, with each bar associated with a curve in the graph.

• **Marked Values.** Sets default markers that will be shown on graphs you display.

• **Number Formatting.** Sets the formatting to be used for numbers displayed on graphs and markers. **Quantities with Units** refers to reported values such as Mean and Standard Deviation that use the units of the graph. **Unitless Quantities** refers to reported statistics such as Skewness and Kurtosis that are not in the units of the values for the graph. Note: If **Currency** format is selected, it is applied only when the Excel cell for the output or input graphed is formatted as Currency.
@RISK’s Application Settings can be saved in a file called **RiskSettings.rsf**. Once saved, this file can be used to set the Application Settings to be used in another installation of @RISK. To do this:

1. Select the Export to File command after clicking the “Save” icon at the bottom of the Application Settings window.
2. Save the file **RiskSettings.rsf**.
3. Place the RiskSettings.rsf in the **RISKxx** folder under the `Program Files\Palisade` (or `Program (x86)\Palisade`) folder on the system where you want to set @RISK’s Application Settings. Typically, you will place the RiskSettings.rsf file in that folder after a new install of @RISK.

If the RiskSettings.rsf file is present when @RISK runs, its application settings will be used and the user will be unable to change them. (The user will still be able to change simulation settings.) The user can change application settings by removing the RiskSettings.rsf file when @RISK isn't running.

The command Import From File can be used to load Application Settings from a **RiskSettings.rsf** not located in the RISKxx folder. Imported settings can be changed as desired, unlike settings used from a RiskSettings.rsf file that is located in the **RISKxx** folder.
Windows Command

Displays the @RISK Windows List

The Windows command (from the Utilities menu) displays a list of all open @RISK windows, and it allows activating, arranging, and closing of these windows. Note that “windows” in this context refers only to @RISK results windows, not the Excel windows that contain your @RISK models.

Double-clicking any window in the list activates it. Any or all windows can be closed by clicking icons with a red Close window icon.
Open Simulation File Command

Opens Simulation Results and Graphs from a .RSK5 File

There might be times when you want to store simulation results in external .RSK5 files as was done in earlier versions of @RISK. You might do this if your simulation is very large, and you do not want to embed that data in your workbook. If you save an .RSK5 file in the same folder, with the same name as your workbook, it will be opened automatically when you open your workbook. Otherwise, you will have to use the Open Simulation File command (from the Utilities menu) to open the .RSK5 file.
Results from simulations (including graphs), can be stored directly in your workbook, in an external .RSK5 file, or in the @RISK Library. Using the Utilities menu Application Settings command, you can also select to have @RISK automatically or never save your simulation results to your workbook. It is important to note that your model, including distribution functions and simulation settings, is always saved when you save your workbook. @RISK’s Excel reports placed in worksheets in Excel are also saved when their Excel workbook is saved. Save Simulation options only affect simulation results and graphs displayed in @RISK graph windows, the Data window, or the Results Summary window.

If desired, @RISK will prompt you to save your simulation results whenever your workbook is saved:

The Options button opens the Save Options dialog where you select how to save results:

![Save @RISK Results dialog](image)

![@RISK Save Options dialog](image)
Options in the **Save Options** dialog include:

- **Workbook Being Saved.** Specifies that @RISK will store all data from the simulation that has been run, including open windows and graphs, in the workbook being saved.

- **External .RSK5 File.** There might be times when you want to store simulation results in external .RSK5 files as was done in earlier versions of @RISK. You might do this if your simulation is very large, and you do not want to embed that data in your workbook. If you save an .RSK5 file in the same folder, with the same name as your workbook, it will be opened automatically when you open your workbook. Otherwise, you will have to use the **Open Simulation File** from the Utilities dropdown list to open the .RSK5 file.

- **Don’t Save.** With this option selected, @RISK will not save simulation results. However, you can always re-run your simulation to view your results again. (This setting doesn’t affect your model. It will be saved, and then when you open the workbook again, you can re-run the simulation to reproduce your results.)

- **Do This Automatically.** This option specifies that you will always save your data to your workbook, or not save results.
Clear @RISK Data Command

Clears the Selected @RISK Data from Open Workbooks

The Clear @RISK Data command (from the Utilities menu) clears the selected @RISK data from all open workbooks.

The following data can be cleared:

- **Simulation Results.** This clears the results of the current simulation from @RISK, as displayed in the active @RISK windows.

- **Settings.** This clears any @RISK settings and the defined Excel names associated with them. This does not clear names entered for @RISK functions; they are stored in cell formulas and not in the Defined Names list in an Excel workbook.

- **Distribution Fitting Definitions.** This clears any definitions of fitted distributions that are shown in the Fit Manager.

- **Spreadsheet Functions.** This removes all @RISK functions from open workbooks, replacing them with their Static value or, if a Static value is not found, the Swap Out value as specified in the Swap Options dialog. However, this is not a Function Swap, as @RISK will not place swap information in your workbook to be used when swapping functions back in, and thus, all model information will be gone.

Selecting all options allow you to remove all @RISK information from open workbooks.
Convert Workbook Command

Converts Workbooks Made with Non-Palisade Software to @RISK Format

The Convert Workbook command (from the Utilities menu) lets you convert spreadsheet models created with Oracle Crystal Ball (Version 7.3 or higher) into @RISK format. To use this feature, however, you must have Crystal Ball and @RISK installed on the same computer.

When you open a Crystal Ball spreadsheet when @RISK is running (and @RISK detects that Crystal Ball is installed), you will automatically be prompted if you want to convert your model.

Conversion Options

At the beginning of the conversion process, you will be presented with the following dialog.

- **Output Workbook Name.** The name of the @RISK workbook that will be created. It will be located on disk in the same folder as the original model.

- **Preserve Assumption Cell Values for Standard Excel Recalculation.** Controls whether the converter will preserve the cell values that are displayed in Excel for assumption (input) cells when a simulation is not running. This is accomplished by adding RiskStatic property functions to all @RISK distribution functions.
• **Remove Crystal Ball Model Definition.** Tells the converter to remove the Crystal Ball model definition as part of the conversion processes. If you uncheck this option, the model will contain both the @RISK and Crystal Ball information.

**The Conversion Summary**  

After a conversion is complete, you will be presented with a summary of the operation, which includes a list of all changes that were made to your workbook, as well an indication of anything that could not be converted or was only partially converted. (A list of limitations is given in the later section *Restrictions and Limitations.*) Read through this list carefully before proceeding; it will indicate areas of your model that might need further attention.

![Conversion Summary](image)

There are two options in this dialog.

• **Add Summary Sheet to Converted Workbook.** If checked, all the information contained in this window will be added to your converted workbook for later reference.

• **Run Comparison Simulations.** If the dialog is dismissed with this button, a set of comparison simulations will be run in both @RISK and in Crystal Ball as an aid in determining the equivalence of the original and the converted models. (See the next section for more information.)
Once the conversion is complete, and if you chose to run a set of comparison simulations, you will see the following dialog.

The only options are the **Number of Simulations** and the **Number of Iterations** to run in the comparison.

When you click OK, Crystal Ball will be launched and run for this many simulations and iterations of the original model, and statistics will be collected for each forecast (output). Then the same process will occur with @RISK and the converted model. Finally, the two sets of statistics are tabulated in a spreadsheet so that you can compare them.
The bottom section of this table shows, for each output and for each simulation, five statistics (mean, standard deviation, skewness, 10th percentile, and 95th percentile.) The averages of these statistics across simulations are shown for each output (the colored values).

If you run at least 10 simulations of 100 iterations each, the comparison utility will perform a two-tailed t-test (with no assumption of equivalent standard deviations) to determine whether it is reasonable to conclude that the @RISK and Crystal Ball statistics are equivalent. If the average statistics are different with a 97.5% significance level, a “warning” is generated, and that statistic is marked in yellow. If the average statistics are different with a 99.5% significance level, a “failure” is generated, and that statistic is marked in red. Given the nature of the t-test under statistical fluctuation, the test can fail even when the @RISK output and Crystal Ball forecast are, in fact, equivalent. For a “warning” this will occur 2.5% of the time. For a “failure” this will occur 0.5% of the time.

In addition, an overall heuristic for the output is generated by combining the results of the t-tests for all five statistics. A score is assigned to the forecast, giving each “warning” a value of 1, and each “failure” a value of 2. The possible outcomes for the heuristic are:
• **PASS:** A score of 0 or 1. The Crystal Ball forecast and @RISK output statistics are considered to be close enough to “pass” the heuristic. These forecasts/outputs are marked in green.

• **WARNING:** A score between 2 and 5. The Crystal Ball forecast and @RISK output statistics are not good enough to “pass” the heuristic, but they are not sufficiently different to “fail.” These forecasts are marked in yellow.

• **FAIL:** A score greater than 6. The Crystal Ball forecast and @RISK output statistics are sufficiently different to “fail” the heuristic. These forecasts are marked in red.

We recommend that you use the comparison tests and heuristic only as a guide. Be sure to carefully examine the statistics of the comparison utility before concluding that your model has been properly or improperly converted. If a model comparison has failed, you might be able to resolve the discrepancy by running more simulations and/or iterations.

**Restrictions and Limitations**

While most Crystal Ball elements can be converted easily, there are a few elements that cannot be converted. Each of these scenarios is marked with an error or warning message in the conversion summary.

• Several lognormal distribution parameter combinations cannot be converted. Lognormals containing a mixture of geometric parameters and percentiles, logarithmic parameters and percentiles, or alternate parameterizations with fixed means and non-fixed locations cannot be converted.

• Student T distributions with a scale factor other than 1, or with percentile arguments, cannot be converted.

• Correlation coefficients that are formulas instead of simple values or references will be converted to static values.

• Currently, only the weighted values and unweighted values “custom” assumptions modes are supported.

• Assumptions and forecasts that are “frozen” will not be converted.

• Assumptions that make references to the same cell they are in must have their cell reference “broken” to avoid a circular reference in the @RISK model.

• Precision settings for forecasts are not converted.

• Forecast filters are not converted.
• Not all CB.GetForeStatFN function options can be converted.
• The CB.GetAssumFN function cannot be converted.
• Crystal Ball decision variables are not converted.
• @RISK cannot call CBBeforeTrial and CBAfterTrial macros.
• Crystal Ball allows you to enter correlations between multiple assumptions, leaving some coefficients undefined (or “blank”). Internally, Crystal Ball uses an undocumented and ad-hoc method to assign non-zero correlation coefficients for these missing values. Because the values Crystal Ball selects for missing coefficients are not the only possibilities, your simulation results are dependent on their arbitrary choice of coefficients. When these “blank” coefficients are encountered by the converter, they are replaced by zeros but marked in red, and a warning is raised in the conversion summary. We recommend you examine these coefficients carefully to decide which values to use.
Unload @RISK Add-In Command

Unloads the @RISK add-in from Excel

The Unload @RISK Add-in command (from the Utilities menu) unloads @RISK, closing all @RISK windows.

Load DecisionTools Add-In Command

Note that there is also a Load DecisionTools Add-In command above the Unload @RISK Add-In command if you have installed the full DecisionTools Suite. This is a handy way to load any of the other add-ins, such as PrecisionTree, while @RISK is running. When you use this option to load other add-ins, @RISK continues to run.
**Color Cells Command**

**Turns coloring of @RISK function cells on and off**

The **Color Cells** command (in the **Utilities** group) lets you color cells in your workbook where @RISK inputs, outputs, statistics functions, and optimization variables are located. This allows you to quickly and easily identify the components of your @RISK model in open workbooks. You can select a color for cell font, border, and/or background.

Once colors are applied to @RISK function cells, cells will be automatically colored (or not) as you enter and remove @RISK functions in the formulas in your spreadsheet.

This command is actually a toggle. If the @RISK cells are already colored, this command removes the coloring.
Thumbnails Command

Turns thumbnail graphs for @RISK function cells on and off

The **Thumbnails** command (in the **Utilities** group) lets you add thumbnail graphs in your workbook where @RISK inputs, outputs, and statistics functions are located. This allows you to quickly see graphs current input distributions and simulation results simply by hovering the mouse over an @RISK cell in your workbook.

A thumbnail graph is a non-editable image of the active graph for an @RISK cell, and it is inserted as a comment in the cell. If you double-click a displayed thumbnail, you can open and edit the graph in either the **Define Distribution** window (for input probability distributions) or the **Browse** window (for simulation results). If you then change the type of graph displayed, add overlays, or otherwise format the graph, the thumbnail for the cell will update with your edits. In addition, if you edit a distribution function in a cell’s formula, the thumbnail for the cell will update to show the new distribution.

Thumbnail graphs are often used with the **Swap Out @RISK** command to display @RISK graphs in Excel when @RISK is not running. This makes it easy to show input distributions and simulation results to users who do not have @RISK installed on their computers. They can simply open the swapped-out workbook and see your @RISK graphs.

**Note:** if you have a pre-existing comment in an @RISK cell, @RISK will not add a thumbnail graph for the cell.
Several options can be used to control the size and location of thumbnail graphs in your workbook.

Options include:

- **Function Types.** Either Outputs and Inputs or just Outputs can be selected for thumbnail graphs.
- **Graph Size.** Controls the size (Small, Medium, or Large) for displayed thumbnail graphs.
- **Only Display in Range.** Show the thumbnail graphs for @RISK cells only in the range of cells selected.
- **Show Statistics Legend.** Specifies that a statistics legend will be included on the right side of the thumbnail graph. This legend includes any customizations added through Application Settings or while editing each individual graph in @RISK.
Help Commands

The commands in the Help menu are straightforward. You can use them to access @RISK help in a number of ways, from the pdf version of this manual to example files and videos. This is also where you can check your @RISK license.
Chapter 7: @RISK Function Reference

Overview

@RISK includes custom functions that can be included in Excel cells and formulas. These functions are used for:

1. **Defining probability distributions** (@RISK distribution functions and distribution property functions).
2. **Defining simulation outputs** (RiskOutput function)
3. **Returning simulation results to your spreadsheet** (@RISK statistics and graphing functions)

This reference chapter describes each of these types of @RISK functions and provides details about their required and optional arguments.

Distribution Functions

Probability distribution functions are used for adding uncertainty to the cells and equations in your spreadsheet model. For example, you could enter the following formula to a cell in your worksheet.

=RiskUniform(10,20)

This specifies that the values for the cell will be generated by a uniform distribution with minimum 10 and maximum 20. This range of values replaces a single fixed value in Excel.

Distribution functions are used by @RISK during a simulation to sample sets of possible values. Each iteration of a simulation samples a new set of values from each distribution function in your worksheet. These values are then used in recalculating your worksheet and generating a new set of results.

As with Excel functions, distribution functions contain two elements, a function name and argument values in parentheses. A typical distribution function is RiskNormal(100,10).
A different distribution function is used for each type of probability distribution. The type of distribution is given by the name of the function. The parameters that specify the distribution are the arguments of the function.

The number and type of arguments required for a distribution function vary by distribution. Some distribution functions require a fixed number of argument. A good example is the RiskNormal function, which always requires two arguments: the mean and the standard deviation. Other distributions requires a variable number of arguments. A good example is the RiskDiscrete function, which requires two arguments, a value and a corresponding probability, for each of the distribution’s possible values.

Like Excel functions, distribution functions can have arguments that reference cells or expressions, such as RiskTriang(B1,1.5*B2,B3). This specifies a triangular distribution with a minimum value from cell B1, a most likely value 1.5 times the value in cell B2, and a maximum value from cell B3.

Distribution functions can also be used in cell formulas, just as are Excel functions. For example, a possible formula is

\[=100+\text{RiskUniform}(10,20)+(1.5\times\text{RiskNormal}(A1,A2))\]

For more about entering distribution functions in Excel, see the Define Distributions section of this manual.

@RISK contains more than 30 types (or families) of distributions to choose from. Unless you know specifically how uncertain values are distributed, it is a good idea to start with some of the simpler (or more recognizable) distribution types, such as uniform, triangular, and normal. If possible, you might want to specify the current cell value as the mean or most likely value of the distribution. The range of the function you are using then reflects the possible variation around the mean or most likely value.

As your models become more complex, you will probably want to choose, or at least experiment with, more complex distribution types to meet your specific modeling needs. You can use this Reference section to help you select and compare distribution types. You can also visit http://www.palisade.com/models/RISKDistributions.asp for Excel example files that describe @RISK’s distribution functions.
A graph of the distribution is often helpful in selecting and specifying
distribution functions. You can use the @RISK Define Distribution
window to display distribution graphs and add distribution functions
to cell formulas. To do this, select the cell where you wish to add a
distribution function and click the Define Distributions icon. For more
information on the Define Distribution window, see the Define
Distributions section in this manual.

Once you better understand the syntax of distribution function
arguments, you can enter the distributions directly in Excel,
bypassing the Define Distribution window.

@RISK (Professional and Industrial versions only) allows you to fit
probability distributions to your data. The distributions from a fit are
then available as input distributions for your spreadsheet model. For
more information on distribution fitting see the Distribution Fitting
section in this manual.

Optional arguments to distribution functions can be entered using
Distribution Property functions. These optional arguments are used
to name an input distribution for reporting and graphing, truncate the
sampling of a distribution, correlate the sampling of a distribution
with other distributions, or keep a distribution from being sampled
during a simulation. These arguments are not required, but they can
be added as needed. Also, a distribution function can include multiple
property functions.

Optional arguments specified with @RISK distribution property
functions are embedded inside a distribution function. Distribution
property functions are entered, just as are standard Excel functions,
and can include cell references and mathematical expressions as
arguments. These property functions are always the last arguments of
a distribution function.

For example, the following function truncates the normal distribution
to a range with a minimum value of 0 and a maximum value of 20:

```
=RiskNormal(10,5,RiskTruncate(0,20))
```

In this case, there won’t be any sampled values below 0 or above 20.
Supplemental functions such as RiskTNormal, RiskTExpon and RiskTLognorm were used in versions of @RISK prior to version 4.0 to truncate distributions such as normal, exponential, and lognormal. These distribution functions can still be used in newer versions of @RISK; however, their functionality has been replaced by the RiskTruncate distribution property function, a more flexible implementation that can be used with any probability distribution. Graphs of these older functions are not displayed in the Define Distribution window. However they are shown in the Model window and can be used in simulations.

Many distribution functions can be specified with desired percentile values. @RISK refers to these as alternate parameters. For example, you might want to specify a normal distribution with a 10\textsuperscript{th} percentile of 20 and a 90\textsuperscript{th} percentile of 50. In this case, it might be easier to assess the percentiles than the usual arguments, the mean and standard deviation.

When you want to use alternate parameters, you should use the Alt form of the distribution function, such as RiskNormalAlt or RiskGammaAlt.

Each parameter to an alternate parameter distribution function requires a pair of arguments in the function. Each pair of arguments specifies:

1) The type of parameter being entered
2) The value for the parameter.

Each argument in a pair is entered directly in the Alt function, such as RiskNormalAlt(arg1type, arg1value, arg2type, arg2value). For example, the following function specifies a normal distribution with a 5\textsuperscript{th} percentile of 67.10 and a 95\textsuperscript{th} percentile of 132.89:

\[=\text{RiskNormalAlt}(5\%, \, 67.10, \, 95\%, \, 132.89)\]
Alternate parameters can be percentiles or standard distribution arguments (or a combination of these). If the type of parameter argument is a label in quotes (such as “\texttt{mu}”), the parameter specified is the standard distribution argument with that name. This allows percentiles to be mixed with standard distribution arguments. For example, the following formula specifies a normal distribution with mean 100 and a 95$^{\text{th}}$ percentile of 132.89:

$$\texttt{=RiskNormalAlt("mu", 100, 95\%, 132.89)}$$

The allowable names for the standard arguments of each distribution can be found in the heading for each function in this chapter, in the Excel Function Wizard in the @RISK Distrib (Alt Param) category, or in the Define Distribution window.

\begin{mdframed}
Note: You can specify Alternate Parameters under the Parameters option for a specific distribution in the Define Distribution window. If your parameters include a standard argument and you click OK, @RISK will write the appropriate name for the standard argument in quotes in the function.
\end{mdframed}

If a type of parameter argument is a value between 0 and 1 (or 0\% to 100\%), the parameter specified is the entered percentile for the distribution.

Some distributions have an additional location parameter when they are specified using alternate parameters. This parameter is typically available for distributions that do not have a location value specified in one of their standard arguments. Location is equivalent to the minimum or 0 perc\% value of the distribution. For example, the Gamma distribution does not have a location value specified through its standard arguments, so a location parameter is available. The normal distribution, on the other hand, does have a location parameter in its standard arguments (the mean), so it does not have a separate location parameter when using alternate parameters. The purpose of this “extra” parameter is to allow you to specify percentiles for shifted distributions, such as a three-parameter Gamma distribution with a location of 10 and two percentiles.

During a simulation @RISK determines the appropriate distribution with percentile values equal to those specified by the alternate parameter, and then it samples from that distribution. Just like all @RISK functions, the specified arguments can be references to other cells or formulas. Also, argument values are allowed to change from iteration to iteration during a simulation.
Alternate percentile parameters to probability distributions can be specified in terms of cumulative *descending* percentiles, as well as the standard cumulative *ascending* percentiles. Each of the Alt forms for probability distribution functions (such as `RiskNormalAlt`) has a corresponding AltD version (such as `RiskNormalAltD`). If the AltD version is used, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.

If you select the **Descending Percentiles** option in @RISK’s Application Settings dialog, all @RISK reports will show cumulative descending percentile values. In addition, when you select the Alternate Parameters option in the Define Distribution window to enter distributions using alternate parameters, cumulative descending percentiles will automatically be shown and AltD forms of probability distribution functions will be entered.

In addition to cumulative descending percentiles for alternate parameters distributions, the @RISK cumulative probability distribution (`RiskCumul`) can also be specified using cumulative descending percentiles. To do this, use the function `RiskCumulD`.
Distribution functions with varying numbers of arguments (such as RiskHistogram, RiskDiscrete, and RiskCumul) require that some of the arguments be entered as arrays. Arrays in Excel are denoted by either enclosing the values of the array in {} brackets or by referencing a contiguous range of cells such as A1:C1. If a function takes a varying number of value/probability pairs, the values will be one array and the probabilities another. The first value in the value array is matched with the first probability in the probability array and so on.

However, you cannot list cell references or names in arrays as you would list constants. For example, you cannot use {A1,B1,C1} to represent the array containing the values in cells A1, B1, and C1. Instead, you must use the cell range reference A1:C1 or enter the values of those cells directly in the arrays as constants, such as {10,20,30}.
@RISK supports the entry of dates in distribution functions, and the display of graphs and statistics using dates. A `RiskIsDate(TRUE)` property function instructs @RISK to display graphs and statistics using dates. @RISK will also display dates in the Distribution Argument panel in the Define Distribution window when date formatting is enabled. You can specify that date formatting should be used for a distribution by selecting **Date Formatting** in the **Parameters** window of the Distribution Argument panel, or enabling Date Formatting in the **Input Properties** dialog. Any of these selections will result in a RiskIsDate property function being placed in your distribution.

Typically date arguments to @RISK distribution functions are entered with references to cells where the desired dates are entered. For example, the following function could reference the date 10/1/2015 in cell A1, 1/1/2016 in cell B1 and 10/10/2016 in cell C1:

```
=RiskTriang(A1,B1,C1,RiskIsDate(TRUE))
```

Date arguments entered directly in @RISK distribution functions must be entered by using an Excel function that converts a date to a value. Several Excel functions are available to do this. For example, the function for a triangular distribution with a minimum value of 10/1/2015, a most likely value of 1/1/2016, and a maximum value of 10/10/2016, is entered as:

```
=RiskTriang(DATEVALUE("10/1/2015"),DATEVALUE("1/1/2016"),
DATEVALUE("10/10/2016"),RiskIsDate(TRUE))
```

Here the Excel DATEVALUE function is used to convert the entered dates to values. The following function uses the Excel DATE and TIME functions to convert the entered dates and times to values:

```
=RiskTriang(DATE(2015,10,4)+TIME(2,27,13),DATE(2015,12,29)+
TIME(2,25,4),DATE(2016,10,10)+TIME(11,46,30),RiskIsDate(TRUE))
```

The advantage of this approach is that the entered dates and times will convert properly if the workbook is moved to a system with different **dd/mm/yy** formatting.

Not all arguments for all functions can logically be specified with dates. For example, functions such as `RiskNormal(mean,stddev)` support a mean entered as a date but not a standard deviation. The **Distribution Argument** panel in the Define Distribution window shows the type of data (dates or numeric) that can be entered for each distribution type when date formatting is enabled.
Some @RISK functions have optional arguments, or arguments that can be used but are not required. For example, the RiskOutput function has only optional arguments. You can use it with 0, 1, or 3 arguments, depending on the information you want to define about the output cell where the function is used. You can do any of the following:

- Just identify the cell as an output, letting @RISK automatically generate a name for you, that is, =RiskOutput().
- Give the output a name you select, such as =RiskOutput("Profit").
- Give the output a name you select and identify it as part of an output range, such as =RiskOutput("Profit","Profit By Year",1).

Any of these forms of the RiskOutput function are allowed because all of its arguments are optional.

When an @RISK function has optional arguments you can add the optional arguments you choose and ignore the rest. However, you must include all required arguments. For example, the mean and standard deviation arguments of the RiskNormal function are required. However, all of the arguments that can be added via distribution property functions are optional and can be entered in any order you like.

In general, the following rules apply:

- Distribution functions with a fixed numbers of arguments will return an error value if an insufficient number of arguments are entered, and it will ignore extra arguments if too many are entered.
- Distribution functions will return an error value if arguments are of the wrong type (number, array, or text).
The @RISK time series functions are **array functions**. This implies that they change the cells where your time series forecast is located *as a group* in each iteration of a simulation. A single time series function is used for the entire range of a time series forecast. As with other Excel array functions, formulas for a cell in the range cannot be edited individually.

To edit a time series function directly in your spreadsheet, you need to select the entire range of the forecast where the array function is located, edit the formula, and press Ctrl+Shift+Enter (all three keys at once) to enter the formula. However, this is not usually necessary because the @RISK Time Series Fit, Batch Fit, and Define tools enter the array functions automatically in the range you select.

**Simulation Output Functions**

Output cells are designated with the RiskOutput function. This function allows easy copying, pasting, and moving of output cells. A RiskOutput function is automatically added to the selected cell’s formula when you click the @RISK Add Output icon. RiskOutput functions optionally allow you to name your simulation outputs and add output cells to output ranges. A typical RiskOutput function might be the following:

```excel
=RiskOutput("Profit")+NPV(.1,H1:H10)
```

In this case, the cell formula prior to its selection as a simulation output contained the formula

```excel
= NPV(.1,H1:H10)
```

The added RiskOutput function designates the cell as a simulation output and gives the output the name *Profit*.

**Simulation Statistics Functions**

@RISK statistics functions return a desired statistic on simulation results or an input distribution. For example, the function `RiskMean(A10)` returns the mean of the simulated distribution for cell A10. These functions can be updated real-time as a simulation is running or just at the end of a simulation (faster).

@RISK statistics functions include all standard statistics plus percentiles and targets. For example, `=RiskPercentile(A10,.99)` returns the 99th percentile of the simulated distribution of the output in cell A10. @RISK statistics functions can be used the way you use any standard Excel function.
Theo Functions for Statistics of an Input Distribution

@RISK statistics functions that return a desired statistic on a simulation input distribution have the identifier Theo in the function name. For example, the function RiskTheoMean(A10) returns the mean of the probability distribution in the cell A10. If multiple distribution functions are present in the formula for a cell referenced in a RiskTheo statistics function, @RISK returns the desired statistic on the last calculated function in the formula.

Note that the Theo function do not require a simulation to provide their results. The return the exact (or theoretical, hence the name) statistics of the probability distribution.

Calculating Statistics on a Subset of a Distribution

@RISK statistics functions can include a RiskTruncate or a RiskTruncateP property function. This will cause the statistic to be calculated on the range specified by the truncation limits. Note: The values returned from @RISK statistics functions only reflect the range set with a RiskTruncate or a RiskTruncateP property function entered in the statistics function itself. Filters set for simulation results and shown in @RISK graphs and reports do not impact the values returned by @RISK statistics functions.

Statistics in Report Templates

Statistics functions can also reference a simulation output or input by name. This allows them to be included in templates used to generate pre-formatted reports in Excel on simulation results. For example, the formula =RiskMean("Profit") will return the mean of the simulated distribution for the output cell named Profit defined in a model.

Note: A cell reference entered in a statistics function does not have to be a simulation output identified with a RiskOutput function.

Graphing Function

A special @RISK function RiskResultsGraph will automatically place a graph of simulation results, wherever it is used, in a spreadsheet. For example, the formula =RiskResultsGraph(A10) will place a graph of the simulated distribution for cell A10 directly in your spreadsheet at the function's location at the end of a simulation. Additional optional arguments to RiskResultsGraph allow you to select the type of graph you want to create, its format, its scaling, and other options.

Supplemental Functions

Additional functions such as RiskCurrentIter, RiskCurrentSim, and RiskStopSimulation are provided for use in the development of macro-based applications using @RISK. These functions return the current iteration and current simulation, respectively, of an executing simulation, or stop a simulation.
Table of Available Functions

This table lists the custom functions that are added to Excel by @RISK.

<table>
<thead>
<tr>
<th>Distribution Function</th>
<th>Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskBernoulli(p)</td>
<td>bernoulli distribution with probability of success p</td>
</tr>
<tr>
<td>RiskBeta(alpha1, alpha2)</td>
<td>beta distribution with shape parameters alpha1 and alpha2</td>
</tr>
<tr>
<td>RiskBetaGeneral(alpha1, alpha2, min, max)</td>
<td>beta distribution with defined minimum and maximum, and shape parameters alpha1 and alpha2</td>
</tr>
<tr>
<td>RiskBetaGeneralAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value, arg4type, arg4value)</td>
<td>beta distribution with four parameters named arg1type to arg4type which can be either “alpha1”, “alpha2”, “min”, “max”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskBetaSubj(minimum, most likely, mean, maximum)</td>
<td>beta distribution with defined minimum, maximum, most likely value, and mean</td>
</tr>
<tr>
<td>RiskBinomial(n, p)</td>
<td>binomial distribution with n trials and probability of success p</td>
</tr>
<tr>
<td>RiskChiSq(v)</td>
<td>Chi-Square distribution with v degrees of freedom</td>
</tr>
<tr>
<td>RiskCompound(dist#1 or value or cellref, dist#2, deductible, limit)</td>
<td>sum of a number of samples from dist#2, where the number of samples drawn from dist#2 is given by the value sampled from dist#1 or by value or cellref. Optionally, deductible is subtracted from each dist#2 sample, and if (dist#2 sample - deductible) exceeds limit, dist#2 sample is set equal to limit.</td>
</tr>
<tr>
<td>RiskCumul(minimum, maximum, X1, X2, ..., Xn, {p1, p2, ..., pn})</td>
<td>cumulative distribution with n points between minimum and maximum, with cumulative ascending probability p for each X value</td>
</tr>
<tr>
<td>RiskCumulD(minimum, maximum, X1, X2, ..., Xn, {p1, p2, ..., pn})</td>
<td>cumulative distribution with n points between minimum and maximum, with cumulative descending probability p for each X value</td>
</tr>
<tr>
<td>RiskDiscrete(X1, X2, ..., Xn, {p1, p2, ..., pn})</td>
<td>discrete distribution with n possible values (X’s) and corresponding probabilities (p’s)</td>
</tr>
<tr>
<td>RiskDoubleTriang(min, m.likely, max, p)</td>
<td>double triangular distribution with minimum, m.likely, and maximum values, along with the probability p of a value falling between min and m.likely</td>
</tr>
<tr>
<td>RiskDuniform(X1, X2, ..., Xn)</td>
<td>discrete uniform distribution with n possible values, the X’s</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
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<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>RiskErf($h$)</td>
<td>error function distribution with variance parameter $h$</td>
</tr>
<tr>
<td>RiskErlang($m$, $beta$)</td>
<td>Erlang distribution with integer shape parameter $m$ and scale parameter $beta$</td>
</tr>
<tr>
<td>RiskExpon($beta$)</td>
<td>exponential distribution with mean $beta$</td>
</tr>
<tr>
<td>RiskExponAlt($arg1type$, $arg1value$, $arg2type$, $arg2value$)</td>
<td>exponential distribution with two parameters named $arg1type$ and $arg2type$ which can be either “beta”, “loc”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskExtvalue($alpha$, $beta$)</td>
<td>extreme value (or Gumbel) distribution with location parameter $alpha$ and scale parameter $beta$</td>
</tr>
<tr>
<td>RiskExtvalueAlt($arg1type$, $arg1value$, $arg2type$, $arg2value$)</td>
<td>extreme value (or Gumbel) distribution with two parameters named $arg1type$ and $arg2type$ which can be either “alpha”, “beta”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskExtValueMin($alpha$, $beta$)</td>
<td>extreme value min distribution with location parameter $alpha$ and scale parameter $beta$</td>
</tr>
<tr>
<td>RiskExtvalueMinAlt($arg1type$, $arg1value$, $arg2type$, $arg2value$)</td>
<td>extreme value min distribution with two parameters named $arg1type$ and $arg2type$ which can be either “alpha”, “beta”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskF($v1$, $v2$)</td>
<td>F distribution with degrees of freedom parameters $v1$ and $v2$</td>
</tr>
<tr>
<td>RiskGamma($alpha$, $beta$)</td>
<td>gamma distribution with shape parameter $alpha$ and scale parameter $beta$</td>
</tr>
<tr>
<td>RiskGammaAlt($arg1type$, $arg1value$, $arg2type$, $arg2value$, $arg3type$, $arg3value$)</td>
<td>gamma distribution with three parameters named $arg1type$, $arg2type$, and $arg3type$ which can be either “alpha”, “beta”, “loc”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskGeneral($minimum$, $maximum$, $X1$, $X2$, $Xn$, $p1$, $p2$, $pn$)</td>
<td>general density function for a probability distribution ranging between $minimum$ and $maximum$ with $n$ values (the $X$'s) and corresponding probabilities (the $p$'s)</td>
</tr>
<tr>
<td>RiskGeometric($p$)</td>
<td>geometric distribution with parameter $p$</td>
</tr>
<tr>
<td>RiskHistogrm($minimum$, $maximum$, $p1$, $p2$, $pn$)</td>
<td>histogram distribution with $n$ classes between $minimum$ and $maximum$ with probabilities the classes (the $p$'s)</td>
</tr>
<tr>
<td>RiskHypergeo($n$, $D$, $M$)</td>
<td>hypergeometric distribution with sample size $n$, $D$ “successes”, and population size $M$</td>
</tr>
<tr>
<td>RiskIntUniform($minimum$, $maximum$)</td>
<td>uniform distribution for integer values between $minimum$ and $maximum$ (inclusive)</td>
</tr>
<tr>
<td>RiskInvGauss($mu$, $lambda$)</td>
<td>inverse gaussian (or Wald) distribution with mean $mu$ and shape parameter $lambda$</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
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<tr>
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</tr>
<tr>
<td>RiskInvGaussAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>inverse gaussian (or Wald) distribution with three parameters named <code>arg1type</code>, <code>arg2type</code>, and <code>arg3type</code> which can be either “mu”, “lambda”, “loc”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskJohnsonSB(alpha1, alpha2, a, b)</td>
<td>Johnson “system bounded” distribution with parameters <code>alpha1</code>, <code>alpha2</code>, <code>a</code>, and <code>b</code></td>
</tr>
<tr>
<td>RiskJohnsonSU(alpha1, alpha2, gamma, beta)</td>
<td>Johnson “system unbounded” distribution with parameters <code>alpha1</code>, <code>alpha2</code>, <code>gamma</code>, and <code>beta</code></td>
</tr>
<tr>
<td>RiskJohnsonMoments(mean, standardDeviation, skewness, kurtosis)</td>
<td>distribution from the Johnson family of distributions (normal, lognormal, JohnsonSB, and JohnsonSU) with moments <code>mean</code>, <code>standardDeviation</code>, <code>skewness</code>, and <code>kurtosis</code></td>
</tr>
<tr>
<td>RiskLaplace(µ, σ)</td>
<td>Laplace distribution with location parameter <code>µ</code> and scale parameter <code>σ</code></td>
</tr>
<tr>
<td>RiskLaplaceAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>Laplace distribution with two parameters named <code>arg1type</code> and <code>arg2type</code> which can be either “µ”, “sigma”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskLevy(a, c)</td>
<td>Levy distribution with location <code>a</code> and continuous scale parameter <code>c</code></td>
</tr>
<tr>
<td>RiskLevyAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>Levy distribution with two parameters named <code>arg1type</code> and <code>arg2type</code> which can be either “a”, “c”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskLogistic(alpha, beta)</td>
<td>logistic distribution with location parameter <code>alpha</code> and scale parameter <code>beta</code></td>
</tr>
<tr>
<td>RiskLogisticAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>logistic distribution with two parameters named <code>arg1type</code> and <code>arg2type</code> which can be either “alpha”, “beta”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskLoglogistic(gamma, beta, alpha)</td>
<td>log-logistic distribution with location parameter <code>gamma</code>, scale parameter <code>beta</code>, and shape parameter <code>alpha</code></td>
</tr>
<tr>
<td>RiskLoglogisticAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>log-logistic distribution with three parameters named <code>arg1type</code>, <code>arg2type</code>, and <code>arg3type</code> which can be either “gamma”, “beta”, “alpha”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskLognorm(mean, standardDeviation)</td>
<td>lognormal distribution with specified <code>mean</code> and <code>standardDeviation</code></td>
</tr>
<tr>
<td>RiskLognormAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>lognormal distribution with three parameters named <code>arg1type</code>, <code>arg2type</code>, and <code>arg3type</code> which can be either “mu”, “sigma”, “loc”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
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</tr>
<tr>
<td>RiskLognorm2(mean, standard deviation)</td>
<td>lognormal distribution generated from the “log” of a normal distribution with specified mean and standard deviation</td>
</tr>
<tr>
<td>RiskMakeInput(formula)</td>
<td>specifies that the calculated value for formula will be treated as a simulation input, just like a distribution function</td>
</tr>
<tr>
<td>RiskNegbin(s,p)</td>
<td>negative binomial distribution with s successes and probability of success p</td>
</tr>
<tr>
<td>RiskNormal(mean, standard deviation)</td>
<td>normal distribution with given mean and standard deviation</td>
</tr>
<tr>
<td>RiskNormalAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>normal distribution with two parameters named arg1type and arg2type which can be either “mu”, “sigma”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskPareto(theta, alpha)</td>
<td>Pareto distribution with parameters theta and alpha</td>
</tr>
<tr>
<td>RiskParetoAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>Pareto distribution with two parameters named arg1type and arg2type which can be either “theta”, “alpha”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskPareto2(b,q)</td>
<td>Pareto distribution (a second version) with parameters b and q</td>
</tr>
<tr>
<td>RiskPareto2Alt(arg1type, arg1value, arg2type, arg2value)</td>
<td>Pareto distribution with two parameters named arg1type and arg2type which can be either “b”, “q”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskPearson5(alpha, beta)</td>
<td>Pearson type V (or inverse gamma) distribution with shape parameter alpha and scale parameter beta</td>
</tr>
<tr>
<td>RiskPearson5Alt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>Pearson type V (or inverse gamma) distribution with three parameters named arg1type, arg2type, and arg3type which can be either “alpha”, “beta”, “loc”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskPearson6(beta, alpha1, alpha2)</td>
<td>Pearson type VI distribution with scale parameter beta and shape parameters alpha1 and alpha2</td>
</tr>
<tr>
<td>RiskPert(minimum, most likely, maximum)</td>
<td>pert distribution with specified minimum, most likely, and maximum values</td>
</tr>
<tr>
<td>RiskPertAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>pert distribution with three parameters named arg1type, arg2type, and arg3type which can be either “min”, “max”, “m. likely”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskPoisson(lambda)</td>
<td>Poisson distribution with mean lambda</td>
</tr>
<tr>
<td>RiskRayleigh(beta)</td>
<td>Rayleigh distribution with scale parameter beta</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
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</tr>
<tr>
<td>RiskRayleighAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>Rayleigh distribution with two parameters named arg1type and arg2type which can be either “beta”, “loc”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskResample(sampMethod, {X1,X2,...Xn})</td>
<td>Samples using sampMethod from a data set with n possible values (the X’s) where each X is equally likely</td>
</tr>
<tr>
<td>RiskSimtable({X1,X2,...Xn})</td>
<td>lists values to be used in each of a series of simulations</td>
</tr>
<tr>
<td>RiskSplice(dist#1 or cellref, dist#2 or cellref, splice point)</td>
<td>specifies a distribution created by splicing dist#1 to dist#2 at the x-value given by splice point</td>
</tr>
<tr>
<td>RiskStudent(nu)</td>
<td>Student’s t distribution with nu degrees of freedom</td>
</tr>
<tr>
<td>RiskTriang(minimum, most likely, maximum)</td>
<td>triangular distribution with given minimum, most likely, and maximum values</td>
</tr>
<tr>
<td>RiskTriangAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>triangular distribution with three parameters named arg1type, arg2type, and arg3type which can be either “min”, “max”, “m. likely”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskTrigen(bottom, most likely, top, bottom percentile, top percentile)</td>
<td>triangular distribution with three points representing a bottom percentile, a most likely value, and a top percentile.</td>
</tr>
<tr>
<td>RiskUniform(minimum, maximum)</td>
<td>uniform distribution between minimum and maximum</td>
</tr>
<tr>
<td>RiskUniformAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>uniform distribution with two parameters named argtype1 and argtype2 which can be either “min”, “max”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
<tr>
<td>RiskVary(base, minimum, maximum, range type, #steps, distribution)</td>
<td>a distribution that ranges between minimum and maximum with a shape given by distribution</td>
</tr>
<tr>
<td>RiskWeibull(alpha, beta)</td>
<td>Weibull distribution with shape parameter alpha and scale parameter beta</td>
</tr>
<tr>
<td>RiskWeibullAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>Weibull distribution with three parameters named arg1type, arg2type, and arg3type which can be either “alpha”, “beta”, “loc”, or a value between 0 and 1 for specifying a percentile</td>
</tr>
</tbody>
</table>

### Distribution Property Function

<table>
<thead>
<tr>
<th>Function</th>
<th>Specifies</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskCategory(categoryName)</td>
<td>Names the category to be used when displaying an input distribution</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>-----------------------</td>
<td>-------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>RiskCollect()</td>
<td>Causes samples to be collected during a simulation for the distribution in which the Collect function is included (when simulation settings specify Collect Distribution Samples for Inputs Marked with Collect)</td>
</tr>
<tr>
<td>RiskConvergence(tolerance, toleranceType, confidenceLevel, useMean, useStdDev, usePercentile, percentile)</td>
<td>Specifies convergence monitoring information for an output</td>
</tr>
<tr>
<td>RiskCopula(copula cell range, position, instance)</td>
<td>Identifies a copula, and a position in the copula for the distribution in which the function is included. The optional instance parameter specifies the particular instance of the copula to be used for correlating this distribution if there are multiple instances.</td>
</tr>
<tr>
<td>RiskCorrmat(matrix cell range, position, instance)</td>
<td>Identifies a matrix of rank correlation coefficients and a position in the matrix for the distribution in which the function is included. The optional instance parameter specifies the instance of the matrix at matrix cell range that will be used for correlating this distribution if there are multiple instances.</td>
</tr>
<tr>
<td>RiskDepC(ID, coefficient)</td>
<td>Identifies dependent variable in correlated sampling pair with rank correlation coefficient and ID identifier string</td>
</tr>
<tr>
<td>RiskFit ProjID, FitID, selected fit result</td>
<td>Links a data set identified by ProjID and FitID and its fit results to the input distribution so the input can be updated when data changes</td>
</tr>
<tr>
<td>RiskIndepC(ID)</td>
<td>Identifies independent distribution in rank correlated sampling pair, where ID is the identifier string</td>
</tr>
<tr>
<td>RiskIsDate(TRUE)</td>
<td>Specifies that the values for the input or output should be displayed as date values in graphs and reports</td>
</tr>
<tr>
<td>RiskIsDiscrete(TRUE)</td>
<td>Specifies that an output should be treated as a discrete distribution when displaying graphs of simulation results and calculating statistics</td>
</tr>
<tr>
<td>RiskLibrary(position, ID)</td>
<td>Specifies that a distribution is linked to a distribution in an @RISK Library with the entered position and ID</td>
</tr>
<tr>
<td>RiskLock()</td>
<td>Blocks sampling of the distribution in which the function is included</td>
</tr>
<tr>
<td>RiskName(input name)</td>
<td>Names the distribution in which the function is included as input name</td>
</tr>
<tr>
<td>RiskSeed(random number generator type, seed value)</td>
<td>Specifies that an input will use its own random number generator of generator type, and that it will be seeded with seed value</td>
</tr>
<tr>
<td>Function Name</td>
<td>Description</td>
</tr>
<tr>
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</tr>
<tr>
<td><strong>RiskShift(shift)</strong></td>
<td>Shifts the domain of the distribution in which the function is included by shift value.</td>
</tr>
<tr>
<td><strong>RiskSixSigma(LSL,USL,target, Long Term Shift, Number of Standard Deviations)</strong></td>
<td>Specifies the lower specification limit, upper specification limit, target value, long term shift, and the number of standard deviations for six sigma calculations for an output.</td>
</tr>
<tr>
<td><strong>RiskStatic(static value)</strong></td>
<td>Defines the static value 1) returned by a distribution function during a standard Excel recalculation and 2) that replaces the @RISK function after @RISK functions are swapped out.</td>
</tr>
<tr>
<td><strong>RiskTruncate(minimum, maximum)</strong></td>
<td>Truncates the distribution in which the function is included so that only values between minimum and maximum are sampled.</td>
</tr>
<tr>
<td><strong>RiskTruncateP(perc% minimum, perc% maximum)</strong></td>
<td>Truncates the distribution in which the function is included so that only values between percentiles perc% minimum and perc% maximum are sampled.</td>
</tr>
<tr>
<td><strong>RiskUnits(units)</strong></td>
<td>Names the units to be used in labeling an input distribution or output.</td>
</tr>
<tr>
<td>Output Function</td>
<td>Specifies</td>
</tr>
<tr>
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</tr>
<tr>
<td>RiskOutput(name,output range name, position in range)</td>
<td>Simulation output cell with name, output range name to which the output belongs, and the position in range (all arguments are optional)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fitting Function</th>
<th>Specifies</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskFitDistribution(data range, data type, distribution list, selector, lower bound, upper bound)</td>
<td>Fits a distribution to data in data range, optionally restricting fitted distributions to those in distribution list. Fitted data has specified data type and best fit is selected using goodness-of-fit test specified by selector</td>
</tr>
<tr>
<td>RiskFitDescription((fit source, distribution style)</td>
<td>Returns text description of best fitting distribution from the fit performed by the function in cell given by fit source</td>
</tr>
<tr>
<td>RiskFitStatistic((fit source, statistic)</td>
<td>Returns statistic from fit performed by the function in cell given by fit source</td>
</tr>
<tr>
<td>RiskFitParameter((fit source, parameter#)</td>
<td>Returns a parameter of best-fitting distribution from fit performed by the function in cell given by fit source</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Project Function</th>
<th>Specifies</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskProjectAddDelay(PrecedingTask, DelayLength, DelayCost)</td>
<td>Adds a new task to a project after PrecedingTask completes with specified DelayLength and DelayCost (in any iteration)</td>
</tr>
<tr>
<td>RiskProjectAddCost(CostToAdd, TimeToAdd)</td>
<td>Adds the cost CostToAdd to a project at the date TimeToAdd (in any iteration)</td>
</tr>
<tr>
<td>RiskProjectRemoveTask(TaskToRemove)</td>
<td>Removes TaskToRemove from a project (in any iteration)</td>
</tr>
<tr>
<td>RiskProjectResourceAdd(Task, Resource, Units)</td>
<td>Assigns a Resource to a Task using the specified Units (in any iteration)</td>
</tr>
<tr>
<td>RiskProjectResourceRemove(Task, Resource)</td>
<td>Removes a Resource assigned to a Task (in any iteration)</td>
</tr>
<tr>
<td>RiskProjectResourceUse(Task, Resource, UsageValue)</td>
<td>In an iteration of a simulation, changes the units of a material Resource (or work for a work resource) used in a Task to UsageValue (in any iteration)</td>
</tr>
<tr>
<td>Time Series Function</td>
<td>Specifies</td>
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<tr>
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</tr>
<tr>
<td>RiskAPARCH((\mu, \Omega, \Delta, \Gamma, A, B, R_0, \Sigma_0, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates an asymmetric power auto-regressive conditional heteroskedastic time series with these parameters</td>
</tr>
<tr>
<td>RiskAR1((\mu, \Sigma, A, R_0, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates an auto-regressive AR(1) time series with these parameters</td>
</tr>
<tr>
<td>RiskAR2((\mu, \Sigma, A_1, A_2, R_0, R_{Neg1}, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates an auto-regressive AR(2) time series with these parameters</td>
</tr>
<tr>
<td>RiskARCH((\mu, \Omega, A, R_0, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates an auto-regressive conditional heteroskedastic time series with these parameters</td>
</tr>
<tr>
<td>RiskARMA((\mu, \Sigma, A_1, B_1, R_0, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates an auto-regressive moving average time series with these parameters</td>
</tr>
<tr>
<td>RiskEGARCH((\mu, \Omega, \Theta, \Gamma, A, B, R_0, \Sigma_0, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates an exponential auto-regressive conditional heteroskedastic time series with these parameters</td>
</tr>
<tr>
<td>RiskGARCH((\mu, \Omega, A, B, R_0, \Sigma_0, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates a generalized auto-regressive conditional heteroskedastic time series with these parameters</td>
</tr>
<tr>
<td>RiskGBM((\mu, \Sigma, \text{Times}, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates a geometric brownian motion time series with these parameters</td>
</tr>
<tr>
<td>RiskGBMJD((\mu, \Sigma, \Lambda, \text{JumpMu}, \text{JumpSigma}, \text{Times}, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates a geometric brownian motion with jump diffusion time series with these parameters</td>
</tr>
<tr>
<td>RiskBMMR((\mu, \Sigma, \Alpha, R_0, \text{Times}, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates a geometric brownian motion with mean reversion time series with these parameters</td>
</tr>
<tr>
<td>RiskBMMRJD((\mu, \Sigma, \Alpha, R_0, \Lambda, \text{JumpMu}, \text{JumpSigma}, \text{Times}, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates a geometric brownian motion with mean reversion and jump diffusion time series with these parameters</td>
</tr>
<tr>
<td>RiskGBMSeasonal((\mu, \Sigma, \text{SeasonalAdj}, \text{AdjType}, \text{AdjIndex}, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates a geometric brownian motion with seasonal adjustment time series with these parameters</td>
</tr>
<tr>
<td>RiskMA1((\mu, \Sigma, B_1, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates a moving average MA(1) time series with these parameters</td>
</tr>
<tr>
<td>RiskMA2((\mu, \Sigma, B_1, B_2, \text{StartValue}, \text{WhatToReturn}))</td>
<td>Calculates a moving average MA(2) time series with these parameters</td>
</tr>
<tr>
<td>Statistics Function</td>
<td>Returns</td>
</tr>
<tr>
<td>---------------------</td>
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</tr>
<tr>
<td><strong>RiskConvergenceLevel</strong>(cellref or output name, Sim#)</td>
<td>Returns the level of convergence (0 to 100) for an output in Sim#. TRUE is returned on convergence.</td>
</tr>
<tr>
<td><strong>RiskCorrel</strong>(cellref1 or output/input1 name, cellref2 or output/input2 name, correlationType, Sim#)</td>
<td>Returns the correlation coefficient using correlationType for the data for the simulated distributions for cellref1 or output/input name1 and cellref2 or output/input name2 in Sim#, where correlationType is either Pearson or Spearman Rank correlation.</td>
</tr>
<tr>
<td><strong>RiskKurtosis</strong>(cellref or output/input name, Sim#)</td>
<td>Kurtosis of the simulated distribution for the entered cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskMax</strong>(cellref or output/input name, Sim#)</td>
<td>Maximum value of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskMean</strong>(cellref or output/input name, Sim#)</td>
<td>Mean of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskMin</strong>(cellref or output/input name, Sim#)</td>
<td>Minimum value of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskMode</strong>(cellref or output/input name, Sim#)</td>
<td>Mode of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskPercentile</strong>(cellref or output/input name, perc%, Sim#)</td>
<td>Both functions return percentile perc% of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskPtoX</strong>(cellref or output/input name, perc%, Sim#)</td>
<td>Both functions return percentile perc% of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskPercentileD</strong>(cellref or output/input name, perc%, Sim#)</td>
<td>Both functions return percentile perc% of the simulated distribution for cellref or output/input name in Sim# (where perc% is a cumulative descending percentile)</td>
</tr>
<tr>
<td><strong>RiskQtoX</strong>(cellref or output/input name, perc%, Sim#)</td>
<td>Both functions return percentile perc% of the simulated distribution for cellref or output/input name in Sim# (where perc% is a cumulative descending percentile)</td>
</tr>
<tr>
<td><strong>RiskRange</strong>(cellref or output/input name, Sim#)</td>
<td>Range of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskSensitivity</strong>(cellref or output name, Sim#, rank, analysisType, returnValueType)</td>
<td>Returns the sensitivity analysis information of the simulated distribution for cellref or output name</td>
</tr>
<tr>
<td><strong>RiskSensitivityStatChange</strong>(cellref or output name, Sim#, rank, numBins, whichStatistic, percentile, returnValueType)</td>
<td>Returns the &quot;change in output statistic&quot; sensitivity analysis information of the simulated distribution for cellref or output name.</td>
</tr>
<tr>
<td><strong>RiskSkewness</strong>(cellref or output/input name, Sim#)</td>
<td>Skewness of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskStdDev</strong>(cellref or output/input name, Sim#)</td>
<td>Standard deviation of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>RiskTarget(cellref or output/input name, target value, Sim#)</td>
<td>Both functions return ascending cumulative probability of target value in the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td>RiskTargetD(cellref or output/input name, target value, Sim#)</td>
<td>Both functions return descending cumulative probability of target value in the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td>RiskXtoP(cellref or output/input name, target value, Sim#)</td>
<td>Both functions return ascending cumulative probability of target value in the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td>RiskXtoQ(cellref or output/input name, target value, Sim#)</td>
<td>Both functions return descending cumulative probability of target value in the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td>RiskVariance(cellref or output/input name, Sim#)</td>
<td>Variance of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td>RiskTheoKurtosis(cellref or distribution function)</td>
<td>Kurtosis of the distribution for the entered cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoMax(cellref or distribution function)</td>
<td>Maximum value of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoMean(cellref or distribution function)</td>
<td>Mean of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoMin(cellref or distribution function)</td>
<td>Minimum value of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoMode(cellref or distribution function)</td>
<td>Mode of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoPtoX(cellref or distribution function, perc%)</td>
<td>Percentile perc% of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoQtoX(cellref or distribution function, perc%)</td>
<td>Percentile perc% of the distribution for cellref or distribution function (perc% is a cumulative descending percentile)</td>
</tr>
<tr>
<td>RiskTheoRange(cellref or distribution function)</td>
<td>Range of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoSkewness(cellref or distribution function)</td>
<td>Skewness of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoStdDev(cellref or distribution function)</td>
<td>Standard deviation of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoXtoP(cellref or distribution function, target value)</td>
<td>Ascending cumulative probability of target value in the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoXtoQ(cellref or distribution function, target value)</td>
<td>Descending cumulative probability of target value in the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoVariance(cellref or distribution function)</td>
<td>Variance of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>Six Sigma Statistics Function</td>
<td>Returns</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td><strong>RiskCp</strong> <em>(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations))</em></td>
<td>Calculates the Process Capability for cellref or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskCPM</strong> <em>(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations))</em></td>
<td>Calculates the Taguchi capability index for cellref or output name in Sim# optionally using the LSL, USL and LongTerm Shift in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskCpk</strong> <em>(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations))</em></td>
<td>Calculates the Process Capability Index for cellref or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskCpkLower</strong> <em>(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations))</em></td>
<td>Calculates the one-sided capability index based on the Lower Specification limit for cellref or output name in Sim# optionally using the LSL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskCpkUpper</strong> <em>(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations))</em></td>
<td>Calculates the one-sided capability index based on the Upper Specification limit for cellref or output name in Sim# optionally using the USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskDPM</strong> <em>(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations))</em></td>
<td>Calculates the defective parts per million for cellref or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskK</strong> <em>(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations))</em></td>
<td>This function calculates a measure of process center for cellref or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskLowerXBound</strong> <em>(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, Long Term Shift, Number of Standard Deviations))</em></td>
<td>Returns the lower X-value for a given number of standard deviations from the mean for cellref or output name in Sim #, optionally using the Number of Standard Deviations in the RiskSixSigma property function.</td>
</tr>
<tr>
<td><strong>RiskPNC</strong> <em>(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations))</em></td>
<td>Calculates the total probability of defect outside the lower and upper specification limits for cellref or output name in Sim# optionally using the LSL, USL and LongTerm Shift in the included RiskSixSigma property function.</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>RiskPNCLower</strong></td>
<td>Calculates the probability of defect outside the lower specification limits for <code>cellref</code> or <code>output name</code> in <code>Sim#</code> optionally using the <code>LSL</code> and <code>LongTerm Shift</code> in the included <code>RiskSixSigma</code> property function.</td>
</tr>
<tr>
<td><strong>RiskPNCUpper</strong></td>
<td>Calculates the probability of defect outside the upper specification limits for <code>cellref</code> or <code>output name</code> in <code>Sim#</code> optionally using the <code>USL</code> and <code>LongTerm Shift</code> in the included <code>RiskSixSigma</code> property function.</td>
</tr>
<tr>
<td><strong>RiskPPMLower</strong></td>
<td>Calculates the number of defects below the lower specification limit for <code>cellref</code> or <code>output name</code> in <code>Sim#</code> optionally using the <code>LSL</code> and <code>LongTerm Shift</code> in the included <code>RiskSixSigma</code> property function.</td>
</tr>
<tr>
<td><strong>RiskPPMUpper</strong></td>
<td>Calculates the number of defects above the upper specification limit for <code>cellref</code> or <code>output name</code> in <code>Sim#</code> optionally using the <code>USL</code> and <code>LongTerm Shift</code> in the included <code>RiskSixSigma</code> property function.</td>
</tr>
<tr>
<td><strong>RiskSigmaLevel</strong></td>
<td>Calculates the Process Sigma level for <code>cellref</code> or <code>output name</code> in <code>Sim#</code> optionally using the <code>USL</code> and <code>LSL</code> and <code>Long Term Shift</code> in the included <code>RiskSixSigma</code> property function. (Note: This function assumes that the output is normally distributed and centered within the specification limits.)</td>
</tr>
<tr>
<td><strong>RiskUpperXBound</strong></td>
<td>Returns the upper X-value for a given number of standard deviations from the mean for <code>cellref</code> or <code>output name</code> in <code>Sim#</code>, optionally using the <code>Number of Standard Deviations</code> in the <code>RiskSixSigma</code> property function.</td>
</tr>
<tr>
<td><strong>RiskYV</strong></td>
<td>Calculates the yield or the percentage of the process that is free of defects for <code>cellref</code> or <code>output name</code> in <code>Sim#</code> optionally using the <code>LSL</code>, <code>USL</code> and <code>Long Term Shift</code> in the included <code>RiskSixSigma</code> property function.</td>
</tr>
<tr>
<td><strong>RiskZlower</strong></td>
<td>Calculates how many standard deviations the Lower Specification Limit is from the mean for <code>cellref</code> or <code>output name</code> in <code>Sim#</code> optionally using the <code>LSL</code> in the included <code>RiskSixSigma</code> property function.</td>
</tr>
<tr>
<td><strong>RiskZMin</strong></td>
<td>Calculates the minimum of Z-Lower and Z-Upper for <code>cellref</code> or <code>output name</code> in <code>Sim#</code> optionally using the <code>USL</code> and <code>LSL</code> in the included <code>RiskSixSigma</code> property function.</td>
</tr>
</tbody>
</table>

*Chapter 7: @RISK Function Reference*
<table>
<thead>
<tr>
<th><strong>RiskZUpper</strong>(<em>cellref or output name, Sim#</em>, RiskSixSigma(<em>LSL, USL, Target, LongTerm Shift, Number of Standard Deviations</em>))</th>
<th>Calculates how many standard deviations the Upper Specification Limit is from the mean for <em>cellref or output name</em> in <em>Sim#</em> optionally using the USL in the included RiskSixSigma property function</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Supplemental Functions</strong></td>
<td><strong>Returns</strong></td>
</tr>
<tr>
<td><strong>RiskCorrectCorrmat</strong>(<em>correlation MatrixRange, adjustmentWeightsMatrixRange</em>)</td>
<td>Returns the corrected correlation matrix for the matrix located in <em>correlationMatrixRange</em> using the adjustment weight matrix located in <em>adjustmentWeightsMatrixRange</em>.</td>
</tr>
<tr>
<td><strong>RiskCurrentIter()</strong></td>
<td>Returns the current iteration number of an executing simulation.</td>
</tr>
<tr>
<td><strong>RiskCurrentSim()</strong></td>
<td>Returns the current simulation number of an executing simulation.</td>
</tr>
<tr>
<td><strong>RiskSimulationInfo</strong>(<em>info to return</em>)</td>
<td>Returns information such as date/time, runtime, etc. about the simulation that was run</td>
</tr>
<tr>
<td><strong>RiskStopRun</strong>(<em>cellRef or formula</em>)</td>
<td>Stops a simulation when the value of <em>cellRef</em> returns TRUE, or the entered <em>formula</em> evaluates to TRUE.</td>
</tr>
<tr>
<td><strong>Graphing Function</strong></td>
<td><strong>Returns</strong></td>
</tr>
<tr>
<td><strong>RiskResultsGraph</strong>(<em>cellRef or output/input name, locationCellRange, graphType, xlFormat, leftDelimiter, rightDelimiter, xMin, xMax, xScale, title, sim#</em>)</td>
<td>Adds a graph of simulation results to a worksheet.</td>
</tr>
</tbody>
</table>
Reference: Distribution Functions

Distribution functions are listed here with their required arguments. Optional arguments may be added to these required arguments using the @RISK Distribution Property functions listed in the next section. You can also visit http://www.palisade.com/models/RISKDistributions.asp for Excel example files that describe @RISK’s distribution functions.
<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskBernoulli}(p) ) specifies a Bernoulli distribution with parameter ( p ). This is used to model an event that either occurs with probability ( p ) (value 1) or doesn't occur with probability ( 1-p ) (value 0).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskBernoulli}(0.1) ) returns a Bernoulli distribution with parameter 0.1. This has a 10% chance of returning 1 and a 90% chance of returning 0. ( \text{RiskBernoulli}(C12) ) returns a Bernoulli distribution with parameter ( p ) taken from cell C12.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>( p ) continuous parameter 0 &lt; ( p ) &lt; 1</td>
</tr>
<tr>
<td>Domain</td>
<td>( x \in {0,1} ) discrete</td>
</tr>
</tbody>
</table>
| Density and Cumulative Distribution Functions | \( f(x) = 1 - p \) for \( x = 0 \)  
\( f(x) = p \) for \( x = 1 \)  
\( f(x) = 0 \) otherwise  
\( F(x) = 0 \) for \( x < 0 \)  
\( F(x) = 1 - p \) for \( 0 \geq x < 1 \)  
\( F(x) = 1 \) for \( x \geq 1 \) |
| Mean | \( p \) |
| Variance | \( p(1-p) \) |
| Skewness | \( \frac{1 - 2p}{[p(1-p)]^{3/2}} \) |
| Kurtosis | \( \frac{p^3 + (1-p)^3}{p(1-p)} \) |
| Mode | 0 if \( p < .5 \)  
1 if \( p > .5 \)  
Bimodal (0,1) if \( p = .5 \) |
Examples

**PMF - Bernoulli(.3)**

**CDF - Bernoulli(.3)**
**RiskBeta**

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskBeta(alpha1, alpha2)</strong> specifies a beta distribution with shape parameters <em>alpha1</em> and <em>alpha2</em>, and possible values from 0 to 1. The Beta distribution is often used as a starting point to derive other distributions (such as the BetaGeneral, PERT, and BetaSubjective). It is intimately related to the Binomial distribution in a Bayesian context, representing the distribution of the uncertainty of the probability <em>p</em> of a Binomial process, based on observations of that process.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskBeta(1,2)</strong> returns a beta distribution with shape parameters 1 and 2. <strong>RiskBeta(C12,C13)</strong> returns a beta distribution with shape parameters <em>alpha1</em> and <em>alpha2</em> taken from cells C12 and C13.</td>
</tr>
</tbody>
</table>
| Guidelines | *\( \alpha_1 \)* continuous shape parameter *\( \alpha_1 > 0 \)*  
*\( \alpha_2 \)* continuous shape parameter *\( \alpha_2 > 0 \)* |
| Domain | *\( 0 \leq x \leq 1 \)* continuous |
| Density and Cumulative Distribution Functions |  
*\( f(x) = \frac{x^{\alpha_1-1}(1-x)^{\alpha_2-1}}{B(\alpha_1, \alpha_2)} \)  
\( F(x) = \frac{B_x(\alpha_1, \alpha_2)}{B(\alpha_1, \alpha_2)} \equiv I_x(\alpha_1, \alpha_2) \)  
Here, B is the Beta Function and B*\(_x\) is the Incomplete Beta Function.* |
| Mean | *\( \frac{\alpha_1}{\alpha_1 + \alpha_2} \) |
| Variance | *\( \frac{\alpha_1\alpha_2}{(\alpha_1 + \alpha_2)^2(\alpha_1 + \alpha_2 + 1)} \) |
| Skewness |  
*\( 2\frac{\alpha_2 - \alpha_1}{\alpha_1 + \alpha_2 + 2\sqrt{\frac{\alpha_1 + \alpha_2 + 1}{\alpha_1\alpha_2}}} \) |
| Kurtosis |  
*\( 3\frac{(\alpha_1 + \alpha_2 + 1)[2(\alpha_1 + \alpha_2)^2 + \alpha_1\alpha_2(\alpha_1 + \alpha_2 - 6)]}{\alpha_1\alpha_2(\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3)} \) |
<table>
<thead>
<tr>
<th>Mode</th>
<th>( \frac{\alpha_1 - 1}{\alpha_1 + \alpha_2 - 2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \alpha_1 \geq 1, \alpha_2 \geq 1 ) or ( \alpha_1 = 1, \alpha_2 &gt; 1 )</td>
</tr>
<tr>
<td>1</td>
<td>( \alpha_1 \geq 1, \alpha_2 &lt; 1 ) or ( \alpha_1 &gt; 1, \alpha_2 = 1 )</td>
</tr>
</tbody>
</table>

**Examples**

CDF - Beta(2,3)

![CDF - Beta(2,3)](image)

PDF - Beta(2,3)

![PDF - Beta(2,3)](image)
**RiskBetaGeneral**

<table>
<thead>
<tr>
<th>Description</th>
<th>\textit{RiskBetaGeneral}(alpha1,alpha2,minimum,maximum) specifies a beta distribution with the defined \textit{minimum} and \textit{maximum} and shape parameters \textit{alpha1} and \textit{alpha2}. The BetaGeneral is directly derived from the Beta distribution by scaling the [0,1] range of the Beta distribution to instead have the specified \textit{minimum} and \textit{maximum} values. (The PERT distribution is a special case of the BetaGeneral distribution.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>\textit{RiskBetaGeneral}(1,2,0,100) returns a beta distribution over the range from 0 to 100 with shape parameters 1 and 2. \textit{RiskBetaGeneral}(C12,C13,D12,D13) returns a beta distribution with its four parameters taken from cells C12, C13, D12, and D13.</td>
</tr>
</tbody>
</table>
| Guidelines | \(\alpha_1\)  continuous shape parameter  
\(\alpha_1 > 0\)  
\(\alpha_2\)  continuous shape parameter  
\(\alpha_2 > 0\)  
\(\text{min}\)  continuous boundary parameter  
\(\text{min} < \text{max}\)  
\(\text{max}\)  continuous boundary parameter |
| Domain | \(\text{min} \leq x \leq \text{max}\)  continuous |
| Density and Cumulative Distribution Functions | \(f(x) = \frac{(x - \text{min})^{\alpha_1-1}(\text{max} - x)^{\alpha_2-1}}{B(\alpha_1, \alpha_2)(\text{max} - \text{min})^{\alpha_1+\alpha_2-1}}\)  
\(F(x) = \frac{B_z(\alpha_1, \alpha_2)}{B(\alpha_1, \alpha_2)} = I_z(\alpha_1, \alpha_2)\)  
with \(z \equiv \frac{x - \text{min}}{\text{max} - \text{min}}\)  
Here, \(B\) is the Beta Function and \(B_z\) is the Incomplete Beta Function. |
<p>| Mean | (\text{min} + \frac{\alpha_1}{\alpha_1 + \alpha_2}(\text{max} - \text{min})) |
| Variance | (\frac{\alpha_1 \alpha_2}{(\alpha_1 + \alpha_2)^2(\alpha_1 + \alpha_2 + 1)}(\text{max} - \text{min})^2) |</p>
<table>
<thead>
<tr>
<th></th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skewness</td>
<td>$2 \frac{\alpha_2 - \alpha_1}{\alpha_1 + \alpha_2 + 2} \sqrt{\frac{\alpha_1 + \alpha_2 + 1}{\alpha_1 \alpha_2}}$</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>$3 \frac{(\alpha_1 + \alpha_2 + 1)(2(\alpha_1 + \alpha_2)^2 + \alpha_1 \alpha_2 (\alpha_1 + \alpha_2 - 6))}{\alpha_1 \alpha_2 (\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3)}$</td>
</tr>
</tbody>
</table>
| Mode     | min: $\frac{\alpha_1 - 1}{\alpha_1 + \alpha_2 - 2} (\max - \min)$, $\alpha_1 > 1$, $\alpha_2 > 1$  
|          | min: $\alpha_1 < 1$, $\alpha_2 \geq 1$ or $\alpha_1 = 1$, $\alpha_2 > 1$  
|          | max: $\alpha_1 \geq 1$, $\alpha_2 < 1$ or $\alpha_1 > 1$, $\alpha_2 = 1$ |

**Examples**

PDF - BetaGeneral(2, 3, 0, 5)
**RiskBetaGeneralAlt, RiskBetaGeneralAltD**

| Description                                                                 | RiskBetaGeneralAlt(*arg1*type, *arg1*value, *arg2*type,*arg2*value,  
| arg3*type,arg3*value, *arg4*type,*arg4*value) specifies a beta  
| distribution with four arguments of the type *arg1*type to *arg4*type.  
| These arguments can be either “alpha1”, “alpha2”, “min”, “max”, or  
| a value between 0 and 1 to specify a percentile.                     |
| Examples                                                                 | RiskBetaGeneralAlt("min",0,10%,1,50%,20,"max",50) returns a  
| beta distribution with a minimum value of 0, a maximum value of  
| 50, a 10th percentile of 1, and a 50th percentile of 20.              |
| Guidelines                                                               | With RiskBetaGeneralAltD, any entered percentile values are  
| cumulative descending percentiles, where the percentile specifies  
| the chance of a value greater than or equal to the entered value.    |
**RiskBetaSubj**

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskBetaSubj}(\text{minimum, m.likely, mean, maximum})$ specifies a beta distribution with a <em>minimum</em> and <em>maximum</em> value as specified. The shape parameters are calculated from the <em>m.likely</em> value and <em>mean</em> parameters. The BetaSubjective distribution is like a BetaGeneral distribution in the sense that the range of the underlying Beta distribution has been scaled. However, its parameterization allows it to be used in cases where you want to use not only a minimum-most likely-maximum parameter set (as in the PERT distribution), but you want to use the mean of the distribution as one of the parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskBetaSubj}(0,1,2,10)$ returns a beta distribution with a minimum of 0, a maximum of 10, a most likely value of 1, and a mean of 2. $\text{RiskBetaSubj}(A1,A2,A3,A4)$ returns a beta distribution with its four parameters taken from cells A1, A2, A3, and A4.</td>
</tr>
</tbody>
</table>
| Definitions | $\text{mid} \equiv \frac{\text{min} + \text{max}}{2}$  

$\alpha_1 \equiv 2 \frac{(\text{mean} - \text{min})(\text{mid} - \text{m.likely})}{(\text{mean} - \text{m.likely})(\text{max} - \text{min})}$  

$\alpha_2 \equiv \alpha_1 \frac{\text{max} - \text{mean}}{\text{mean} - \text{min}}$ |
| Parameters | min \hspace{1cm} continuous boundary parameter  

min < max  

m.likely \hspace{1cm} continuous parameter  

min < m.likely < max  

mean \hspace{1cm} continuous parameter  

min < mean < max  

max \hspace{1cm} continuous boundary parameter  

mean > mid \hspace{1cm} if m.likely > mean  

mean < mid \hspace{1cm} if m.likely < mean  

mean = mid \hspace{1cm} if m.likely = mean |
| Domain | $\text{min} \leq x \leq \text{max}$ \hspace{1cm} continuous |
### Density and Cumulative Distribution Functions

$$f(x) = \frac{(x - \text{min})^{\alpha_1 - 1}(\text{max} - x)^{\alpha_2 - 1}}{B(\alpha_1, \alpha_2)(\text{max} - \text{min})^{\alpha_1 + \alpha_2 - 1}}$$

$$F(x) = \frac{B_z(\alpha_1, \alpha_2)}{B(\alpha_1, \alpha_2)} = I_z(\alpha_1, \alpha_2) \quad \text{with} \quad z \equiv \frac{x - \text{min}}{\text{max} - \text{min}}$$

Here, $B$ is the Beta Function and $B_z$ is the Incomplete Beta Function.

<table>
<thead>
<tr>
<th>Mean</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>(\frac{(\text{mean} - \text{min})(\text{max} - \text{mean})(\text{mean} - \text{m.likely})}{2 \cdot \text{mid} + \text{mean} - 3 \cdot \text{m.likely}})</td>
</tr>
<tr>
<td>Skewness</td>
<td>(\frac{2(\text{mid} - \text{mean})}{</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>(\frac{3(\alpha_1 + \alpha_2 + 1)[2(\alpha_1 + \alpha_2)^2 + \alpha_1 \alpha_2 (\alpha_1 + \alpha_2 - 6)]}{\alpha_1 \alpha_2 (\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3)})</td>
</tr>
<tr>
<td>Mode</td>
<td>m.likely</td>
</tr>
</tbody>
</table>
Examples

CDF - BetaSubj(0, 1, 2, 5)

PDF - BetaSubj(0, 1, 2, 5)
### RiskBinomial

**Description**

*RiskBinomial*({eqn}n, p\) specifies a binomial distribution with parameters {eqn}n\) and {eqn}p\). This is usually used to model the number of “successes” in {eqn}n\) independent, identical trials, where {eqn}p\) is the probability of success on each trial.

For example, *RiskBinomial*(10,20%) represents the number of discoveries of oil from a portfolio of 10 prospects, where each prospect has a 20% chance of having oil. An important modelling application is when {eqn}n=1\), so that there are two possible outcomes (0 or 1), where the 1 has probability {eqn}p\), and the 0 has probability 1-{eqn}p\). (In this case, it is equivalent to the RiskBernoulli function with parameter {eqn}p\).) With {eqn}p=0.5\), it is equivalent to tossing a fair coin. For other values of {eqn}p\), the distribution can be used to model event risk, that is, the occurrence or nonoccurrence event, or to transform registers of risks into simulation models to aggregate the risks.

**Examples**

- *RiskBinomial*(5,0.25) returns a binomial distribution generated from 5 trials with a 25% probability of success on each trial.
- *RiskBinomial*(C10*3,B10) returns a binomial distribution generated from the number of trials taken from the value in cell C10 times 3. The probability of success on each trial is taken from cell B10.

**Guidelines**

- {eqn}n\) must be a positive integer less than or equal to 32,767.
- {eqn}p\) must be between 0 and 1.

**Parameters**

- {eqn}n\) discrete “count” parameter, {eqn}n \geq 0\) *
- {eqn}p\) continuous “success” probability, {eqn}0 < p < 1\) *

*{eqn}n = 0, p = 0\) and {eqn}p = 1\) are supported but give degenerate distributions.

**Domain**

- {eqn}0 \leq x \leq n\) discrete integers

**Mass and Cumulative Distribution Functions**

- \(f(x) = \binom{n}{x} p^x (1-p)^{n-x}\)
- \(F(x) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}\)

**Mean**

- np

**Variance**

- np(1 − p)
## Skewness

\[
\frac{(1 - 2p)}{\sqrt{np(1 - p)}}
\]

## Kurtosis

\[
3 - \frac{6}{n} + \frac{1}{np(1 - p)}
\]

## Mode

(bimodal) \( p(n + 1) - 1 \) and \( p(n + 1) \) if \( p(n + 1) \) is integral

(unimodal) largest integer less than \( p(n + 1) \) otherwise

## Examples

![PMF - Binomial(8, .4)](image-url)
RiskChiSq

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskChiSq}(v)$ specifies a Chi-Square distribution with $v$ degrees of freedom. This distribution is generally used in statistical hypothesis testing to test a population variance.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskChiSq}(5)$ returns a Chi-Square distribution with 5 degrees of freedom. $\text{RiskChiSq}(A7)$ returns a Chi-Square distribution with the degrees of freedom parameter taken from cell A7.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>$v$ must be a positive integer.</td>
</tr>
<tr>
<td>Parameters</td>
<td>$v$ discrete shape parameter $v &gt; 0$</td>
</tr>
<tr>
<td>Domain</td>
<td>$0 \leq x &lt; +\infty$ continuous</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>$f(x) = \frac{1}{2^{v/2} \Gamma(v/2)} e^{-x/2} x^{(v/2)-1}$ $F(x) = \frac{\Gamma_{x/2}(v/2)}{\Gamma(v/2)}$</td>
</tr>
<tr>
<td>Mean</td>
<td>$v$</td>
</tr>
<tr>
<td>Variance</td>
<td>$2v$</td>
</tr>
<tr>
<td>Skewness</td>
<td>$\sqrt{8} \sqrt{v}$</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>$3 + \frac{12}{v}$</td>
</tr>
<tr>
<td>Mode</td>
<td>$v - 2$ if $v \geq 2$ $0$ if $v = 1$</td>
</tr>
</tbody>
</table>

Here, $\Gamma$ is the Gamma Function, and $\Gamma_x$ is the Incomplete Gamma Function.
Examples

PDF - ChiSq(5)

CDF - ChiSq(5)
## RiskCompound

**Description**  
The `RiskCompound` function is used to return the sum of a random number of independent random values from a specified distribution. Specifically, it returns the sum of a number of samples from `dist#2` (or a `cellref` to this distribution), where the number of samples drawn from `dist#2` is given by the value sampled from `dist#1` (or `value` or the value in `cellref`). Typically, `dist#1` is called the frequency distribution and `dist#2` is the called severity distribution. Optionally, `deductible` is subtracted from each `dist#2` sample and if `(dist#2 sample - deductible)` exceeds `limit`, the `dist#2` sample is set equal to `limit`.  

RiskCompound is evaluated each iteration of a simulation. The first argument's value is calculated using a sample from `dist#1` (or `value` or the value taken from `cellRef`). Then this number of samples are drawn from `dist#2` and summed. This sum is the return value for the RiskCompound function.

### Examples

- **Example 1:**  
  ```  
  RiskCompound(RiskPoisson(5),RiskLognorm(10000,10000))  
  ```  
  Returns the sum of a number of samples drawn from `RiskLognorm(10000,10000)`, where the number of samples in the sum is the value sampled from `RiskPoisson(5)`.

### Guidelines

- `dist#1`, but not `dist#2`, can be correlated. RiskCompound itself cannot be correlated.
- `deductible` and `limit` are optional arguments.
- If `(dist#2 sample - deductible)` exceeds `limit`, the sample for `dist#2` is set equal to `limit`.
- `dist#1`, `dist#2`, and RiskCompound itself can include property functions (but `RiskCorrmat`, as noted above).

Input distribution functions `dist#1` or `dist#2`, along with any distribution functions in cells referenced in the RiskCompound function, are not displayed in sensitivity analysis results for outputs affected by the RiskCompound function. The RiskCompound function itself, however, includes sensitivity analysis results. Those results include the effects of `dist#1`, `dist#2`, and any distribution functions in cells referenced in a RiskCompound function.

- `dist#1` argument is an integer value. If the distribution function or formula entered for `dist#1` returns a non-integer value, it is truncated. This is the same behavior as Excel functions with arguments that are integer values (such as `INDEX`). You can use an Excel `ROUND` function to round `dist#1` as desired.

- `dist#2` argument can contain only a single @RISK distribution function, a constant value, or a cell reference. It cannot be an expression or formula. If you want to use a formula to calculate a severity value, you can enter that formula in a separate cell and reference that cell (as discussed below).

- `dist#2` can be a reference to a cell that contains a distribution function or a formula. If a formula is entered, this formula will be recalculated each time a severity value is needed. For example, the severity formula for cell A10 and compound function in A11 could be entered as follows:
In this case, the “sample” for the severity distribution would be generated by evaluating the formula in A10. In each iteration, this formula would be evaluated the number of times specified by the sample drawn from the frequency distribution. Note, however, that the formula entered needs to have fewer than 256 characters. If more complex calculations are required, a user-defined function (UDF) can be entered in the formula to be evaluated. In addition, all @RISK distributions to be sampled in the severity calculation must be entered in the cell’s formula (for example, in the formula for cell A10 above) and not referenced in other cells.

It is important to note that a single distribution of simulation results is not available for the severity distribution or severity calculation after a run. No entry is made for the severity distribution in the Results Summary window, and a Browse window graph cannot be displayed for the severity distribution. This is because the severity distribution can be sampled any number of times during a single iteration, versus one time for other input distributions.
## RiskCumul

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskCumul}(\text{minimum}, \text{maximum}, {X1, X2, \ldots, Xn}, {p1, p2, \ldots, pn}) ) specifies a cumulative distribution with ( n ) points. The range of the cumulative curve is set by the ( \text{minimum} ) and ( \text{maximum} ) arguments. Each point on the cumulative curve has a value ( X ) and a probability ( p ). Points on the cumulative curve are specified with increasing value and increasing probability. Any number of points can be specified for the curve. Note that even though the ( X, p ) pairs &quot;define&quot; the distribution, any value between ( \text{minimum} ) and ( \text{maximum} ) can be returned.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
<th>( \text{RiskCumul}(0, 10, {1, 5, 9}, {0.1, 0.7, 0.9}) ) returns a cumulative curve with 3 data points and a range of 0 to 10. The first point on the curve is 1 with a cumulative probability of 0.1 (10% of the distribution values are less than or equal to 1, 90% are greater). The second point on the curve is 5 with a cumulative probability 0.7 (70% of the distribution values are less than or equal to 5, 30% are greater). The third point on the curve is 9 with a cumulative probability of 0.9 (90% of the distribution values are less than or equal to 9, 10% are greater). ( \text{RiskCumul}(100, 200, \text{A1:C1}, \text{A2:C2}) ) returns a cumulative distribution with 3 data points and a range of 100 to 200. Cells A1 through C1 hold the values of each data point, and cells A2 through C2 hold the cumulative probabilities. Note that curly braces are not required when cell ranges are used as entries to the function.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
<th>The points on the curve must be specified in order of increasing value ( X1 &lt; X2 &lt; \ldots &lt; Xn ). The cumulative probabilities ( p ) for ( p ) must be between 0 and 1 and in order of increasing probability ( p1 &lt;= p2 &lt;= \ldots &lt;= pn ). ( \text{minimum} ) must be less than ( X1 ) and ( \text{maximum} ) must be greater than ( Xn ). The maximum number of ( X, p ) pairs is 2,147,483,647.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>min</td>
</tr>
<tr>
<td>-----------</td>
<td>-----</td>
</tr>
<tr>
<td></td>
<td>min &lt; max</td>
</tr>
<tr>
<td>max</td>
<td>continuous parameter</td>
</tr>
<tr>
<td>{x} = {x₁, x₂, ..., x₉}</td>
<td>array of continuous parameters</td>
</tr>
<tr>
<td></td>
<td>min ≤ xᵢ ≤ max</td>
</tr>
<tr>
<td>{p} = {p₁, p₂, ..., p₉}</td>
<td>array of continuous parameters</td>
</tr>
<tr>
<td></td>
<td>0 ≤ pᵢ ≤ 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domain</th>
<th>min ≤ x ≤ max</th>
<th>continuous</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Density and Cumulative Distribution Functions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \frac{p_{i+1} - p_i}{x_{i+1} - x_i} )</td>
<td>for ( x_i ≤ x &lt; x_{i+1} )</td>
</tr>
<tr>
<td>( F(x) = p_i + \left( p_{i+1} - p_i \right) \left( \frac{x - x_i}{x_{i+1} - x_i} \right) )</td>
<td>for ( x_i ≤ x ≤ x_{i+1} )</td>
</tr>
</tbody>
</table>

With the assumptions:
The arrays are ordered from left to right
The \( i \) index runs from 0 to \( N+1 \), with two extra elements:
\( x_0 ≡ \min, p_0 ≡ 0 \) and \( x_{N+1} ≡ \max, p_{N+1} ≡ 1 \).

<table>
<thead>
<tr>
<th>Mean</th>
<th>No Closed Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Skewness</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Mode</td>
<td>No Closed Form</td>
</tr>
</tbody>
</table>
CDF - Cumul(0,5,{1,2,3,4},{.2,.3,.7,.8})

PDF - Cumul(0,5,{1,2,3,4},{.2,.3,.7,.8})
## RiskCumulD

### Description

*RiskCumulD*(minimum, maximum, \{X1, X2, ..., Xn\}, \{p1, p2, ..., pn\}) specifies a cumulative distribution with \(n\) points. The range of the cumulative curve is set by the *minimum* and *maximum* arguments. Each point on the cumulative curve has a value \(X\) and a probability \(p\). Points on the cumulative curve are specified with increasing value and decreasing probability. Each probability entered is a cumulative descending probabilities, that is, the probability of a value greater than the entered \(X\) value. Any number of points may be specified for the curve. Note that even though the \(X,p\) pairs “define” the distribution, any value between *minimum* and *maximum* can be returned.

### Examples

*RiskCumulD*(0, 10, \{1, 5, 9\}, \{0.9, 0.3, 0.1\}) returns a cumulative curve with 3 data points and a range of 0 to 10. The first point on the curve is 1 with a cumulative descending probability of 0.9 (10% of the distribution values are less than or equal to 1, 90% are greater). The second point on the curve is 5 with a cumulative descending probability 0.3 (70% of the distribution values are less than or equal to 5, 30% are greater). The third point on the curve is 9 with a cumulative descending probability of 0.1 (90% of the distribution values are less than or equal to 9, 10% are greater).

*RiskCumulD*(100, 200, A1:C1, A2:C2) returns a cumulative distribution with 3 data points and a range of 100 to 200. Cells A1 through C1 hold the values of each data point, and cells A2 through C2 hold the cumulative probability at each of the 3 points in the distribution. Note that curly braces are not required when cell ranges are used as entries to the function.

### Guidelines

The points on the curve must be specified in order of increasing value \((X1 < X2 < ... < Xn)\).

The cumulative probabilities \(p\) for must be between 0 and 1 and in order of decreasing probability \((p1 \geq p2 \geq ... \geq pn)\).

*minimum* must be less than \(X1\) and *maximum* must be greater than \(Xn\). The maximum number of \(X,p\) pairs is 2,147,483,647.

### Parameters

- **min**: continuous parameter, \(\text{min} < \text{max}\)
- **max**: continuous parameter
- **\{x\} = \{x_1, x_2, ..., x_N\]**: array of continuous parameters, \(\text{min} \leq x_i \leq \text{max}\)
- **\{p\} = \{p_1, p_2, ..., p_N\]**: array of continuous parameters, \(0 \leq p_i \leq 1\)

### Domain

\(\text{min} \leq \text{x} \leq \text{max}\), continuous
<table>
<thead>
<tr>
<th>Density and Cumulative Distribution Functions</th>
<th>( f(x) = \frac{p_i - p_{i+1}}{x_{i+1} - x_i} ) for ( x_i \leq x &lt; x_{i+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(x) = 1 - p_i + (p_i - p_{i+1})\left(\frac{x - x_i}{x_{i+1} - x_i}\right) ) for ( x_i \leq x \leq x_{i+1} )</td>
<td></td>
</tr>
<tr>
<td>The arrays are assumed to be ordered from left to right. The ( i ) index runs from 0 to ( N+1 ), with two extra elements: ( x_0 = \text{min}, p_0 = 1 ) and ( x_{N+1} = \text{max}, p_{N+1} = 0 ).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean</th>
<th>No Closed Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Skewness</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Mode</td>
<td>No Closed Form</td>
</tr>
</tbody>
</table>

| Examples | CDF - CumulD(0,5,[1,2,3,4],[8,7,3,2]) |

![Graph showing CDF - CumulD(0,5,[1,2,3,4],[8,7,3,2])](image)
**RiskDiscrete**

| Description | *RiskDiscrete*(*X1*,*X2*,…,*Xn*),{*p1*,*p2*,…,*pn*}) specifies a general discrete distribution with *n* possible outcomes. Each possible outcome has a value *X* and a probability weight *p* that specifies the outcome's likelihood of occurrence. The probability weights can sum to any value, but internally, @RISK normalizes them so that they sum to 1. This distribution is very flexible and can be used for any discrete set of possibilities. |
| Examples | *RiskDiscrete*({0, 5},{1,1}) returns a discrete distribution with two equally probable values, 0 and 5. Although the weights are 1 and 1, @RISK will normalize them to probabilities, 0.5 and 0.5. *RiskDiscrete*(A1:C1,A2:C2) returns a discrete distribution with three possible outcomes. Cells A1 through C1 hold the possible values, and cells A2 through C2 hold the probability weights. |
| Guidelines | The probability weights must be nonnegative. |
| Parameters | *x* = {*x1*,*x2*,…,*xN*} array of continuous parameters |
| | *p* = {*p1*,*p2*,…,*pN*} array of continuous parameters |
| Domain | *x* ∈ {*x*} discrete |
| Mass and Cumulative Distribution Functions | $f(x) = p_i$ for $x = x_i$ |
|                                         | $f(x) = 0$ for $x \not\in \{x\}$ |
|                                         | $F(x) = 0$ for $x < x_1$ |
|                                         | $F(x) = \sum_{i=1}^{s} p_i$ for $x_s \leq x < x_{s+1}$, $s < N$ |
|                                         | $F(x) = 1$ for $x \geq x_N$ |

The arrays are assumed to be ordered from left to right. The $p$ array is assumed to be normalized so that they sum to 1.

### Mean

$$\sum_{i=1}^{N} x_i p_i \equiv \mu$$

### Variance

$$\sum_{i=1}^{N} (x_i - \mu)^2 p_i \equiv V$$

### Skewness

$$\frac{1}{V^{3/2}} \sum_{i=1}^{N} (x_i - \mu)^3 p_i$$

### Kurtosis

$$\frac{1}{V^2} \sum_{i=1}^{N} (x_i - \mu)^4 p_i$$
<table>
<thead>
<tr>
<th>Mode</th>
<th>The x-value corresponding to the highest p-value.</th>
</tr>
</thead>
</table>

**Examples**

**CDF - Discrete({1,2,3,4},{2,1,2,1})**

<table>
<thead>
<tr>
<th>x</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**PMF - Discrete({1,2,3,4},{2,1,2,1})**

| x  | 0.00 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 |
|----|------|------|------|------|------|------|------|------|----|----|----|----|----|----|----|----|----|----|
| p  | 0.36 | 0.30 | 0.25 | 0.20 | 0.15 | 0.10 | 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
RiskDoubleTriang

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskDoubleTriang}(min, m.\text{likely}, max, p) ) specifies a double triangular distribution with minimum, most likely, and maximum values, along with the probability ( p ) of a value falling between ( min ) and ( m.\text{likely} ). The double triangular distribution allows additional probability information to be specified as compared with the standard triangular distribution RiskTriang. Essentially, this distribution “splices” two triangular distributions, one from ( min ) to ( m.\text{likely} ), the other from ( m.\text{likely} ) to ( max ), and the parameter ( p ) is used to guarantee that the two parts of the distribution are normalized so that the total area under the density curve is 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskDoubleTriang}(1, 2, 3, 0.1) ) returns a double triangular distribution with a minimum possible value of 1, a most likely of 2, and a maximum of 3. The probability of a value in the range 1 to 2 is 0.1 or 10%. ( \text{RiskDoubleTriang}(A10, B10, 500, 0.4) ) returns a triangular distribution with a \textit{minimum} value taken from cell A10, a \textit{most likely} value taken from cell B10 and a \textit{maximum} value of 500. The probability of a value in the range with a minimum from cell A10 and a maximum from cell B10 is 0.4.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
<tr>
<td>Parameters</td>
<td>( min ) continuous boundary parameter ( min &lt; max ) * ( m.\text{likely} ) continuous mode parameter ( min &lt; m.\text{likely} &lt; max ) ( max ) continuous boundary parameter ( p ) probability of lower section ( 0 &lt; p &lt; 1 )</td>
</tr>
<tr>
<td>Domain</td>
<td>( min \leq x \leq max ) continuous</td>
</tr>
<tr>
<td>Mass and Cumulative Distribution Functions</td>
<td>( f(x) = \frac{2p(x - min)}{(m.\text{likely} - min)^2} ) ( min \leq x \leq m.\text{likely} ) ( f(x) = \frac{2(1 - p)(max - x)}{(max - m.\text{likely})^2} ) ( m.\text{likely} \leq x \leq max ) ( F(x) = \frac{p(x - min)^2}{(m.\text{likely} - min)^2} ) ( min \leq x \leq m.\text{likely} )</td>
</tr>
<tr>
<td>Mean</td>
<td>( \frac{(p)(\text{min}) + 2(\text{m.likely}) + (1 - p)(\text{max})}{6} )</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Variance</td>
<td>Complicated</td>
</tr>
<tr>
<td>Skewness</td>
<td>Complicated</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>Complicated</td>
</tr>
<tr>
<td>Mode</td>
<td>( \text{m.likely} )</td>
</tr>
</tbody>
</table>

**Function Reference**

\[
F(x) = 1 - \frac{(1 - p)(\max - x)^2}{(\max - \text{m.likely})^2} \quad \text{m.likely} \leq x \leq \max
\]

**Examples**

![PDF - DoubleTriang(0,.5,1,.4)](attachment.png)
### RiskDUniform

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskDUniform}({X_1, X_2, \ldots, X_n}) ) specifies a discrete distribution with ( n ) equally likely outcome values, the ( X )'s. Note that the ( X )'s do not have to be consecutive integers; any set of ( X ) values is allowed. You use the RiskIntUniform function if the possible ( X ) values are consecutive integers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskDUniform}({1, 2.1, 4.45, 99}) ) returns a discrete uniform distribution with 4 possible values: 1, 2.1, 4.45, and 99. ( \text{RiskDUniform}(A1:A5) ) returns a discrete uniform distribution with 5 possible outcomes. The possible outcomes have the values taken from cells A1 through A5.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
<tr>
<td>Parameters</td>
<td>( {x} = {x_1, x_2, \ldots, x_n} ) array of continuous parameters</td>
</tr>
<tr>
<td>Domain</td>
<td>( x \in {x} ) discrete</td>
</tr>
<tr>
<td>Mass and Cumulative Distribution Functions</td>
<td>( f(x) = \frac{1}{N} ) for ( x \in {x} )</td>
</tr>
<tr>
<td></td>
<td>( f(x) = 0 ) for ( x \not\in {x} )</td>
</tr>
<tr>
<td></td>
<td>( F(x) = 0 ) for ( x &lt; x_1 )</td>
</tr>
<tr>
<td></td>
<td>( F(x) = \frac{i}{N} ) for ( x_i \leq x &lt; x_{i+1} )</td>
</tr>
<tr>
<td></td>
<td>( F(x) = 1 ) for ( x \geq x_N )</td>
</tr>
<tr>
<td>Mean</td>
<td>( \frac{1}{N} \sum_{i=1}^{N} x_i \equiv \mu )</td>
</tr>
</tbody>
</table>

This assumes that the \( \{x\} \) array is ordered.
| **Variance** | \[
\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2 \equiv V
\] |
| **Skewness** | \[
\frac{1}{NV^{3/2}} \sum_{i=1}^{N} (x_i - \mu)^3
\] |
| **Kurtosis** | \[
\frac{1}{NV^2} \sum_{i=1}^{N} (x_i - \mu)^4
\] |
| **Mode** | Not uniquely defined |
| **Examples** | CDF - DUniform({1,5,8,11,12}) |

![CDF - DUniform({1,5,8,11,12})](image)
PMF - DUniform((1,5,8,11,12))
**RiskErf**

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskErf}(h)$ specifies an error function with variance parameter $h$. The error function distribution is derived from a normal distribution.</th>
</tr>
</thead>
</table>
| Examples    | $\text{RiskErf}(5)$ returns an error function with variance parameter 5.  
$\text{RiskErf}(A7)$ returns an error function with variance parameter from cell A7. |
| Guidelines  | $h$ must be positive. |
| Parameters  | $h$ continuous inverse scale parameter $h > 0$ |
| Domain      | $-\infty < x < +\infty$ continuous |
| Density and Cumulative Distribution Functions | $f(x) = \frac{h}{\sqrt{\pi}} e^{-(hx)^2}$  
$F(x) = \Phi(\sqrt{2hx}) = \frac{1 + \text{erf}(hx)}{2}$ |

Here, $\Phi$ is called the cumulative distribution function for the Normal(0,1), and erf is the Error Function.

<table>
<thead>
<tr>
<th>Mean</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>$\frac{1}{2h^2}$</td>
</tr>
<tr>
<td>Skewness</td>
<td>0</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3</td>
</tr>
<tr>
<td>Mode</td>
<td>0</td>
</tr>
</tbody>
</table>
### RiskErlang

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskErlang}(m, \beta)$ generates an Erlang distribution with integer shape parameter $m$ and scale parameter $\beta$.</th>
</tr>
</thead>
</table>

#### Examples
- **$\text{RiskErlang}(5, 10)$** returns an Erlang distribution with shape parameter 5 and scale parameter 10.
- **$\text{RiskErlang}(A1, A2/6.76)$** returns an Erlang distribution with shape parameter value taken from cell A1 and scale parameter equaling the value in cell A2 divided by 6.76.

#### Guidelines
- $m$ must be a positive integer.
- $\beta$ must be positive.

#### Parameters
- $m$ integral shape parameter $m > 0$
- $\beta$ continuous scale parameter $\beta > 0$

#### Domain
- $0 \leq x < +\infty$ continuous

#### Density and Cumulative Distribution Functions
- $f(x) = \frac{1}{\beta (m-1)! \left(\frac{x}{\beta}\right)^{m-1}} e^{-x/\beta}$
- $F(x) = \frac{\Gamma_{x/\beta}(m)}{\Gamma(m)} = 1 - e^{-x/\beta} \sum_{i=0}^{m-1} \frac{(x/\beta)^i}{i!}$

Here, $\Gamma$ is the Gamma Function and $\Gamma_x$ is the Incomplete Gamma Function.

#### Mean
- $m\beta$

#### Variance
- $m\beta^2$

#### Skewness
- $\frac{2}{\sqrt{m}}$

#### Kurtosis
- $3 + \frac{6}{m}$
<table>
<thead>
<tr>
<th>Mode</th>
<th>$\beta(m - 1)$</th>
</tr>
</thead>
</table>

**Examples**

CDF - Erlang(2,1)

PDF - Erlang(2,1)
**RiskExpon**

| Description | RiskExpon(beta) specifies an exponential distribution with parameter beta, the mean of the distribution. This distribution is the continuous time equivalent of the Geometric distribution. It is often used to model the waiting time for the first occurrence of a process which is continuous in time and of constant intensity. It can be used in similar applications to the Geometric distribution (queueing, maintenance, breakdown modelling, and others), but it is unrealistic in some practical applications because of the constant intensity property (also known as the memoryless property). |
| Examples | RiskExpon(5) returns an exponential distribution with mean 5.  
RiskExpon(A1) returns an exponential distribution with mean from cell A1. |
| Guidelines | beta must be positive. |
| Parameters | $\beta$ continuous scale parameter $\beta > 0$ |
| Domain | $0 \leq x < +\infty$ continuous |
| Density and Cumulative Distribution Functions | $f(x) = \frac{e^{-x/\beta}}{\beta}$  
$F(x) = 1 - e^{-x/\beta}$ |
| Mean | $\beta$ |
| Variance | $\beta^2$ |
| Skewness | 2 |
| Kurtosis | 9 |
| Mode | 0 |
Examples

CDF - Expon(1)

PDF - Expon(1)
## RiskExponAlt, RiskExponAltD

| Description | RiskExponAlt(arg1type, arg1value, arg2type, arg2value) specifies an exponential distribution with two arguments of the type arg1type and arg2type. These parameters can be either “beta”, “loc”, or a value between 0 and 1 to specify a percentile. |
| Examples | RiskExponAlt("beta", 1, 95%, 10) returns an exponential distribution with beta parameter 1 and a 95% percentile of 10. Note that this distribution will be shifted (the minimum won’t be 0) to achieve the two specified parameters. |
| Guidelines | With RiskExponAltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value. |
### RiskExtValue

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskExtValue}(\alpha, \beta) ) specifies an extreme value distribution with location parameter ( \alpha ) and shape parameter ( \beta ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskExtValue}(1,2) ) returns an extreme value distribution with location parameter 1 and shape parameter 2. ( \text{RiskExtValue}(\text{A1},\text{B1}) ) returns an extreme value distribution with location parameter taken from cell A1 and shape parameter taken from cell B1.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>( \beta ) must be positive.</td>
</tr>
<tr>
<td>Parameters</td>
<td>( \alpha ) continuous location parameter ( \beta ) continuous scale parameter ( \beta &gt; 0 )</td>
</tr>
<tr>
<td>Domain</td>
<td>( -\infty &lt; x &lt; +\infty ) continuous</td>
</tr>
</tbody>
</table>
| Density and Cumulative Distribution Functions | \[
f(x) = \frac{1}{b} \left( \frac{1}{e^{z+\exp(-z)}} \right)\]
\[
F(x) = \frac{1}{e^{\exp(-z)}} \quad \text{where} \quad z = \frac{(x - \alpha)}{\beta}
\]
Where \( \alpha = \alpha \), \( \beta = \beta \) |
| Mean | \( a + b \Gamma'(1) \approx a + .577b \) Here, \( \Gamma'(x) \) is the derivative of the Gamma Function. |
| Variance | \( \frac{\pi^2b^2}{6} \) |
| Skewness | \( \frac{12\sqrt{6}}{\pi^3} \zeta(3) \approx 1.139547 \) |
| Kurtosis | 5.4 |
| Mode | \( a \) |
Examples

PDF - ExtValue(0, 1)

CDF - ExtValue(0, 1)
### RiskExtValueAlt, RiskExtValueAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskExtValueAlt}(\text{arg1type}, \text{arg1value}, \text{arg2type}, \text{arg2value})$ specifies an extreme value distribution with two arguments of the type $\text{arg1type}$ and $\text{arg2type}$. These parameters can be either “alpha”, “beta”, or a value between 0 and 1 to specify a percentile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskExtvalueAlt}(5%, 10, 95%, 100)$ returns an extreme value distribution with 5th percentile 10 and 95th percentile 100.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>With $\text{RiskExtValueAltD}$, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>

---

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**RiskExtValueMin**

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskExtValueMin}(\text{alpha}, \text{beta}) ) specifies an extreme value “min” distribution with location parameter ( \text{alpha} ) and shape parameter ( \text{beta} ).</th>
</tr>
</thead>
</table>
| Examples | \( \text{RiskExtValueMin}(1,2) \) returns an extreme value min distribution with location parameter 1 and shape parameter 2.  
\( \text{RiskExtValueMin}(A1,B1) \) returns an extreme value min distribution with location and shape parameters taken from cells A1 and B1. |
| Guidelines | \( \text{beta} \) must be positive. |
| Parameters | \( \text{alpha} \)  continuous location parameter  
\( \text{beta} \)  continuous scale parameter  
\( \text{beta} > 0 \) |
| Domain | \( -\infty < x < +\infty \)  continuous |
| Density and Cumulative Distribution Functions | \( f(x) = \frac{1}{b} \left( e^{z - \exp(-z)} \right) \)  
\( F(x) = 1 - \exp(-e^z) \)  
\( \text{where } z \equiv \frac{(x - a)}{b} \)  
Here, \( a = \text{alpha}, b = \text{beta} \) |
| Mean | \( a - b\Gamma'(1) \approx a - 0.577b \)  
Here, \( \Gamma'(x) \) is the derivative of the Gamma Function. |
| Variance | \( \frac{\pi^2 b^2}{6} \) |
| Skewness | \( \frac{-12\sqrt{-6}}{\pi^3} \zeta(-3) \approx -1.139547 \) |
| Kurtosis | 5.4 |
| Mode | \( a \) |
Examples

PDF - ExtValueMin(0,1)

CDF - ExtValueMin(0,1)
# RiskExtValueMinAlt, RiskExtValueMinAltD

| Description | \( \text{RiskExtValueMinAlt}(\text{arg1type, arg1value, arg2type, arg2value}) \) specifies an extreme value min distribution with two arguments of the type \( \text{arg1type} \) and \( \text{arg2type} \). These parameters can be either “alpha”, “beta”, or a value between 0 and 1 to specify a percentile. |
| Examples | \( \text{RiskExtvalueMinAlt}(5\%, 10, 95\%, 100) \) returns an extreme value min distribution with 5\(^{th}\) percentile and 95\(^{th}\) percentile 100. |
| Guidelines | With \( \text{RiskExtValueMinAltD} \), any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value. |
### RiskF

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskF}(v_1, v_2) ) specifies an F distribution has two degrees of freedom, ( v_1 ) and ( v_2 ). The F distribution is used primarily in statistical hypothesis testing to test the equality of two population variances.</th>
</tr>
</thead>
</table>
| Examples | \( \text{RiskF}(1,2) \) returns an F distribution with degrees of freedom parameters 1 and 2.  
\( \text{RiskF}(C_{12}, C_{13}) \) returns an F distribution with degrees of freedom parameters taken from cells C12 and C13. |
| Guidelines | \( v_1 \) and \( v_2 \) must be positive integers. |
| Parameters | \( v_1 \) first degrees of freedom  
\( v_2 \) second degrees of freedom |
| Domain | \( x \geq 0 \) continuous |
| Density and Cumulative Distribution Functions | \[
\begin{align*}
  f(x) &= \left( \frac{v_1}{v_2} \right)^{v_1/2} \frac{v_1-2}{x^{v_1/2}} \frac{1}{B(v_1/2, v_2/2)} \left[ 1 + \frac{v_1}{v_2} x \right]^{-(v_1+v_2)/2} \\
  F(x) &= I_{1+\frac{v_2}{v_1 x + v_2}} \left( \frac{v_1}{2}, \frac{v_2}{2} \right)
\end{align*}
\]
Here, \( B \) is the Beta Function and \( I \) is the Regularized Incomplete Beta Function. |
| Mean | \[
  \frac{v_2}{v_2 - 2} \quad \text{for} \quad v_2 > 2
\]
| Variance | \[
\frac{2v_2(v_1+v_2-2)}{v_1(v_2-2)^2(v_2-4)} \quad \text{for} \quad v_2 > 4
\]
| **Skewness** | \[
\frac{(2v_1 + v_2 - 2)}{(v_2 - 6)} \cdot \sqrt{\frac{8(v_2 - 4)}{v_1(v_1 + v_2 - 2)}} \quad \text{for } v_2 > 6
\] |
| **Kurtosis** | \[
3 + 12 \left[ \frac{(v_2 - 2)^2(v_2 - 4) + v_1(v_1 + v_2 - 2)(5v_2 - 22)}{v_1(v_2 - 6)(v_2 - 8)(v_1 + v_2 - 2)} \right]
\] for \(v_2 > 8\) |
| **Mode** | \[
\frac{v_2(v_1 - 2)}{v_1(v_2 + 2)} \quad \text{for } v_1 > 2
\]
\[
0 \quad \text{for } v_1 \leq 2
\] |

**Examples**

![PDF - F(4,3)](attachment:pdf.png)
CDF - F(4,3)
**RiskGamma**

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskGamma}(\alpha,\beta)$ specifies a gamma distribution with shape parameter $\alpha$ and scale parameter $\beta$. The Gamma distribution often represents the distribution of interarrival times for several events from a Poisson process. More generally, it can be used to model a nonnegative value when skewness is desired. In a Bayesian context, it can be used to represent the distribution of possible values for the rate of a Poisson process, based on observations of the process.</th>
</tr>
</thead>
</table>
| Examples | $\text{RiskGamma}(2.5,1)$ returns a gamma distribution with shape parameter 2.5 and scale parameter 1.  
$\text{RiskGamma}(C12,C13)$ returns a gamma distribution with shape and scale parameters taken from cells C12 and C13. |
| Guidelines | $\alpha$ and $\beta$ must be positive. |
| Parameters | $\alpha$ continuous shape parameter $\alpha > 0$  
$\beta$ continuous scale parameter $\beta > 0$ |
| Domain | $0 < x < +\infty$ continuous |
| Density and Cumulative Distribution Functions | $f(x) = \frac{1}{\beta \Gamma(\alpha)} \left( \frac{x}{\beta} \right)^{\alpha-1} e^{-x/\beta}$  
$F(x) = \frac{\Gamma_{x/\beta}(\alpha)}{\Gamma(\alpha)}$  
Here, $\Gamma$ is the Gamma Function and $\Gamma_{x}$ is the Incomplete Gamma Function. |
<p>| Mean | $\beta \alpha$ |
| Variance | $\beta^2 \alpha$ |
| Skewness | $\frac{2}{\sqrt{\alpha}}$ |</p>
<table>
<thead>
<tr>
<th>Kurtosis</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 + \frac{6}{\alpha}$</td>
<td></td>
</tr>
<tr>
<td><strong>Mode</strong></td>
<td></td>
</tr>
<tr>
<td>$\beta(\alpha - 1)$</td>
<td>if $\alpha \geq 1$</td>
</tr>
<tr>
<td>0</td>
<td>if $\alpha &lt; 1$</td>
</tr>
<tr>
<td><strong>Examples</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image1" alt="CDF - Gamma(4,1)" /></td>
<td></td>
</tr>
<tr>
<td><img src="image2" alt="PDF - Gamma(4,1)" /></td>
<td></td>
</tr>
</tbody>
</table>
## RiskGammaAlt, RiskGammaAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskGammaAlt}(\text{arg1type, arg1value, arg2type, arg2value, arg3type, arg3value})$ specifies a gamma distribution with three arguments of the type $\text{arg1type}$ to $\text{arg3type}$. These arguments can be either “alpha”, “beta”, “loc”, or a value between 0 and 1 to specify a percentile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskGammaAlt}(\text{&quot;alpha&quot;,1,&quot;beta&quot;,5,95%,10})$ returns a gamma distribution with shape parameter 1, scale parameter 5, and 95th percentile 10. Note that this distribution will be shifted (minimum value won’t be 0) to achieve the specified parameters.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>$\text{alpha}$ and $\text{beta}$ must be positive. With $\text{RiskGammaAltD}$, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>
RiskGeneral

Description

RiskGeneral(minimum, maximum, {X1, X2, ..., Xn}, {p1, p2, ..., pn}) specifies a generalized probability distribution defined by a density curve specified at n points. Each point has a value X and a probability weight p that specifies the relative height of the density curve at that X value. The probability weights can sum to any value, but internally, @RISK normalizes them so that they sum to 1.

Examples

RiskGeneral(0, 10, {2, 5, 7, 9}, {1, 2, 3, 1}) returns a general probability distribution density function with four points. The distribution ranges from 0 to 10 with the height of the curve specified at the values 2, 5, 7, and 9. The relative heights of the curve at these values are 1, 2, 3, and 1. The curve intersects the X-axis at 0 and 10.

RiskGeneral(100, 200, A1:C1, A2:C2) returns a general probability distribution with three data points and a range of 100 to 200. Cells A1 through C1 hold the X values, and cells A2 through C2 hold the p values. Note that curly braces are not required when cell ranges are used as array entries to the function.

Guidelines

The p values must be nonnegative.
The X values must be entered in increasing order and must fall within the minimum-maximum range of the distribution.
minimum must be less than maximum.

Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>continuous parameter</td>
</tr>
<tr>
<td>max</td>
<td>continuous parameter</td>
</tr>
<tr>
<td>{x} = {x1, x2, ..., xn}</td>
<td>array of continuous parameters</td>
</tr>
<tr>
<td>{p} = {p1, p2, ..., pn}</td>
<td>array of continuous parameters</td>
</tr>
</tbody>
</table>

Domain

min ≤ x ≤ max  continuous
### Density and Cumulative Distribution Functions

The density function is given by:

\[
    f(x) = p_i + \left[ \frac{x - x_i}{x_{i+1} - x_i} \right] (p_{i+1} - p_i) \quad \text{for} \; x_i \leq x \leq x_{i+1}
\]

The cumulative distribution function is given by:

\[
    F(x) = F(x_i) + (x - x_i) \left[ p_i + \frac{(p_{i+1} - p_i)(x - x_i)}{2(x_{i+1} - x_i)} \right]
\]

for \( x_i \leq x \leq x_{i+1} \)

The \( p \) and \( X \) arrays are assumed to be ordered from left to right. The \( p \) array is normalized so that the area under the density curve is 1. The \( i \) index runs from 0 to \( N+1 \), with two extra elements: \( x_0 \equiv \min, p_0 \equiv 0 \) and \( x_{N+1} \equiv \max, p_{N+1} \equiv 0 \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Closed Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td></td>
</tr>
<tr>
<td>Mode</td>
<td></td>
</tr>
</tbody>
</table>
Examples

CDF - General(0,5,{1,2,3,4},{2,1,2,1})

PDF - General(0,5,{1,2,3,4},{2,1,2,1})
## RiskGeomet

### Description

*RiskGeomet*(\( p \)) specifies a geometric distribution with parameter \( p \). This distribution is usually used to model the number of trials before the first “success” in a sequence of independent, identical trials with probability \( p \) of success on each trial. For example, it could be used in a maintenance model to represent the number of months before a car breaks down. However, because the distribution requires a constant probability of breakdown per trial, other models, where the probability of breakdown increases with age, might be more realistic.

### Examples

- **RiskGeomet(0.25)** returns a geometric distribution with a 25% probability of success on each trial.
- **RiskGeomet(A18)** returns a geometric distribution with a probability of success on each trial taken from cell A18.

### Guidelines

\( p \) must be between 0 and 1 (0 isn’t allowed, 1 is allowed).

### Parameters

- **\( p \)**: continuous “success” probability, \( 0 < p \leq 1 \)

### Domain

- \( 0 \leq x < +\infty \): discrete integers

### Mass and Cumulative Distribution Functions

\[
f(x) = p(1 - p)^x \]
\[
F(x) = 1 - (1 - p)^{x+1} \]

### Mean

\[
\frac{1}{p} - 1 \]

### Variance

\[
\frac{1 - p}{p^2} \]

### Skewness

\[
\frac{(2 - p)}{\sqrt{1 - p}} \quad \text{for} \ p < 1
\]

Not Defined \quad \text{for} \ p = 1

---

Reference: Distribution Functions
<table>
<thead>
<tr>
<th>Kurtosis</th>
<th>$9 + \frac{p^2}{1-p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>for $p &lt; 1$</td>
</tr>
<tr>
<td></td>
<td>Not Defined</td>
</tr>
<tr>
<td></td>
<td>for $p = 1$</td>
</tr>
<tr>
<td>Mode</td>
<td>0</td>
</tr>
<tr>
<td>Examples</td>
<td></td>
</tr>
<tr>
<td></td>
<td>![CDF - Geomet(.5)]</td>
</tr>
<tr>
<td></td>
<td>![PMF - Geomet(.5)]</td>
</tr>
</tbody>
</table>
### RiskHistogrm

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskHistogrm}(\text{minimum}, \text{maximum}, {p_1, p_2, \ldots, p_n})$ specifies a user-defined histogram distribution with a range defined by the specified $\text{minimum}$ and $\text{maximum}$ values. This range is divided into $n$ classes. Each class has a weight $p$ reflecting the probability of occurrence of a value within the class. These weights can be any nonnegative values to indicate relative likelihoods, but @RISK will normalize them so that they sum to 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskHistogrm}(10, 20, {1, 2, 3, 2, 1})$ returns a histogram distribution with $\text{minimum}$ value 10 and a $\text{maximum}$ value 20. This range is divided into 5 equal-length classes because there are 5 probability weights, 1, 2, 3, 2 and 1. The normalized probabilities would be 1/9, 2/9, 3/9, 2/9, and 1/9. $\text{RiskHistogrm}(A1, A2, B1:B3)$ returns a histogram distribution with $\text{minimum}$ and $\text{maximum}$ values taken from cells A1 and A2. This range is divided into 3 equal-length classes because there are 3 probability weights, taken from cells B1 through B3.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>All $p$ values must be nonnegative.</td>
</tr>
<tr>
<td>Parameters</td>
<td>$\text{min}$ continuous parameter $\text{min} &lt; \text{max}$ *</td>
</tr>
<tr>
<td></td>
<td>$\text{max}$ continuous parameter</td>
</tr>
<tr>
<td></td>
<td>${p} = {p_1, p_2, \ldots, p_N}$ array of continuous parameters $p_i \geq 0$</td>
</tr>
<tr>
<td></td>
<td>* $\text{min} = \text{max}$ is supported for modelling convenience, but yields a degenerate distribution.</td>
</tr>
<tr>
<td>Domain</td>
<td>$\text{min} \leq x \leq \text{max}$ continuous</td>
</tr>
</tbody>
</table>
### Density and Cumulative Distribution Functions

<table>
<thead>
<tr>
<th>Density and Cumulative Distribution Functions</th>
<th>( f(x) = p_i ) for ( x_i \leq x &lt; x_{i+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( F(x) = F(x_i) + p_i \left( \frac{x - x_i}{x_{i+1} - x_i} \right) ) for ( x_i \leq x \leq x_{i+1} )</td>
</tr>
<tr>
<td></td>
<td>( x_i = \min + i \left( \frac{\max - \min}{N} \right) )</td>
</tr>
</tbody>
</table>

Here, the \( \{p\} \) array has been normalized so that the area under the histogram is 1.

<table>
<thead>
<tr>
<th>Mean</th>
<th>No Closed Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Skewness</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Mode</td>
<td>Not Uniquely Defined.</td>
</tr>
</tbody>
</table>
Examples

CDF - Histogrm(0,5,{6,5,3,4,5})

PDF - Histogrm(0,5,{6,5,3,4,5})
**RiskHypergeo**

| **Description** | **RiskHypergeo**(n,D,M) specifies a hypergeometric distribution with sample size n, number of “successes” in the population D, and population size M. This distribution is typically used in surveys when sampling from a population composed of “successes” and “failures” is performed without replacement (the same population member can be sampled only once). If the population contains D successes and M-D failures, this distribution represents the number of successes in a sample of size n. (If the sample size is small relative to the population size, this distribution can be approximated well by a Binomial distribution with parameters n and D/M, which is appropriate for sampling with replacement.) |
| **Examples** | **RiskHypergeo**(50,10,1000) returns a hypergeometric distribution with sample size 50, and 10 “successes” in the population of size 1000. **RiskHypergeo**(A6,A7,A8) returns a hypergeometric distribution with its three parameters taken from cells A6, A7, and A8. |
| **Guidelines** | All arguments, n, D and M, must be positive integers. Both n and D must be less than or equal to M. |

| **Parameters** | n | the number of draws | integer | 0 ≤ n ≤ M |
| | D | the number of "successes" | integer | 0 ≤ D ≤ M |
| | M | the total number of items | integer | M ≥ 0 |

| **Domain** | max(0,n+D-M) ≤ x ≤ min(n,D) | discrete integers |

| **Mass and Cumulative Distribution Functions** | \( f(x) = \binom{D}{x} \binom{M-D}{n-x} \binom{M}{n} \) | \( F(x) = \sum_{i=1}^{x} \binom{D}{x} \binom{M-D}{n-x} \binom{M}{n} \) |

<p>| <strong>Mean</strong> | ( \frac{nD}{M} ) for M &gt; 0 |
| | 0 for M = 0 |</p>
<table>
<thead>
<tr>
<th></th>
<th>Formula</th>
</tr>
</thead>
</table>
| **Variance**   | \[
\frac{nD \left[ (M - D)(M - n) \right]}{M^2 \left[ \frac{(M - n)}{M - 1} \right]} \]
   for \(M > 1\)

|                | 0 \quad \text{for } M = 1 \quad \text{for } M = 1 |
| **Skewness**   | \[
\frac{(M - 2D)(M - 2n)}{M - 2} \sqrt{\frac{M - 1}{nD(M - D)(M - n)}} \]
   for \(M > 2, M > D > 0, M > n > 0\)

|                | \text{Not Defined} \quad \text{otherwise} |
| **Kurtosis**   | \[
\frac{M^2(M - 1)}{n(M - 2)(M - 3)(M - n)} \left[ \frac{M(M + 1) - 6n(M - n)}{D(M - D)} + \frac{3n(M - n)(M + 6)}{M^2} - 6 \right] \]
   for \(M > 3, M > D > 0, M > n > 0\)

|                | \text{Not Defined} \quad \text{otherwise} |
| **Mode**       | (bimodal) \(x_m\) and \(x_{m-1}\) \quad \text{if } x_m \text{ is integral} |

|                | (unimodal) biggest integer less than \(x_m\) \quad \text{otherwise} |

\[
x_m \equiv \frac{(n + 1)(D + 1)}{M + 2} \quad \text{where} \quad x_m \text{ is integral}\
\]
Examples

CDF - HyperGeo(6,5,10)

PMF - HyperGeo(6,5,10)
## RiskIntUniform

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskIntUniform(minimum, maximum) specifies a discrete distribution on the integers from minimum to maximum, where each of the integers in this range is equally likely.</th>
</tr>
</thead>
</table>
| Examples    | RiskIntUniform(10,20) returns a uniform distribution with equally likely values 10, 11, ..., 20.  
RiskIntUniform(A1+90,B1) returns a uniform distribution with a minimum value equaling the value in cell A1 plus 90 and a maximum value taken from cell B1. |
| Guidelines  | minimum must be less than maximum, and both must be integers. |
| Parameters  | min discrete boundary parameter min < max  
max discrete boundary probability |
| Domain      | min ≤ x ≤ max discrete integers |
| Mass and Cumulative Distribution Functions |  
\[f(x) = \frac{1}{\text{max} - \text{min} + 1}\]  
\[F(x) = \frac{x - \text{min} + 1}{\text{max} - \text{min} + 1}\] |
| Mean        | \[\frac{\text{min} + \text{max}}{2}\] |
| Variance    | \[\frac{\Delta(\Delta + 2)}{12}\] where \(\Delta = (\text{max} - \text{min})\) |
| Skewness    | 0 |
| Kurtosis    | \(\frac{9}{5} \cdot \left( \frac{n^2 - 7/3}{n^2 - 1} \right)\) where \(n = (\text{max} - \text{min} + 1)\) |
| Mode        | Not uniquely defined |
Examples

CDF - IntUniform(0, 8)

PMF - IntUniform(0, 8)
### RiskInvgauss

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskInvgauss(mu, lambda) specifies an inverse Gaussian distribution with mean mu and shape parameter lambda.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskInvgauss(5,2) returns an inverse Gaussian distribution with mean 5 and shape parameter 2. RiskInvgauss(B5,B6) returns an inverse Gaussian distribution with mean taken from cell B5 and shape parameter taken from cell B6.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Both mu and lambda must be positive.</td>
</tr>
</tbody>
</table>
| Parameters  | µ  continuous parameter  µ > 0  
λ  continuous parameter  λ > 0 |
| Domain      | x > 0  continuous |
| Density and Cumulative Distribution Functions | f(x) = \( \frac{\lambda}{\sqrt{2\pi x^3}} e^{-\frac{\lambda(x-\mu)^2}{2\mu^2x}} \)  
F(x) = \( \Phi \left[ \sqrt{\frac{\lambda}{x}} \left( \frac{x}{\mu} - 1 \right) \right] + e^{2\lambda/\mu} \Phi \left[ -\sqrt{\frac{\lambda}{x}} \left( \frac{x}{\mu} + 1 \right) \right] \)  
Here, \( \Phi(z) \) is the cumulative distribution function of a Normal(0,1). |
| Mean        | µ |
| Variance    | \( \frac{\mu^3}{\lambda} \) |
| Skewness    | \( 3\sqrt{\frac{\mu}{\lambda}} \) |
### Kurtosis

\[ 3 + 15 \frac{\mu}{\lambda} \]

### Mode

\[ \mu \left[ \sqrt{1 + \frac{9\mu^2}{4\lambda^2}} - \frac{3\mu}{2\lambda} \right] \]

### Examples

**PDF - InvGauss(1,2)**

![PDF - InvGauss(1,2) Graph](image-url)
CDF - InvGauss(1,2)
## RiskInvgaussAlt, RiskInvgaussAltD

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskInvgaussAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value) specifies an inverse Gaussian distribution with three arguments of the type arg1type to arg3type. These arguments can be either “mu”, “lambda”, “loc”, or a value between 0 and 1 to specify a percentile.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskInvgaussAlt(&quot;mu&quot;,10,5%,1,95%,25) returns an inverse Gaussian distribution with a mu value 10, 5th percentile 1, and 95th percentile 25.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both mu and lambda must be positive. With RiskInvgaussAltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>
### RiskJohnsonMoments

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskJohnsonMoments(\textit{mean, standardDeviation, skewness, kurtosis}) specifies one of four distributions functions (all members of the Johnson system) that matches the specified \textit{mean}, \textit{standard deviation}, \textit{skewness}, and \textit{kurtosis}. This resulting distribution is either a JohnsonSU, JohnsonSB, lognormal, or normal distribution.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskJohnsonMoments(10,20,4,41) returns a distribution from the Johnson family that has mean 10, standard deviation 20, skewness 4, and kurtosis 41. RiskJohnsonMoments(A6,A7,A8,A9) returns a distribution from the Johnson family that takes the four parameters from cells A6 to A9.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>\textit{standard deviation} must be positive. \textit{kurtosis} must be greater than 1.</td>
</tr>
<tr>
<td>Parameters</td>
<td>$\mu$ \hspace{1em} continuous location parameter</td>
</tr>
<tr>
<td></td>
<td>$\sigma$ \hspace{1em} continuous scale parameter $\sigma &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>$s$ \hspace{1em} continuous shape parameter</td>
</tr>
<tr>
<td></td>
<td>$k$ \hspace{1em} continuous shape parameter $k &gt; 1$</td>
</tr>
<tr>
<td></td>
<td>$k - s^2 \geq 1$</td>
</tr>
<tr>
<td>Domain</td>
<td>$-\infty &lt; x &lt; +\infty$ \hspace{1em} continuous</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>See entries for each Johnson system distribution</td>
</tr>
<tr>
<td>Mean</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Variance</td>
<td>$\sigma^2$</td>
</tr>
<tr>
<td>Skewness</td>
<td>$s$</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>$k$</td>
</tr>
<tr>
<td>Mode</td>
<td>No Closed Form</td>
</tr>
</tbody>
</table>

Reference: Distribution Functions
RiskJohnsonSB

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskJohnsonSB</strong>(alpha1,alpha2,a,b) specifies a Johnson “system bounded” distribution with shape parameters alpha1 and alpha2, and boundary (min and max) parameters a and b.</th>
</tr>
</thead>
</table>
| Examples | **RiskJohnsonSB**(10,20,1,2) returns a JohnsonSB distribution with parameters 10, 20, 1, and 2.  
**RiskJohnsonSB**(A6,A7,A8,A9) returns a JohnsonSB distribution with parameters taken from cells A6 to A9. |
| Guidelines | b must be greater than a. |
| Parameters | alpha1 continuous shape parameter  
alpha2 continuous shape parameter  
a continuous boundary parameter  
b continuous boundary parameter  
alpha2 > 0  
b > a |
| Domain | a ≤ x ≤ b continuous |
| Density and Cumulative Distribution Functions |  
f(x) = \frac{\alpha_2 (b - a)}{\sqrt{2\pi} (x - a)(b - x)} \times e^{-\frac{1}{2}\left[\alpha_1 + \alpha_2 \ln\left(\frac{x - a}{b - x}\right)\right]}  
F(x) = \Phi\left[\alpha_1 + \alpha_2 \ln\left(\frac{x - a}{b - x}\right)\right]  
Here, \Phi is the cumulative distribution function of a standard Normal(0, 1). |
| Mean | Closed Form exists but is extremely complicated. |
| Variance | Closed Form exists but is extremely complicated. |
| Skewness | Closed Form exists but is extremely complicated. |
| Kurtosis | Closed Form exists but is extremely complicated. |
| Mode | No Closed Form. |
Examples

PDF - JohnsonSB(2,2,0,1)

CDF - JohnsonSB(2,2,0,1)
### RiskJohnsonSU

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskJohnsonSU((alpha1, alpha2, gamma, beta)) specifies a Johnson “system unbounded” distribution with shape parameters (alpha1) and (alpha2), location parameter (gamma), and scale parameter (beta).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskJohnsonSU(10,20,1,2)</strong> returns a JohnsonSU distribution with parameters 10, 20, 1, and 2. &lt;br&gt;<strong>RiskJohnsonSU(A6,A7,A8,A9)</strong> returns a JohnsonSU distribution with parameters taken from cells A6 to A9.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Both (alpha2) and (beta) must be positive.</td>
</tr>
<tr>
<td>Parameters</td>
<td>(alpha1) continuous shape parameter &lt;br&gt;(alpha2) continuous shape parameter (alpha2 &gt; 0) &lt;br&gt;(gamma) continuous location parameter &lt;br&gt;(beta) continuous scale parameter (beta &gt; 0)</td>
</tr>
<tr>
<td>Domain</td>
<td>(-\infty &lt; x &lt; +\infty) continuous</td>
</tr>
<tr>
<td>Definitions</td>
<td>(\theta = \exp\left(\frac{1}{\alpha_2}\right)^2) &lt;br&gt;(r = \frac{\alpha_1}{\alpha_2})</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>(f(x) = \frac{\alpha_2}{\beta \sqrt{2\pi(1 + z^2)}} \times e^{-\frac{1}{2}\left[\alpha_1 + \alpha_2 \sinh^{-1}(z)\right]^2}) &lt;br&gt;(F(x) = \Phi\left(\alpha_1 + \alpha_2 \sinh^{-1}(z)\right)) &lt;br&gt;Here, (z = \frac{(x - \gamma)}{\beta}) &lt;br&gt;and (\Phi) is the cumulative distribution function of a standard Normal(0,1).</td>
</tr>
<tr>
<td>Mean</td>
<td>(\gamma - \beta \sqrt{\theta} \sinh(r))</td>
</tr>
<tr>
<td>Variance</td>
<td>(\frac{\beta^2}{2} (\theta - 1)(\theta \cosh(2r) + 1))</td>
</tr>
</tbody>
</table>
### Skewness

\[
-\frac{1}{4} \sqrt{\theta} (\theta - 1)^2 \left[ \theta (\theta + 2) \sinh(3r) + 3 \sinh(r) \right] \\
\left[ \frac{1}{2} (\theta - 1) (\theta \cosh(2r) + 1) \right]^{3/2}
\]

### Kurtosis

\[
\frac{1}{8} (\theta - 1)^2 \left[ \theta^2 (\theta^4 + 2\theta^3 + 3\theta^2 - 3) \cosh(4r) + 4\theta^2 (\theta + 2) \cosh(2r) + 3(2\theta + 1) \right] \\
\left[ \frac{1}{2} (\theta - 1) (\theta \cosh(2r) + 1) \right]^2
\]

### Mode

*No Closed Form.*

### Examples

![PDF - JohnsonSU(2, 2, 0, 1)](image-url)
**RiskLaplace**

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskLaplace</strong> <em>(µ, σ)</em> specifies a Laplace distribution with location parameter <em>µ</em> and scale parameter <em>σ</em>. The Laplace distribution is sometimes called a “double exponential distribution” because it resembles two exponential distributions placed back to back around the location parameter.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Guidelines</strong></td>
<td><em>σ</em> must be positive.</td>
</tr>
<tr>
<td><strong>Parameters</strong></td>
<td><em>µ</em> continuous location parameter</td>
</tr>
<tr>
<td></td>
<td><em>σ</em> continuous scale parameter <em>σ</em> &gt; 0 * *</td>
</tr>
<tr>
<td></td>
<td><em>σ</em> = 0 is supported but gives a degenerate distribution with <em>x</em> = <em>µ</em>.</td>
</tr>
<tr>
<td><strong>Domain</strong></td>
<td><em>−∞ &lt; x &lt; +∞</em> continuous</td>
</tr>
<tr>
<td><strong>Density and Cumulative Distribution Functions</strong></td>
<td>( f(x) = \frac{1}{\sqrt{2} \sigma} e^{-\sqrt{2} \left</td>
</tr>
<tr>
<td></td>
<td>( F(x) = \begin{cases} \frac{1}{2} e^{-\sqrt{2} \left</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td><em>µ</em></td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td><em>σ</em>²</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>0</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>6</td>
</tr>
<tr>
<td><strong>Mode</strong></td>
<td><em>µ</em></td>
</tr>
</tbody>
</table>
**RiskLaplaceAlt, RiskLaplaceAltD**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskLaplaceAlt(<em>arg1type, arg1value, arg2type, arg2value</em>) specifies a Laplace distribution with two arguments of the type <em>arg1type</em> and <em>arg2type</em>. These arguments can be either “mu”, “sigma”, or a value between 0 and 1 to represent a percentile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskLaplaceAlt (5%,1,95%,100) returns a Laplace distribution with 5th percentile 1 and 95th percentile 100.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>With RiskLaplaceAltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
<tr>
<td>Description</td>
<td>\textbf{RiskLevy} ((a,c)) specifies a Levy distribution with location parameter (a) and scale parameter (c).</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Guidelines</td>
<td>(c) must be positive.</td>
</tr>
<tr>
<td>Parameters</td>
<td>(a) continuous location parameter (c) continuous scale parameter (c &gt; 0)</td>
</tr>
<tr>
<td>Domain</td>
<td>(a \leq x &lt; +\infty) continuous</td>
</tr>
</tbody>
</table>
| Density and Cumulative Distribution Functions | \(f(x) = \sqrt{c \over 2\pi} \frac{e^{-\frac{c}{2(x-\mu)}}}{(x - \mu)^{3/2}}\)  
\(F(x) = 1 - \text{erf} \left( \frac{\sqrt{c}}{\sqrt{2(x - \mu)}} \right)\)  
Here, \(\text{erf}\) is the \textit{Error Function}. |
| Mean | Does not exist. |
| Variance | Does not exist. |
| Skewness | Does not exist. |
| Kurtosis | Does not exist. |
| Mode | \(\mu + \frac{c}{3}\) |
RiskLevyAlt, RiskLevyAltD

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskLevyAlt</strong>(arg1type, arg1value, arg2type, arg2value) specifies a Levy distribution with two arguments of the type arg1type and arg2type. These arguments can be either “a”, “c”, or a value between 0 and 1 to specify a percentile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskLevyAlt</strong> (5%, 1, 95%, 100) returns a Levy distribution with 5th percentile 1 and 95th percentile 100.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>With <strong>RiskLaplaceAltD</strong>, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>
## RiskLogistic

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskLogistic}(\alpha, \beta)$ specifies a logistic distribution with parameters $\alpha$ and $\beta$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskLogistic}(10, 20)$ returns a logistic distribution with parameters 10 and 20. $\text{RiskLogistic}(A6, A7)$ returns a logistic distribution with parameters taken from cells A6 and A7.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>$\beta$ must be positive.</td>
</tr>
<tr>
<td>Parameters</td>
<td>$\alpha$ continuous location parameter</td>
</tr>
<tr>
<td></td>
<td>$\beta$ continuous scale parameter $\beta &gt; 0$</td>
</tr>
<tr>
<td>Domain</td>
<td>$-\infty &lt; x &lt; +\infty$ continuous</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f(x) = \frac{\text{sech}^2\left(\frac{1}{2}\left(\frac{x - \alpha}{\beta}\right)\right)}{4\beta}$</td>
</tr>
<tr>
<td></td>
<td>$F(x) = \frac{1 + \tanh\left(\frac{1}{2}\left(\frac{x - \alpha}{\beta}\right)\right)}{2}$</td>
</tr>
<tr>
<td></td>
<td>Here, “sech” is the Hyperbolic Secant Function and “tanh” is the Hyperbolic Tangent Function.</td>
</tr>
<tr>
<td>Mean</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Variance</td>
<td>$\frac{\pi^2\beta^2}{3}$</td>
</tr>
<tr>
<td>Skewness</td>
<td>0</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.2</td>
</tr>
<tr>
<td>Mode</td>
<td>$\alpha$</td>
</tr>
</tbody>
</table>
Examples

PDF - Logistic(0,1)

CDF - Logistic(0,1)
**RiskLogisticAlt, RiskLogisticAltD**

<table>
<thead>
<tr>
<th>Description</th>
<th><em>RiskLogisticAlt</em>(arg1type, arg1value, arg2type, arg2value) specifies a logistic distribution with two arguments of the type arg1type and arg2type. These arguments can be either “alpha”, “beta”, or a value between 0 and 1 to specify a percentile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><em>RiskLogisticAlt</em>(5%,1,95%,100) returns a logistic distribution with 5th percentile 1 and 95th percentile 100.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>With <em>RiskLogisticAltD</em>, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>
### RiskLogLogistic

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskLogLogistic}(\gamma, \beta, \alpha)$ specifies a log-logistic distribution with location parameter $\gamma$, shape parameter $\alpha$, and scale parameter $\beta$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guidelines</td>
<td>Both $\alpha$ and $\beta$ must be positive.</td>
</tr>
<tr>
<td>Parameters</td>
<td>$\gamma$ continuous location parameter $\beta$ continuous scale parameter $\beta &gt; 0$ $\alpha$ continuous shape parameter $\alpha &gt; 0$</td>
</tr>
<tr>
<td>Definitions</td>
<td>$\theta \equiv \frac{\pi}{\alpha}$</td>
</tr>
<tr>
<td>Domain</td>
<td>$\gamma \leq x &lt; +\infty$ continuous</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>$f(x) = \frac{\alpha t^{\alpha-1}}{\beta(1 + t^{\alpha})^2}$ $F(x) = \frac{1}{1 + \left(\frac{1}{t}\right)^\alpha}$ with $t \equiv \frac{x - \gamma}{\beta}$</td>
</tr>
<tr>
<td>Mean</td>
<td>$\beta \theta \csc(\theta) + \gamma$ for $\alpha &gt; 1$</td>
</tr>
<tr>
<td>Variance</td>
<td>$\beta^2 \theta \left[2 \csc(2\theta) - \theta \csc^2(\theta)\right]$ for $\alpha &gt; 2$</td>
</tr>
<tr>
<td>Skewness</td>
<td>$\frac{3 \csc(3\theta) - 6 \theta \csc(2\theta) \csc(\theta) + 2 \theta^2 \csc^3(\theta)}{\sqrt{\theta \left[2 \csc(2\theta) - \theta \csc^2(\theta)\right]^3}}$ for $\alpha &gt; 3$</td>
</tr>
</tbody>
</table>
### Kurtosis

\[
\frac{4 \csc(4\theta) - 12 \csc(3\theta) \csc(\theta) + 12\theta^2 \csc(2\theta) \csc^2(\theta) - 3\theta^3 \csc^4(\theta)}{\theta \left[2 \csc(2\theta) - 3 \csc^2(\theta) + \theta \csc^3(\theta) - \csc^4(\theta)\right]^2}
\]

for \(\alpha > 4\)

### Mode

\[
\gamma + \beta \left[\frac{\alpha - 1}{\alpha + 1}\right]^{1/\alpha}
\]

\[
\gamma
\]

for \(\alpha > 1\)

for \(\alpha \leq 1\)

### Examples

![PDF - LogLogistic(0,1,5)](image)
**RiskLogLogisticAlt, RiskLogLogisticAltD**

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskLogLogisticAlt}(\text{arg1type, arg1value, arg2type, arg2value, arg3type, arg3value}) ) specifies a log-logistic distribution with three arguments of the type \text{arg1type} to \text{arg3type}. These arguments can be either “gamma”, “beta”, “alpha”, or a value between 0 and 1 to specify a percentile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskLogLogisticAlt(&quot;gamma&quot;,5,&quot;beta&quot;,2,90%,10}) ) returns a log-logistic distribution location parameter 5, shape parameter 2, and 90(^{th}) percentile 10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>With ( \text{RiskLogLogisticAltD} ), any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>
### RiskLognorm

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskLognorm}(\text{mean}, \text{standard deviation}) ) specifies a lognormal distribution with parameters \textit{mean} and \textit{standard deviation}. The arguments for this form of the lognormal distribution specify the actual mean and standard deviation of the distribution. This distribution is often used to model the product of several random variables, just as the normal distribution is used to model the sum of several random variables. For example, it is often used to model the future value of an asset (the product of the current value and a number of percentage changes). It is also used in the oil industry to model reserves. The lognormal distribution has a number of desirable properties for modeling real world processes. It is skewed, and it has a positive and unbounded range.</th>
</tr>
</thead>
</table>
| Examples | \( \text{RiskLognorm}(10,20) \) returns a lognormal distribution with mean 10 and standard deviation 20.  
\( \text{RiskLognorm}(\text{C10*3.14}, \text{B10}) \) returns a lognormal distribution mean equal to the value in cell C10 times 3.14 and standard deviation equal to the value in cell B10. |
| Guidelines | Both \textit{mean} and \textit{standard deviation} must be positive. |
| Parameters | \( \mu \) continuous parameter \( \mu > 0 \)  
\( \sigma \) continuous parameter \( \sigma > 0 \) |
<p>| Domain | ( 0 \leq x &lt; +\infty ) continuous |</p>
<table>
<thead>
<tr>
<th>Density and Cumulative Distribution Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = \frac{1}{x\sqrt{2\pi}\sigma'} e^{-\frac{1}{2}\left[\ln x - \mu'\right]^2}$</td>
</tr>
<tr>
<td>$F(x) = \Phi\left(\frac{\ln x - \mu'}{\sigma'}\right)$</td>
</tr>
<tr>
<td>$\mu' \equiv \ln \left[\frac{\mu^2}{\sqrt{\sigma^2 + \mu^2}}\right]$ and $\sigma' \equiv \sqrt{\ln \left[1 + \left(\frac{\sigma}{\mu}\right)^2\right]}$</td>
</tr>
<tr>
<td>Here, $\Phi(z)$ is the cumulative distribution function of a Normal(0,1).</td>
</tr>
</tbody>
</table>

| Mean | $\mu$ |
| Variance | $\sigma^2$ |
| Skewness | $\left(\frac{\sigma}{\mu}\right)^3 + 3\left(\frac{\sigma}{\mu}\right)$ |
| Kurtosis | $\omega^4 + 2\omega^3 + 3\omega^2 - 3$ with $\omega \equiv 1 + \left(\frac{\sigma}{\mu}\right)^2$ |
| Mode | $\frac{\mu^4}{(\sigma^2 + \mu^2)^{3/2}}$ |
Examples

PDF - Lognorm(1,1)

CDF - Lognorm(1,1)
**RiskLognormAlt, RiskLognormAltD**

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskLognormAlt</strong>(arg1type, arg1value, arg2type,arg2value, arg3type,arg3value) specifies a lognormal distribution function with three type-value pairs. Each type can be either &quot;mu&quot;, &quot;sigma&quot;, &quot;loc&quot;, or a value between 0 and 1 to specify a percentile. Note that “mu” is the mean of the distribution before applying any shift, “sigma” is the standard deviation of the distribution, and “loc” is the minimum value of the distribution, that is, the amount (positive or negative) by which the distribution is shifted right from its base position with its minimum at x=0. Therefore, the mean of the shifted distribution is mu+loc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskLognormAlt</strong>(&quot;mu&quot;, 2, &quot;sigma&quot;, 5, 95%, 30) returns a lognormal distribution starting with mu parameter 2 and standard deviation 5, then shifted so that the 95th percentile is 30.</td>
</tr>
</tbody>
</table>
| Guidelines | *mu* and *sigma*, if specified, must be positive. Arguments specified as percentiles must lead to a lognormal distribution with positive *mu* and standard deviation; otherwise the distribution returns #VALUE. However, the actual mean of the distribution can be negative if there is a negative value of loc.  
With **RiskLognormAltD**, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value. |
## RiskLognorm2

**Description**  
*RiskLognorm2*(mean of corresponding normal dist., std. dev. of normal) specifies a lognormal distribution, where the parameters are the mean and standard deviation of the corresponding normal distribution. In this form of the distribution, a normally distributed value is first sampled, and then the lognormal value is the exponential of this sampled value.

**Examples**  
- **RiskLognorm2(10,0.5)** returns the exponential of a normally distributed sample with mean 10 and standard deviation 0.5.  
- **RiskLognorm2(C10*3.14,B10)** returns the exponential of a normally distributed sample with mean equal to 3.14 times the value in cell C10 and standard deviation equal to the value in cell B10.

**Guidelines**  
*standard deviation* must be positive.

**Parameters**  

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>µ</td>
<td>continuous parameter</td>
<td>0 ≤ x &lt; +∞ continuous</td>
</tr>
<tr>
<td>σ</td>
<td>continuous parameter</td>
<td>σ &gt; 0</td>
</tr>
</tbody>
</table>

**Domain**  
0 ≤ x < +∞  continuous

**Density and Cumulative Distribution Functions**

\[
f(x) = \frac{1}{x \sqrt{2\pi \sigma}} e^{-\frac{1}{2} \left( \ln x - \mu \right)^2 / \sigma^2} \\
F(x) = \Phi \left( \frac{\ln x - \mu}{\sigma} \right)
\]

Here, \( \Phi(z) \) is the cumulative distribution function of a Normal(0,1).

**Mean**  
\[
\bar{x} = \frac{\mu + \sigma^2}{e^{\sigma^2/2}}
\]

**Variance**  
\[
\text{Var}(x) = e^{2\mu + \sigma^2} \omega(1-\omega)
\]

with  \( \omega = e^{\sigma^2} \)

**Skewness**  
\[
\text{Skew}(x) = \frac{(1+2\omega)^{1/2}}{(\omega-1)^{1/2}}
\]

with  \( \omega = e^{\sigma^2} \)

**Kurtosis**  
\[
\text{Kurt}(x) = \frac{\omega^4 + 2\omega^3 + 3\omega^2 - 3}{\omega^4 - 3\omega^3 + 3\omega^2 - 3}
\]

with  \( \omega = e^{\sigma^2} \)
<table>
<thead>
<tr>
<th>Mode</th>
<th>$e^{\mu - \sigma^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Examples</strong></td>
<td></td>
</tr>
</tbody>
</table>

**CDF - Lognorm2(0,1)**

**PDF - Lognorm2(0,1)**
**RiskMakeInput**

<table>
<thead>
<tr>
<th>Description</th>
<th><em>RiskMakeInput</em>(formula) specifies that the calculated value for <em>formula</em> will be treated as a simulation input, the same way as a distribution function. This function allows the results of Excel calculations (or a combination of distribution functions) to be treated as a single “input” in a sensitivity analysis.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><em>RiskMakeInput</em> <em>(RiskNormal(10,1)+RiskTriang(1,2,3)+A5)</em> specifies that the sum of the samples from the distributions <em>RiskNormal(10,1)</em> and <em>RiskTriang(1,2,3)</em>, plus the value of cell A5, will be treated as a simulation input by @RISK. An entry for the distribution for this formula will be shown in the Inputs tab of the Results Summary window, and it will be used in sensitivity analyses for outputs dependent on it.</td>
</tr>
</tbody>
</table>
| Guidelines | Distributions that precede, or “feed into”, a RiskMakeInput function are not included in a sensitivity analysis of the outputs they impact. This avoids double counting of their impacts.  
The RiskMakeInput function does not have to be a precedent of an output to be included in its sensitivity analysis; only the distributions that precede the RiskMakeInput do. For example, you can add a single RiskMakeInput function that averages a set of distributions. Each distribution in the averaged set might be precedents of an output. They will be replaced in the sensitivity analysis for that output by the RiskMakeInput function.  
The following distribution property functions can be included in a RiskMakeInput function: RiskName, RiskCollect, RiskCategory, RiskStatic, RiskUnits, RiskSixSigma, RiskConvergence, RiskIsDiscrete, and RiskIsDate. RiskMakeInput functions cannot include RiskCorrmat or RiskCopula property functions because they are not sampled in the same way as standard distribution functions.  
No graph is available of a RiskMakeInput function, prior to simulation, in the Define Distribution window or the Model window.  
RiskTheo functions cannot be used with inputs specified with RiskMakeInput. |
### RiskNegbin

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskNegbin}(s,p)$ specifies a negative binomial distribution parameters $s$ and $p$. This distribution is typically used to model the number of failures before achieving $s$ successes in independent, identical trials, each with probability $p$ of success. (The geometric distribution is a special case with $s=1$.) It is sometimes used in models of quality control and production testing, breakdown, and maintenance modelling.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskNegbin}(5,0.25)$ returns a negative binomial distribution with 5 successes and a 25% probability of success on each trial. $\text{RiskNegbin}(A6,A7)$ returns a negative binomial distribution with the number of successes taken from cell A6 and the probability of success taken from cell A7.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>$s$ must be a positive integer less than or equal to 32,767. $p$ must be between 0 and 1 (0 is disallowed, 1 is allowed).</td>
</tr>
</tbody>
</table>
| Parameters | $S$ the number of successes discrete parameter $s \geq 0$  
$p$ probability of a single success continuous parameter $0 < p \leq 1$ |
| Domain | $0 \leq x < +\infty$ discrete integers |
| Mass and Cumulative Distribution Functions | $f(x) = \binom{s + x - 1}{x} p^s (1-p)^x$  
$F(x) = p^s \sum_{i=0}^{x} \binom{s + i - 1}{i} (1-p)^i$  
Here, $\binom{a}{b}$ is the Binomial Coefficient. |
<p>| Mean | $\frac{s(1-p)}{p}$ |
| Variance | $\frac{s(1-p)}{p^2}$ |</p>
<table>
<thead>
<tr>
<th>Skewness</th>
<th>[ \frac{2 - p}{\sqrt{s(1 - p)}} ] for ( s &gt; 0, p &lt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td>[ 3 + \frac{6 + \frac{p^2}{s}}{s(1 - p)} ] for ( s &gt; 0, p &lt; 1 )</td>
</tr>
<tr>
<td>Mode</td>
<td>(bimodal) ( z ) and ( z + 1 ) integer ( z &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td>(unimodal) ( 0 ) ( z &lt; 0 )</td>
</tr>
<tr>
<td></td>
<td>(unimodal) smallest integer greater than ( z ) otherwise</td>
</tr>
<tr>
<td></td>
<td>[ z \equiv \frac{s(1 - p) - 1}{p} ]</td>
</tr>
</tbody>
</table>

**Examples**

PDF - NegBin(3, 6)
### RiskNormal

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskNormal(mean, standard deviation) specifies a normal distribution with parameters mean and standard deviation. This is the traditional &quot;bell shaped&quot; curve. The use of the normal distribution can often be justified by a mathematical result called the Central Limit Theorem. This states essentially that the sum of many random quantities is approximately normally distributed, regardless of the distributions of the quantities in the sum. Therefore, the normal distribution can be used to represent the uncertainty of a model's input whenever the input is the result of many random processes acting together in an additive manner. Examples could include the total number of goals scored in a soccer season or the amount of oil in the world (assuming that there are many reservoirs of approximately equal size, each with an uncertain amount of oil). Two potential drawbacks of the normal distribution for real applications are (1) it is symmetric, not skewed, and (2) it allows negative values. However, if the mean is positive and is at least 3 or 4 times larger than the standard deviation, the probability of a negative value is fairly negligible. (In any case, the RiskTruncate property function can be used to ensure that no negative values are sampled.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskNormal(10,2) returns a normal distribution with mean 10 and standard 2. RiskNormal(SQRT(C101),B10) returns a normal distribution with mean equal to the square root of the value in cell C101 and standard deviation taken from cell B10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>standard deviation must be positive.</td>
</tr>
<tr>
<td>Parameters</td>
<td>$\mu$ continuous location parameter</td>
</tr>
<tr>
<td></td>
<td>$\sigma$ continuous scale parameter $\sigma &gt; 0$ *</td>
</tr>
<tr>
<td></td>
<td>*$\sigma = 0$ is supported but gives a degenerate distribution with $x = \mu$.</td>
</tr>
<tr>
<td>Domain</td>
<td>$-\infty &lt; x &lt; +\infty$ continuous</td>
</tr>
</tbody>
</table>
Density and Cumulative Distribution Functions

\[ f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \]

\[ F(x) \equiv \Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{2} \left[ \text{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right) + 1 \right] \]

Here, \( \Phi \) is called the cumulative distribution for \( \text{N}(0,1) \), and \( \text{erf} \) is the error function.

Mean \( \mu \)

Variance \( \sigma^2 \)

Skewness 0

Kurtosis 3

Mode \( \mu \)

Examples

![PDF - Normal(0,1)](Image)
RiskNormalAlt, RiskNormalAltD

Description

RiskNormalAlt(arg1type, arg1value, arg2type, arg2value) specifies a normal distribution with two arguments of the type arg1type and arg2type. These arguments can be either "mu", "sigma", or a value between 0 and 1 to specify a percentile.

Examples

RiskNormalAlt(5%,1,95%,10) returns a normal distribution with 5th percentile 1 and 95th percentile 10.

Guidelines

With RiskNormalAltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.
**RiskPareto**

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskPareto}(\theta, \alpha) ) specifies a Pareto distribution with shape parameter ( \theta ) and scale parameter ( \alpha ).</th>
</tr>
</thead>
</table>
| Examples | \( \text{RiskPareto}(5, 5) \) returns a Pareto distribution with shape parameter 5 and scale parameter 5.  
\( \text{RiskPareto}(A10, A11 + A12) \) returns a Pareto distribution with shape parameter taken from cell A10 and scale parameter equal to the sum of the values in cells A11 and A12. |
| Guidelines | Both \( \theta \) and \( \alpha \) must be positive. |
| Parameters |  
\[ \begin{align*}  
\theta & \quad \text{continuous shape parameter} \quad \theta > 0 \\
\alpha & \quad \text{continuous scale parameter} \quad \alpha > 0 
\end{align*} \] |
| Domain | \( \alpha \leq x < +\infty \) \hspace{1cm} \text{continuous} |
| Density and Cumulative Distribution Functions |  
\[ f(x) = \frac{\theta \alpha}{x^{\theta+1}} \]  
\[ F(x) = 1 - \left( \frac{a}{x} \right)^\theta \]  
Here, \( a = \alpha \) |
| Mean |  
\[ \frac{a \theta}{\theta - 1} \]  
for \( \theta > 1 \) |
| Variance |  
\[ \frac{\theta a^2}{(\theta - 1)^2(\theta - 2)} \]  
for \( \theta > 2 \) |
| Skewness |  
\[ 2 \frac{\theta + 1}{\theta - 3} \sqrt[\theta - 2]{\frac{\theta - 2}{\theta}} \]  
for \( \theta > 3 \) |
| Kurtosis |  
\[ \frac{3(\theta - 2)(3\theta^2 + \theta + 2)}{\theta(\theta - 3)(\theta - 4)} \]  
for \( \theta > 4 \) |
<table>
<thead>
<tr>
<th>Mode</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Examples</strong></td>
<td></td>
</tr>
<tr>
<td><strong>PDF - Pareto(2,1)</strong></td>
<td><img src="image" alt="PDF - Pareto(2,1)" /></td>
</tr>
<tr>
<td><strong>CDF - Pareto(2,1)</strong></td>
<td><img src="image" alt="CDF - Pareto(2,1)" /></td>
</tr>
</tbody>
</table>
**RiskParetoAlt, RiskParetoAltD**

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskParetoAlt}(\text{arg1type, arg1value, arg2type, arg2value}) ) specifies a Pareto distribution with two arguments of the type ( \text{arg1type} ) and ( \text{arg2type} ). These arguments can be either “theta”, “alpha”, or a value between 0 and 1 to specify a percentile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskParetoAlt}(5%, 1, 95%, 4) ) returns a Pareto distribution with 5(^{th}) percentile 1 and 95(^{th}) percentile 4.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>With ( \text{RiskParetoAltD} ), any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>
## RiskPareto2

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskPareto2</strong>((b, q)) specifies a “Pareto2” distribution with scale parameter (b) and shape parameter (q). This is a different parametrization of the Pareto distribution, where all nonnegative values are possible.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskPareto2</strong>((5, 5)) returns a Pareto2 distribution with parameters 5 and 5. <strong>RiskPareto2</strong>((A10, A11 + A12)) returns a Pareto distribution with scale parameter taken from cell A10 and shape parameter equal to the sum of the values in cells A11 and A12.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Both (b) and (q) must be positive.</td>
</tr>
</tbody>
</table>
| Parameters | \(b\) continuous scale parameter \(b > 0\)  
\(q\) continuous shape parameter \(q > 0\) |
| Domain | \(0 \leq x < +\infty\) continuous |
| Density and Cumulative Distribution Functions |  
\[f(x) = \frac{qb^q}{(x + b)^{q+1}}\]  
\[F(x) = 1 - \frac{b^q}{(x + b)^q}\] |
| Mean | \(\frac{b}{q - 1}\) for \(q > 1\) |
| Variance | \(\frac{b^2q}{(q - 1)^2(q - 2)}\) for \(q > 2\) |
| Skewness | \(2\left[\frac{q + 1}{q - 3}\right]\sqrt{\frac{q - 2}{q}}\) for \(q > 3\) |
| Kurtosis | \(\frac{3(q - 2)(3q^2 + q + 2)}{q(q - 3)(q - 4)}\) for \(q > 4\) |
| Mode | 0 |
Examples

PDF - Pareto2(3,3)

CDF - Pareto2(3,3)
<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskPareto2Alt}(\text{arg1type, arg1value, arg2type, arg2value}) ) specifies a Pareto2 distribution with two arguments of the type ( \text{arg1type} ) and ( \text{arg2type} ). These arguments can be either “b”, “q”, or a value between 0 and 1 to specify a percentile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskPareto2Alt}(5%,0.05,95%,5) ) returns a Pareto2 distribution with 5(^{th}) percentile 0.05 and 95(^{th}) percentile 5.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>With ( \text{RiskPareto2AltD} ), any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>
## RiskPearson5

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskPearson5</strong>(<em>alpha</em>,<em>beta</em>) specifies a Pearson type V distribution with shape parameter <em>alpha</em> and scale parameter <em>beta</em>.</th>
</tr>
</thead>
</table>
| Examples    | **RiskPearson5(1,1)** returns a Pearson type V distribution shape parameter 1 and scale parameter 1.  
**RiskPearson5(C12,C13)** returns a Pearson type V distribution with shape and scale parameters taken from cells C12 and C13. |
| Guidelines  | Both *alpha* and *beta* must be positive.                                                                                                                                                        |
| Parameters  | $\alpha$ continuous shape parameter $\alpha > 0$  
$\beta$ continuous scale parameter $\beta > 0$ |
| Domain      | $0 \leq x < +\infty$ continuous |
| Density and Cumulative Distribution Functions | 
$f(x) = \frac{1}{\beta \Gamma(\alpha)} \cdot \frac{e^{-\beta/x}}{(x/\beta)^{\alpha+1}}$  
F(x) has no closed form. |
| Mean        | $\frac{\beta}{\alpha - 1}$ for $\alpha > 1$ |
| Variance    | $\frac{\beta^2}{(\alpha - 1)^2(\alpha - 2)}$ for $\alpha > 2$ |
| Skewness    | $\frac{4\sqrt{\alpha - 2}}{\alpha - 3}$ for $\alpha > 3$ |
| Kurtosis    | $\frac{3(\alpha + 5)(\alpha - 2)}{(\alpha - 3)(\alpha - 4)}$ for $\alpha > 4$ |
| Mode        | $\frac{\beta}{\alpha + 1}$ |
Examples

PDF - Pearson5(3, 1)

CDF - Pearson5(3, 1)
**RiskPearson5Alt, RiskPearson5AltD**

| Description | RiskPearson5Alt(arg1type, arg1value, arg2type,arg2value, arg3type,arg3value) specifies a Pearson type V distribution with three arguments of the type arg1type to arg3type. These arguments can be either “alpha”, “beta”, “loc”, or a value between 0 and 1 to specify a percentile. |
| Guidelines | With RiskPearson5AltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value. |
### RiskPearson6

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskPearson6}(\alpha_1, \alpha_2, \beta) ) specifies a Pearson type VI distribution with shape parameters ( \alpha_1 ) and ( \alpha_2 ), and scale parameter ( \beta ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskPearson6}(5, 1, 2) ) returns a Pearson type VI distribution with shape parameters 5 and 1, and scale parameter 2.  ( \text{RiskPearson6}(D3, E3, F3) ) returns a Pearson type VI distribution with shape parameters taken from cells D3 and E3, and scale parameter from cell F3.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>All parameters, ( \alpha_1 ), ( \alpha_2 ), and ( \beta ), must be positive.</td>
</tr>
<tr>
<td>Parameters</td>
<td>( \alpha_1 ) continuous shape parameter ( \alpha_1 &gt; 0 )  ( \alpha_2 ) continuous shape parameter ( \alpha_2 &gt; 0 )  ( \beta ) continuous scale parameter ( \beta &gt; 0 )</td>
</tr>
<tr>
<td>Domain</td>
<td>( 0 \leq x &lt; +\infty ) continuous</td>
</tr>
</tbody>
</table>
| Density and Cumulative Distribution Functions | \( f(x) = \frac{1}{\beta B(\alpha_1, \alpha_2)} \times \frac{(x/\beta)^{\alpha_1-1}}{(1 + x/\beta)^{\alpha_1 + \alpha_2}} \)

\( F(x) \) has no closed form.

Here, \( B \) is the Beta Function.

<p>| Mean       | ( \frac{\beta \alpha_1}{\alpha_2 - 1} ) for ( \alpha_2 &gt; 1 ) |
| Variance   | ( \frac{\beta^2 \alpha_1 (\alpha_1 + \alpha_2 - 1)}{(\alpha_2 - 1)^2 (\alpha_2 - 2)} ) for ( \alpha_2 &gt; 2 ) |
| Skewness   | ( 2 \sqrt{\frac{\alpha_2 - 2}{\alpha_1 (\alpha_1 + \alpha_2 - 1)}} \left[ \frac{2\alpha_1 + \alpha_2 - 1}{\alpha_2 - 3} \right] ) for ( \alpha_2 &gt; 3 ) |</p>
<table>
<thead>
<tr>
<th>Kurtosis</th>
<th>[ \frac{3(\alpha_2 - 2)}{(\alpha_2 - 3)(\alpha_2 - 4)} \left[ \frac{2(\alpha_2 - 1)^2}{\alpha_1(\alpha_1 + \alpha_2 - 1)} + (\alpha_2 + 5) \right] ] for ( \alpha_2 &gt; 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>[ \frac{\beta(\alpha_1 - 1)}{\alpha_2 + 1} ] for ( \alpha_1 &gt; 1 ) [ 0 ] otherwise</td>
</tr>
<tr>
<td>Examples</td>
<td><img src="image" alt="PDF - Pearson6(3,3,1)" /></td>
</tr>
</tbody>
</table>

Reference: Distribution Functions
**RiskPert**

| Description | **RiskPert**(minimum, m.likely, maximum) specifies a PERT distribution with specified minimum, most likely, and maximum values. The PERT distribution (meaning Program Evaluation and Review Technique) is similar to a triangular distribution in that it has the same set of easily understandable parameters, but it might be preferred to the triangular distribution because of its curved density. Technically, it is a special case of the BetaGeneral) distribution. |
| Examples | **RiskPert**\((0,2,10)\) returns a PERT distribution with minimum 0, maximum 10, and most likely value 2. **RiskPert** \((A1,A2,A3)\) specifies a PERT distribution with its three parameters taken from the cells A1, A2, and A3. |
| Guidelines | minimum must be less than maximum. most likely must be greater than or equal to minimum and less than or equal to maximum. |
| Definitions | \[ \mu = \frac{\text{min} + 4 \cdot \text{m.likely} + \text{max}}{6} \quad \alpha_1 = 6 \left[ \frac{\mu - \text{min}}{\text{max} - \text{min}} \right] \] \[ \alpha_2 = 6 \left[ \frac{\text{max} - \mu}{\text{max} - \text{min}} \right] \] |
| Parameters | min continuous boundary parameter \(\text{min} < \text{max}\) m.likely continuous parameter \(\text{min} \leq \text{m.likely} \leq \text{max}\) max continuous boundary parameter |
| Domain | \(\text{min} \leq x \leq \text{max}\) continuous |
## Density and Cumulative Distribution Functions

\[
f(x) = \frac{(x - \min)^{\alpha_1 - 1}(\max - x)^{\alpha_2 - 1}}{B(\alpha_1, \alpha_2)(\max - \min)^{\alpha_1 + \alpha_2 - 1}}
\]

\[
F(x) = \frac{B_z(\alpha_1, \alpha_2)}{B(\alpha_1, \alpha_2)} = I_z(\alpha_1, \alpha_2)
\]

where \( z = \frac{x - \min}{\max - \min} \)

Here, \( B \) is the Beta Function and \( B_z \) is the Incomplete Beta Function.

### Mean

\[
\mu = \frac{\min + 4 \cdot \text{m.likely} + \max}{6}
\]

### Variance

\[
\frac{(\mu - \min)(\max - \mu)}{7}
\]

### Skewness

\[
\frac{\min + \max - 2\mu}{4} \sqrt{\frac{7}{(\mu - \min)(\max - \mu)}}
\]

### Kurtosis

\[
\frac{21}{(\alpha_1 \cdot \alpha_2)}
\]

### Mode

\( \text{m.likely} \)

## Examples

![PDF - Pert(0,1,3)](image-url)
## RiskPertAlt, RiskPertAltD

### Description

RiskPertAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value) specifies a PERT distribution with three arguments of the type arg1type to arg3type. These arguments can be either “min”, “m.likely”, “max”, or a value between 0 and 1 to specify a percentile.

### Examples

RiskPertAlt("min", 2, "m. likely", 5, 95%, 30) returns a PERT distribution with minimum 2, most likely value 5, and 95th percentile 30.

### Guidelines

With RiskPertAltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.
**RiskPoisson**

**Description**  
*RiskPoisson*(\(\lambda\)) specifies a Poisson distribution parameter \(\lambda\), the mean. The Poisson distribution is a discrete distribution that returns only nonnegative integer values. It is often used to model the number of events that occur in a given time period where the rate of occurrences is constant. For example, it is often used in insurance modeling and financial markets to model the number of events (e.g., earthquakes, fires, stock market crashes) that might occur in a given period. It can also be applied to processes over other domains, e.g., spatial).

**Examples**  
*RiskPoisson*(5) returns a Poisson distribution with mean 5.  
*RiskPoisson*(A6) returns a Poisson distribution with mean taken from cell A6.

**Guidelines**  
\(\lambda\) must be positive.

**Parameters**  
\(\lambda\) mean number of successes  
continuous \(\lambda > 0\)  
*\(\lambda = 0\) is supported but it returns a degenerate distribution with \(x = 0\).*

**Domain**  
\(0 \leq x < +\infty\)  
discrete integers

**Mass and Cumulative Distribution Functions**

\[
f(x) = \frac{\lambda^x e^{-\lambda}}{x!}
\]

\[
F(x) = e^{-\lambda} \sum_{n=0}^{x} \frac{\lambda^n}{n!}
\]

**Mean**  
\(\lambda\)

**Variance**  
\(\lambda\)

**Skewness**  
\[\frac{1}{\sqrt{\lambda}}\]

**Kurtosis**  
\[3 + \frac{1}{\lambda}\]

**Mode**  
(bimodal) \(\lambda\) and \(\lambda - 1\) (bimodal)  
if \(\lambda\) is an integer  
(unimodal) largest integer less than \(\lambda\) otherwise
Examples

CDF - Poisson(3)

PMF - Poisson(3)
### RiskRayleigh

<table>
<thead>
<tr>
<th>Description</th>
<th>$RiskRayleigh(beta)$ specifies a Rayleigh distribution with parameter $beta$, the mode.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$RiskRayleigh(3)$ specified a Rayleigh distribution with mode 3. $RiskRayleigh(C7)$ specifies a Rayleigh distribution with mode taken from the value in cell C7.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>$beta$ must be positive.</td>
</tr>
<tr>
<td>Parameters</td>
<td>$beta$ continuous scale parameter $beta &gt; 0$</td>
</tr>
<tr>
<td>Domain</td>
<td>$0 \leq x &lt; +\infty$ continuous</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>$f(x) = \frac{x}{b^2} e^{-\frac{1}{2} \left( \frac{x}{b} \right)^2}$ $F(x) = 1 - e^{-\frac{1}{2} \left( \frac{x}{b} \right)^2}$ Here, $b = beta$</td>
</tr>
<tr>
<td>Mean</td>
<td>$b \sqrt{\frac{\pi}{2}}$</td>
</tr>
<tr>
<td>Variance</td>
<td>$b^2 \left( 2 - \frac{\pi}{2} \right)$</td>
</tr>
<tr>
<td>Skewness</td>
<td>$\frac{2(\pi - 3)\sqrt{\pi}}{(4 - \pi)^{3/2}} \approx 0.6311$</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>$\frac{32 - 3\pi^2}{(4 - \pi)^2} \approx 3.2451$</td>
</tr>
<tr>
<td>Mode</td>
<td>$b$</td>
</tr>
</tbody>
</table>
### RiskRayleighAlt, RiskRayleighAltD

| Description | *RiskRayleighAlt*(arg1type, arg1value, arg2type, arg2value) specifies a Rayleigh distribution with two arguments of the type *arg1type* and *arg2type*. These arguments can be either “beta”, “loc”, or a value between 0 and 1 that specifies a percentile. |
| Examples | *RiskRayleighAlt*(5%, 1, 95%, 10) returns a Rayleigh distribution with 5\(^{th}\) percentile 1 and 95\(^{th}\) percentile 10. |
| Guidelines | With *RiskRayleighAltD*, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value. |
# RiskResample

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskResample}(\text{sampMethod}, {X_1, X_2, \ldots, X_n}) ) samples from a data set with ( n ) equally likely values, the ( X )'s. ( @RISK ) will sample from the ( X ) values using the sampling method indicated by the ( \text{sampMethod} ) index. Available sampling methods are order, random with replacement, and random without replacement.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskResample}(2, {1, 2.1, 4.45, 99}) ) returns a value from a data set with the 4 possible values 1, 2.1, 4.45 and 99. Samples will be randomly drawn with replacement from these 4 values. ( \text{RiskResample}(1, \text{A1:A500}) ) returns a value from a data set with 500 possible values. The possible values are taken from cells A1 through A500. The “order” sampling method will be used.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>( \text{sampMethod} ) can be 1-order, 2-random with replacement, or 3-random without replacement. A RiskLibrary property function can be included with a Resample function to link the ( X ) data to a simulation output that is stored in the ( @RISK ) Library. A RiskLibrary property function causes ( @RISK ) to update the Resample ( X ) data with the current data stored for the simulation output at the start of each simulation. Therefore, if a new version of the simulation containing the output has been saved in the ( @RISK ) Library, ( @RISK ) will automatically update the RiskResample function with the new data for that output prior to simulating. When using the random with or without replacement methods, you should also consider the sampling type defined under Simulation Settings. When the number of possible values and the number of iterations are the same, all values will be used in both the random with replacement and random without replacement methods when the sampling type is set to Latin Hypercube, because each stratification will be covered exactly once. Using the sampling type set to Monte Carlo will result in duplicate values when the RiskResample sampling method is random with replacement. A RiskCorrmat or RiskCopula property function can be used with RiskResample only when the sampling method is random with replacement. RiskCorrmat and RiskCopula cannot be used with the other sampling methods.</td>
</tr>
</tbody>
</table>
### RiskSimtable

**Description**

RiskSimtable\([\{val1, val2, ..., valn\}]\) specifies a list of values that will be used sequentially in individual simulations executed during a run of simulations. The number of simulations should be set to \(n\) (the @RISK ribbon). Then simulation 1 will use the first argument’s value from each RiskSimtable function in the model, simulation 2 will use the second argument’s value, and so on.

**Examples**

- **RiskSimtable**\([\{10, 20, 30, 40\}]\) should be used with the number of simulations set to 4. Then the value 10 will be used in simulation 1, 20 will be used in simulation 2, and so on.
- **RiskSimtable**\([A1:A6]\) should be used with the number of simulations set to 6. Then the value in cell A1 will be used in simulation 1, the value in cell A2 will be used in simulation 2, and so on.

**Guidelines**

- Any number of arguments can be entered.
- The number of simulations should ideally be set to the number of arguments in each RiskSimtable function in the model. If there are more arguments than simulations, the extra arguments will be ignored. If there are fewer arguments than simulations, @RISK will report errors for the extra simulations.
<table>
<thead>
<tr>
<th>Description</th>
<th>\texttt{RiskSplice(dist1 or cellref,dist2 or cellref,splice point)} specifies a distribution created by splicing dist1 to dist2 at the x-value given by splice point. Samples below the splice point are drawn from dist1 and above from dist2. The resulting distribution is treated as a single input distribution in a simulation and can be correlated with other distributions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>\texttt{RiskSplice(RiskNormal(1,1),RiskPareto(1,1),2)} splices a normal distribution with mean 1 and standard deviation 1 to a Pareto distribution with parameters 1 and 1 at the splice point 2.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>dist1 and dist2 may cannot be correlated, but the RiskSplice distribution can be correlated with other distributions. dist1, dist2, and RiskSplice itself can include property functions (except for RiskCorrmat,k as noted above). dist1 and/or dist2 can be referenced by a cell that contains a distribution function. The two parts of the spliced distribution will be re-weighted so that the total area under the spliced density will be 1.</td>
</tr>
<tr>
<td>Description</td>
<td>( \text{RiskStudent}(nu) ) specified a Student's t distribution with ( nu ) degrees of freedom. This distribution is usually used in statistical hypothesis testing to test a population mean when the population variance is unknown (or the difference between two population means when the population variances are unknown).</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
| Examples | \( \text{RiskStudent}(10) \) returns a Student's t distribution with 10 degrees of freedom. 
\( \text{RiskStudent}(J2) \) returns a Student's t distribution with degrees of freedom equal to the value in cell J2. |
| Guidelines | \( nu \) must be a positive integer. |
| Parameters | \( \nu \) the degrees of freedom \( \text{integer} \ \ \ \nu > 0 \) |
| Domain | \(-\infty < x < +\infty \) continuous |
| Density and Cumulative Distribution Functions | 

\[
f(x) = \frac{1}{\sqrt{\pi \nu}} \frac{\Gamma \left( \frac{\nu + 1}{2} \right)}{\Gamma \left( \frac{\nu}{2} \right)} \left[ \frac{\nu}{\nu + x^2} \right]^{\frac{\nu + 1}{2}}
\]

\[
F(x) = \frac{1}{2} \left[ 1 + I_s \left( \frac{1}{2}, \frac{\nu}{2} \right) \right]
\]

\( s \equiv \frac{x^2}{\nu + x^2} \)

Here, \( \Gamma \) is the Gamma Function and \( I_s \) is the Incomplete Beta Function. |
<p>| Mean | 0 ( \text{for } \nu &gt; 1^* ) |
| Variance | ( \frac{\nu}{\nu - 2} ) ( \text{for } \nu &gt; 2 ) |
| Skewness | 0 ( \text{for } \nu &gt; 3^* ) |
| Kurtosis | ( 3 \left( \frac{\nu - 2}{\nu - 4} \right) ) ( \text{for } \nu &gt; 4 ) |</p>
<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskTriang}(\text{minimum}, \text{m.likely}, \text{maximum}) ) specifies a triangular distribution defined by a minimum, a most likely value, and a maximum. It can be skewed in either direction, depending on the position of ( \text{m.likely} ) relative to ( \text{minimum} ) and ( \text{maximum} ). This distribution is perhaps the most readily understandable distribution for basic risk models. Its possible drawback for real applications is that it allows no possible values outside the min-max range.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskTriang}(100,200,300) ) returns a triangular distribution with minimum value 100, most likely value 200 and maximum value 300. ( \text{RiskTriang}(A10/90,B10,500) ) specifies a triangular distribution with minimum value equal to the value in cell A10 divided by 90, most likely value taken from cell B10, and maximum value 500.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>( \text{minimum} ) must be less than or equal to ( \text{m.likely} ). ( \text{m.likely} ) must be less than or equal to ( \text{maximum} ). ( \text{minimum} ) must be less than ( \text{maximum} ).</td>
</tr>
<tr>
<td>Parameters</td>
<td>( \text{min} ) continuous boundary parameter ( \text{min} &lt; \text{max} ) * ( \text{m.likely} ) continuous mode parameter ( \text{min} \leq \text{m.likely} \leq \text{max} ) ( \text{max} ) continuous boundary parameter *( \text{min} = \text{max} ) is supported but gives a degenerate distribution.</td>
</tr>
<tr>
<td>Domain</td>
<td>( \text{min} \leq x \leq \text{max} ) continuous</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>( f(x) = \frac{2(x - \text{min})}{(\text{m.likely} - \text{min})(\text{max} - \text{min})} \quad \text{min} \leq x \leq \text{m.likely} )</td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>( f(x) = \frac{2(\text{max} - x)}{(\text{max} - \text{m.likely})(\text{max} - \text{min})} \quad \text{m.likely} \leq x \leq \text{max} )</td>
</tr>
<tr>
<td></td>
<td>( F(x) = \frac{(x - \text{min})^2}{(\text{m.likely} - \text{min})(\text{max} - \text{min})} \quad \text{min} \leq x \leq \text{m.likely} )</td>
</tr>
<tr>
<td></td>
<td>( F(x) = 1 - \frac{(\text{max} - x)^2}{(\text{max} - \text{m.likely})(\text{max} - \text{min})} \quad \text{m.likely} \leq x \leq \text{max} )</td>
</tr>
<tr>
<td>Mean</td>
<td>( \frac{\text{min} + \text{m.likely} + \text{max}}{3} )</td>
</tr>
<tr>
<td>Variance</td>
<td>( \frac{\text{min}^2 + \text{m.likely}^2 + \text{max}^2 - (\text{max})(\text{m.likely}) - (\text{m.likely})(\text{min}) - (\text{max})(\text{min})}{18} )</td>
</tr>
<tr>
<td>Skewness</td>
<td>( \frac{2\sqrt{2}}{5} \frac{f(f^2 - 9)}{(f^2 + 3)^{3/2}} \quad \text{where} \ f = \frac{2(\text{m.likely} - \text{min})}{\text{max} - \text{min}} - 1 )</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.4</td>
</tr>
<tr>
<td>Mode</td>
<td>\text{m.likely}</td>
</tr>
</tbody>
</table>
Examples

PDF - Triang(0,3,5)

CDF - Triang(0,3,5)
## RiskTriangAlt, RiskTriangAltD

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>RiskTriangAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</code> specifies a triangular distribution with three arguments of the type <code>arg1type</code> to <code>arg3type</code>. These arguments can be either “min”, “m.likely”, “max”, or a value between 0 and 1 to specify a percentile.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>RiskTriangAlt(&quot;min&quot;,2,&quot;m. likely&quot;,5,95%,30)</code> returns a triangular distribution with minimum 2, and most likely value 5, and 95th percentile 30.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>With <code>RiskTriangAltD</code>, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>
## RiskTrigen

**Description**  
*RiskTrigen(bottom value, m.likely value, top value, bottom perc., top perc.)* specifies a triangular distribution with three points: one at the most likely value and two at the specified bottom and top percentiles. The parameters `bottom perc.` and `top perc.` are values between 0 and 100. They represent the cumulative probabilities for the values `top value` and `bottom value`. This is sometimes useful because it allows values below `bottom value` or above `top value` to be possible values for the distribution, that is, these two parameters don’t specify the “absolute” lowest and highest values for the distribution. Note that RiskTrigen is actually a shortcut for a special form of RiskTriangAlt. For example:

RiskTrigen(0,1,2,10,90)  
Is the same as  
RiskTriangAlt(10%,0,“m. likely”,1,90%,2)

### Examples

<table>
<thead>
<tr>
<th>RiskTrigen(100,200,300,10,90)</th>
<th>returns a triangular distribution with 10th percentile 100, most likely value 200, and 90th percentile 300.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskTrigen(A10/90,B10,500,30,70)</td>
<td>returns a triangular distribution with 30th percentile equal to the value in cell A10 divided by 90, most likely value taken from cell B10, and 70th percentile 500.</td>
</tr>
</tbody>
</table>

### Guidelines

- `bottom value` must be less than or equal to `m.likely value`.
- `m.likely value` must be less than or equal to `top value`.
- `bottom perc.` must be less than `top perc.`
**RiskUniform**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskUniform(minimum, maximum) specifies a uniform distribution over the range from minimum to maximum. Every value in this range is equally likely. This distribution is sometimes called the “no knowledge” distribution, as in, “we have no idea what the uncertain value will be, except that it will be between 50 and 100 (for example). This makes it unrealistic in many real applications, where the relative likelihoods fall away on either side of some modal value. Nevertheless, the uniform distribution is important because it is used by random number algorithms to generate samples from other distributions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskUniform(10,20) returns a uniform distribution with minimum 10 and maximum 20. <strong>RiskUniform(A1+90,B1)</strong> returns a uniform distribution with minimum equal to the value in cell A1 plus 90, and maximum taken from cell B1.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>minimum must be less than maximum.</td>
</tr>
<tr>
<td>Parameters</td>
<td>min continuous boundary parameter min &lt; max *</td>
</tr>
<tr>
<td></td>
<td>max continuous boundary parameter *min = max is supported but gives a degenerate distribution.</td>
</tr>
<tr>
<td>Domain</td>
<td>min ≤ x ≤ max continuous</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>( f(x) = \frac{1}{\text{max} - \text{min}} )</td>
</tr>
<tr>
<td></td>
<td>( F(x) = \frac{x - \text{min}}{\text{max} - \text{min}} )</td>
</tr>
<tr>
<td>Mean</td>
<td>( \frac{\text{max} + \text{min}}{2} )</td>
</tr>
<tr>
<td>Variance</td>
<td>( \frac{(\text{max} - \text{min})^2}{12} )</td>
</tr>
<tr>
<td>Skewness</td>
<td>0</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.8</td>
</tr>
<tr>
<td>Mode</td>
<td>Not uniquely defined</td>
</tr>
<tr>
<td>---------------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td><strong>Examples</strong></td>
<td></td>
</tr>
<tr>
<td><strong>PDF - Uniform(0,1)</strong></td>
<td>![PDF Uniform(0,1)]</td>
</tr>
<tr>
<td><strong>CDF - Uniform(0,1)</strong></td>
<td>![CDF Uniform(0,1)]</td>
</tr>
</tbody>
</table>
### RiskUniformAlt, RiskUniformAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskUniformAlt}(\text{arg1type}, \text{arg1value}, \text{arg2type}, \text{arg2value}) ) specifies a uniform distribution with two arguments of the type \text{arg1type} and \text{arg2type}. These arguments can be either “min”, “max”, or a value between 0 and 1 to specify a percentile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskUniformAlt}(5%,1,95%,10) ) returns a uniform distribution with 5(^{th}) percentile 1 and 95(^{th}) percentile 10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>With ( \text{RiskUniformAltD} ), any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>
## RiskVary

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskVary</strong>(base, minimum, maximum, range type, #steps, distribution) specifies a distribution with a range defined by minimum and maximum. If desired, arguments for range type, #steps, and distribution can be entered. If not entered, the default range type, steps and distribution are used. The base value is the value returned by the function when a simulation is not underway. Typically, this is the value that was used in the spreadsheet prior to entering the Vary function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskVary</strong>(100,-10,10,0,8,&quot;Triang&quot;) specifies a distribution with base value 100, a -10% to +10% range, a percentage range type, 8 steps, and a triangular distribution across the min-max range. <strong>RiskVary</strong>(100,A1,B1) specifies a distribution with a range from the value in cell A1 to the value in cell B1, around the base value 100. The default range type, #Steps, and distribution are used.</td>
</tr>
<tr>
<td>Guidelines</td>
<td><strong>maximum</strong> must be greater than <strong>base</strong>.  <strong>base</strong> must be greater than <strong>minimum</strong>  <strong>Range Type</strong> = 0 indicates a +/- percentage change from base case is defined by <strong>minimum</strong> and <strong>maximum</strong> (e.g., -20% and +20%). Each percentage should be entered as an absolute percentage value (such as -20 instead of -0.2).  <strong>Range Type</strong> = 1 indicates a +/- actual change is defined by <strong>minimum</strong> and <strong>maximum</strong> (e.g., -150 and +150).  <strong>Range Type</strong> = 2 indicates that <strong>minimum</strong> and <strong>maximum</strong> are the actual minimum and maximum values in the range.  <strong>#Steps</strong> must be a positive integer. This argument is used only when conducting a what-if analysis with TopRank.  <strong>distribution</strong> must be “Normal”, “Triang”, “Trigen”, “Uniform”, or “Pert”, with the distribution name surrounded by quotes. The defaults are: <strong>Range Type</strong> is +/- percentage change, <strong>#Steps</strong> =5, and <strong>distribution</strong> = Triang</td>
</tr>
</tbody>
</table>
## RiskWeibull

### Description
RiskWeibull(\(alpha, beta\)) specifies a Weibull distribution with shape parameter \(alpha\) and scale parameter \(beta\). This distribution is often used in reliability studies to model the lifetime of a device. When \(alpha\) is greater than 1, the usual case, the distribution has the “wearout” property where the older the device is, the more likely it is to fail in the near future. The opposite is true when \(alpha\) is less than 1, which usually makes this parameter value unrealistic. When \(alpha\) equals 1, this distribution is equivalent to an exponential distribution, with a constant failure rate.

### Examples
- **RiskWeibull(3,20)** returns a Weibull distribution with shape parameter 3 and scale parameter 20.
- **RiskWeibull(D1,D2)** generates a Weibull distribution with shape parameter taken from cell D1 and scale parameter taken from cell D2.

### Guidelines
Both \(alpha\) and \(beta\) must be positive.

### Parameters
- \(\alpha\) continuous shape parameter \(\alpha > 0\)
- \(\beta\) continuous scale parameter \(\beta > 0\)

### Domain
\(0 \leq x < +\infty\) continuous

### Density and Cumulative Distribution Functions
\[
f(x) = \frac{\alpha x^{\alpha-1}}{\beta^\alpha} e^{-\left(\frac{x}{\beta}\right)^\alpha}
\]
\[
F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^\alpha}
\]

### Mean
\[
\beta \Gamma\left(1 + \frac{1}{\alpha}\right)
\]
Here, \(\Gamma\) is the Gamma Function.

### Variance
\[
\beta^2 \left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \Gamma^2\left(1 + \frac{1}{\alpha}\right)\right]
\]
### Skewness

\[
\frac{\Gamma\left(1+\frac{3}{\alpha}\right) - 3\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1+\frac{1}{\alpha}\right) + 2\Gamma^3\left(1+\frac{1}{\alpha}\right)}{\left[\Gamma\left(1+\frac{2}{\alpha}\right) - \Gamma^2\left(1+\frac{1}{\alpha}\right)\right]^{3/2}}
\]

### Kurtosis

\[
\frac{\Gamma\left(1+\frac{4}{\alpha}\right) - 4\Gamma\left(1+\frac{3}{\alpha}\right)\Gamma\left(1+\frac{1}{\alpha}\right) + 6\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma^2\left(1+\frac{1}{\alpha}\right) - 3\Gamma^4\left(1+\frac{1}{\alpha}\right)}{\left[\Gamma\left(1+\frac{2}{\alpha}\right) - \Gamma^2\left(1+\frac{1}{\alpha}\right)\right]^2}
\]

### Mode

\[
\beta\left(1-\frac{1}{\alpha}\right)^{1/\alpha}
\]

for \(\alpha > 1\)

0

for \(\alpha \leq 1\)
Examples

PDF - Weibull(2,1)

CDF - Weibull(2,1)
### RiskWeibullAlt, RiskWeibullAltD

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskWeibullAlt</strong>(<em>arg1type, arg1value, arg2type,arg2value, arg3type,arg3value</em>) specifies a Weibull distribution with three arguments of the type <em>arg1type</em> to <em>arg3type</em>. These arguments can be either “alpha”, “beta”, “loc”, or a value between 0 and 1 to specify a percentile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskWeibullAlt</strong>(&quot;alpha&quot;,1,&quot;beta&quot;,1,95%,3) specifies a Weibull distribution with shape parameter 1, scale parameter 1, and 95th percentile 3. Note that this distribution will be shifted (minimum value won’t be 0) to achieve the specified parameters.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>With <strong>RiskWeibullAltD</strong>, any entered percentile values are cumulative descending percentiles, where the percentile specifies the probability of a value greater than or equal to the entered value.</td>
</tr>
</tbody>
</table>
Reference: Distribution Property Functions

The following functions are used to add optional arguments to distribution functions. The arguments can be added as needed. These optional arguments are specified with @RISK distribution property functions that are embedded inside a distribution function.

### RiskCategory

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCategory(category name) names the category to be used when displaying an input distribution. This name defines the grouping in which an input will appear in the @RISK Model window’s inputs list and in any reports that include simulation results for the input.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTriang(10,20,30,RiskCategory(&quot;Prices&quot;)) places the probability distribution RiskTriang(10,20,30) in the category “Prices”.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>The specified category name must be entered in quotes. Any valid cell references can be used to define a category name.</td>
</tr>
</tbody>
</table>
### RiskCollect

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCollect() identifies specific distribution functions whose samples are collected during a simulation and whose:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• statistics are displayed</td>
</tr>
<tr>
<td></td>
<td>• data points are available</td>
</tr>
<tr>
<td></td>
<td>• sensitivities and scenario values are calculated</td>
</tr>
</tbody>
</table>

When RiskCollect is used and Inputs Marked With Collect is selected for Collect Distribution Samples in the Simulation Settings dialog, only functions identified by RiskCollect are displayed in the Results Summary window.

Earlier versions of @RISK had the RiskCollect function entered by placing it in the cell formula, immediately preceding the distribution function for which samples will be collected, e.g.:

\[=\text{RiskCollect()}+\text{RiskNormal}(10,10)\]

RiskCollect is typically used when a large number of distribution functions are present in a simulated worksheet, but sensitivities and scenario analyses are desired on only a pre-identified subset of important distributions. It can also be used to bypass Windows memory constraints that might prevent sensitivity analysis on all inputs in a large simulation model.

| Examples | RiskNormal(10,2,RiskCollect()) collects samples from the probability distribution RiskNormal(10,2). |

| Guidelines | The Inputs Marked With Collect option in the Simulation Settings dialog must be checked for RiskCollect functions to take effect. |
### RiskConvergence

| Description | \( \text{RiskConvergence}(\text{tolerance},\text{toleranceType},\text{confidenceLevel}, \text{useMean},\text{useStdDev},\text{usePercentile},\text{percentile}) \) specifies convergence monitoring information for a specific output. \( \text{tolerance} \) is the +/- tolerance amount desired, \( \text{toleranceType} \) specifies the type of tolerance value entered (1 for +/- actuals, 2 for +/- percentage or relative), \( \text{confidenceLevel} \) specifies the confidence level for the estimate, \( \text{useMean}, \text{useStdDev}, \text{usePercentile} \) is set to TRUE to select the monitoring statistic desired, and \( \text{percentile} \) enters the percentile to monitor when \( \text{usePercentile} \) is set to TRUE. \( \text{RiskConvergence} \) returns FALSE if the output has not converged, and TRUE if it has converged. |
| Examples | \( \text{RiskOutput}(:,:,\text{RiskConvergence}(3\%,2,95\%,\text{TRUE})) \) specifies a +/- 3% tolerance with a 95% confidence level, where the monitored statistic is the mean. |
| Guidelines | This property function overrides any default convergence monitoring specified in the Simulation Settings dialog. The RiskConvergence property function is available only for simulation outputs. |
**RiskCopula**

| Description | *RiskCopula*(copula definition range, position, instance) identifies a distribution function of an input as being “attached” to a copula to allow multivariate correlation. This function identifies a *copula definition range*, the *position* in the copula, and optionally a unique copula *instance*.  

Copulas are typically defined through @RISK’s Define Copula command. However, the same type of correlation can be entered directly in a cell formula using the RiskCopula function. The copula identified by the *copula definition range* is an array of cells whose format is different for each type of copula.  

The *position* argument is an integer from 1 to the number of distributions being correlated. For example, if *position* is 2, this indicates the second variable being correlated.  

The *instance* argument is optional and is used when multiple groups of correlated inputs use the same copula. *Instance* is an integer or string argument, and all inputs in a correlated group of inputs share the same instance value or string. If string arguments are used, they must be inside quotes.  

See the Define Copula Command section of this manual for more detailed information about copulas. |
| Examples | *RiskNormal*(10,10, RiskCopula(MyCopula,1)) specifies that the Normal(10,10) distribution will be “attached” to position 1 of the copula called MyCopula. |
| Guidelines | None. |
| **Description** | **RiskCorrmat(matrix cell range,position,instance)** identifies a distribution function of an input as being “attached” to a correlation matrix to allow multivariate correlation. This function identifies a *matrix definition range*, the *position* in the matrix, and optionally a unique correlation *instance*.

Correlation matrices are typically defined through @RISK’s Define Correlation Matrix command. However, the same type of correlation can be entered directly in a cell formula with the RiskCorrmat function.

The matrix identified by the *matrix cell range* is a matrix of rank correlation coefficients. Each element in the matrix contains a correlation coefficient. The number of distribution functions correlated by the matrix equals the number of rows or columns in the matrix.

The *position* argument is an integer from 1 to the number of distributions being correlated. For example, if *position* is 2, this indicates the second variable being correlated.

The *instance* argument is optional and is used when multiple groups of correlated inputs use the same correlation matrix. *Instance* is an integer or string argument, and all inputs in a correlated group of inputs share the same instance value or string. If string arguments are used, they must be inside quotes.

See the **Define Correlations** section of this manual for more detailed information about correlations and correlation matrices. |
| **Examples** | **RiskNormal(10,10, RiskCorrmat(C10:G14,1,"Matrix 1"))** specifies that the Normal(10,10) distribution is the first (position 1) of five correlated inputs, with the 5x5 matrix of correlation coefficients located in the range C10:G14. |
| **Guidelines** | See the **Define Correlations** section of this manual. |
## RiskDepC

| Description | RiskDepC\((ID, coefficient)\) designates a dependent variable in a correlated pair of inputs. The \(ID\) is the string used to identify the independent variable it is being correlated with. This string must be inside quotes. This same \(ID\) must be used in the RiskIndepC function for the independent variable. The \(coefficient\) entered is the rank-order correlation coefficient between the two variables. This is the "old" way to correlate two variables in @RISK, and it has been kept for backward compatibility. However, a better way is to use a correlation matrix (or a copula). See the Define Correlations section of this manual for more detailed information about correlations, correlation matrices, and copulas. |
| Examples | RiskNormal(100,10, RiskDepC("Price",0.5)) specifies that the RiskNormal(100,10) distribution will be correlated with the distribution identified by RiskIndepC("Price"). The correlation between these two variables is 0.5. |
| Guidelines | \(coefficient\) must be from -1 to +1. \(ID\) must be the same string of characters used to identify the independent variable in the RiskIndepC function. \(ID\) can be a reference to a cell that contains an identifier string. |
**RiskFit**

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskFit}($fit name,selected fit result$)$ links a data set, and its fit results, to the input distribution the RiskFit function is used in. The fit name in quotes is the name of the fit given when the data was fit using the Fit Distributions to Data command. The selected fit result in quotes is a string used to identify the type of fit result to select. The RiskFit function is used to link an input to the fit results for a data set, so that when the data is changed, the input distribution selected from the fit can be updated. The selected fit result can be any of the following: AIC, BIC, ChiSq, AD, KS, RMSErr, or a distribution name such as “Normal”. The RiskFit function “hot-links” a distribution function to a data set and the fit of that data set. The data used in a fit is in a range in Excel. When the fitted data changes and a simulation is started, the following actions take place: @RISK re-runs the fit using the current settings on the fit tab where the fit was originally run. The distribution function that includes the RiskFit function that references the fit is changed to reflect the new fit results. The changed function replaces the original one in Excel. For example, if the distribution function’s RiskFit argument specifies “AIC” for selected fit result, the new best-fitting distribution based on the AIC criterion will replace the original best fit. This new function will also include the same RiskFit function as the original one. For more information, see the Distribution Fitting section of this manual.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskNormal}(2.5, 1, \text{RiskFit}($&quot;Price Data&quot;, &quot;$AIC&quot;)$)$ specifies that the best fitting distribution with the AIC criterion for the fitted data associated with the fit named Price Data is a normal distribution with mean 2.5 and standard deviation 1.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
**RiskIndepC**

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskIndepC</strong>(ID) designates an independent variable in a pair of correlated inputs. The ID is the string used to identify the dependent variable it is being correlated with. This string must be inside quotes. This same ID must be used in the RiskDepC function for the dependent variable. The correlation coefficient is not specified here; it is specified in the corresponding RiskDepC function. This is the “old” way to correlate two variables in @RISK, and it has been kept for backward compatibility. However, a better way is to use a correlation matrix (or a copula). See the Define Correlations section of this manual for more detailed information about correlations, correlation matrices, and copulas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskNormal(10,10, RiskIndepC(&quot;Price&quot;))</strong> identifies the Normal(10,10) distribution as the distribution of the independent variable “Price”. There should be a corresponding RiskDepC function for the dependent variable in the pair.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>ID must be the same string of characters used to identify the dependent variable in the RiskDepC function. ID can be a reference to a cell that contains an identifier string.</td>
</tr>
<tr>
<td><strong>RiskIsDiscrete</strong></td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td><strong>Description</strong></td>
<td><em>RiskIsDiscrete</em>(TRUE) specifies that the corresponding output should be treated as a discrete distribution for displaying graphs of simulation results and calculating statistics. If <em>RiskIsDiscrete</em> is not entered, @RISK will attempt to detect whether an output’s distribution is best treated as discrete.</td>
</tr>
<tr>
<td><strong>Examples</strong></td>
<td><em>RiskOutput</em>(,,<em>RiskIsDiscrete</em>(TRUE))+NPV(.1,C1:C10) specifies that the output distribution of NPV should be treated as a discrete distribution.</td>
</tr>
<tr>
<td><strong>Guidelines</strong></td>
<td>None.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>RiskIsDate</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description</strong></td>
</tr>
</tbody>
</table>
| **Examples**     | *RiskOutput*(,,*RiskIsDate*(TRUE)) specifies that the output distribution will be displayed using dates, regardless of the cell formatting in Excel.  
*RiskTriang*(DATE(2015,10,4),DATE(2015,12,29),DATE(2016,10,10),*RiskIsDate*(TRUE)) specifies a triangular distribution with minimum date 10/4/2015, most likely date 12/29/2015, and maximum date 10/10/2016. |
| **Guidelines**   | *RiskIsDate*(FALSE) causes @RISK to display graphs and reports for the input or output in values, not dates, even if the cell where the function is located in Excel has date formatting. |
## RiskLibrary

| **Description** | *RiskLibrary*(position, ID) specifies that the corresponding distribution is linked to a distribution in an @RISK Library with the specified position and ID. Each time a simulation is run, the distribution function will update with the current definition of the distribution in the @RISK Library with the specified ID. |
| **Examples** | RiskNormal(5000,1000,RiskName("Sales Volume / 2015"),RiskLibrary(2,"LV6W59J5"),RiskStatic(0.46)) specifies that the entered distribution is taken from the @RISK Library with the position 2 and the ID LV6W59J5. The current definition of this library distribution is RiskNormal(10,10, RiskName("Sales Volume / 2015")), but this will change when the distribution in the library changes. |
| **Guidelines** | A RiskStatic value is not updated from the @RISK Library because it is unique to the model where the library distribution is used. |

## RiskLock

| **Description** | *RiskLock()* keeps a distribution from being sampled in a simulation. Locking an input distribution causes it to return the same value (the “static” value) in each iteration. |
| **Examples** | RiskNormal(10,2,RiskLock()) stops samples from being drawn from this normal distribution. |
| **Guidelines** | The optional argument Lock_Mode can take the value TRUE (lock to the value returned by the distribution function in Excel during a standard recalculation), FALSE (don’t lock the distribution function), or LockValue (specifies a value the distribution function will return in each iteration of the simulation). When no argument is entered for RiskLock, the behavior is the same as when Lock_Mode = TRUE. |
### RiskName

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskName(<em>input name</em>) names the corresponding input distribution. This name will appear in the @RISK Model window and in any reports and graphs that include simulation results for the input.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskTriang(10,20,30,RiskName(&quot;Price&quot;)) specifies the name Price for the RiskTriang(10,20,30) input distribution.</td>
</tr>
<tr>
<td>RiskTriang(10,20,30,RiskName(A10)) specifies the name contained in the cell A10 for the RiskTriang(10,20,30) input distribution.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>input name</em> must be entered in quotes.</td>
</tr>
<tr>
<td>Any valid cell references can be used to point to a name.</td>
</tr>
</tbody>
</table>

### RiskSeed

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskSeed(<em>random number generator type</em>,<em>seed value</em>) specifies that an input will use its own random number generator of the specified type, and that it will be seeded with <em>seed value</em>. Seeding an individual input is useful when the same distribution is shared across models using the @RISK Library and a reproducible set of samples is desired for the input in every model.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskBeta(10,2,RiskSeed(1,100)) specifies that the input RiskBeta(10,2) will use its own Mersenne Twister random number generator (type 1), and it will use the seed 100.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input distributions that use RiskSeed always have their own reproducible stream of random numbers. The Initial Seed, set in the Simulation Settings Sampling tab, affects only the random numbers generated for input distributions that do not have an independent seed specified using the RiskSeed property function.</td>
</tr>
<tr>
<td>The <em>random number generator type</em> is specified as value between 1 and 8, where 1=MersenneTwister, 2=MRG32k3a, 3=MWC, 4=KISS, 5=LFIB4, 6=SWB, 7=KISS_SWB, 8=RAN3I. For more information on the available random number generators, see the Simulation Settings section of this manual.</td>
</tr>
<tr>
<td>Seed value is an integer between 1 and 2,147,483,647.</td>
</tr>
<tr>
<td>The RiskSeed function has no effect when an input is correlated with other inputs.</td>
</tr>
</tbody>
</table>
### RiskShift

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskShift(shift amount) shifts the domain of the corresponding distribution by shift amount. This function is automatically entered when a fit result includes a shift factor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskBeta(10,2,RiskShift(100)) shifts the domain of the RiskBeta(10,2) distribution by 100. Its possible values were from 0 to 1; now they will be from 100 to 101.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskSixSigma

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskSixSigma(LSL, USL, Target, Long Term Shift, Number of Standard Deviations) specifies the lower specification limit (LSL), the upper specification limit (USL), the target value, the long term shift, and the number of standard deviations for six sigma calculations for the corresponding output. These values are used in calculating six sigma statistics displayed in the results window and graphs for the output.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskOutput(A10,,,RiskSixSigma(0.88,0.95,0.915,1.5,6)) specifies an LSL of 0.88, a USL of 0.95, target value of 0.915, a long term shift of 1.5, and a number of standard deviations of 6 for the output in cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>The parameters entered in the RiskSixSigma property function for an output are read at the start of a simulation. If you change the property function values you need to re-run the simulation to update six sigma statistics displayed in the results window and on graphs for the output.</td>
</tr>
</tbody>
</table>
### RiskStatic

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskStatic</strong>(static value) defines the static value 1) returned by a distribution function during a standard Excel recalculation and 2) that replaces the @RISK function after @RISK functions are swapped out.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>**RiskBeta(10,2,<strong>RiskStatic(9.5))</strong> specifies that the static value for the RiskBeta(10,2) distribution will be 9.5.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskTruncate

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskTruncate</strong>(minimum,maximum) truncates the corresponding input distribution. Truncating a distribution restricts samples drawn from the distribution to values within the entered minimum-maximum range. Truncated forms of specific distributions available in earlier versions of @RISK (such as RiskTnormal and RiskTlognorm) are still supported.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>**RiskTriang(10,20,30,<strong>RiskTruncate(13,27))</strong> restricts the samples drawn from the RiskTriang(10,20,30) distribution to a minimum possible value of 13 and a maximum possible value of 27.</td>
</tr>
</tbody>
</table>
| Guidelines | minimum must be less than or equal to maximum.  
To enter a distribution that is truncated on only one side, leave the argument for the unbounded side blank, as in **RiskNormal(10,1,**RiskTruncate(5,))**. This sets the minimum to 5 but leaves the maximum unbounded. |
### RiskTruncateP

**Description**  
*RiskTruncateP*(perc% minimum, perc% maximum) truncates the corresponding input distribution to percentiles of the original distribution. Truncating a distribution restricts samples drawn from the distribution to values within the entered *minimum-maximum* range. Truncated forms of specific distributions available in earlier versions of @RISK (such as RiskTnormal and RiskTlognorm) are still supported.

**Examples**  
*RiskTriang*(10,20,30,RiskTruncate(0.01,0.99)) restricts the samples drawn from the RiskTriang(10,20,30) distribution to the range between the 1st percentile and 99th percentile of the original distribution.

**Guidelines**  
perc% minimum must be less than or equal to perc% maximum.  
perc% minimum and perc% maximum must be between 0 and 1.  
Distribution functions that contain a RiskTruncateP property function cannot be displayed in the Define Distribution window.  
As with RiskTruncate, to enter a distribution that is truncated on only one side, leave the argument for the unbounded side blank.

### RiskUnits

**Description**  
*RiskUnits*(units) names the units to be used in labeling the corresponding input distribution or output. This name will appear in the @RISK Model window and in any reports and graphs that include simulation results for the input or output.

**Examples**  
*RiskTriang*(10,20,30,RiskUnits("Dollars")) specifies the name Dollars for the units described by the RiskTriang(10,20,30) distribution.  
*RiskTriang*(10,20,30, RiskUnits(A10)) specifies the name contained in the cell A10 for the units described by the RiskTriang(10,20,30) distribution.

**Guidelines**  
units must be entered in quotes.  
Any valid cell references can be used to define a units name.  
If RiskUnits is used as a property function for a RiskOutput function, it must come after the name, range name, and position arguments of RiskOutput. For example, if you are using RiskOutput with no name, range name, or position arguments, you would enter *RiskOutput(,,RiskUnits("MyUnits"))*.
Reference: Output Functions

Output cells are defined using the RiskOutput function. This function allows easy copying, pasting, and moving of output cells. RiskOutput functions are automatically added when you select one or more cells and click the Add Output icon on the @RISK ribbon. RiskOutput functions optionally allow you to name your simulation outputs, and to add individual output cells to output ranges.

The property functions RiskUnits, RiskConvergence, RiskSixSigma, and RiskIsDiscrete can be used with RiskOutput functions.

RiskOutput

| Description | RiskOutput(output cell name, output range name, position# in range) specifies that the corresponding cell will be treated as an output in a simulation model. Its three arguments are all optional. The simple form =RiskOutput() is sufficient to designate a single-cell output, where @RISK will create the output name from labels in adjacent cells. Alternatively, RiskOutput can be used with a single argument, as in =RiskOutput ("Profit"), to designate a single-cell output with the specified name. For a multiple-cell output range, the form =RiskOutput ("Profit1", "Profit", 1) is used. However, the first argument can be omitted if you want to let @RISK automatically generate a name for each output cell in the range. In this case, the individual output cells would have names Profit1, Profit2, and so on. The easiest way to generate RiskOutput functions is by clicking the Add Output icon on the @RISK ribbon. This changes a formula such as =NPV(0.1,G1…G10) to =RiskOutput()+NPV(0.1,G1…G10) Alternatively, you can attach a RiskOutput function to any cell formula by typing directly in the formula. |
| Examples | =RiskOutput("Profit 2011", "Annual Profit", 1)+NPV(0.1,G1…G10) identifies corresponding the cell as a simulation output, gives it the name Profit 2011, and makes it the first cell in a multiple-cell output range named Annual Profit. However, the naming might be confusing. It might be better to omit the first argument. Then the first output cell would be named Annual Profit 1, the second would be named Annual Profit 2, and so on. The choice is yours. |
| Guidelines | If names are entered directly in the RiskOutput function, the entered output cell name and output range name must be inside quotes. Names can also be identified by referencing cells with appropriate labels.  
*Position# in range* must be a positive integer.  
Any property functions must follow the first three arguments of the RiskOutput function. For example, if you add a RiskUnits property function to a default RiskOutput function, you would need to enter  
=RiskOutput(,,,RiskUnits("MyUnits"))  
If you are using RiskOutput with a property function such as RiskSixSigma, the Property Functions section of this Reference section describes the arguments of the property function. If the @RISK Insert Function command is used to enter RiskOutput in Six Sigma format, simply click in the formula bar on the displayed RiskSixSigma property function to enter its arguments or to view help on the RiskSixSigma property function. |
Statistics functions return desired statistics on simulation results for 1) a specified cell or 2) a simulation output or input. These functions are updated in real time as a simulation is running or at the end of a simulation. Statistics functions located in template sheets used for creating custom reports are updated only when a simulation is completed.

If a cell reference is entered as the first argument of a statistics function, the cell does not have to be a simulation output identified with a RiskOutput function.

If a name is entered instead of a cell reference, @RISK first checks for an output with the entered name. If none exists, @RISK looks for an input probability distribution with the entered name and, if one is found, it returns the appropriate statistic for the samples drawn for that input. It is up to you to insure that unique names are given to outputs and inputs referenced in statistics functions.

The Sim# argument entered selects the simulation for which a statistic will be returned when multiple simulations are run. This argument is optional and can be omitted for single-simulation runs.

Statistics functions that calculate a statistic of a distribution for a simulation result can include a RiskTruncate or a RiskTruncateP property function. This will cause the statistic to be calculated on the min-max range specified by the truncation limits. For example, if you want to calculate statistics on a percentile range of a distribution, you can use RiskTruncateP, as in

\[ =\text{RiskMean}(A1,\text{RiskTruncateP}(0.9,1)) \]

In this case, the mean of the data in the top 10% of the simulated values for cell A1 will be returned by the RiskMean function.

@RISK’s statistics functions can be updated either at the end of a simulation or with each iteration during a simulation. (The latter is the default in @RISK 5.5 or later.) In most cases, statistics do not need to be updated until the end of a simulation, when you will view the results of the simulation. However, if the calculations in your model require a new statistic to be returned in each iteration (for example, when a custom convergence calculation has been entered using Excel formulas), the Each Iteration option should be used. You use the Update Statistic Functions option on the Sampling tab of the Simulation Settings dialog to control this.
### RiskConvergenceLevel

**Description**

\[ \text{RiskConvergenceLevel}(\text{cellref or output name}, Sim\#) \]

returns the convergence level (0 to 100) for \text{cellref or output name}. The function value TRUE is returned on convergence.

**Examples**

\[ \text{RiskConvergenceLevel}(A10) \]

returns the convergence level for cell A10.

**Guidelines**

A RiskConvergence property function needs to be entered for \text{cellref or output name}, or convergence monitoring needs to be enabled in the Simulation Settings dialog, for this function to return a convergence level.

### RiskCorrel

**Description**

\[ \text{RiskCorrel}(\text{cellref1 or output/input name1}, \text{cellref2 or output/input name2}, \text{correlationType}, Sim\#) \]

returns the correlation coefficient using \text{correlationType} for the simulated input or output data from simulation \text{Sim\#} in the cells referenced by the first two arguments. The \text{correlationType} is either Pearson or Spearman Rank correlation.

**Examples**

\[ \text{RiskCorrel}(A10, A11, 1) \]

returns the Pearson correlation coefficient for the simulated input or output data in cells A10 and A11. No Sim\# argument indicates that there is a single simulation.

\[ \text{RiskCorrel} ("Profit","Sales",2,3) \]

returns the Spearman Rank correlation coefficient for the simulated “Profit” and “Sales” data in simulation 3.

**Guidelines**

correlationType is 1 for Pearson correlation or 2 for Spearman Rank correlation.

All iterations that contain ERR, or are filtered out in either input/output cell, are removed, and the correlation coefficient is calculated from the remaining data.

You can enter a RiskTruncate or RiskTruncateP property function for each distribution to base the correlation on only the truncated data. The first RiskTruncate function is used for \text{cellref1 or output/input name1}, and the second RiskTruncate function is used for \text{cellref2 or output/input name2}. 
### RiskData

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskData(cellref or output/input name,iteration#,Sim#) returns the simulated data value(s) in iteration iteration# of simulation Sim# for cellref. RiskData can optionally be entered as an array formula, where iteration# is the iteration to be returned in the first cell in the array formula range. The data values for each subsequent iteration will be filled into cells in the range where the array formula is entered.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskData(A10,1) returns the simulated data value for cell A10 from iteration #1. No Sim# argument indicates that there is a single simulation. RiskData(&quot;Profit&quot;,100,2) returns the simulated data value for the output named “Profit” from iteration 100 of simulation 2 (of multiple simulations).</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskKurtosis

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskKurtosis(cellref or output/input name,Sim#) returns the kurtosis of the simulated distribution for cellref. You can use the RiskTruncate property function to optionally restrict the range of the simulated distribution for calculating the statistic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskKurtosis(A10) returns the kurtosis of the simulated distribution for cell A10. RiskKurtosis(&quot;Profit&quot;,2) returns the kurtosis of the simulated distribution for the output cell named “Profit” for simulation 2 (of multiple simulations).</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskMax

**Description**

RiskMax\((cell\text{ref or output/input name}, Sim\#)\) returns the maximum value of the simulated distribution for \(cell\text{ref}\). You can use the RiskTruncate property function to optionally restrict the range of the simulated distribution for calculating the statistic.

**Examples**

- RiskMax\((A10)\) returns the maximum of the simulated distribution for cell A10.
- RiskMax\("Profit\") returns the maximum of the simulated distribution for the output cell named “Profit”.

**Guidelines**

None.

### RiskMean

**Description**

RiskMean\((cell\text{ref or output/input name}, Sim\#)\) returns the mean value of the simulated distribution for \(cell\text{ref}\). You can use the RiskTruncate property function to optionally restrict the range of the simulated distribution for calculating the statistic.

**Examples**

- RiskMean\((A10)\) returns the mean of the simulated distribution for cell A10.
- RiskMean\("Price\") returns the mean of the simulated distribution for the output cell named “Price”.

**Guidelines**

None.

### RiskMin

**Description**

RiskMin\((cell\text{ref or output/input name}, Sim\#)\) returns the minimum value of the simulated distribution for \(cell\text{ref}\). You can use the RiskTruncate property function to optionally restrict the range of the simulated distribution for calculating the statistic.

**Examples**

- RiskMin\((A10)\) returns the minimum of the simulated distribution for cell A10.
- RiskMin\("Sales\") returns the minimum value of the simulated distribution for the output cell named “Sales”.

**Guidelines**

None.
## RiskMode

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskMode(cellref or output/input name, Sim#) returns the mode of the simulated distribution for cellref. You can use the RiskTruncate property function to optionally restrict the range of the simulated distribution for calculating the statistic.</th>
</tr>
</thead>
</table>
| Examples    | RiskMode(A10) returns the mode of the simulated distribution for cell A10.  
RiskMode("Sales") returns the mode of the simulated distribution for the output cell in the current model named “Sales”. |
| Guidelines  | None. |

## RiskPercentile, RiskPtoX, RiskPercentileD, RiskQtoX

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskPercentile(cellref or output/input name, percentile, Sim#) or RiskPtoX(cellref or output/input name, percentile, Sim#) returns the value of the entered percentile of the simulated distribution for cellref. You can use the RiskTruncate property function to optionally restrict the range of the simulated distribution for calculating the statistic.</th>
</tr>
</thead>
</table>
| Examples    | RiskPercentile(C10,0.99) returns the 99th percentile of the simulated distribution for cell C10.  
RiskPercentile(C10,A10) returns the percentile value from cell A10 of the simulated distribution for cell C10. |
| Guidelines  | percentile must be between 0 and 1.  
RiskPercentileD and RiskQtoX are similar, but they take a cumulative descending percentile value.  
RiskPercentile and RiskPtoX are alternate names for the same function. Similarly for RiskPercentileD and RiskQtoX. |
### RiskRange

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskRange(cellref or output/input name, Sim#) returns the length of the minimum-maximum range of the simulated distribution for cellref. You can use the RiskTruncate property function to optionally restrict the range of the simulated distribution for calculating the statistic.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskRange(A10) returns the length of the range of the simulated distribution for cell A10.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskSensitivity

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskSensitivity(cellref or output name, Sim#, rank, analysisType, returnValueType) returns the sensitivity analysis information of the simulated distribution for cellref or output name. The rank argument specifies the rank in the sensitivity analysis for the input whose results are desired, where 1 is the top ranking, or most important, input. The analysisType argument selects the type of analysis desired: 1 for regression, 2 for regression-mapped values and 3 for correlation. The returnValueType selects the type of data to be returned: 1 for input name/cell reference/distribution function, 2 for the sensitivity coefficient or value, 3 for equation coefficient, 4 for equation constant and 5 for R-squared (returnValueTypes 3 through 5 are for regression only).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskSensitivity(A10,1,1,1,1) returns a description of the top ranking input for a regression sensitivity analysis on the simulation results for cell A10.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskSensitivityStatChange

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskSensitivityStatChange</strong>(cellref or output name, Sim#, rank, numBins, whichStatistic, percentile, returnType) returns the “change in output statistic” sensitivity analysis information of the simulated distribution for cellref or output name. The rank argument specifies the rank in the sensitivity analysis for the input whose results are desired, where 1 is the top ranking, or most important, input. The numBins argument specifies the number of equal-sized bins the samples for each input will be divided into. The whichStatistic argument specifies the statistic that will be calculated for the output in this analysis. If whichStatistic is a percentile, percentile is the percentile value to use. The returnType selects the type of data to be returned: 1 for input name/cell reference/distribution function, 2 for the minimum statistic value for a bin, and 3 for the maximum statistic for a bin.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskSensitivityStatChange</strong>(A10,1,1,20,1,0,1) returns a description of the top ranking input for a change in output statistic sensitivity analysis on the simulation results for cell A10. The mean is the statistic used in the analysis and the input samples are divided into 20 equal-sized bins.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>numBins must be a positive integer. whichStatistic is 1=mean, 9=mode, 10=percentile. percentile must be between 0 and 1.</td>
</tr>
</tbody>
</table>

### RiskSkewness

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskSkewness</strong>(cellref or output/input name, Sim#) returns the skewness of the simulated distribution for cellref. You can use the RiskTruncate property function to optionally restrict the range of the simulated distribution for calculating the statistic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskSkewness</strong>(A10) returns the skewness of the simulated distribution for cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
<tr>
<td><strong>RiskStdDev</strong></td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td></td>
</tr>
<tr>
<td><strong>Description</strong></td>
<td><em>RiskStdDev</em>(cellref or output/input name, Sim#) returns the standard deviation of the simulated distribution for <em>cellref</em>. You can use the RiskTruncate property function to optionally restrict the range of the simulated distribution for calculating the statistic.</td>
</tr>
<tr>
<td><strong>Examples</strong></td>
<td><em>RiskStdDev</em>(A10) returns the standard deviation of the simulated distribution for cell A10.</td>
</tr>
<tr>
<td><strong>Guidelines</strong></td>
<td>None.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>RiskTarget, RiskXtoP, RiskTargetD, RiskXtoQ</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description</strong></td>
</tr>
<tr>
<td><strong>Examples</strong></td>
</tr>
<tr>
<td><strong>Guidelines</strong></td>
</tr>
</tbody>
</table>
RiskVariance

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskVariance(cellref or output/input name, Sim#) returns the variance of the simulated distribution for cellref. You can use the RiskTruncate property function to optionally restrict the range of the simulated distribution for calculating the statistic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskVariance(A10) returns the variance of the simulated distribution for cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

RiskTheoKurtosis

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoKurtosis(cellref or distribution function) returns the kurtosis of the last distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTheoKurtosis(A10) returns the kurtosis of the last distribution function in cell A10. RiskTheoKurtosis(RiskNormal(10,1)) returns the kurtosis of the RiskNormal(10,1) distribution.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

RiskTheoMax

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoMax(cellref or distribution function) returns the maximum value of the last distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTheoMax(A10) returns the maximum of the last distribution function in cell A10. RiskTheoMax(RiskNormal(10,1)) returns the maximum of the RiskNormal(10,1) distribution. Actually, this will return an error because the normal distribution has no theoretical maximum.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskTheoMean

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoMean(cellref or distribution function) returns the mean value of the last distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
</table>
| Examples    | RiskTheoMean(A10) returns the mean of the last distribution function in cell A10.  
RiskTheoMean(RiskNormal(10,1)) returns the mean of the RiskNormal(10,1) distribution. |
| Guidelines  | None.                                                                                                                                                                           |

### RiskTheoMin

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoMin(cellref or distribution function) returns the minimum value of the last distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
</table>
| Examples    | RiskTheoMin(A10) returns the minimum of the last distribution function in cell A10.  
RiskTheoMin(RiskNormal(10,1)) returns the minimum of the RiskNormal(10,1) distribution. Actually, this will return an error because the normal distribution has no theoretical minimum. |
| Guidelines  | None.                                                                                                                                                                           |

### RiskTheoMode

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoMode(cellref or distribution function) returns the mode of the last distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
</table>
| Examples    | RiskTheoMode(A10) returns the mode of the last distribution function in cell A10.  
RiskTheoMode(RiskNormal(10,1)) returns the mode of the RiskNormal(10,1) distribution. |
| Guidelines  | None.                                                                                                                                                                           |
RiskTheoPercentile, RiskTheoPtoX, RiskTheoPercentileD, RiskTheoQtoX

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoPercentile(cellref or distribution function, percentile) or RiskTheoPtoX(cellref or distribution function, percentile) returns the value of the entered percentile of the last distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guidelines</td>
<td>percentile must be between 0 and 1. RiskTheoPercentileD and RiskTheoQtoX are similar, but their percentile is a cumulative descending value. RiskTheoPercentile and RiskTheoPtoX are alternate names for the same function. Similarly for RiskTheoPercentileD and RiskTheoQtoX.</td>
</tr>
</tbody>
</table>

RiskTheoRange

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoRange(cellref or distribution function) returns the length of the minimum-maximum range of the last distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTheoRange(A10) returns the length of the range of the distribution function in cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

None.
### RiskTheoSkewness

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoSkewness(cellref or distribution function) returns the skewness of the last distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTheoSkewness(A10) returns the skewness of the last distribution function in cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskTheoStdDev

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoStdDev(cellref or distribution function) returns the standard deviation of the last distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTheoStdDev(A10) returns the standard deviation of the last distribution function in cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskTheoTarget, RiskTheoXtoP, RiskTheoTargetD, RiskTheoXtoQ

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoXtoP(cellref or distribution function, targetValue) returns the cumulative probability for targetValue in the last distribution function in the formula in cellref or the entered distribution function. The cumulative probability returned is the probability of a value less than or equal to targetValue occurring.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTheoXtoP(C10,1000) returns the cumulative probability of the value 1000 using the last distribution function in cell C10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>RiskTheoTargetD and RiskTheoXtoQ are similar, but they return a cumulative descending probability. RiskTheoTarget and RiskTheoXtoP are alternate names for the same function. Similarly for RiskTheoTargetD and RiskTheoXtoQ.</td>
</tr>
</tbody>
</table>

Reference: Statistics Functions
### RiskTheoVariance

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskTheoVariance}$(cellref or distribution function) returns the variance of the last distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskTheoVariance}(A10)$ returns the variance of the last distribution function in cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskTheoXtoY

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskTheoXtoY}$(cellref or distribution function, xValue) returns the probability for xValue in the last distribution function in the formula in cellref or the entered distribution function. For a continuous distribution, the value returned is the probability density value at xValue. For a discrete distribution, the value returned is the probability value at xValue.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskTheoXtoY}(C10,1000)$ returns the probability density value of the value 1000 using the (continuous) distribution in cell C10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
## Reference: Fitting Functions

### RiskFitDistribution

| Description | **RiskFitDistribution**(data range, data type, distribution list, selector, lower bound, upper bound) fits a distribution to data in data range, optionally restricting fitted distributions to those in distribution list. Fitted data has specified data type and best fit is selected using the criterion specified by selector.  
RiskFitDistribution fits data interactively and returns samples from the best fitting distribution during a simulation. It operates the same as an @RISK distribution function for the best fit that is entered in a cell. It can be correlated, named, or include property functions, just as with standard @RISK distribution functions.  
RiskFitDistribution automatically updates the fitted distribution when fitted data changes in Excel. Using this capability, you can have fitted distributions automatically update as new data is received or data changes during a simulation. |

| Examples | **RiskFitDistribution**(BatchFit!$B$10:$B$210, 1, {"Normal","Weibull"},"AIC") fits the data located in the range BatchFit!$B$10:$B$210 and returns the best fitting Weibull or normal distribution. The best fit is selected using the AIC criterion. |

| Guidelines | data type is 1=continuous samples, 2=discrete samples, 3=counted discrete samples, 4=unnormalized XY values, 5=normalized XY values or 6=X and P values. These are the equivalent of the Data Set Type options in the Distribution Fit dialog.  
distribution list lists the names of the distributions to fit, in quotes. If multiple distribution types are desired, curly braces should be used, as in{"Normal","Weibull"}. Only distribution types available in the Distribution Fit dialog may be used.  
selector specifies the goodness of fit criterion to use in selecting the best fit. Allowable values are "AIC", "BIC","ChiSq","KS", and “AD”.  
lower bound and upper bound specify the limits for the fitted distribution. Use “INF” or “-INF” to indicate infinity. Use “Bounded” to represent the Fitting dialog option Bounded but Unknown.  
All arguments to RiskFitDistribution except for data range are optional. If omitted, defaults for optional arguments are data type 1 (continuous samples), selector="AIC", all distributions will be tried during fitting, and lower and upper bound are unsure. |
### RiskFitDescription

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskFitDescription</strong>(fit source,distribution style) returns a text description of the best fitting distribution from the fit performed by the RiskFitDistribution function in the fit source cell.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskFitDescription</strong>(B9) returns the description of the best fitting distribution for the fit performed by the RiskFitDistribution function in cell B9.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>The formula in the fit source cell must contain a RiskFitDistribution function. &lt;br&gt; <em>distribution style</em> can be TRUE (or omitted) for @RISK distribution function format such as RiskNormal(10.1,3.22). It can be FALSE for a more readable format, such as LogLogistic: gamma=-1.384 beta=104.1, alpha=2.0912.</td>
</tr>
</tbody>
</table>

### RiskFitParameter

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskFitParameter</strong>(fit source,parameter#) returns a parameter of the best fitting distribution from the fit performed by the RiskFitDistribution function in the fit source cell.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskFitParameter</strong>(B9,1) returns the first parameter of the best fitting distribution for the fit performed by the RiskFitDistribution function in cell B9.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>The formula in the fit source cell must contain a RiskFitDistribution function. &lt;br&gt; <em>parameter#</em> can be a value between 1 and the number of arguments for the best fitting distribution.</td>
</tr>
</tbody>
</table>

### RiskFitStatistic

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskFitStatistic</strong>(fit source,statistic) returns a statistic from the fit performed by the RiskFitDistribution function in the fit source cell.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskFitDescription</strong>(B9,&quot;ChiSq&quot;) returns the chi-square statistic for the best fit for the fit performed by the RiskFitDistribution function in cell B9.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>The formula in the fit source cell must contain a RiskFitDistribution function. &lt;br&gt; <em>statistic</em> can be &quot;AIC&quot;, &quot;BIC&quot;,&quot;ChiSq&quot;,&quot;KS&quot;, &quot;AD&quot;, or &quot;RMSError&quot;.</td>
</tr>
</tbody>
</table>
# ProjectFieldVal

<table>
<thead>
<tr>
<th>Description</th>
<th>ProjectFieldVal returns the value of a field directly from Microsoft Project to the related cell in Excel. This is useful to allow @RISK distributions (when a simulation is not running) to return the same value for a field as is shown in Microsoft Project. Otherwise, you might see the mean of a distribution in Excel, which might not be the same as the value in Project. ProjectFieldVal can be also used to allow a % variation around the deterministic estimate entered in the schedule in Microsoft Project. Then, even if the value in Microsoft Project is later updated or changed, the same distribution can be used to describe the uncertainty.</th>
</tr>
</thead>
</table>
| Examples | =RiskPert(53.1,59,80,RiskStatic(ProjectFieldVal))  
If this function is entered in an Excel cell associated the duration of a task, the value shown in Excel when a simulation is not running (the “static” value) will be the value entered in the matching Duration field in Microsoft Project. |
| Guidelines | ProjectFieldVal must be added to a cell associated with a task or resource field for a project that has been imported into Excel by @RISK. This cell must be a reference to a cell in the Tasks or Resources worksheet for a project.  
ProjectFieldVal is a defined name added to Excel by @RISK and takes no arguments. |
RiskProjectAddDelay

| Description | $\text{RiskProjectAddDelay}(\text{PrecedingTask}, \text{DelayLength}, \text{DelayCost})$ adds a new task to a project after $\text{PrecedingTask}$ completes. This task has the specified length $\text{DelayLength}$ and cost $\text{DelayCost}$. This function is used to add an additional task to the project being simulated only in iterations when a risk event occurs. RiskProjectAddDelay is only active during a simulation, and adds a new task only to iterations where the arguments $\text{DelayLength}$ and $\text{DelayCost}$ are positive. |
| Examples | $\text{RiskProjectAddDelay}(\text{Tasks!B10}, 10, 10000)$ adds a task following the task located in row 10 of the Tasks worksheet. The new task has a length of 10 (using the duration units of task located in row 10) and a cost of 10000. $\text{RiskProjectAddDelay}(\text{Tasks!B10}, \text{RiskTriang}(5, 10, 15), \text{RiskNormal}(10000, 1000))$ adds a task following the task located in row 10 of the Tasks worksheet. The new task has a length of the value sampled from the distribution $\text{RiskTriang}(5, 10, 15)$, (using the duration units of task located in row 10) and a cost sampled from the distribution $\text{RiskNormal}(10000, 1000)$. |
| Guidelines | $\text{Preceding task}$ must be a reference to a cell in the Tasks worksheet for a project. The row where the cell is located determines the task to be used as preceding task. $\text{DelayCost}$ is assigned to the new task and is added to the cost rollup of the project in Microsoft Project. At the start of each iteration any tasks added in the prior iteration by $\text{RiskProjectAddDelay}$ are removed and the project is reset to its original tasks. In an iteration where a task is added, the successor of $\text{Preceding task}$ is changed to the new task added by $\text{RiskProjectAddDelay}$. The successors of the new task are set to the original successors of $\text{Preceding task}$. If multiple $\text{RiskProjectAddDelay}$ functions reference the same preceding task and are active on the same iteration, the tasks they add follow the preceding task in series. That is, the second delay task will follow the first delay, and so on. This allows a block of tasks to be added when a risk event occurs. $\text{DelayLength}$ and $\text{DelayCost}$ must be nonnegative. RiskProject functions are active only during a simulation and not during single Excel recalculations when the sampling type is Monte Carlo. |
### RiskProjectAddCost

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskProjectAddCost(CostToAdd, TimeToAdd) adds a new cost to a project at the date TimeToAdd. This function is used to add an additional cost to the project being simulated in iterations when a risk event occurs. <strong>RiskProjectAddCost</strong> is only active during a simulation, and it adds a new cost only to iterations where the argument CostToAdd is positive. The cost is added to the project in the workbook where the function is located.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskProjectAddCost (10000,DATE(2015,1,1)) adds a new cost of 10000 on Jan 1 2015. RiskProjectAddCost (RiskNormal(10000,1000), RiskUniform(DATE(2013,1,1),DATE(2016,1,1),RiskIsDate(TRUE))) adds a new cost sampled from the distribution RiskNormal(10000,1000) on the date sampled from the distribution RiskUniform(DATE(2013,1,1),DATE(2016,1,1), RiskIsDate(TRUE))).</td>
</tr>
<tr>
<td>Guidelines</td>
<td>CostToAdd must be positive. At the start of each iteration, any costs added in the prior iteration by <strong>RiskProjectAddCost</strong> are removed and the project is reset. RiskProject functions are active only during a simulation and not during single Excel recalcuations when the sampling type is Monte Carlo.</td>
</tr>
</tbody>
</table>

### RiskProjectRemoveTask

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskProjectRemoveTask(tasksToRemove) removes task(s) from a project being simulated in a given iteration. This is used if you want to not execute certain tasks in the project being simulated when a risk event occurs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskProjectRemoveTask(Tasks!B10) removes the task located in row 10 of the Tasks worksheet.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>tasksToRemove must be a reference to cell(s) in the Tasks worksheet for a project. The row where the cell(s) is located determines the task to be removed. At the start of each iteration, any tasks removed in the prior iteration by <strong>RiskProjectRemoveTask</strong> are added back as the project is reset to its original tasks. RiskProject functions are active only during a simulation and not during single Excel recalcuations when the sampling type is Monte Carlo.</td>
</tr>
</tbody>
</table>
### RiskProjectResourceAdd

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskProjectResourceAdd</strong> <em>(Task, Resource, Units)</em> assigns a Resource to a Task. This function is used to change the resources that are assigned to a task in each iteration of a simulation. Costs calculated in Project will reflect the changed usage in each iteration of a simulation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>IF(RiskUniform(0,1)&gt;0.5), RiskProjectResourceAdd <em>(Tasks!B10, Resources!B7,1)</em> assigns the resource in row 7 of the Resources worksheet to the Task located in row 10 of the Tasks worksheet when the Excel IF function evaluates to TRUE in an iteration of a simulation. A units value of 1 is used, indicating that the new resource is 100% allocated to the task.</td>
</tr>
</tbody>
</table>
| Guidelines | *Task* must be a reference to cell(s) in the Tasks worksheet for a project.  
*Resource* must be a reference to cell(s) in the Resources worksheet for a project.  
*Units* must be nonnegative.  
Tasks use their default resource assignments (that is, those that exist prior to simulation) if a RiskProjectResourceAdd function is not evaluated in a given iteration.  
RiskProject functions are active only during a simulation and not during single Excel recalcuations when the sampling type is Monte Carlo. |
<table>
<thead>
<tr>
<th>Description</th>
<th>RiskProjectResourceRemove(Task, Resource) removes a Resource assigned to a Task. This function is used to change the resources that are assigned to a task in each iteration of a simulation. Costs calculated in Project will reflect the changed usage each iteration of a simulation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>IF(RiskUniform(0,1)&gt;.5), RiskProjectResourceRemove(Tasks!B10, Resources!B7) removes the resource in row 7 of the Resources worksheet that is assigned to the Task located in row 10 of the Tasks worksheet when the Excel IF function evaluates to TRUE in an iteration of a simulation.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Task must be a reference to cell(s) in the Tasks worksheet for a project. Resource must be a reference to cell(s) in the Resources worksheet for a project. Tasks use their default resource assignments (that is, those that exist prior to simulation) if a RiskProjectResourceRemove function is not evaluated in a given iteration. RiskProject functions are active only during a simulation and not during single Excel recalcuations when the sampling type is Monte Carlo.</td>
</tr>
</tbody>
</table>
### RiskProjectResourceUse

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskProjectResourceUse (Task, Resource, UsageValue) applies UsageValue to a Resource assigned to a Task. This function is used to change the amount of units of a material resource (or amount of work of a work resource) that is assigned to a task in each iteration of a simulation. Costs calculated in Project will reflect the changed usage in each iteration of a simulation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskProjectResourceUse (Tasks!B10, Resources!B7, RiskUniform(10,50)) sets the usage for the resource in row 7 of the Resources worksheet that is assigned to the task located in row 10 of the Tasks worksheet. The usage value is sampled from the RiskUniform(10,50) distribution.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Task must be a reference to cell(s) in the Tasks worksheet for a project. Resource must be a reference to cell(s) in the Resources worksheet for a project. UsageValue must be nonnegative. UsageValue is applied to the resource units assigned to Task if Resource is a material resource or resource work assigned to Task if Resource is a work resource. If there is no existing assignment in the project of Resource to Task, @RISK will add one prior to the simulation, apply the units samples during the run, and clear the assignment at the end of the run. RiskProject functions are active only during a simulation and not during single Excel recalculation when the sampling type is Monte Carlo.</td>
</tr>
</tbody>
</table>
There are three groups of @RISK Time Series functions: **ARMA** (autoregressive moving average) processes, **GBM** (geometric Brownian motion) and its variations, and the **ARCH** (autoregressive conditional heteroskedasticity) process and its variations.

The ARMA processes are arguably the best known. They were developed by Box and Jenkins several decades ago, and they have been applied in a wide variety of settings. They have also been implemented in many statistical software packages. The theory of ARMA processes is based on stationarity, which means that the distribution of the time series variable is constant through time. In particular, its mean and variance are constant through time. When stationarity does not hold, it is common to transform the series, usually through logarithms, differencing, and/or deseasonalizing, to induce stationarity. Then an ARMA process is applied to the transformed process.

In general, ARMA processes are characterized by two integer values, p and q, where p is the number of autoregressive terms and q is the number of moving average terms. The only versions implemented in @RISK are the most common versions, where p+q is less than or equal to 2. These include AR(1), AR(2), MA(1), MA(2), and ARMA(1,1).

The GBM process and its variations are continuous-time processes. They have been used most extensively in financial applications, such as for pricing options. In these applications, the time series variable is sometimes the price of a security, and it is sometimes the change in price (the return). Unlike ARMA processes, there is often no assumption of stationarity. For example, if the time series variable is a security price, there might well be an upward drift, meaning that the price tends to increase through time. However, GBM processes have the Markov (memoryless) property, meaning that if the current value is known, the past is irrelevant for predicting the future.

The discretized versions of GBM implemented in @RISK include the basic GBM process and GBM with jump diffusion (GBMJD). Non-geometric Brownian motion processes include BM with mean reversion (BMMR), and GBM with mean reversion and jump diffusion (BMMRJD).

The ARCH process and its variations were developed more recently to account for changes in volatility observed in financial variables, and they have been applied primarily in financial models. They are based on autoregressive (AR) processes with a constant mean, but the
volatility is modeled separately to allow for nonstationary variance. (The term “heteroskedasticity” means nonconstant variance.)

Like the ARMA processes, the ARCH process is characterized by an integer value $q$, and its variations are characterized by two integer values $p$ and $q$. Here, $p$ is again the number of autoregressive terms, and $q$ is the number of terms involving “error” terms (deviations from the mean). The only versions implemented in @RISK have $p$ and $q$ equal to 1: ARCH(1), GARCH(1,1), EGARCH(1,1), and APARCH(1,1).

Note that the parametrization of these processes varies from one time series reference to another. The parametrization used here is fairly standard, but you might need to “translate” symbols from your favorite time series reference.
### RiskAR1

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskAR1(μ, σ, a₁, Y₀) generates a first-order autoregressive (AR1) process with mean μ, volatility parameter σ, autoregressive coefficient a₁, and value Y₀ at time 0. An AR1 process is a common model for a time series because it is simple and often provides a good fit. It is characterized by an autocorrelation function (ACF) that decreases geometrically and a partial autocorrelation function (PACF) that cuts off to 0 after lag 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskAR1(100, 40, 0.8, 490) generates an AR1 process with mean 100, variance $40^2 / (1 - 0.8^2) = 66.7^2$, autoregressive coefficient 0.8, and value 490 at time 0. RiskAR1(C10, C11, C12, C13) generates an AR1 process with parameters taken from cells C10 to C13.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>$</td>
</tr>
<tr>
<td>Technical Details</td>
<td>Define $N_i$ = a sample from a Normal(0,1) distribution $\varepsilon_i = \sigma N_i$ Then $(Y_i - \mu) = a₁(Y_{i-1} - \mu) + \varepsilon_i$ The mean and variance are $E(Y_i) = \mu$ and $Var(Y_i) = \sigma^2 / (1 - a₁^2)$</td>
</tr>
</tbody>
</table>
## RiskAR2

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskAR2(μ, σ, a₁, a₂, Y₀, Y₋₁) generates a second-order autoregressive (AR2) process with mean μ, volatility parameter σ, autoregressive coefficients a₁ and a₂, and values Y₀ and Y₋₁ at times 0 and -1. An AR2 process is characterized by an autocorrelation function (ACF) that decreases geometrically or according to damped sine waves, and a partial autocorrelation function (PACF) that cuts off to 0 after lag 2.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskAR2(100, 40, 0.6, 0.2, 490, 495) generates an AR2 process with mean 100, variance $40^2 / (1 - 0.6^2 - 0.2^2) = 51.6^2$, autoregressive coefficients 0.6 and 0.2, and values 490 and 495 at times 0 and -1. RiskAR1(C10, C11, C12, C13, C14) generates an AR2 process with parameters taken from cells C10 to C14.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>$a₁ + a₂ &lt; 1, a₂ - a₁ &lt; 1, and -1 &lt; a₂ &lt; 1$ are necessary conditions for stationarity.</td>
</tr>
<tr>
<td>Technical Details</td>
<td>Define $N_t$ = a sample from a Normal(0,1) distribution $\varepsilon_t = \sigma N_t$ Then $(Y_t - \mu) = a_1(Y_{t-1} - \mu) + a_2(Y_{t-2} - \mu) + \varepsilon_t$ The mean and variance are $E(Y_t) = \mu$ and $Var(Y_t) = \sigma^2 / (1 - a₁^2 - a₂^2)$</td>
</tr>
</tbody>
</table>
RiskMA1

| Description | RiskMA1(μ, σ, b₁, ε₀) generates a first-order moving average (MA1) process with mean μ, volatility parameter σ, moving average coefficient b₁, and initial error term ε₀. An MA1 process is a common model for a time series because it is simple and often provides a good fit. It is characterized by an autocorrelation function (ACF) that cuts off to 0 after lag 1 and a partial autocorrelation function (PACF) that decreases geometrically. |
| Examples | RiskMA1(500, 40, 0.5, 10) generates an MA1 process with mean 500, variance 40²(1 + 0.5²) = 44.7², moving average coefficient 0.5, and initial error term 10. RiskMA1(C10, C11, C12, C13) generates an MA1 process with parameters taken from cells C10 to C13. |
| Technical Details | Define 
Nₜ = a sample from a Normal(0,1) distribution 
εₜ = σNₜ 
Then 
Yₜ = μ + b₁εₜ₋₁ + εₜ 
The mean and variance are 
E(Yₜ) = μ 
and 
Var(Yₜ) = σ²(1 + b₁²) |
**RiskMA2**

| Description | RiskMA2(μ, σ, b₁, b₂, ε₀, ε₋₁) generates a second-order moving average (MA2) process with mean μ, volatility parameter σ, moving average coefficients b₁ and b₂, and initial error terms ε₀ and ε₋₁. An MA2 process is characterized by an autocorrelation function (ACF) that cuts off to 0 after lag 2, and a partial autocorrelation function (PACF) that decreases geometrically or according to damped sine waves. |
| Examples | RiskMA2(500, 40, 0.4, -0.2, 10, -5) generates an MA2 process with mean 500, variance $40^2 (1 + 0.4^2 + (-0.2)^2) = 43.8^2$, moving average coefficients 0.4 and -0.2, and initial error terms 10 and -5. RiskMA2(C10, C11, C12, C13, C14, C15) generates an MA2 process with parameters taken from cells C10 to C15. |
| Technical Details | Define $N_t$: a sample from a Normal(0,1) distribution $\varepsilon_t = \sigma N_t$ Then $Y_t = \mu + b_1 \varepsilon_{t-1} + b_2 \varepsilon_{t-2} + \varepsilon_t$ The mean and variance are $E(Y_t) = \mu$ and $Var(Y_t) = \sigma^2 (1 + b_1^2 + b_2^2)$ |
**RiskARMA11**

| Description | **RiskARMA11** \( (\mu, \sigma, a_1, b_1, Y_0, \varepsilon_0) \) generates a first-order autoregressive moving average (ARMA11) process with mean \( \mu \), volatility parameter \( \sigma \), autoregressive coefficient \( a_1 \), moving average coefficient \( b_1 \), value \( Y_0 \) at time 0, and initial error term \( \varepsilon_0 \).

An ARMA11 is characterized by an ACF that decreases geometrically and a PACF that is similar to the PACF of an MA1 process. |
| --- | --- |

| Examples | **RiskARMA11(100, 40, 0.8, -0.2, 490, 10)** generates an ARMA11 process with mean 100, variance 
\[ 40^2 (1 + (-0.2)^2 + 2(0.8)(-0.2)) / (1 - 0.8^2) = 56.6^2, \]
autoregressive coefficient 0.8, moving average coefficient -0.2, value 490 at time 0, and initial error term 10. **RiskARMA11(C10, C11, C12, C13, C14, C15)** generates an ARMA11 process with parameters taken from cells C10 to C15. |
| --- | --- |

| Guidelines | \( |a_1| < 1 \) is a necessary condition for stationarity. |
| --- | --- |

| Technical Details | Define 
\[ N_t = \text{a sample from a Normal}(0,1) \] distribution 
\[ \varepsilon_t = \sigma N_t \]

Then 
\[ (Y_t - \mu) = a_1(Y_{t-1} - \mu) + b_1 \varepsilon_{t-1} + \varepsilon_t \]

The mean and variance are 
\[ E(Y_t) = \mu \]
and 
\[ Var(Y_t) = \sigma^2 (1 + b_1^2 + 2a_1b_1) / (1 - a_1^2) \] |
### RiskGBM

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskGBM</strong>((\mu, \sigma, \text{transform}, \text{integrate})) generates a geometric Brownian motion (GBM) process with location parameter (\mu), volatility parameter (\sigma) at time 0. A geometric Brownian motion is a continuous-time stochastic process in which the logarithm of the series follows a Brownian motion, also called a Wiener process. In a financial context, the series is typically the price of a security, which is lognormally distributed. In this case, the “log return” of the series, essentially the change in the price, is normally distributed. The RiskGBM function requires an exponential transformation and first order integrations. This is done by using the RiskTSTransform and RiskTSIntegrate property functions.</th>
</tr>
</thead>
</table>
| Examples | **RiskGBM**(0.01, 0.05, **RiskTSTransform**(1,0), **RiskTSIntegrate**(1,1)) generates a GBM process with drift 1% and volatility 5%.  
**RiskGBM**(C10, C11, **RiskTSTransform**(1,0), **RiskTSIntegrate**(1,1)) generates a GBM process with parameters taken from cells C10 and C11. |
| Technical Details | Define \(N_t\) = a sample from a Normal(0,1) distribution. Then for any \(t \geq 0, \ T > 0\),  
\[ Y_{t+T} = Y_t \exp\left(\left(\mu - \sigma^2 / 2\right)T + N_{t+T}\sigma\sqrt{T}\right) \]  
The discrete equivalent of this is  
\[ Y_i = Y_{i-1} \exp\left(\left(\mu - \sigma^2 / 2\right) + N_i\sigma\right) \]  
The conditional mean and variance of \(Y_{t+T}\) given \(Y_t\), are  
\[ E(Y_{t+T}) = Y_t \exp(\mu T) \]  
and  
\[ Var(Y_{t+T}) = Y_t^2 \exp(2\mu T)\left[\exp(\sigma^2 T) - 1\right] \]  
If this is in a financial context and \(Y_t\) is the price of a security at time \(t\), then the term inside the square brackets in the equation for \(Y_{t+T}\), the “log return” of the security, is normally distributed with mean \((\mu - \sigma^2 / 2)T\) and variance \(\sigma^2 T\). |
### Description

RiskBMMR(μ, σ, α, Y₀) generates a Brownian motion with mean reversion process with long-term mean parameter μ, volatility parameter σ, speed of reversion parameter α, and value Y₀ at time 0.

Unlike standard Brownian Motion, a mean reverting model will tend toward a long-run equilibrium mean. When the series is above this level, it will tend to decrease, and vice versa. The parameter α governs the speed of this reversion, with larger levels implying a quicker reversion.

This process was originally proposed by Vasicek in 1977 as a model for interest rates. It is typically not a good model for stock prices because there is typically no reason to believe that stock prices revert to some long-term mean. However, interest rates cannot rise indefinitely because of economic forces; they tend to revert back to some long-term mean value.

### Examples

RiskBMMR(0.01, 0.05, 0.2, 0.015) generates a mean reverting Brownian motion process with long-term mean 1%, volatility 5%, speed of reversion rate 0.2, and value 1.5% at time 0.

RiskBMMR(C10, C11, C12, C13) generates a mean reverting Brownian motion process with parameters taken from cells C10 to C13.

### Technical Details

Define \( N_t = \) sample from a Normal(0,1) distribution

Then for any \( t \geq 0, T > 0 \),

\[
Y_{t+T} = \left[ \mu + e^{-\alpha T} (Y_t - \mu) \right] + N_{t+T} \sigma \sqrt{\frac{1-e^{-2\alpha T}}{2\alpha}}
\]

\( r_t = \left( \mu - \frac{\sigma^2}{2} \right) t + N_t \sigma \sqrt{t} \)

The discrete equivalent of this is

\[
Y_t = \left[ \mu + e^{-\alpha} (Y_{t-1} - \mu) \right] + N_t \sigma \sqrt{\frac{1-e^{-2\alpha}}{2\alpha}}
\]

The conditional mean and variance of \( Y_{t+T} \) given \( Y_t \), are

\[
E(Y_{t+T}) = \mu + e^{-\alpha T} (Y_t - \mu) \rightarrow \mu \quad \text{as} \quad T \rightarrow \infty \quad \text{and}
\]

\[
Var(Y_{t+T}) = \sigma^2 \frac{1-e^{-2\alpha T}}{2\alpha} \rightarrow \sigma^2 / 2\alpha \quad \text{as} \quad T \rightarrow \infty
\]
## RiskGBMJD

| Description | RiskGBMJD \((\mu, \sigma, \lambda, \mu_j, \sigma_j, \text{transform, integrate})\) generates a geometric Brownian motion with jump diffusion process with drift parameter \(\mu\), volatility parameter \(\sigma\), jump rate \(\lambda\), and normal parameters of jump size \(\mu_j\) and \(\sigma_j\).

This process is typically used in a financial context to model a return, such as the change in a stock price, when random shocks occur. Specifically, it is assumed that shocks occur according to a Poisson process with rate \(\lambda\), and that at such times, there is a jump in the process that is normally distributed with parameters \(\mu_j\) and \(\sigma_j\).

| Examples | RiskGBMJD(0.01, 0.05, 0.1, 0.015, 0.025, RiskTSTransform(1,0), RiskTSIntegrate(1,1)) generates a jump diffusion GBM process with drift 1%, volatility 5%, jump rate 0.1, jump mean 1.5%, and jump standard deviation 2.5%.

RiskGBMJD(C10, C11, C12, C13, C14, RiskTSTransform(1,0), RiskTSIntegrate(1,1)) specifies a jump diffusion geometric Brownian motion function with parameters taken from cells C10 to C14.

| Technical Details | Define 
\(N_t = \text{sample from a Normal}(0,1)\) distribution 
\(K_t = \text{sample from a Poisson}(\lambda t)\) distribution 

Then for any \(t \geq 0\),
\[
Y_t = (\mu - \frac{\sigma^2}{2})t + K_t\mu_j + N_t\sqrt{\sigma^2 t + \sigma_j^2}K_t
\]
\[
r_t = (\mu - \frac{\sigma^2}{2})t + N_t\sigma \sqrt{t}
\]

Again, this is typically a model for the return of a security. The price \(P_t\) is then found from \(\ln(P_t) - \ln(P_0) = Y_t\), or equivalently,
\[
P_t = P_0 \exp(Y_t).
\]
# RiskBMMRJD

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskBMMRJD($\mu, \sigma, \alpha, \lambda, \mu_j, \sigma_j, Y_0$) generates a Brownian motion process with mean reversion and jump diffusion. It combines the RiskBMMR and jump diffusion.</th>
</tr>
</thead>
</table>
| Examples    | RiskBMMRJD(0.01, 0.05, 0.2, 0.1, 0.015, 0.025, 0.015) generates a Brownian motion process with mean reversion and jump diffusion with drift 1%, volatility 5%, speed of reversion 0.2, jump rate 0.1, jump size mean 1.5%, jump size standard deviation 2.5%, and value 1.5% at time 0.  
  RiskBMMRJD(C10, C11, C12, C13, C14, C15, C16) generates a Brownian motion process with mean reversion and jump diffusion with parameters taken from cells C10 to C16. |
| Technical Details | There is no simple form for this process. |
## RiskARCH1

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskARCH1(μ, ω, b₁, Y₀) generates a first-order autoregressive conditional heteroskedasticity (ARCH1) process with mean μ, volatility parameter ω, error coefficient b₁, and value Y₀ at time 0. ARCH processes are used when there is reason to believe that the variance of the process varies through time.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskARCH1(50, 10, 0.5, 49) generates an ARCH1 process with mean 50, volatility parameter 10, error coefficient 0.5, and value 49 at time 0. RiskARCH1(C10, C11, C12, C13) specifies an ARCH1 process with parameters taken from cells C10 to C13.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>a₁ &gt; 0</td>
</tr>
<tr>
<td>Technical Details</td>
<td>Define ( N_t ) = a sample from a Normal(0,1) distribution Then ( Y_t = \mu + \sigma_t N_t ) where ( \sigma_t ) is modeled as ( \sigma_t^2 = \omega + b_1 (Y_{t-1} - \mu)^2 ) The idea is that ( Y_t ) is normally distributed with mean ( \mu ) and variance ( \sigma_t^2 ), but this variance, conditional on the previous value of the process, is a weighted combination of the volatility parameter ( \omega ) and the previous squared deviation from the mean.</td>
</tr>
</tbody>
</table>
### RiskGARCH11

| Description | \( \text{RiskGARCH11}(\mu, \omega, b_1, a_1, Y_0, \sigma_0) \) generates a Generalized ARCH process with mean \( \mu \), volatility parameter \( \omega \), error coefficient \( b_1 \), autoregressive coefficient \( a_1 \), value \( Y_0 \) at time 0, and initial standard deviation \( \sigma_0 \). This is a generalization of the original ARCH model, where the model for the conditional variance at time \( t \) is a weighted combination of three terms: the volatility parameter \( \omega \), the previous squared deviation from the mean, and the previous variance. This process has been more successful than the ARCH1 process for fitting financial data. |
|---|
| Examples | \( \text{RiskGARCH11}(50, 10, 0.25, 0.35, 49, 2) \) generates a GARCH11 process with mean 50, volatility parameter 10, error coefficient 0.25, autoregressive coefficient 0.35, value 49 at time 0, and initial standard deviation 2. \( \text{RiskGARCH11}(C10, C11, C12, C13, C14, C15) \) generates a GARCH11 process with parameters taken from cells C10 to C15. |
| Guidelines | \( a_1 \geq 0, b_1 \geq 0 \), at least one of \( a_1 \) or \( b_1 \) positive, \( \omega > 0 \) |
| Technical Details | Define \( N_t \) = a sample from a Normal(0,1) distribution  
Then  
\( Y_t = \mu + \sigma_t N_t \)  
where \( \sigma_t \) is modeled as  
\( \sigma_t^2 = \omega + b_1 (Y_{t-1} - \mu)^2 + a_1 \sigma_{t-1}^2 \) |
RiskEGARCH11

| **Description** | **RiskEGARCH11** (μ, ω, θ, γ, b₁, a₁, Y₀, σ₀) generates an Exponential GARCH process with mean μ, volatility parameter ω, parameters θ and γ, error coefficient b₁, autoregressive parameter a₁, value Y₀ at time 0, and initial standard deviation σ₀. This version of GARCH allows negative values (of the logs) in the variance equation, and there are now no restrictions on the parameters a₁ and b₁. |
| **Examples** | **RiskEGARCH11(50, 10, 0.1, 0.25, 0.35, 49, 2)** generates an EGARCH11 process with mean 50, volatility parameter 10, theta parameter 0.1, gamma parameter 0.25, autoregressive coefficient 0.35, value 49 at time 0, and initial standard deviation 2. **RiskEGARCH11(C10, C11, C12, C13, C14, C15, C16, C17)** generates an EGARCH11 process with parameters taken from cells C10 to C17. |
| **Technical Details** | Define \( N_t, N_\tilde{t} \) = a sample from a Normal(0,1) distribution. Then \( Y_t = \mu + \sigma_t N_t \) where \( \sigma_t \) is modeled as \( \ln(\sigma_t^2) = \omega + b_1 g(N_{t-1}) + a_1 \ln(\sigma_{t-1}^2) \) with \( g(N_t) = \theta N_t + \gamma \left( |N_t| - E(|N_t|) \right) \) Note that \( E(|N_t|) = \sqrt{2/\pi} \). |
## RiskAPARCH11

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskAPARCH11(µ, ω, δ, γ, b₁, a₁, Y₀, σ₀) generates an Asymmetric Power GARCH process with mean µ, volatility parameter ω, parameter δ and γ, error coefficient b₁, autoregressive parameter a₁, value Y₀ at time 0, and initial standard deviation σ₀. In this variation of the basic GARCH model, the parameter δ plays the role of a Box-Cox transformation for the conditional variance, and the parameter γ provides a so-called leverage effect. This process has proved very promising for fitting financial data because of its generality, and it includes both ARCH1 and GARCH11 as special cases.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskAPARCH11(50, 10, 0.75, 0.2, 0.25, 0.35, 49, 2) generates an APARCH11 process with mean 50, volatility parameter 10, power 0.75, leverage parameter 0.2, error coefficient 0.25, autoregressive coefficient 0.35, initial value 49, and initial standard deviation 2. RiskAPARCH11(C10, C11, C12, C13, C14, C15, C16, C17) generates an APARCH11 process with parameters taken from cells C10 to C17.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₁ ≥ 0, b₁ ≥ 0, at least one of a₁ or b₁ positive, ω &gt; 0, −1 &lt; γ &lt; 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technical Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Define Nₜ, Nₜ = a sample from a Normal(0,1) distribution. Then Yₜ = µ + σₜNₜ where σₜ is modeled as σₜ⁻δ = ω + b₁[Yₜ₋₁ − µ</td>
</tr>
</tbody>
</table>

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Chapter 7: @RISK Function Reference 707
## Reference: Time Series Property Functions

### RiskTSTransform

| Description | RiskTSTransform(transform function type, transform shift) specifies that a time series function will have the specified transformation applied to the process result. 
The *transform function type* is the type of transformation function to apply, and *transform shift* is the data shift to apply. |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskGBM(0.2, 0.05, RiskTSTransform(1, 5)) will exponentiate the process result, and subtract 5 from each item in the transformed process.</td>
</tr>
</tbody>
</table>
| Guidelines | The RiskTSTransform property function is automatically added to fitted time series functions if the original data was transformed using a Logarithmic or Square Root transformation. 
The *transform function type* is specified as a value between 0 and 2, where. 0=None, 1=Exponentiate, and 2=Square. 
The *transform shift* is subtracted from the process result after the transformation function is applied. |

### RiskTSIntegrate

| Description | RiskTSIntegrate(integration order, C1, C2) specifies that a time series function will have the specified integration applied to the process result. 
The *integration order* is the number of integrations to apply. *C1* is the first starting constant and *C2* is the second starting constant. |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskGBM(0.2, 0.05, RiskTSIntegrate(2,10,15)) will apply a second-order integration to the time series process using 10 as the first constant and 15 for second constant.</td>
</tr>
</tbody>
</table>
| Guidelines | The RiskTSIntegrate property function is automatically added to fitted time series functions if the original data was transformed using Detrending. 
The *integration order* is specified as a value between 0 and 2, where. 0=None, 1=First order, and 2=Second order. 
The *C1* parameter is the first constant to be added, and *C2* is the second constant to be added. |
### RiskTSSeasonality

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskTSSeasonality</strong>(seasonality type, seasonal period, {seasonal terms}, starting index) specifies that a time series function will have the specified seasonality applied to the process result. The <em>seasonality type</em> specifies the seasonality method, <em>seasonal period</em> is the seasonal period size, {seasonal terms} is the seasonality definition string, and <em>starting index</em> is the index within the seasonal period where the seasonalization begins.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskGBM(0.2, 0.05, RiskTSSeasonality(3,4,{0.1, 0.3, 0.5, 0.2}, 1)) will apply additive seasonality to the time series process, using 4 periods, beginning with period 1.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>The RiskTSSeasonality property function is automatically added to fitted time series functions if the original data was transformed using Deseasonalization. The <em>seasonality type</em> is specified as a value between 0 and 3, where 0=None, 1=First order seasonal differencing, 2=Second order seasonal differencing, and 3=Additive seasonality. The <em>seasonal period</em> is the number of seasonal periods. For example, it is 4 for quarterly data, 12 for monthly data, or 24 for hourly data. The {seasonal terms}, is the seasonality definition. For seasonal integration, this provides the integration constants, and for additive seasonality it provides the additive terms. The <em>starting index</em> is the starting index in the seasonal period for additive seasonality. For example, the time series for a process with monthly seasonal data would have a <em>starting index</em> of 5 if the process began in May instead of January.</td>
</tr>
</tbody>
</table>
### RiskTSSync

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTSSync(synchronization type, data cell range) specifies that a time series function will be synchronized to the referenced data cell range, and that startup values will be provided to the process. The synchronization type specifies whether to synchronize to the first value or the last value of the referenced data cell range.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskAR1(0.2, 0.05, 0.25, 0.1, RiskTSSync(2, B1:B72)) synchronizes the time series function to the last value of the data found in cells B1 to B72.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>The RiskTSSync property function is automatically added to fitted time series functions. For most of the time series functions, there are parameters that “start up” the process. These include Y0, Y-1, e0, e-1, and Sigma0. These parameters provide initial states for autoregression and averaging. Start up values are also required when using trending in transformations, or when using seasonal integration. Note that RiskTSSync will only synchronize the “start up” parameters, but it will not provide a new fit. The synchronization type is specified as a value between 0 and 2, where 0=None, 1= First value of dataset, and 2= Last value of dataset. The data cell range is range of the data that is to be synchronized to the time series process. When fitting, this is the original fitting data range.</td>
</tr>
</tbody>
</table>
Reference: Six Sigma Functions

Six Sigma functions return desired Six Sigma statistics on simulation results for a specified cell, usually (but not necessarily) and output cell. These functions are updated in real time as a simulation runs. (If they are entered in custom report template sheets, they are updated only when a simulation is completed.)

All of the statistics functions in this section rely on a RiskSixSigma property function to supply values such as LSL and USL. This property function must be included in either the RiskOutput function for an output cell or in the Six Sigma statistics function itself (or both).

For example, suppose cell A10 is an output cell with the formula

=RiskOutput(“Width”,,RiskSixSigma(90,96,93,1.5,6))+A6-A5

Then you could use the following statistics function in any cell to return the Cpk statistic for the the simulation results on width:

=RiskCpk(A10)

This function doesn’t require a RiskSixSigma property function because it refers to an output cell with a RiskSixSigma property function in its RiskOutput function. However, suppose the formula in cell A10 is simply

=RiskOutput(“Width”)+A6-A5

Then you must supply the RiskCpk statistics function with a RiskSixSigma property function, as in

=RiskCpk(A10,RiskSixSigma(90,96,93,1.5,6))

Actually, the only required arguments for RiskSixSigma are the values needed to calculate Cpk: LSL, USL, and Target.

It is possible to include the RiskSixSigma property function in both the RiskOutput function and the statistics function. In this case, the parameters in the latter override the parameters in the former.

If a cell reference is entered as the first argument of a Six Sigma statistics function, the cell does not have to be a simulation output identified with a RiskOutput function. In this case, the statistics function will require a RiskSixSigma property function.
## RiskCp

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskCp}(\text{cellref or output name, Sim#}, \text{RiskSixSigma}(\text{LSL, USL, Target, LongTerm Shift, Number of Standard Deviations})) ) calculates the Cp process capability index for cellref or output name in Sim#.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskCp}(A10) ) returns the Cp index for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list LSL and USL, the values required to calculate Cp. ( \text{RiskCp}(A10, ,\text{RiskSixSigma}(100,120,110,1.5,6)) ) returns the Cp index for the output cell A10, using LSL 100 and USL 120. The last three arguments, 110, 1.5, and 6, are ignored for this calculation.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

## RiskCpm

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskCpm}(\text{cellref or output name, Sim#}, \text{RiskSixSigma}(\text{LSL, USL, Target, LongTerm Shift, Number of Standard Deviations})) ) returns the Taguchi capability index for cellref or output name in Sim#. This function is essentially the same as the RiskCpk function, but it permits a target value outside the specification limits.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskCpm}(A10) ) returns the Taguchi capability index for cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list LSL, USL, and the target, the values required to calculate Cpm. ( \text{RiskCpm}(A10, ,\text{RiskSixSigma}(100, 120, 110, 0, 6)) ) returns the Taguchi capability index for cell A10, using USL 120, LSL 100, and target 110. The last two arguments, 0 and 6, are ignored for this calculation.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskCpk

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCpk(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the Cpk process capability Index for cellref or output name in Sim#. This function is similar to the RiskCp function, but it adjusts the Cpk for a mean that isn’t necessarily halfway between LSL and USL. The formula for Cpk is either (USL-Mean) / (3 x sigma) or (Mean-LSL) / (3 x sigma), whichever is the smaller.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskCpk(A10) returns the Cpk index for output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list LSL and USL, the values required to calculate Cpk. RiskCpk(A10, ,RiskSixSigma(100,120,110,1.5,6)) returns the Process Capability Index for the output cell A10, using LSL 100 and USL 120. The last three arguments, 110, 1.5, and 6, are ignored for this calculation.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskCpkLower

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCpkLower(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the one-sided Cpk capability index based on the lower specification limit for cellref or output name in Sim#.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskCpkLower(A10) returns the one-sided Cpk capability index based on the lower specification limit for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list LSL, the value required to calculate the one-sided Cpk. RiskCpkLower(A10,,RiskSixSigma(100,120,110,1.5,6)) returns the one-sided Cpk capability index for the output cell A10, using LSL 100. The last four arguments, 120, 110, 1.5, and 6, are ignored for this calculation.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskCpkUpper

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCpkUpper(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the one-sided Cpk capability index based on the Upper Specification limit for cellref or output name in Sim#.</th>
</tr>
</thead>
</table>
| Examples    | RiskCpkUpper(A10) returns the one-sided Cpk capability index based on the upper specification limit for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list USL, the value required to calculate the one-sided Cpk.  
RiskCpkUpper(A10, ,RiskSixSigma(100, 120, 110, 1.5, 6)) returns the one-sided Cpk capability index for the output cell A10, using an USL of 100. All arguments except the second, 120, are ignored for this calculation. |
| Guidelines  | None. |

### RiskDPM

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskDPM(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the defective parts per million for cellref or output name in Sim#.</th>
</tr>
</thead>
</table>
| Examples    | RiskDPM(A10) returns the defective parts per million for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list LSL and USL, the values required to calculate DPM.  
RiskDPM(A10, ,RiskSixSigma(100, 120, 110, 1.5, 6)) returns the defective parts per million for the output cell A10, using LSL 100 and USL 120. The last three arguments, 110, 1.5, and 6, are ignored for this calculation. |
| Guidelines  | None. |
**RiskK**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskK(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, Long Term Shift, Number of Standard Deviations)) calculates a measure of process center for cellref or output name in Sim#.</th>
</tr>
</thead>
</table>
| Examples    | RiskK(A10) returns a measure of process center for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list LSL and USL, the values required to calculate the K statistic.  
RiskK(A10, ,RiskSixSigma(100,120,110,1.5,6)) returns a measure of process center for the output cell A10, using LSL 100 and USL 120. The last three arguments, 110, 1.5, and 6, are ignored for this calculation. |
| Guidelines  | None. |

**RiskLowerXBound**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskLowerXBound(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, Long Term Shift, Number of Standard Deviations)) returns the lower X-value for a specified number of standard deviations from the mean for cellref or output name in Sim#.</th>
</tr>
</thead>
</table>
| Examples    | RiskLowerXBound(A10) returns the lower X-value for a specified number of standard deviations from the mean for cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list Number of Standard Deviations, the value required to calculate the lower X bound.  
RiskLowerXBound(A10,, RiskSixSigma(100, 120, 110, 1.5, 6)) returns the lower X-value for cell A10, using 6 standard deviations from the mean. The first four arguments, 100, 120,110, and 1.5, are ignored for this calculation. |
| Guidelines  | None. |
## RiskPNC

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskPNC</strong>(<em>cellref or output name</em>, <em>Sim</em>, <em>RiskSixSigma(LSL, USL, Target, Long Term Shift, Number of Standard Deviations)</em>) calculates the total probability of defect outside the lower and upper specification limits for <em>cellref or output name</em> in <em>Sim</em>.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskPNC(A10)</strong> returns the probability of defect outside the lower and upper specification limits for the output cell A10. In this case, a <em>RiskSixSigma</em> property function is required in the <em>RiskOutput</em> function in cell A10, and it must list LSL, USL, and Long Term Shift, the values required to calculate the PNC statistic. <strong>RiskPNC(A10, ,RiskSixSigma(100,120,110,1.5,6))</strong> returns the probability of defect outside the lower and upper specification limits for the output cell A10, using LSL 100, USL 120, LongTerm Shift 1.5. The other two arguments, 110 and 6, are ignored for this calculation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>None.</td>
</tr>
</tbody>
</table>

## RiskPNCLower

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskPNCLower</strong>(<em>cellref or output name</em>, <em>Sim</em>, <em>RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)</em>) calculates the probability of defect outside the lower specification limit for <em>cellref or output name</em> in <em>Sim</em>.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskPNCLower (A10)</strong> returns the probability of defect outside the lower specification limit for the output cell A10. In this case, a <em>RiskSixSigma</em> property function is required in the <em>RiskOutput</em> function in cell A10, and it must list LSL and Long Term Shift, the values required to calculate the lower PNC statistic. <strong>RiskPNCLower(A10, ,RiskSixSigma(100,120,110,1.5,6))</strong> returns the probability of defect outside the lower specification limit for the output cell A10, using LSL 100 and LongTerm Shift 1.5. The other three arguments, 120, 110, and 6, are ignored for this calculation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskPNCUpper

**Description**

RiskPNCUpper(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the probability of defect outside the upper specification limit for cellref or output name in Sim#.

**Examples**

RiskPNCUpper(A10) returns the probability of defect outside the upper specification limit for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list USL and Long Term Shift, the values required to calculate the upper PNC statistic.

RiskPNCUpper(A10, ,RiskSixSigma(100,120,110,1.5,6)) returns the probability of defect outside the upper specification limits for the output cell A10, using USL 120 and LongTerm Shift 1.5. The other three arguments, 100, 110, and 6, are ignored for this calculation.

**Guidelines**

None.

### RiskPPMLower

**Description**

RiskPPMLower(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the number of defects below the lower specification limit for cellref or output name in Sim#.

**Examples**

RiskPPMLower(A10) returns the number of defects below the lower specification limit for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list LSL and Long Term Shift, the values required to calculate the lower PPM statistic.

RiskPPMLower(A10, ,RiskSixSigma(100,120,110,1.5,6)) returns the number of defects below the lower specification limit for the output cell A10, using LSL 100 and LongTerm Shift 1.5. The other three arguments, 120, 110, and 6, are ignored for this calculation.

**Guidelines**

None.
## RiskPPMUpper

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskPPMUpper(\textit{cellref or output name}, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the number of defects above the upper specification limit for \textit{cellref or output name} in Sim#.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskPPMUpper(A10) returns the number of defects above the upper specification limit for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list USL and Long Term Shift, the values required to calculate the upper PPM statistic.</td>
</tr>
<tr>
<td>RiskPPMUpper(A10, ,RiskSixSigma(100, 120, 110, 1.5, 6)) returns the number of defects above the upper specification limit for the output cell A10, using USL 120 and LongTerm Shift 1.5. The other three arguments, 100, 110, and 6, are ignored for this calculation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>None.</td>
</tr>
</tbody>
</table>

## RiskSigmaLevel

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskSigmaLevel(\textit{cellref or output name}, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the process Sigma level for \textit{cellref or output name} in Sim#. (Note: This function assumes that the output is normally distributed and centered within the specification limits.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskSigmaLevel(A10) returns the process Sigma level for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list LSL, USL, and Long Term Shift, the values required to calculate the Sigma statistic.</td>
</tr>
<tr>
<td>RiskSigmaLevel(A10, ,RiskSixSigma(100, 120, 110, 1.5, 6)) returns the process Sigma level for the output cell A10, using LSL 100, USL 120, and Long Term Shift 1.5. The other two arguments, 110 and 6, are ignored for this calculation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskUpperXBound

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskUpperXBound(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, Long Term Shift, Number of Standard Deviations)) returns the upper X-value for a specified number of standard deviations from the mean for cellref or output name in Sim#.</th>
</tr>
</thead>
</table>
| Examples    | RiskUpperXBound(A10) returns the upper X-value for a specified number of standard deviations from the mean for cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list Number of Standard Deviations, the value required to calculate the upper X bound.  
RiskUpperXBound(A10, ,RiskSixSigma(100, 120, 110, 1.5, 6)) returns the upper X-value for cell A10, using 6 standard deviations from the mean. The first four arguments, 100, 120, 110, and 1.5, are ignored for this calculation. |
| Guidelines  | None. |

### RiskYV

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskYV(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations)) calculates the yield, or the percentage of the process, that is free of defects for cellref or output name in Sim#.</th>
</tr>
</thead>
</table>
| Examples    | RiskYV(A10) returns the yield or the percentage of the process that is free of defects for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list LSL, USL, and Long Term Shift, the values required to calculate the YV statistic.  
RiskYV(A10, ,RiskSixSigma(100,120,110,1.5,6)) returns the yield or the percentage of the process that is free of defects for the output cell A10, using LSL 100, USL 120, and LongTerm Shift 1.5. The other two arguments, 110 and 6, are ignored for this calculation. |
| Guidelines  | None. |
### RiskZlower

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskZlower(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the number of standard deviations the lower specification limit is from the mean for cellref or output name in Sim#.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskZlower(A10) returns the number of standard deviations the lower specification limit is from the mean for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list USL, the value required to calculate the lower Z statistic. RiskZlower(A10, ,RiskSixSigma(100, 120, 110, 1.5, 6)) returns the number of standard deviations the lower specification limit is from the mean for the output cell A10, using LSL 100. The last four arguments, 120, 110, 1.5, and 6, are ignored for this calculation.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskZMin

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskZMin(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the minimum of Z-Lower and Z-Upper for cellref or output name in Sim#.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskZMin(A10) returns the minimum of Z-Lower and Z-Upper for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list LSL and USL, the values required to calculate the ZMin statistic. RiskZMin(A10, ,RiskSixSigma(100, 120, 110, 1.5, 6)) returns the minimum of Z-Lower and Z-Upper for the output cell A10, using USL 120 and LSL 100. The last three arguments, 110, 1.5, and 6, are ignored for this calculation.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskZUpper

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskZUpper</strong>(<em>cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)</em>) calculates the number of standard deviations the upper specification limit is from the mean for <em>cellref or output name</em> in Sim#.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskZUpper(A10)</strong> returns the number of standard deviations the upper specification limit is from the mean for the output cell A10. In this case, a RiskSixSigma property function is required in the RiskOutput function in cell A10, and it must list USL, the value required to calculate the upper Z statistic.</td>
</tr>
<tr>
<td><strong>RiskZUpper(A10, ,RiskSixSigma(100,120,110,1.5,6))</strong> returns the number of standard deviations the upper specification limit is from the mean for the output cell A10, using USL 120. All arguments except the second, 120, are ignored for this calculation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>None.</td>
</tr>
</tbody>
</table>
Reference: Supplemental Functions

The following functions return information on the status of a running simulation or correlations used in a simulation.

### RiskCorrectCorrmat

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCorrectCorrmat(correlationMatrixRange,adjustmentWeightsMatrixRange) returns the corrected correlation matrix for the matrix located in correlationMatrixRange using the adjustment weight matrix located in adjustmentWeightsMatrixRange. An invalid matrix specifies inconsistent simultaneous relationships between three or more inputs and must be corrected prior to simulation. The returned matrix is a valid correlation matrix, that is, all diagonal entries are 1, the off-diagonal entries are in the range -1 to 1, inclusive, and the matrix is positive-definite (the smallest eigenvalue is positive). If the adjustmentWeightsMatrixRange is specified, the correlations have been optimized so they are as close as possible to the originally-specified correlations, taking into account the weights.</th>
</tr>
</thead>
</table>
| Guidelines | adjustmentWeightsMatrixRange is an optional argument. This is an array formula that returns an array with the corrected correlation matrix. To enter it:
1) Select a range with the same number of rows and columns as the original correlation matrix.
2) Enter the formula
   =RiskCorrectCorrmat(CorrelationMatrixRange,AdjustmentWeightsMatrixRange)
3) Press Ctrl+Shift+Enter (all three keys at once) to enter the formula as an array formula. |

### RiskCurrentIter

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCurrentIter() returns the current iteration number of an executing simulation. No arguments are required.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>None.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskCurrentSim

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskCurrentSim}()$ returns the current simulation number. No arguments are required.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>None.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskSimulationInfo

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskSimulationInfo}(\text{info to return})$ returns information such as date/time, runtime, iterations, or simulations for a simulation that was run.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskSimulationInfo}(1)$ returns the date and time that the simulation whose results are active in @RISK was run.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>$\text{info to return}$ can be: 1= date/time, 2= runtime (in seconds), 3= number of simulations run, 4= number of iterations run. Date/time info is returned as a value that displays as a date when the cell with the RiskSimulationInfo function is formatted as a date.</td>
</tr>
</tbody>
</table>

### RiskStopRun

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskStopRun}(\text{cellRef or formula})$ stops a simulation when the value of cellRef returns TRUE, or the entered formula evaluates to TRUE. Use this function in conjunction with the RiskConvergenceLevel function to stop a simulation when simulation results for cellRef have converged.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskStopRun}(A1)$ stops a simulation when the value in cell A1 is TRUE.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
Reference: Graphing Function

The @RISK function **RiskResultsGraph** automatically places a graph of simulation results wherever it is used in a spreadsheet. For example, the formula =**RiskResultsGraph** (A10) will place a graph of the simulated distribution for A10 in your spreadsheet at the function's location at the end of a simulation. Additional arguments to RiskResultsGraph allows you to select the type of graph you want to create, its format, scaling, and other options. This function can also be called from the @RISK macro language to generate graphs in Excel in custom @RISK applications.

### RiskResultsGraph

| Description | **RiskResultsGraph**(cellRef or output/input name, locationCellRange,graphType,xlFormat,leftDelimiter,rightDelimiter,xMin,xMax,xScale,title,sim#) adds a graph of simulation results to a worksheet. The graph generated is the same as generated in the Results Summary window. Many arguments to this function are optional. If optional arguments are not entered, the function creates a graph using the current default settings in the Results Summary window for any omitted argument. |
| Examples | **RiskResultsGraph**(A10) generates a graph of the simulation results for cell A10 as a graph in @RISK format at the location of the function, using the default graph type (histogram, cumulative ascending, or cumulative descending).

**RiskResultsGraph**(A10,C10:M30,1,TRUE,1,99) generates a graph of the simulation results for cell A10 in the range C10:M30 as a histogram in Excel format, and sets the left and right delimiters at the 1% and 99% percentile values, respectively. |
| Guidelines | cellRef is any valid Excel cell reference with one or more cells. Either a cellRef or an output/input name argument must be included in a RiskResultsGraph function. When cellRef is entered, the results to be graphed depend on the following:

- If there is a RiskOutput function in cellRef, the simulation results for this output will be graphed.

- If there is no RiskOutput function in cellRef, but there is a distribution function, the function will graph the collected samples for this input.

- If there is no RiskOutput function and no distribution function in cellRef, a RiskOutput function is automatically added and this output is graphed.

- If there are multiple cells in cellRef, an overlay graph is created for the simulation results for each cell in cellRef. Each overlay has the same graphType. |
locationCellRange is any valid Excel cell range. The created graph will be located in and sized according to this cell range.

graphType (optional) is one of the following constants:

- 0 for histogram
- 1 for cumulative ascending graph
- 2 for cumulative descending graph
- 3 for tornado graph of regression sensitivity results
- 4 for tornado graph of correlation sensitivity results
- 5 for a summary graph of 1) the output range that includes cellRef or 2) the results for each cell in cellRef (where cellRef is a multi-cell range)
- 6 for a box plot of 1) the output range that includes cellRef or 2) the results for each cell in cellRef (where cellRef is a multi-cell range)
- 7 for a graph of a theoretical distribution function
- 8 for a histogram of a simulated input overlaid with its theoretical distribution
- 9 for a histogram with a cumulative ascending overlay
- 10 for a histogram with a cumulative descending overlay
- 11 for tornado graph of mapped value sensitivity results
- 12 for a scatter plot graph of the results for each cell in cellRef (where cellRef is a multi-cell range)
- 13 for histogram using relative frequency
- 14 for tornado graph of change in output statistic
- 15 for a spider graph

xlFormat (optional) specifies whether the graph will be created as an Excel format chart. Enter TRUE for an Excel format chart; enter FALSE, or leave blank, for an @RISK format chart.

leftDelimiter (optional) specifies the location of the left delimiter on the graph in % (for histograms and cumulative graphs only). leftDelimiter is a value from 0 to 100.

rightDelimiter (optional) specifies the location of the right delimiter on the graph in % (for histograms and cumulative graphs only). rightDelimiter is a value from 0 to 100.

xMin (optional) specifies the minimum value for the X-axis in unscaled units (for histograms and cumulative graphs only). If used, it must be used in conjunction with xMax.

xMax (optional) specifies the maximum value for the X axis in unscaled units for histograms and cumulative graphs only. If used, it must be used in conjunction with xMin.

xScale (optional) specifies the scale factor for the X-axis (for histograms and cumulative graphs only). xScale is an integer value representing the power of 10 used to convert the X-axis values when labeling the axis. For example, a value of 3 for xScale specifies that values should be shown in thousands.

title (optional) specifies the graph title shown on the graph. A string in quotes, or cell reference containing the title, can be entered.

sim# (optional) specifies simulation number for which results are to be graphed when multiple simulations are run.
Reference:  @RISK for Excel Developers Kit (XDK)

@RISK for Excel includes a powerful API (application programming interface) for use in automating @RISK and building custom applications with @RISK using VBA, VB, C or other programming languages. For more information on this API, see the items under Developer Kit (XDK) on the @RISK Help menu.
Appendix A: Distribution Fitting

Overview

@RISK allows you to fit probability distributions to your data (Professional and Industrial versions only). Fitting is done when you have a set of collected data that you want to use as the basis for an input distribution in your spreadsheet. For example, you might have collected historical data on a product price and you might want to create a distribution of possible future prices based on this data.

To fit distributions to data using @RISK, there are five steps that you should consider:

- Define Input Data
- Specify Distributions to Fit
- Run the Fit
- Interpret the Results
- Use the Results of a Fit

These steps are each discussed in the earlier Distribution Fitting section of this manual, and further details are discussed in this appendix.
Define Input Data

@RISK allows you to analyze three kinds of data for distribution fitting: sample, density, and cumulative. @RISK supports up to 10,000,000 data points for each of these types. The available data types are shown in the Data tab of the Fit Distributions to Data dialog.

Sample Data

Sample (or observation) data is a set of values drawn randomly from a large population. Distributions are fit to sample data to estimate the properties of the population.

Sample data is either continuous or discrete. Continuous sample data can take on any value over a continuous range, whereas discrete data is limited to integer values. Discrete data can be entered in two formats. In the “standard” format, you enter each data point individually. In the “counted” format, the data is entered in pairs, where the first value is the sampled value and the second is the number of samples drawn with that value.
Data requirements for sample data include:

- You must have at least five data values.
- Discrete data values must be integers.
- All sample values must fall in the range -1E+37 \(\leq x \leq +1E+37\), or else should be dates.

**Density Data**

Density data is a set of \((x,y)\) points that describe the probability density function of a continuous distribution. Distributions are fit to density data to give the best representation of the curve points using a theoretical probability distribution.

Because all probability distribution functions must have unit area, @RISK automatically scales your y-values so that the area under the density curve described by the data is 1. The points you specify are isolated points on a continuum, so linear interpolation between these points is used to calculate the normalization factor. In certain cases, such as fitting to data generated from a mathematical function already known to be normalized, it is undesirable to have @RISK apply its own normalization. In such cases, you can turn this feature off.

Data requirements for density data include:

- You must have at least three \((x,y)\) data pairs.
- All x-values must be in the range from -1E+37 to +1E+37 or be dates.
- All x-values must be distinct.
- All y-values must be in the range from 0 to +1E+37.
- At least one y-value must be positive.
Cumulative Data

Cumulative data is a set of (x,p) points that describe a continuous cumulative distribution function. The p-value associated with a given x-value is the probability of obtaining a value less than or equal to x. Distributions are fit to cumulative data to give the best representation of the curve points using a theoretical probability distribution.

To calculate statistics and display graphs of your cumulative data, @RISK needs to know where the input minimum and maximum are (the points with p=0 and p=1). If you do not explicitly supply these points, @RISK will linearly interpolate them from the data. In general, it is recommended that you always include the p=0 and p=1 points in your data set, if possible.

Data requirements for cumulative data include:

- You must have at least three (x,p) data pairs.
- All x-values must be in the range from -1E+37 to +1E+37 or be dates.
- All x-values must be distinct.
- All p-values must be in the range from 0 to 1.
- Increasing x-values must always correspond to increasing p-values.

Filtering Your Data

You can further refine your input data by applying an input filter. Filtering tells @RISK to ignore outliers, based on criteria you specify, without requiring you to explicitly remove them from your data set. For example, you might want to analyze only positive x-values, or you might want to filter values that lie outside two standard deviations from the mean.

Select Distributions to Fit

After you define your data set, you must specify the distributions you want @RISK to attempt to fit. There are three general questions you must answer to do this.
Continuous versus Discrete Distributions

For sample data, you should first decide if your data is continuous or discrete. Discrete distributions always return integer values. For example, suppose you have a set of data describing the number of failures in a series of 100 trial batches. You would fit only discrete distributions to this data because partial failures are not allowed. In contrast, continuous data can take on any value in a range. For example, suppose you have a set of data describing the height, in inches, of 300 people. You would fit continuous distributions to this data because heights are not restricted to integer values.

If you specify that your data is discrete, all your data values must be integers. Keep in mind, however, that the converse is not true. Just because you have all integer data values does not mean you have to fit only discrete distributions. In the previous example, the height data set might be rounded to the nearest inch, but fitting to continuous distributions is still appropriate.

@RISK does not support the fitting of discrete distributions to density and cumulative curve data.

You can specify whether your data set is continuous or discrete in the Data tab of the Fit Distributions to Data dialog.

Estimated Parameters versus Predefined Distributions

Generally, you will want @RISK to estimate the parameters of your distributions. However, in some cases, you might want to specify exactly which distributions to use. For example, you might want to have @RISK compare two competing distributions and indicate which describes your data best.

Predefined distributions can be set in the Distributions to Fit tab of the Fit Distributions to Data dialog.
Domain Limits

For continuous data sets (sample or curve data) you can specify how you want @RISK to treat the upper and lower limits of the distributions. For both limits there are four choices: **Fixed Bound**, Bounded but Unknown, Open and Unsure.

![@RISK - Fit Distributions to Data](image)

**Fixed Bound**

If you specify a fixed bound, you are telling @RISK that the limit of the distribution must be the value you specify. For example, if you have a data set of the times between arrivals of customers in a queue, you might want to fit distributions that have a fixed lower bound of zero, since it is impossible to have a negative time between events.

**Bounded But Unknown**

If you specify an unknown bound, you are telling @RISK that the limit of the distribution has a finite bound (that is, it does not extend to plus or minus infinity). Unlike a fixed bound, however, you do not know what the actual value of the limit is. You want @RISK to choose the value for you as it performs its fit.

**Open**

If you specify an open bound, you are telling @RISK that the limit of the distribution must extend to minus infinity (for a lower bound) or plus infinity (for an upper bound).

**Unsure**

This is the default option. It is the combination of an unknown bound and an open bound. The limits of distributions that are non-
asymptotic are treated as in the unknown bound case, while asymptotic distributions are still included as in the open bound case.

Note, not all distributions functions are compatible with all the possible choices. For example, you cannot specify a fixed or unknown lower bound for the Normal distribution because it asymptotically extends to minus infinity.

**Fixed Parameters**

It is possible to set certain distribution parameters to fixed values, instead of having the fitting algorithm determine them. For example, imagine you know you want to fit a normal distribution to a set of data, but you only want the standard deviation of that distribution to be determined by the software; you want its mean to have a particular fixed value.

It is important to note that the fixed bounds (see “Domain Limits” above) are, in a sense, also a type of fixed parameter. In that case, however, the fixed limits apply universally to all distribution types.

**Suppress Questionable Fits**

The Suppress Questionable Fits option indicates that fits that are mathematically valid, but that fail various common sense heuristics should be rejected as possible fits. For example, it is often possible to fit normal-like data to a BetaGeneral distribution with very large shape parameters and artificially wide minimum and maximum parameters. Although this can lead to a good fit from a mathematical viewpoint, the fit is questionable from a practical standpoint.
Run the Fit

To start the fitting process, you click the Fit button in the Fit Distributions to Data dialog.

For each of the distributions specified in the previous step, @RISK will try to find the set of parameters that provide the closest match between the distribution function and your data. Keep in mind, however, that @RISK does not produce an absolute answer, but rather identifies a distribution that most likely produced your data. Always evaluate your @RISK results quantitatively and qualitatively, examining both the comparison graphs and statistics before accepting a fit as “good.”

@RISK uses two methods to calculate the best distributions for your data. For sample data, distribution parameters are usually estimated using Maximum Likelihood Estimators (MLEs) or slight modifications of MLEs. For density and cumulative data (collectively known as curve data), the method of least squares is used to minimize the root-mean square error between the curve points and the theoretical function.

Sample Data — Maximum Likelihood Estimators (MLEs)

The MLEs of a distribution are the parameters of that function that maximize the joint probability of obtaining the given data set.

For any density distribution \( f(x) \) with one parameter \( \alpha \), and a corresponding set of \( n \) sampled values \( X_i \), an expression called the likelihood can be defined:

\[
L = \prod_{i=1}^{n} f(X_i, \alpha)
\]

To find the MLE, the procedure maximizes \( L \) with respect to \( \alpha \) by setting the first derivative to 0 and solving for \( \alpha \):

\[
\frac{dL}{d\alpha} = 0
\]

This method can be easily generalized to distributions with more than one parameter.
An exponential function with a fixed lower bound of zero has only one adjustable parameter, and its MLE can be calculated easily. The distribution’s density function is:

\[ f(x) = \frac{1}{\beta} e^{-x/\beta} \]

and the likelihood function is:

\[ L(\beta) = \prod_{i=1}^{n} \frac{1}{\beta} e^{-X_i/\beta} = \beta^{-n} \exp\left(-\frac{1}{\beta} \sum_{i=1}^{n} X_i\right) \]

Algebraically, it is easier to maximize the natural log of the likelihood:

\[ l(\beta) = \ln L(\beta) = -n \ln(\beta) - \frac{1}{\beta} \sum_{i=1}^{n} X_i \]

This is permitted because the parameter that maximizes the log of the likelihood also maximizes the likelihood itself. To maximize the log of the likelihood, simply set its derivative with respect to \( \beta \) to zero:

\[ \frac{d}{d\beta} l(\beta) = -\frac{n}{\beta} + \frac{1}{\beta^2} \sum_{i=1}^{n} X_i \]

This equals zero when:

\[ \beta = \frac{\sum_{i=1}^{n} X_i}{n} \]

Therefore, when @RISK tries to fit your data to the best exponential distribution with a fixed lower bound of 0, it uses the mean of the data as the MLE for \( \beta \).
**Modifications to the MLE Method**

For some distributions, the MLE method does not work. For example, a 3-parameter gamma distribution (a gamma distribution whose lower bound is allowed to vary) cannot always be fitted using MLEs. In these cases, @RISK resorts to a hybrid algorithm that combines the standard MLE approach with a moment-matching procedure.

In certain distributions, a strict MLE method produces heavily biased parameters for small sample sizes. For example, the MLE of the “shift” parameter of an exponential distribution, and the minimum and maximum parameters of the uniform distribution, are heavily biased for small sample sizes. Where possible, @RISK corrects for the bias.

**Curve Data: The Method of Least Squares**

The root-mean square error (RMSErr) between set of n curve points \((X_i, Y_i)\) and a theoretical distribution function \(f(x)\) with one parameter \(\alpha\) is:

\[
RMSErr = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (f(x_i, \alpha) - y_i)^2}
\]

The value of \(\alpha\) that minimizes this value is called the least squares fit. This value essentially minimizes the “distance” between the theoretical curve and the data. The formula above is easily generalized to more than one parameter.

This method is used to calculate the best distribution for both density and cumulative curve data.

**Parametric Bootstrap**

Some calculations (parameter confidence intervals, p-value, and critical value calculations) require the use of a parametric bootstrap. Effectively, this takes each fitted distribution and generates a large number of new sample data sets from it, each with the same size as the original data set. It then refits these new data sets and tabulates information about each of the resampled fits.

See the discussions below for a description of parameter confidence intervals, p-values, and critical values to understand when you would want this information.

By default, the parametric bootstrap option in turned off in @RISK. For large data sets, this procedure is time-consuming, so if you do not need the information the parametric bootstrap provides, we recommend you leave this feature turned off.
Interpret the Results

Once @RISK has completed the fitting process, you should review its results. @RISK provides a powerful array of graphs, statistics, and reports to help you evaluate fits and select the best choice for your models.

@RISK ranks all the fitted distributions using one or more fit statistics. For sample data, you can choose to rank fits by their Akaike information criteria (AIC), Bayesian information criteria (BIC), chi-squared statistic (Chi-Squared), Anderson-Darling (AD) statistic, or Kolmogorov-Smirnov (KS) statistic. (The AD and KS statistics apply only to continuous fits.) Each of these statistics is discussed in more detail later in this section.

For density and cumulative curve data, the fits are ranked by their RMSErr values.

Graphs

@RISK provides four types of graphs to help you visually assess the quality of your fits.

A comparison graph superimposes the input data and fitted distribution on the same graph, allowing you to visually compare them, either as density or cumulative curves. This graph allows you to determine if the fitted distribution matches the input data in specific areas. For example, it might be important to have a good match near the mean or in the tails.
**P-P Graphs**

Probability-Probability (P-P) graphs plot the distribution of the input data \((P_i)\) versus the distribution of the result \((F(x_i))\). If the fit is "good," the plot will be nearly linear. P-P graphs are available for fits to sample data only.

**Q-Q Graphs**

Quantile-Quantile (Q-Q) graphs plots percentile values of the input distribution \((x_i)\) versus percentile values of the result \((F^{-1}(P_i))\). If the fit is "good," the plot will be nearly linear. Q-Q graphs are available for fits to continuous sample data only.
Basic Statistics and Percentiles

@RISK reports basic statistics (mean, variance, mode, etc.) for each fitted distribution, which can easily be compared to the same statistics for the input data.

@RISK allows you to compare percentiles between distributions and the input data. For example, perhaps the 5th and 95th percentile values are important to you. This can be done in two ways. First, all @RISK graphs have a set of “delimiters” that allow you to visually set different targets, or percentiles. Second, the @RISK graphs can display percentiles you select in the legend to the right of the graph.
**Fit Statistics**

For each fit, @RISK reports one or more fit statistics: the Akaike Information Criteria (AIC), the Bayesian Information Criteria (BIC), the Chi-Squared, Kolmogorov-Smirnov (K-S), Anderson-Darling (A-D), and Root-Mean Squared Error (RMSErr). These statistics measure how well the distribution fits the input data and how confident you can be that the data could have been produced by the distribution function. For each of these statistics, a smaller value represents a better the fit.

The RMSErr statistic is used solely for curve (density or cumulative) data. The A-D and K-S tests are used for continuous sample data only. The AIC, BIC, and Chi-Sq statistics can be used for both continuous and discrete sample data.

Fit statistics are used for two related, but distinct purposes. **Model selection** is the process of picking one particular fitted distribution type over another, whereas **fit validation** is the process of determining whether a fitted distribution is a “good” fit for the data.

For curve data, the RMSErr statistic serves both of these roles.

For sample data, the so-called “classical” goodness-of-fit statistics, the Chi-Sq, K-S, and A-D statistics, were originally developed as tests for fit validation; they were not directly developed as a tool for deciding between alternate distributions. Nevertheless, they are often used in this way, and in the case where the number of data values is very large, using these statistics for model selection is often acceptable.

More recently, the so-called “Information Criteria” were developed, and were specially designed for model selection. They are better suited for this task because they take into account, among other things, the number of free parameters of the fitted distribution. To understand why this is important, consider the hypothetical case where a normal distribution and a beta-general distribution are both good fits to a particular set of data. All things being equal, the normal distribution is preferable because it has only two adjustable parameters, whereas the beta-general has four. This is called the principle of **parsimony** (explaining the most with the least).

We recommend that you use the AIC or BIC for model selection unless you have a specific reason for not doing so. It is important to note, however, that these statistics do not provide a measure of the absolute goodness of a particular fit. That is, the actual values of the AIC and BIC statistic do not have meaning, except in relative terms, when you compare one proposed distribution type to another. Either
could point to a particular distribution as providing the best fit among the competitors, but it is still possible that none provide a good fit in an absolute sense.

The AIC and BIC statistics are calculated from the log-likelihood function by the simple expressions:

\[ \text{AIC} = 2k - 2 \ln L \]

\[ \text{BIC} = k \ln n - 2 \ln L \]

Here, \( L \) is the likelihood function, \( k \) is the number of parameters estimated for the fit, and \( n \) is the number of sampled points.

The AIC and BIC are clearly very similar. The theoretical underpinnings of both rely on Bayesian analysis, and the two different forms come from differing assumptions for the Bayesian “priors.”

The AIC tends to penalize the number of parameters less than the BIC. There has been much discussion in the literature about which one is more appropriate, and the jury appears to still be out. Our recommendation is to use the AIC unless you have reasons to do otherwise.

The chi-squared statistic is the best known goodness-of-fit statistic. It can be used with both continuous and discrete sample data. To calculate the chi-squared statistic, you first must break up the x-axis domain into several “bins”. The chi-squared statistic is then defined as:

\[ \chi^2 = \sum_{i=1}^{K} \left( \frac{N_i - E_i}{E_i} \right)^2 \]

where

\( K = \text{the number of bins} \)

\( N_i = \text{the observed number of samples in the } i^{th} \text{ bin} \)

\( E_i = \text{the expected number of samples in the } i^{th} \text{ bin} \).

A weakness of the chi-squared statistic is that there are no clear guidelines for selecting the number or location of the bins. In some situations, you can reach different conclusions from the same data depending on how you define the bins.

Some of the arbitrariness of the bin selection can be removed by telling @RISK to use equiprobable bins. In this case, @RISK will
adjust the bin sizes based on the fitted distribution, trying to make each bin contain an equal amount of probability. For continuous distributions this is straightforward. For discrete distributions, however, @RISK will be able to make the bins only approximately equal.

@RISK allows you full control of how bins are defined for the chi-squared test. These settings are on the **Chi-Sq Binning** tab of the Fit Distributions to Data dialog.

Another fit statistic that can be used for continuous sample data is the Kolmogorov-Smirnov statistic, defined as

\[
D_n = \sup \left[ \left| F_n(x) - \hat{F}(x) \right| \right]
\]

where

\[
n = \text{total number of data points}
\]

\[
\hat{F}(x) = \text{the fitted cumulative distribution function}
\]

\[
F_n(x) = \frac{N_x}{n}
\]

\[
N_x = \text{the number of } X_i \text{'s less than } x.
\]

The K-S statistic does not require binning, which makes it less arbitrary than the chi-squared statistic. A weakness of the K-S statistic is that it does not detect tail discrepancies very well.
The final fit statistic that can be used with continuous sample data is the Anderson-Darling statistic, defined as:

$$A_n^2 = n \int_{-\infty}^{+\infty} \left[ F_n(x) - \hat{F}(x) \right]^2 \Psi(x) \hat{f}(x) \, dx$$

where

- $n$ = total number of data points
- $\Psi^2 = \frac{1}{\hat{F}(x) [1 - \hat{F}(x)]}$
- $\hat{f}(x)$ = the hypothesized density function
- $\hat{F}(x)$ = the hypothesized cumulative distribution function
- $F_n(x) = \frac{N_x}{n}$
- $N_x$ = the number of $X_i$'s less than $x$.

Like the K-S statistic, the A-D statistic does not require binning. But unlike the K-S statistic, which focuses in the middle of the distribution, the A-D statistic highlights differences between the tails of the fitted distribution and input data.

For density and cumulative curve data, the only fit statistic used is the Root-Mean Squared Error. This is the same quantity that @RISK minimizes to determine the distribution parameters during its fitting process. It is a measure of the “average” squared error between the input and fitted curve.
Fit Validation Using P-Values and Critical Values

The “classical” goodness-of-fit statistics (Chi-Squared, K-S, and A-D statistics) each report a measure of the deviation of the fitted distribution from the input data. As mentioned earlier, the smaller the fit statistic is, the better the fit. But how small a value is needed for a “good” fit? For fits to sample data, this section explains how P-values and critical values can be used to analyze the “goodness” of a fit.

For the discussion below, suppose a distribution has been fitted to a set of \( N \) sampled values, and this results in a corresponding fit statistic, \( s \). In classical statistical terms, the goal is to run a hypothesis test, where the null hypothesis is that fitted distribution is the distribution responsible for generating the data. Therefore, the burden of proof is on showing that the fitted distribution could not have generated the data.

The test asks the following question: How likely is it that a new set of \( N \) samples drawn from the fitted distribution would generate a fit statistic greater than or equal to \( s \)? This probability is referred to as the P-value and is sometimes called the “observed significance level” of the test. As the P-value decreases to 0, there is less and less confidence that the fitted distribution could possibly have generated the observed data. Conversely, as the P-value approaches 1, there is no basis to reject the hypothesis that the fitted distribution actually generated the data set. In short, a small P-value indicates a poor fit, and a large P-value indicates a good fit.

Often the same question around by specifying a particular level of significance to use, usually denoted by \( \alpha \). This value is the probability that the null hypothesis will incorrectly be rejected. Given this significance level, what is the largest value of \( s \) that would lead to accepting the fitted distribution? This value of \( s \) is called the “critical value” of the fit statistic at the \( \alpha \) level of significance. Any fit that has a value of \( s \) above the critical value is rejected, while fits with values of \( s \) below the critical value are accepted. Typically, critical values depend on the type of distribution fit, the particular fit statistic being used, the sample size, and the significance level.
The Chi-Squared, K-S, and A-D statistics were developed in the pre-computer era. To calculate P-values and critical values, statisticians needed to determine the sampling distribution that these fit statistics follow. (In fact, the Chi-Squared test gets its name from that statistic’s particular sampling distribution.) However, in general their analyses were limited to the cases where there were no adjustable parameters. Various approximations and assumptions were made to extend these analyses to the cases where one or more parameters were adjustable, but in some cases this was unacceptable.

Fortunately, these limitations can be overcome now by using the technique of parametric bootstrapping. As described earlier in this section, the bootstrap actually draws a large number of new samples from the fitted distribution and refits them, and thus is able to generate a sampling distribution for the fit statistics, and thus directly compute the P-values and critical values.

Be aware, then, that P-value and critical value calculations can only be performed if you turn on the parametric bootstrapping fitting option.
Confidence Intervals for Fitted Parameters

Often you would like information on how uncertain a particular fitted parameter is. For example, if @RISK reports a RiskNormal(6.5,1.2) as the best fitting normal distribution, it is reasonable to ask how certain that estimated mean of 6.5 is.

Using the technique of parametric bootstrapping there is a way to estimate this uncertainty. As discussed previously, the parametric bootstrap takes the fitted distribution and resamples a large number of new data sets and refits them to the same distribution type. It then can create what is called a sampling distribution for that parameter. Effectively this gives you an estimate for the spread of that parameter.

Typically this spread is phrased in terms of a confidence interval. For example, a 95% confidence interval indicates that you are 95% confident that the true (but unknown) parameter falls within the interval. Such confidence intervals for the two gamma distribution parameters appear at the top of the above screenshot.

Be aware that confidence interval calculations can only be performed if you turn on the parametric bootstrapping fitting option.
Exporting Graphs and Reports

Once you have analyzed the results of your calculation, you may wish to export the results to another program. Of course, you can always copy and paste any @RISK graph or report into Excel, or another Windows application, via the clipboard. In addition, using the Chart in Excel command, @RISK allows you to create a copy of the current @RISK graph in Excel’s native chart format.

Using Fitted Distributions in Excel

Often you will want to use the result of a fit in an @RISK model. Clicking Write to Cell places a fit result in your model as a new distribution function.

Selecting Update and Refit at the Start of Each Simulation causes @RISK, at the start of each simulation, to automatically refit your data when it has changed and place the new resulting distribution function in your model.
Appendix B: Optimization

Introduction

Palisade offers two optimization add-ins, RISKOptimizer and Evolver. RISKOptimizer, which is totally integrated into @RISK, combines optimization and simulation to find optimal solutions to models with uncertainty. Evolver is a separate add-in in the Palisade DecisionTools Suite. It is intended for optimization of deterministic model, that is, models with no uncertainty. This appendix provides background information on the optimization methods used by RISKOptimizer. Because there is some overlap between RISKOptimizer and Evolver methods, the discussion also includes some mention of Evolver.

Optimization Methods

Traditional Excel-based optimization has focused on deterministic models. Such models are comprised of:

- An output or “target” cell that you want to minimize or maximize.
- A set of input or “adjustable cells” whose values you control.
- A set of constraints that need to be satisfied, often specified with expressions such as TotalCost<=100 or A11>=0.

During an optimization, the adjustable cells are changed across allowable ranges you specify. For each possible set of adjustable cell values, the model is recalculated and a new value for the target cell is generated. When the optimization is complete, an optimal solution (a combination of adjustable cell values) has been found. This solution is the combination of adjustable cell values that yields the optimal value for the target cell while satisfying all of the constraints.

Some optimization models are much more difficult to solve than others. For difficult problems, such as a model for finding the shortest route between 200 cities, it is not feasible to examine every possible solution. Such an approach would require weeks or months of calculations on the fastest computers.
To solve such problems, it is necessary to search through a subset of all possible solutions. This is accomplished with an algorithm. An algorithm is a step-by-step description of how to solve a problem.

One type of algorithm is a hill-climbing algorithm:

1) Start at any initial point.
2) Move away from this point in some direction to get to a new point.
3) If the target value at the new point is better, stay and repeat step 2. If it is lower, go back to the previous point and try again.

Numerous hill-climbing variations have been developed and implemented in optimization software, but the devil is in the details, especially in step 2. It isn’t at all clear how to choose the direction or how far to move in that direction to get the next solution. Unfortunately, a specific algorithm that works well on one problem or type of problem does not always work well on other problems.

Hill-climbing algorithms have one other critical drawback. They tend to find only local optimal solutions, not global optimal solutions. Imagine a terrain where there are many hilltops but only one that is highest. This highest hilltop represents the desired solution, the global maximum, but because hilltop algorithms tend to search only locally when making the move in step 2, they can easily find a lower hilltop, a local maximum, and then stop, never reaching the global maximum.

Although hill-climbing algorithms can be modified so that they don’t necessarily get stuck at local hilltops, they are naturally most successful on problems guaranteed to have a single local (and global) hilltop. There are many such problems, but there are also many that do not have this attractive property.
Types of Models

Several different types of models are typically optimized.

**Linear Models**

In linear models, the target function and all constraint functions are linear functions of the inputs. If a model is represented algebraically, it is easy to identify whether the model is linear: all expressions will be sums of products of constants and variables, where “variables” refer to the adjustable cells. With complex spreadsheet models, it is not always that easy to spot linearity or the lack of it. However, an advantage of the Palisade add-ins is that you don’t need to verify that a problem is, or is not, linear. The software will figure it out.

Linear models have been fairly easy to solve since the advent of computers and the invention by George Dantzig of the simplex method. A simple linear model can be solved most quickly and accurately with a linear programming algorithm. Palisade's Evolver can solve linear models (with no uncertainty) efficiently. Its Industrial version can handle an unlimited number of variables and constraints.

**Nonlinear Models**

There are very few real-world models that are truly linear. Many real problems can be approximated well by linear models, as has been documented in hundreds of successful applications of linear programming. However, many problems are inherently nonlinear. This can be due to economies of scale, complex chemical interactions in oil blending, nonlinearities in financial portfolio measures of risk, and many others. Such models cannot be represented realistically by linear models. The nonlinearity is inherent and must be dealt with. Unfortunately, these models are much more difficult to solve than linear models.

To deal with nonlinear models, neither RISKOptimizer nor Evolver uses hill-climbing algorithms. Rather, they use global optimization methods: the Genetic Algorithm and the OptQuest engine. This lets them jump around in the solution space, examining many combinations of input values without getting stuck a local optimal solution. These methods retain in computer memory the information on previously attempted solutions, and then they use that information to better guess which scenarios are likely to be successful.
Many problems require the use of lookup tables and databases. For example, in choosing the quantities of different materials to buy, you might need to look up the prices charged for different quantities.

Tables and databases make models discontinuous (non-smooth). This makes it difficult for hill-climbing to find optimal solutions. To optimize these types of non-smooth models, RISKOptimizer and Evolver have offered the Genetic Algorithms since their initial releases. In version 6, the OptQuest optimization engine was added, representing state-of-the-art methods in non-smooth optimization. Fortunately, the Palisade products do not require you to identify a model as non-smooth. The optimization algorithms in the Palisade software work well on both smooth and non-smooth models, and the software can choose the most appropriate algorithm automatically.

There is a large class of models that are very different from the models discussed so far. Models where the outputs involve changing the order of existing input variables, or grouping subsets of the inputs, are called **combinatorial** models. These problems are usually very difficult to solve because they often require exponential time; that is, the amount of time needed to solve a problem with 4 variables might be $4 \times 3 \times 2 \times 1$, and doubling the number of variables to 8 raises the solving time to $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$, or a factor of 1,680. The number of variables doubles, but the number of possible solutions that must be checked increases 1,680 times. For example, choosing the starting lineup for a baseball team is a combinatorial problem. For 9 players, you can choose one out of the 9 as the first batter. You can then choose one out of the remaining 8 as the second batter, one of the remaining 7 will be the third, and so on. There are thus $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ (9 factorial) ways to choose a lineup of 9 players. This is about 362,880 different orderings. Now if you double the number of players, there are 18 factorial possible lineups, or $6,402,373,705,000,000$ possible lineups!

RISKOptimizer and Evolver’s algorithms, Genetic Algorithm and OptQuest, are both capable of intelligently searching through the possible permutations. This is much more practical than searching through all possibilities, and it is much more efficient than examining purely random permutations.
Genetic Algorithms

RISKOptimizer uses genetic algorithms as one of its optimization methods. The genetic algorithms used are adapted from Evolver, Palisade’s add-in for deterministic models. This section provides background information on genetic algorithms to give insights on how they are used for optimizing simulation models.

The first genetic algorithms were developed in the early 1970s by John Holland at the University of Michigan. Holland was impressed by the ease in which biological systems could perform tasks that eluded even the most powerful super-computers. For example, animals can flawlessly recognize objects, understand and translate sounds, and generally navigate through a dynamic environment almost instantaneously.

For decades, scientists have promised to replicate these capabilities in machines, but this has proved to be very difficult. Most scientists agree that any complex biological system that exhibits these qualities has evolved to get that way.

Evolution has produced systems with amazing capabilities through relatively simple, self-replicating building blocks that follow a few simple rules:

1) Evolution takes place at the level of the chromosome. The organism doesn’t evolve, but only serves as the vessel in which the genes are carried and passed along. It is the chromosomes that are dynamically changing with each re-arrangement of genes.

2) Nature tends to make more copies of chromosomes which produce a more “fit” organism. If an organism survives long enough, and is healthy, its genes are more likely to be passed along to a new generation of organisms through reproduction. This principle is often referred to as “survival of the fittest.” Remember that “fittest” is a relative term; an organism only needs to be fit in comparison to others in the current population to be “successful.”
3) **Diversity must be maintained in the population.** Seemingly random mutations occur frequently in nature, and these ensure variation in the organisms. These genetic mutations often result in a useful or even vital feature for a species’ survival. With a wider spectrum of possible combinations, a population is also less susceptible to a common weakness that could destroy them all (such as a virus) or other problems associated with inbreeding.

Once evolution is broken down into these fundamental building blocks, it becomes easier to apply these techniques to the computational world.

Holland began applying these properties of evolution to simple strings of numbers that represented chromosomes. He first encoded his problem into binary strings (rows of 1s and 0s) to represent the chromosomes, and then he had the computer generate many of these “bit” strings to form a whole population of them. A fitness function was programmed that could evaluate and rank each bit string, and those strings that were deemed most “fit” would exchange data with others through a “crossover” routine to create “offspring” bit strings. Holland even subjected his digital chromosomes to a “mutation” operator, which injected randomness into the resulting “offspring” chromosomes to retain diversity in the population. This fitness function replaced the role of death in the biological world to determine which strings were good enough to continue breeding and which would no longer be kept in memory.

```
"Chromosome" 1
1010011010
"Gene"

"Chromosome" 2
1110100100

Offspring "Chromosome"
1010000100
```

The program kept a given number of these “chromosomes” in memory, and this entire “population” of strings continued to evolve until they maximized the fitness function. The result was then decoded back to its original values to reveal the solution.
Holland’s original genetic algorithm was quite simple, yet remarkably robust in finding optimal solutions to a wide variety of problems. Many custom programs today solve very large and complex real-world problems using only slightly modified versions of this original genetic algorithm.

As interest swelled in academic circles, as serious computational power began moving its way into mainstream desktop machines, standards like Microsoft Windows and Excel made design and maintenance of complex models easier. The use of real numbers rather than bit string representations eliminated the difficult task of encoding and decoding chromosomes.

The popularity of the genetic algorithm is now growing exponentially, with seminars, books, magazine articles, and knowledgeable consultants popping up everywhere. The International Conference of Genetic Algorithms is already focusing on practical applications, a sign of maturity that eludes other “artificial intelligence” technologies. Many Fortune 500 companies, from brokerage firms to power plants, phone companies, restaurant chains, automobile manufacturers and television networks, employ genetic algorithms regularly to solve real-world problems.

Consider a simple example of evolution in the biological world (on a small scale). Here, “evolution” means any change in the distribution or frequency of genes in a population. Of course, the interesting thing about evolution is that it tends to lead to populations that are constantly adapting to their environments.

Specifically, consider a population of mice. These mice exhibit two sizes, small and large, and they exhibit two colors, light or dark. Our population consists of the following eight mice:
One day, cats move into the neighborhood and start eating mice. It turns out that darker mice and smaller mice are harder for the cats to find. Therefore, different mice have different odds of avoiding the cats long enough to reproduce. This affects the nature of the next generation of mice. Assuming the old mice die soon after reproducing, the next generation of mice looks like this:

Notice that large mice, light mice, and especially large, light mice, are having trouble surviving long enough to reproduce. This continues in the next generation:

Now the population consists mostly of small, dark mice, because these mice are better suited for survival in this environment than other kinds of mice. Similarly, as the cats begin to go hungry with less mice to eat, perhaps those cats who prefer a lunch of grass will be better adapted, and pass along their grass-loving gene to a new generation of cats. This is the central concept of “survival of the fittest.” More precisely, it could be phrased “survival until reproduction.” In evolutionary terms, being the healthiest bachelor in the population is worthless because you must reproduce for your genes to influence future generations.
Now consider a problem with two variables, X and Y, that produce a result Z. If you calculated and plotted the resulting Z for every possible X and Y values, you would see a solution “landscape” emerge. If you are trying to find the maximum “Z”, the peaks of the function are “good” solutions, and the valleys are “bad” ones.

When a genetic algorithm is used to maximize the function, it starts by creating several possible solutions or scenarios at random (the black dots), rather than just one starting point. It then calculates the function’s output for each scenario and plots each scenario as one dot. Next it ranks all of the scenarios by height, from best to worst. It keeps the scenarios from the top half and discards the others.

First, create a whole “population” of possible solutions. Some will be better (higher) than others.

Next, rank them all and keep the solutions that yield better results.

Each of the three remaining scenarios duplicates itself, bringing the number of scenarios back up to six. Now comes the interesting part: Each of the six scenarios is composed of two adjustable values, X and Y. The scenarios pair off with each other at random. Each scenario exchanges the first of its two adjustable values with the corresponding value from its partner. For example:

<table>
<thead>
<tr>
<th></th>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>3.4, 5.0</td>
<td>2.6, 5.0</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>2.6, 3.2</td>
<td>3.4, 3.2</td>
</tr>
</tbody>
</table>
This operation is called **crossover**. When the six scenarios randomly mate and perform crossover, there is a new set of scenarios:

![Diagram of crossover](image)

In the above example, the original three scenarios, a, b, and c, paired up with the duplicates, A, B, C, to form the pairs aB, bC, bA. These pairs then switched values for the first adjustable cell, which is equivalent in the diagram to exchanging the x and y coordinates between pairs of dots. The population of scenarios has just lived through a generation, with its cycle of “death” and “birth.”

Notice that some of the new scenarios result in lower height than any in the original generation. However, one scenario has moved high up on the tallest hill, indicating progress. If the population evolves for another generation, it might lead to the following:

![Diagram of subsequent generation](image)

You can see how the average performance of the population of scenarios increases over the last generation. In this example, there is not much room left for improvement. This is because there are only two genes per organism, only six organisms, and no way for new genes to be created. This means there is a limited **gene pool**. The gene pool is the collection of all the genes of all organisms in the population.
Genetic algorithms can be made much more powerful by replicating more of the inherent strength of evolution in the biological world; increasing the number of genes per organism, increasing the number of organisms in a population, and allowing for occasional random mutations. In addition, the algorithms can choose the scenarios that will live and reproduce with a random element that has a slight bias towards those that perform better, instead of simply choosing the best performers to breed.

All of these techniques stimulate genetic refinement, and help to maintain diversity in the gene pool, keeping all kinds of genes available in case they turn out to be useful in different combinations. RISKOptimizer and Evolver automatically implement all of these techniques.

**OptQuest**

The OptQuest Engine incorporates metaheuristics to guide its search algorithm toward better solutions. This approach remembers which solutions worked well and recombines them into new, better solutions. In addition, this technique does not get trapped in local solutions or get thrown off course by noisy (uncertain) model data. The OptQuest Engine combines Tabu search, scatter search, integer programming, and neural networks into a single, composite search algorithm that provide maximum efficiency in identifying new scenarios.

**Linear Constraints**

OptQuest generates solutions that almost always meet all the specified linear constraints, and time is saved by not evaluating invalid solutions. (OptQuest can occasionally generate a solution that does not meet a linear constraint, due to the fact that computers cannot handle calculations with infinite precision.)

The Product Mix with Uncertainty 1.xls example demonstrates OptQuest linear constraint handling. All the constraints are linear, and all the solutions generated by OptQuest will be valid. More specifically, the “SumProduct” formula in the constrained cell expresses a linear function of the adjustable cells. Other constrained cells are also linearly dependent on the adjustable cells.

**Nonlinear Constraints**

OptQuest handles nonlinear constraints efficiently, including situations in which the original values of adjustable cells are invalid (they do not satisfy the specified constraints). The Genetic Algorithm generally requires the original adjustable cell values to satisfy the constraints. If the original solution is invalid, the “Constraint Solver” tool finds a valid solution, providing a starting point for an
optimization that uses the Genetic Algorithm. OptQuest does not require the use of the Constraint Solver. If the original solution is invalid, OptQuest can start the optimization by generating a sequence of invalid solutions. However, during this stage, it collects information about how much each solution falls short of meeting the constraints, with the goal of finding valid solutions.

**RISKOptimizer Constraints**

Realistic models often have a number of constraints that must be satisfied. For example, in the tutorial for the Airline Revenue Management example, the constraint is that the probability of the profit exceeding $15000 should be greater than 5%.

A scenario that meets all the constraints in a model is called a valid solution. Sometimes it is difficult to find valid solutions for a model, much less find the optimal valid solution. This might be because the problem is very complex and only has a few valid solutions, or because the problem is over-specified (there are too many constraints, or some constraints conflict with others) and there are no valid solutions.

There are three basic kinds of constraints: range constraints, or min-max ranges placed on adjustable cells, hard constraints, which must always be met, and soft constraints which you would like to be satisfied as much as possible, but which can be violated to some extent to improve fitness. These are discussed in the earlier RISKOptimizer section of this manual, so this discussion is not repeated here.

**Multiple Goal Problems**

You can specify only one cell in the target cell field of RISKOptimizer, but you can still solve for multiple goals. There are two possible approaches.

The first approach is to optimize the weighted sum of the goals. This is straightforward, but it often involves tricky trade-offs (comparing apples and oranges). Obviously, the weights you use can bias the final solution toward one goal versus the others.

The second approach is to choose one of the goals as the target and then add constraints on the others. For example, you might maximize mean profit, while constraining the mean pollution level to be no greater than a specified value. Then you could run RISKOptimizer multiple times, each with a different upper bound on the mean.
pollution level. Indeed, this is exactly the approach used to create an efficient frontier with RISKOptimizer (and Evolver).

**Improving Speed**

When you use RISKOptimizer to solve a problem, you are using both the RISKOptimizer library of compiled routines to control the process and Excel’s spreadsheet evaluation function to examine different scenarios. A large percentage of the time used by RISKOptimizer is actually used by Excel as it recalculates your spreadsheet. There are a number of things that you can do to speed up RISKOptimizer optimization and Excel’s recalculation process.

- The speed of RISKOptimizer is directly related to the speed of your computer processor. If you double the speed of your processor, RISKOptimizer will be able to evaluate twice as many trials in the same amount of time.

- You can experiment with different simulation stopping conditions. Initial tests of a model should be done with a low fixed number of iterations per simulation.

- Try to avoid redrawing in your window. Drawing graphics and numbers on the screen takes time, sometimes more than half the time spent optimizing. If you have charts or graphs on the sheet, they will slow down the recalculation time significantly. You can tell Excel not to spend time drawing while RISKOptimizer is solving a problem by turning off the “Show Excel Recalculations” option in the View tab of the Simulation Settings dialog, or by selecting “Minimize Excel at Start of Simulation” in the same tab. You can see how much faster your problem is working by watching the status bar.

You can set tighter ranges for the adjustable cells. This creates a smaller area in which RISKOptimizer must search for solutions. However, make sure that your ranges allow enough freedom for RISKOptimizer to explore all realistic solutions.
How RISKOptimizer’s Genetic Algorithm Optimization is Implemented

This section describes more specifically how RISKOptimizer’s Genetic Algorithm optimization is implemented.

**NOTE:** You do not need to know this material to use RISKOptimizer.

The majority of RISKOptimizer’s genetic algorithm technology such as the recipe and order solving methods are based on academic work in the genetic algorithm field over the last decade or two. However, most of the descendant solving methods included with RISKOptimizer, and the multiple groups of adjustable cells, backtracking, strategy, and probability features are unique to RISKOptimizer.

RISKOptimizer uses a steady-state approach. This means that only one organism is replaced at a time, rather than an entire “generation” being replaced. This steady-state technique has been shown to work as well or better than the generational replacement method. To determine the equivalent number of “generations” RISKOptimizer has run, you can divide the number of individual trials it has explored by the size of the population.

**Selection**

When a new organism is to created, two parents are chosen from the current population. Organisms that have high fitness scores are more likely to be chosen as parents.

In RISKOptimizer, parents are chosen with a rank-based mechanism. Instead of some genetic algorithm systems, where a parent’s chance of being selected for reproduction is directly proportional to its fitness, a ranking approach offers a smoother selection probability curve. This prevents good organisms from completely dominating the evolution from an early point.

**Crossover**

Because each solving method adjusts the variables in different ways, RISKOptimizer employs a different crossover routine optimized for each type of problem.

The basic recipe solving method performs crossover using a uniform crossover routine. This means that instead of chopping the list of variables in a given scenario at some point and dealing with each of the two blocks (called “single-point” or “double-point” crossover), two groups are formed by randomly selecting items to be in one group or another. Traditional x-point crossovers can bias the search with the irrelevant position of the variables, whereas the uniform crossover method is considered better at preserving schema, and can generate any schema from the two parents.
The order solving method performs crossover using a similar algorithm to the order crossover operator described in L. Davis’ *Handbook of Genetic Algorithms*. This selects items randomly from one parent, finds their place in the other parent, and copies the remaining items into the second parent in the same order as they appear in the first parent. This preserves some of the sub-orderings in the original parents while creating some new sub-orderings.

**Mutation**

Like crossover, mutation methods are customized for each of the different solving methods. The basic recipe solving method performs mutation by looking at each variable individually. A random number between 0 and 1 is generated for each of the variables in the organism, and if a variable gets a number that is less than or equal to the mutation rate (for example, 0.06), that variable is mutated. The amount and nature of the mutation is automatically determined by a proprietary algorithm. Mutating a variable involves replacing it with a randomly generated value (within its valid min-max range).

To preserve all the original values, the order solving method performs mutation by swapping the positions of some variables in the organism. The number of swaps performed is increased or decreased proportionately to the increase and decrease of the mutation rate setting (from 0 to 1).

**Replacement**

Because RISKOptimizer uses a rank-ordered rather than generational replacement method, the worst-performing organisms are always replaced with the new organism that is created by selection, crossover, and mutation, regardless of its fitness “score”.

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Mutation Replacement
Hard constraints are implemented with Palisade’s proprietary “backtracking” technology. If a new offspring violates some externally imposed constraints, RISKOptimizer backtracks towards one of the parents of the child, changing the child until it falls within the valid solution space.
Troubleshooting / Q&A

This section answers some commonly asked questions regarding RISKOptimizer and keeps you up to date on common questions, problems and suggestions. After reading through this section, you can call Palisade customer support at the numbers listed in the beginning of this manual.

Q: Why am I having trouble getting a valid answer from RISKOptimizer?
A: Make sure that the RISKOptimizer dialog is set up correctly. Most of the problems are associated with the setting of the variables. Each group of adjustable cells should be exclusive, in that no single cell or range of cells is being treated with more than one solving method.

Q: Can RISKOptimizer deal with concepts or categories instead of just numbers?
A: RISKOptimizer can indirectly deal with any kind of data. You can use a lookup table in Excel to translate between integers and strings of text. RISKOptimizer (like all computer programs) ultimately can only deal with numbers, but your interface can use those numbers to represent and display any strings.

Q: Even though I’m filling in the dialogs the same way, and letting RISKOptimizer run the same amount of time, why does RISKOptimizer sometimes find different solutions?
A: As is the case with natural selection in the biological world, the RISKOptimizer genetic algorithm will not always follow the same path when searching for solutions (unless you use a fixed random number generator seed). Ironically, it is this “unpredictability” that allows RISKOptimizer to solve more types of problems, and often find better solutions, than traditional techniques. RISKOptimizer’s genetic algorithm engine is not just executing a series of pre-programmed commands, or plugging values through a mathematical formula. It is efficiently experimenting with many random hypothetical scenarios simultaneously, and then refining the search through many “survival-of-the-fittest” operators that also contain random elements. A similar reason can be given in the case of OptQuest, the other optimization algorithm in RISKOptimizer. OptQuest experiments with different possible solutions, keeping track of the results of the experiments performed so far. The randomness involved in the
process keeps it from returning a locally optimal solution if there is a better globally optimal solution.

It is possible to specify settings that will make two RISKOptimizer optimizations return identical results. To do this, “Fixed” needs to be selected for “Initial Seed” in the “Sampling” tab of the Simulation Settings dialog. In addition, the “Trials” or “Progress” stopping condition needs to be selected in the Runtime tab of RISKOptimizer’s Optimization Settings dialog.

Q: Why is the best solution found not changing?
A: You might have specified the wrong target cell in the RISKOptimizer Model dialog, a cell that is not affected by the adjustable cell values. A proper target cell has a formula that depends, directly or indirectly, on the adjustable cells.

Q: Some of the cells in my spreadsheet model contain “####” symbols.
A: If the cell is too small to display all of its contents, it will display several #### signs. Increase the width of the cell.

Q: RISKOptimizer is working OK, but is there any simple way to get better results?
A: Consider loosening the constraints in the problem, including variable ranges. You can also change some of your hard constraints to soft constraints via penalty functions. Finally, remember that the longer you let RISKOptimizer explore the possibilities, the more likely it is to find the optimal solution.

The more scenarios RISKOptimizer can run through, the better. Speed optimization by turning off the “Show Excel Recalculations” option in the View tab of the Simulation Settings dialog.
Appendix C: @RISK and Six Sigma

A key application of @RISK is Six Sigma and quality analysis. Whether it’s in DMAIC, Design for Six Sigma (DFSS), Lean projects, Design of Experiments (DOE), or other areas, uncertainty and variable lies at the core of any Six Sigma analysis. @RISK uses Monte Carlo simulation to identify, measure, and root out the causes of variability in your production and service processes. A full suite of capability metrics gives you the calculations you need to step through any Six Sigma method quickly and accurately. Charts and tables clearly show Six Sigma statistics, making it easy and effective to illustrate this powerful technique to management. The Industrial edition of @RISK adds RISKOptimizer to your Six Sigma analyses for optimization of project selection, resource allocation, and more.

Industries ranging from engine manufacturing to precious metals to airlines and consumer goods are using @RISK every day to improve their processes, enhance the quality of their products and services, and save millions. This guide will walk you through the @RISK Six Sigma functions, statistics, charts and reports to show you how @RISK can be put to work at any stage of a Six Sigma project. Example case studies round out the guide, giving you pre-built models you can adapt to your own analyses.

The standard features of @RISK, such as entering distribution functions, fitting distributions to data, running simulations and performing sensitivity analyses, are also applicable to Six Sigma models.
Overview of @RISK and Six Sigma Methodologies

In today’s competitive business environment, quality is more important than ever. Enter @RISK, the perfect companion for any Six Sigma or quality professional. This powerful solution allows you to quickly analyze the effect of variation within processes and designs.

In addition to Six Sigma and quality analysis, @RISK can be used to analyze any situation in which there is uncertainty. Applications include analysis of NPV, IRR, and real options, cost estimation, portfolio analysis, oil and gas exploration, insurance reserves, pricing, and much more. To learn more about @RISK in other applications, and the use of @RISK in general, refer to the @RISK User’s Guide included with your software.

What is Six Sigma?

Six Sigma is a set of practices to systematically improve processes by reducing process variation and thereby eliminating defects. A defect is defined as nonconformity of a product or service to its specifications. While the particulars of the methodology were originally formulated by Motorola in the mid-1980s, Six Sigma was heavily inspired by six preceding decades of quality improvement methodologies such as quality control, TQM, and Zero Defects. Like its predecessors, Six Sigma asserts the following:

- Continuous efforts to reduce variation in process outputs is key to business success
- Manufacturing and business processes can be measured, analyzed, improved and controlled
- Succeeding at achieving sustained quality improvement requires commitment from the entire organization, particularly from top-level management

Six Sigma is driven by data, and frequently refers to “X” and “Y” variables. X variables are independent input variables that affect the dependent output variables, Y. Six Sigma focuses on identifying and controlling variation in X variables to maximize quality and minimize variation in Y variables.
The term Six Sigma (or in symbols, $6\sigma$) is very descriptive. The Greek letter sigma ($\sigma$) signifies standard deviation, an important measure of variation. The variation of a process refers to how tightly all outcomes are clustered around the mean. The probability of creating a defect can be estimated and translated into a “Sigma level.” The higher the Sigma level, the better the performance. **Six Sigma refers to having six standard deviations between the average of the process center and the closest specification limit or service level.** That translates to fewer than 3.4 defects per one million opportunities (DPMO). The chart below illustrates Six Sigma graphically.

*Six sigmas – or standard deviations – from the mean.*

The cost savings and quality improvements that have resulted from Six Sigma corporate implementations are significant. Motorola has reported $17$ billion in savings since implementation in the mid 1980s. Lockheed Martin, GE, Honeywell, and many others have experienced tremendous benefits from Six Sigma.
The Importance of Variation

Too many Six Sigma practitioners rely on static models that don’t account for inherent uncertainty and variability in their processes or designs. In the quest to maximize quality, it’s vital to consider as many scenarios as possible.

That’s where @RISK comes in. @RISK uses Monte Carlo simulation to analyze thousands of different possible outcomes, showing you the likelihood of each occurring. Uncertain factors are defined with probability distribution functions that describe the possible range of values your inputs could take. @RISK allows you to define Upper and Lower Specification Limits and Target values for each output, and it includes a wide range of Six Sigma statistics and capability metrics on the outputs.

@RISK Industrial edition also includes RISKOptimizer, which combine the power of Monte Carlo simulation with genetic algorithm-based optimization. This gives you the ability to tackle optimization problems like that have inherent uncertainty, such as:

- resource allocation to minimize cost
- project selection to maximize profit
- optimize process settings to maximize yield or minimize cost
- optimize tolerance allocation to maximize quality
- optimize staffing schedules to maximize service
Six Sigma Methodologies

@RISK can be used in a variety of Six Sigma and related analyses. The three principal areas of analysis are:

- Six Sigma / DMAIC / DOE
- Design for Six Sigma (DFSS)
- Lean or Lean Six Sigma

Six Sigma / DMAIC

When most people refer to Six Sigma, they are in fact referring to the DMAIC methodology. The DMAIC methodology should be used when a product or process is in existence but is not meeting customer specification or is not performing adequately.

DMAIC focuses on evolutionary and continuous improvement in manufacturing and services processes, and is almost universally defined as comprising of the following five phases: Define, Measure, Analyze, Improve and Control:

1. Define the project goals and customer (internal and external Voice of Customer or VOC) requirements
2. Measure the process to determine current performance
3. Analyze and determine the root cause(s) of the defects
4. Improve the process by eliminating defect root causes
5. Control future process performance

Design for Six Sigma (DFSS)

DFSS is used to design or re-design a product or service from the ground up. The expected process Sigma level for a DFSS product or service is at least 4.5 (no more than approximately 1 defect per thousand opportunities), but can be 6 Sigma or higher depending on the product. Producing such a low defect level from product or service launch means that customer expectations and needs (Critical-To-Qualities or CTQs) must be completely understood before a design can be completed and implemented. Successful DFSS programs can reduce unnecessary waste at the planning stage and bring products to market more quickly.
Unlike the DMAIC methodology, the steps of DFSS are not universally recognized or defined; almost every company or training organization will define DFSS differently. One popular Design for Six Sigma methodology is called DMADV, and retains the same number of letters, number of phases, and general feel as the DMAIC acronym. The five phases of DMADV are defined as: Define, Measure, Analyze, Design and Verify:

1. **Define** the project goals and customer (internal and external VOC) requirements
2. **Measure** and determine customer needs and specifications; benchmark competitors and industry
3. **Analyze** the process options to meet the customer needs
4. **Design** (detailed) the process to meet the customer needs
5. **Verify** the design performance and ability to meet customer needs

**Lean or Lean Six Sigma**

“Lean Six Sigma” is the combination of Lean manufacturing (originally developed by Toyota) and Six Sigma statistical methodologies in a synergistic tool. Lean deals with improving the speed of a process by reducing waste and eliminating non-value added steps. Lean focuses on a customer “pull” strategy, producing only those products demanded with “just in time” delivery. Six Sigma improves performance by focusing on those aspects of a process that are critical to quality from the customer perspective and eliminating variation in that process. Many service organizations, for example, have already begun to blend the higher quality of Six Sigma with the efficiency of Lean into Lean Six Sigma.

Lean utilizes “Kaizen events” -- intensive, typically week-long improvement sessions -- to quickly identify improvement opportunities and goes one step further than a traditional process map in its use of value stream mapping. Six Sigma uses the formal DMAIC methodology to bring measurable and repeatable results.

Both Lean and Six Sigma are built around the view that businesses are composed of processes that start with customer needs and should end with satisfied customers using your product or service.
@RISK and Six Sigma

Whether it’s in DMIAC, Design of Experiments, or Lean Six Sigma, uncertainty and variability lie at the core of any Six Sigma analysis. @RISK uses Monte Carlo simulation to identify, measure, and root out the causes of variability in your production and service processes. Each of the Six Sigma methodologies can benefit from @RISK throughout the stages of analysis.

@RISK and DMAIC

@RISK is useful at each stage of the DMAIC process to account for variation and hone in on problem areas in existing products.

1. **Define.** Define your process improvement goals, incorporating customer demand and business strategy. Value-stream mapping, cost estimation, and identification of CTQs (Critical-To-Qualities) are all areas where @RISK can help narrow the focus and set goals. Sensitivity analysis in @RISK zooms in on CTQs that affect your bottom-line profitability.

2. **Measure.** Measure current performance levels and their variations. Distribution fitting and over 35 probability distributions make defining performance variation accurate. Statistics from @RISK simulations can provide data for comparison against requirements in the Analyze phase.

3. **Analyze.** Analyze to verify relationship and cause of defects, and attempt to ensure that all factors have been considered. Through @RISK simulation, you can be sure all input factors have been considered and all outcomes presented. You can pinpoint the causes of variability and risk with sensitivity and scenario analysis, and analyze tolerance. Use @RISK’s Six Sigma statistics functions to calculate capability metrics which identify gaps between measurements and requirements. Here we see how often products or processes fail and get a sense of reliability.
4. **Improve.** Improve or optimize the process based upon the analysis using techniques like Design of Experiments. Design of Experiments includes the design of all information-gathering exercises where variation is present, whether under the full control of the experimenter or not. Using @RISK simulation, you can test different alternative designs and process changes. @RISK is also used for reliability analysis and - using RISKOptimizer - resource optimization at this stage.

5. **Control.** Control to ensure that any variances are corrected before they result in defects. In the Control stage, you can set up pilot runs to establish process capability, transition to production and thereafter continuously measure the process and institute control mechanisms. @RISK automatically calculates process capability and validates models to make sure that quality standards and customer demands are met.

**@RISK and Design for Six Sigma (DFSS)**

One of @RISK’s main uses in Six Sigma is with DFSS at the planning stage of a new project. Testing different processes on physical manufacturing or service models or prototypes can be cost prohibitive. @RISK allows engineers to simulate thousands of different outcomes on models without the cost and time associated with physical simulation. @RISK is helpful at each stage of a DFSS implementation in the same way as the DMAIC steps. Using @RISK for DFSS gives engineers the following benefits:

- Experiment with different designs / Design of Experiments
- Identify CTQs
- Predict process capability
- Reveal product design constraints
- Cost estimation
- Project selection – using RISKOptimizer to find the optimal portfolio
- Statistical tolerance analysis
- Resource allocation – using RISKOptimizer to maximize efficiency
@RISK and Lean Six Sigma

@RISK is the perfect companion for the synergy of Lean manufacturing and Six Sigma. “Quality only” Six Sigma models may fail when applied to reducing variation in a single process step, or to processes which do not add value to the customer. For example, an extra inspection during the manufacturing process to catch defective units may be recommended by a Six Sigma analysis. The waste of processing defective units is eliminated, but at the expense of adding inspection which is in itself waste. In a Lean Six Sigma analysis, @RISK identifies the causes of these failures. Furthermore, @RISK can account for uncertainty in both quality (ppm) and speed (cycle time) metrics.

@RISK provides the following benefits in Lean Six Sigma analysis:

- Project selection – using RISKOptimizer to find the optimal portfolio
- Value stream mapping
- Identification of CTQs that drive variation
- Process optimization
- Uncover and reduce wasteful process steps
- Inventory optimization – using RISKOptimizer to minimize costs
- Resource allocation – using RISKOptimizer to maximize efficiency
Using @RISK for Six Sigma

@RISK’s standard simulation capabilities have been enhanced for use in Six Sigma modeling through the addition of four key features. These are:

1. **RiskSixSigma** property function for entering specification limits and target values for simulation outputs.

2. **Six Sigma statistics functions**, including process capability indices such as RiskCpk, RiskCpm and others which return Six Sigma statistics on simulation results directly in spreadsheet cells.

3. **New columns in the Results Summary window** that provide Six Sigma statistics on simulation results in table form.

4. **Markers** on graphs of simulation results that display specification limits and the target value.

The standard features of @RISK, such as entering distribution functions, fitting distributions to data, running simulations and performing sensitivity analyses, are also applicable to Six Sigma models.
RiskSixSigma Property Function

In an @RISK simulation the RiskOutput function identifies a cell in a spreadsheet as a simulation output. A distribution of possible outcomes is generated for every output cell selected. These probability distributions are created by collecting the values calculated for a cell for each iteration of a simulation.

When Six Sigma statistics are to be calculated for an output, the RiskSixSigma property function should be entered as an argument to the RiskOutput function. This property function specifies the lower specification limit, upper specification limit, target value, long term shift, and the number of standard deviations for the six sigma calculations for an output. These values are used in calculating six sigma statistics displayed in the Results window and on graphs for the output. For example:

= RiskOutput("Part Height", RiskSixSigma(0.88, 0.95, 0.915, 1.5, 6))

specifies an LSL of 0.88, a USL of 0.95, target value of 0.915, long term shift of 1.5, and a number of standard deviations of 6 for the output Part Height. You can also use cell referencing in the RiskSixSigma property function.

These values are used in calculating Six Sigma statistics displayed in the Results window and as markers on graphs for the output.

When @RISK detects a RiskSixSigma property function in an output, it automatically displays the available Six Sigma statistics on the simulation results for the output in the Results Summary window and adds markers for the entered LSL, USL and Target values to graphs of simulation results for the output.
A typical output cell formula with the RiskSixSigma property function is the following:

\[ \text{RiskOutput}(\text{C14},,\text{RiskSixSigma}(\text{C4,C5,C6,0,6})) + \text{RiskNormal(C10,C11)} \]

You can type the RiskOutput function, together with the RiskSixSigma function, directly into the cell’s formula or you can have @RISK help you in its Add Output dialog:

![@RISK - Add/Edit Output: Cell C15](image)

From here, you can click the \( fx \) button to get the Output Properties dialog. Then you can fill out the Six Sigma tab.

![Output Properties: C15](image)

This will add the RiskOutput function, together with the RiskSixSigma function, to the cell’s formula.

An equivalent method is to choose the the RiskOutput (Six-Sigma Format) from the Output group on @RISK’s Insert Function dropdown list.

The default settings for an output to be used in Six Sigma calculations are set on the Six Sigma tab. These properties include:
Appendix C: @RISK and Six Sigma

- **Calculate Capability Metrics for This Output.** Specifies that capability metrics will be displayed in reports and graphs for the output. These metrics will use the entered LSL, USL and Target values.

- **LSL, USL and Target.** Sets the LSL (Lower Specification Limit), USL (Upper Specification Limit) and Target values for the output.

- **Use Long Term Shift and Shift.** Specifies an optional shift for calculation of long-term capability metrics.

- **Upper/Lower X Bound.** The number of standard deviations to the right or the left of the mean for calculating the upper or lower X-axis values.

Only outputs that contain a RiskSixSigma property function will display Six Sigma markers and statistics in graphs and reports. @RISK Six Sigma statistics functions in Excel worksheets can reference any output cell that contains a RiskSixSigma property function. This is explained in more detail in the earlier Reference: Six Sigma Functions section of this manual.

**Note:** All graphs and reports in @RISK use the LSL, USL, Target, Long Term Shift and the Number of Standard Deviations values from RiskSixSigma property functions that existed at the start of a simulation. If you change the specification limits for an output (and its associated RiskSixSigma property function), you need to re-run the simulation to view changed graphs and reports.
Six Sigma Statistics Functions

@RISK includes a set of Six Sigma statistics functions. These are discussed in detail in the earlier Reference: Six Sigma Functions section of this manual, so that information is not repeated here.

The following screenshot provides an example. Cell C15 contains a RiskOutput function with a RiskSixSigma property function:

=RiskOutput(C14,,RiskSixSigma(C4,C5,C6,0,6))
+RiskNormal(C10,C11)

Then the green cells in column C contain the following Six Sigma statistics functions:

=RiskCpk(C15)
=RiskPNC(C15)
=RiskDPM(C15)

These statistics functions, like other @RISK statistics functions, show relevant results only after a simulation has been run. They rely on the parameter values (LSL, USL, and so on) in the RiskSixSigma property function in cell C15 for their values.

Note also in this screenshot how the graph of the output in C15 shows the LSL, Target, and USL as markers. These markers derive their information from the RiskSixSigma property function in cell C15.
Six Sigma and the Results Summary Window

As always, @RISK’s Results Summary window summarizes the results of your model and displays thumbnail graphs and summary statistics for your simulated output cells and input distributions. However, when @RISK detects a RiskSixSigma property function in an output, it automatically displays the available Six Sigma statistics on the simulation results for the output in the table.

The Results Summary window columns can be customized to select which statistics you want to display on your results. The Columns icon at the bottom of the window (second from the right) displays the Columns for Table dialog. The list contain the usual statistics, plus a number of statistics relevant to Six Sigma.

If you like, you can click the Edit and Export button at the bottom of the Results Summary window to export the results to Excel worksheets.
Six Sigma Markers on Graphs

As mentioned earlier, when @RISK detects a `RiskSixSigma` property function in an output, it adds markers for the entered LSL, USL and Target values to graphs of simulation results for the output.

You can remove these markers if you like from the Markers tab of the Graph Options dialog, or you can add additional markers.
Appendix D: Sampling Methods

Sampling is used in an @RISK simulation to generate possible values from probability distribution functions. These sets of possible values are then used to evaluate your Excel worksheet model. Because of this, sampling is the basis for the hundreds or thousands of “what-if” scenarios @RISK calculates for your model. Each set of samples represents a possible combination of input values that could occur. Choosing a sampling method affects both the quality of your results and the time necessary to run the simulations.

What is Sampling?

Sampling is the process by which values are randomly drawn from input probability distributions. Probability distributions are represented in @RISK by probability distribution functions, and sampling is performed by the @RISK program. Sampling in a simulation is done, repetitively, with one sample drawn every iteration from each input probability distribution. With enough iterations, the sampled values for a probability distribution become distributed in a manner that approximates the known input probability distribution. The statistics of the sampled distribution (mean, standard deviation, and others) approximate the theoretical statistics for the input distribution. In addition, the graph of the sampled distribution will look like a graph of the theoretical input distribution.

Statisticians and practitioners have developed several techniques for drawing random samples. The important factor to examine when evaluating sampling techniques is the number of iterations required to accurately recreate an input distribution through sampling. Accurate results for output distributions depend on a complete sampling of input distributions. If one sampling method requires more iterations and longer simulation runtimes than another to approximate input distributions, it is a less “efficient” method.
The two methods of sampling used in @RISK, Monte Carlo sampling and Latin Hypercube sampling, differ in the number of iterations required until sampled values approximate input distributions to any degree of accuracy. Monte Carlo sampling often requires a larger number of samples, especially if the input distribution is highly skewed or has some outcomes of low probability. Latin Hypercube sampling forces the samples drawn to correspond more closely with the input distribution, and it converges faster.

**Cumulative Distribution**

It is often helpful, when reviewing different sampling methods, to first understand the concept of a cumulative distribution. Any probability distribution may be expressed in cumulative form. A cumulative curve is scaled from 0 to 1 on the Y-axis, with Y-axis values representing the cumulative probability up to the corresponding X-axis value.

In this cumulative curve, the 0.5 cumulative value is the point of 50% cumulative probability (0.5 = 50%). Fifty percent of the values in the distribution fall below this median value and 50% are above. The 0 cumulative value is the minimum value (0% of the values will fall below this point) and the 1.0 cumulative value is the maximum value (100% of the values will fall below this point).
Why is this cumulative curve so important to understanding sampling? The 0 to 1 scale of the cumulative curve is the range of the possible random numbers generated during sampling. In a typical sampling sequence, the computer will generate a random number between 0 and 1, with any number in the range equally likely to occur. This number is then used to select a value from the cumulative curve. In the example graph, the value sampled for the distribution shown would be X1 if a random number of 0.5 was generated during sampling. Because the shape of the cumulative curve is based on the shape of the input probability distribution, it is more probable that more likely outcomes will be sampled. The more likely outcomes are in the range where the cumulative curve is the steepest.

**Monte Carlo Sampling**

Monte Carlo sampling refers to the traditional technique for generating random or pseudo-random numbers to sample from a probability distribution. The term Monte Carlo was introduced during World War II, as a code name for the simulation of problems associated with the development of the atomic bomb. Today, Monte Carlo techniques are applied to a wide variety of complex problems involving random behavior. A wide variety of algorithms are available for generating random samples from different types of probability distributions.

Actually, the term Monte Carlo is used in two ways in @RISK. It is used to describe the overall simulation process, as in "Monte Carlo simulation." This distinguishes @RISK-type simulations from other types of simulation, such as discrete-event simulations. However, the second use of the term, "Monte Carlo sampling," is more restrictive and is relevant to the current discussion. This term applies only to the way random values are generated during a simulation. Another way to say it is that you can perform either Monte Carlo sampling or Latin Hypercube sampling in a Monte Carlo simulation.

When Monte Carlo sampling is performed, each draw from a uniform distribution over 0 to 1 is independent of each other draw, with one draw per iteration for each input distribution. Each of these draws provides a Y value for the cumulative curve, which is then mapped to the relevant X value as shown in the above graph. This is a perfectly acceptable procedure in the "long run," but in the "short run" (a small number of draws), it can lead to clustering, where not all of the relevant distribution is represented.

The following graph illustrates this clustering. The five generated Y values just happen to be close to each other. This leads to five X
values close to one another as well. The rest of the X values haven’t been sampled at all.

Clustering becomes especially pronounced when a distribution includes low probability outcomes, which could have a major impact on your results. It is important to include the effects of these low probability outcomes. To do this, these outcomes must be sampled, but if their probability is low enough, a small number of Monte Carlo iterations might not sample sufficient quantities of these outcomes for inclusion in your simulation model. This problem has led to the development of stratified sampling techniques such as the Latin Hypercube sampling used in @RISK.

**Latin Hypercube Sampling**

Latin Hypercube sampling is a more recent development in sampling technology. It is designed to avoid the clustering that can occur with Monte Carlo sampling. The key to Latin Hypercube sampling is stratification of the input probability distributions. Stratification divides the cumulative curve into equal intervals on the cumulative probability scale (0 to 1). A sample is then randomly taken from each interval or “stratification” of the input distribution. Sampling is forced to represent values in each interval. Therefore, all values in the input distribution have a better chance at being sampled.

In the following graph, the Y axis has been divided into five intervals. During sampling, for every five draws, one of the draws will fall in each of the five intervals; they can’t cluster as in the Monte Carlo method. With Latin Hypercube, the samples are guaranteed to more accurately reflect the distribution of values in the input probability distribution.
Latin Hypercube and Low Probability Outcomes

When using the Latin Hypercube technique to sample from multiple variables, it is important to maintain independence between variables. The values sampled, for one variable, need to be independent of those sampled for another (unless, of course, you explicitly want them correlated). This independence is maintained by randomly selecting the interval to draw a sample from for each variable. In a given iteration, Variable #1 may be sampled from strata #4, Variable #2 may be sampled from strata #2, and so on. This preserves randomness and independence, and avoids unwanted correlation between variables.
Latin Hypercube offers great benefits in terms of increased sampling efficiency and faster runtimes (due to fewer iterations). These gains are especially noticeable in a PC-based simulation environment such as @RISK. Latin Hypercube also aids the analysis of situations, where low probability outcomes are represented in input probability distributions. By forcing the sampling of the simulation to include the outlying events, Latin Hypercube sampling assures they are accurately represented in your simulation outputs.

When low probability outcomes are very important, it often helps to run an analysis that simulates just the contribution to the output distribution from the low probability events. In this case, the model simulates only the occurrence of low probability outcomes. This enables you to isolate those outcomes and directly study the results they generate.

The concept of convergence is used to test a sampling method. At the point of convergence, the output distributions are stable in the sense that additional iterations do not markedly change the shape or statistics of the sampled distribution. The sample mean versus the true mean is typically a measure of convergence, but skewness, percentiles, and other statistics can be used as well.

@RISK provides a good environment for testing the speed at which the two available sampling techniques converge on an input distribution. You can run an equal number of iterations with each of the sampling techniques, while selecting an input distribution function as a simulation output. Using the built-in Convergence Monitoring capability in @RISK, you will see how many iterations it takes the mean, standard deviation, and percentiles to stabilize. It should be evident that Latin Hypercube sampling converges faster to the true distributions than Monte Carlo sampling.

**More about Sampling Techniques**

The academic and technical literature has addressed both Monte Carlo and Latin Hypercube sampling. Any of the references to simulation in the Recommended Readings give an introduction to Monte Carlo sampling. References which specifically address Latin Hypercube sampling are included in a separate section.

For a hand-on comparison of the two sampling methods, you can view the two @RISK example files, Latin Hypercube Sample.xlsx and Monte Carlo vs Latin Hypercube Sampling.xlsx.
Appendix E: Using @RISK With Other DecisionTools

Palisade’s DecisionTools Suite is a complete set of decision analysis solutions for Microsoft Windows. With the introduction of DecisionTools, Palisade brings you a decision-making suite whose components combine to take full advantage of the power of your spreadsheet software.

The DecisionTools Suite

The DecisionTools Suite focuses on providing advanced tools for any decision, from risk analysis, to sensitivity analysis, to distribution fitting. Software packaged with the DecisionTools Suite includes:

- @RISK — risk analysis using Monte-Carlo simulation
- TopRank® — sensitivity analysis
- PrecisionTree® — decision analysis with decision trees and influence diagrams
- NeuralTools® — neural networks in Excel
- Evolver® — genetic optimization in Excel
- StatTools® — statistics in Excel
- BigPicture® — smart drawing in Excel

While the tools listed above can be purchased and used separately, they become more powerful when used together. This chapter explains many of the ways the components of the DecisionTools suite interact, and how they will make your decision making easier and more effective.
Purchasing Information

All of the software mentioned here, including the DecisionTools Suite, can be purchased directly from Palisade Corporation. To place an order or receive more information, please contact the technical sales department at Palisade Corporation using one of the following methods:

- **Telephone:** (800) 432-7475 (U.S. only) or (607) 277-8000
  Mon-Fri. from 8:30 AM to 5:00 PM, EST
- **Fax:** (607) 277-8001
- **E-mail:** sales@palisade.com
- **Visit us on the Web:** at [http://www.palisade.com](http://www.palisade.com)
- **Or, mail a letter to:**
  
  Technical Sales  
  Palisade Corporation  
  798 Cascadilla St  
  Ithaca, NY 14850  
  USA

To contact Palisade Europe:

- **E-mail:** sales@palisade-europe.com.
- **Telephone:** +44 1895 425050 (UK).
- **Fax:** +44 1895 425051 (UK).
- **Or, mail a letter to:**
  
  Palisade Europe  
  31 The Green  
  West Drayton  
  Middlesex  
  UB7 7PN  
  United Kingdom

If you want to contact Palisade Asia-Pacific:

- **Email us at** sales@palisade.com.au
- **Telephone us at** + 61 2 9252 5922 (AU).
- **Fax us at** + 61 2 9252 2820 (AU).
- **Mail us a letter to:**
  
  Palisade Asia-Pacific Pty Limited  
  Suite 404, Level 4  
  20 Loftus Street  
  Sydney NSW 2000  
  Australia
Palisade’s DecisionTools Case Study

Excelsior Electronics currently makes desktop computers. They are working on a laptop computer, the Excelsior 5000, and want to know whether the company will profit from this venture. They plan to build a spreadsheet model which spans the next two years. The model should take into account production costs, marketing, shipping, price per unit, units sold, and so on. The bottom line for the planning horizon is overall profit. Excelsior expects some initial setbacks on this venture, but as long as these setbacks are not too bad and profits are up by the end of two years, they will go ahead with the E5000.

The company executive first uses BigPicture to develop a “map,” where the various elements of the problem and their relationships are shown graphically. This map serves as a basis for a discussion of the problem from a high-level view, so that everyone understands the main issues.

Once a basic spreadsheet model of the problem is developed in Excel, TopRank is used on the model to identify the critical variables. The overall profit cell is selected as the critical output, and an automatic what-if analysis is run with TopRank. The results quickly show there are five variables (out of many more) that have the most impact on profit: price per unit, marketing costs, build time, price of memory, and price of CPU chips. Excelsior decides to concentrate on these five input variables.

Distribution functions are needed to replace the five variables in the spreadsheet model. Normal distributions are used for price per unit and build time, based on internal decisions and information from Excelsior’s manufacturing division.

Research is performed to collect weekly price quotes for memory and CPU’s over the past two years. This data is fed into @RISK’s distribution fitting, and distributions are fitted to the data. Confidence level information confirms that the distributions are good fits, and the resulting @RISK distribution functions are entered into the model.
Once all the @RISK functions are in place, the profit cell is selected as an @RISK output cell, and a simulation is run. Overall, the results look promising. Although there will be losses initially, there is an 85% chance the company will make an acceptable profit. In fact, there is a 25% chance that the venture will generate even more revenue than had initially been assumed. Therefore, Excelsior 5000 project is given the go-ahead.

Excelsior Electronics had assumed they would sell, and distribute, the Excelsior 5000 themselves. However, they could use various catalogs and computer warehouses to distribute their product. A decision tree model is built using PrecisionTree, taking into account unit prices, sales volume, and other critical factors for direct sales versus catalog sales. A decision analysis is run, and PrecisionTree suggests using catalogs and warehouses. Excelsior Electronics puts that plan into full motion.
TopRank®

Palisade’s TopRank add-in is the ultimate what-if tool for spreadsheets, from Palisade Corporation. TopRank greatly enhances the standard what-if and data table capabilities found in Excel. In addition, it leads naturally to powerful risk analysis with its companion add-in, @RISK.

**TopRank and What-if Analysis**

TopRank runs an automated what-if sensitivity analysis to help you identify which spreadsheet value(s) or variable(s) affects your bottom-line results the most. You can have TopRank automatically try any number of values for a variable (as in an Excel data table) and tell you the results calculated at each value. You can also have TopRank try all possible combinations of values for a set of variables (a multi-way what-if analysis), giving you the results calculated for each combination.

TopRank is a spreadsheet add-in for Microsoft Excel. It can be used with any pre-existing, or new, spreadsheet model. To set up your what-if analyses, TopRank adds new custom “Vary” functions to Excel’s function set. These functions specify how the values in your spreadsheet will be varied in a what-if analysis; for example, +10% and -10%, +1000 and -500, or according to a table of values you’ve entered.

TopRank can also run a fully automatic what-if analysis. It uses powerful auditing technology to find all possible variables in your spreadsheet model that could affect your bottom-line outputs. It can then vary these variables automatically and identify which are most significant in determining your results.

TopRank applications are the same as spreadsheet applications. If you can build your model in a spreadsheet, you can use TopRank to analyze it. Businesses use TopRank to identify the critical factors — price, up front investment amount, sales volume, or overhead — that most affect the success of their new product. Engineers use TopRank to identify the individual product components whose quality most affects final product production rates. A loan officer can have TopRank quickly run a model at any possible interest rate, loan principle amount, and down payment combinations, and review results for each possible scenario. Whether your application is in business, science, engineering, accounting, or another field, TopRank can work for you to identify the critical variables that affect your results.
TopRank Modeling Features

Why TopRank?

As an add-in to Microsoft Excel, TopRank links directly to your spreadsheet to add what-if analysis capabilities. The TopRank system provides all the necessary tools for conducting a what-if analysis on any spreadsheet model. And TopRank works in a style you are familiar with: Excel style menus and functions.

You probably already perform what-if analysis in Excel in an ad hoc manner, by changing input values in various cells and see how results change. You might even use the more structured Excel data tables to vary one or inputs in a systematic manner to see their effect on results. The advantage of TopRank is that it performs these tasks automatically and analyzes their results for you. It instantly performs what-ifs on all input variables in your spreadsheet model that could affect your results, instead of requiring you to individually change values and recalculate. It then shows the what-if results in easily understandable reports and graphs.

Multi-Way What-if Analysis

TopRank also runs data table combinations automatically, without requiring you to set up tables in your spreadsheet. You can combine more than two variables in its multi-way what-if analysis, and then you can rank the combinations of input values by their affect on your results. You can perform these sophisticated and automated analyses quickly, as TopRank keeps track of all the values and combinations it tries, and their results, separate from your spreadsheet. By taking an automated approach, TopRank gives you what-if results, including multi-way what-if results, almost instantly. Even the least experienced modeler can get powerful analysis results.

TopRank Functions

TopRank defines variations in input variables by using functions. To do this, TopRank adds a set of new “Vary” functions to the Excel function set, each of which specifies a particular type of variation for your variables. These functions include:

- **RiskVary** and **RiskAutoVary** functions which, during a what-if analysis, change the value of an input variable across a +/- range you define.

- **RiskVaryTable** functions which, during a what-if analysis, substitute each of a table of values for in input variable.

TopRank Pro also includes over 30 probability distribution functions found in @RISK. These functions can be used, along with “Vary” functions, to describe variation in spreadsheet values.
TopRank functions are entered wherever you want to try different values in a what-if analysis. The functions can be added to any number of cells in a spreadsheet, and they can include cell references and expressions as arguments. This provides extreme flexibility in defining variation in input variables in spreadsheet models.

TopRank can even automatically enter “Vary” functions for you. You can use this powerful feature to quickly analyze your spreadsheet model, without manually identifying variables to vary and typing in functions.

When automatically entering “Vary” functions, TopRank traces back through your spreadsheet and finds all possible input cells that could affect the result cell you identify. When it finds such an input cell, it substitutes in a RiskAutoVary function with the default variation parameters (such as -10% to +10%) you’ve selected. With a set of RiskAutoVary functions inserted, TopRank can then run its what-if analysis, ranking the variables in order of importance.

With TopRank, you can step through your RiskVary and RiskAutoVary functions and change the variation each function specifies. The default is to use a -10% to +10% variation, but change this as you like. You can also select to not vary an input variable in case its value is fixed and could never be changed.

During its analysis TopRank individually changes values for each “Vary” function and recalculates your spreadsheet model using each new value. Each time it recalculates, it collects the new value calculated in each result cell. This process of changing value and recalculating is repeated for each “Vary” and “VaryTable” function. The number of recalculations performed depends on the number of “Vary” functions entered, the number of steps (i.e., values across the min-max range) you want TopRank to try for each function, the number of “VaryTable” functions entered, and the values in each table used.

TopRank ranks all varied values by their impact on each result cell, or output you’ve selected. Impact is defined as the amount of change in the output value that was calculated when the input value was changed. If, for example, the result of your spreadsheet model was 100 prior to changing values, and the result was 150 when an input changed, there is a +50% change in results caused by changing the input.

TopRank results can be view graphically in a tornado, spider or sensitivity graph. These graphs summarize your results to easily show the most important inputs for your results.
Using @RISK with TopRank

What-if analysis is often the first analysis performed on a spreadsheet model. Its results lead to a further refinement of the model, additional analyses, and ultimately, a final decision based on the best possible model. Risk analysis, a powerful analytical technique available using TopRank's companion product, @RISK, is often the next analysis performed on a spreadsheet after a what-if analysis.

Moving from What-if to Simulation

A what-if analysis initially identifies the important inputs in your model. You can then focus on these important inputs and better estimate what their values could be. Usually, however, there are several of these important uncertain inputs, and, in reality, they could all vary at the same time. To analyze an uncertain model such as this, you need risk analysis with Monte Carlo simulation. Risk analysis varies all uncertain inputs simultaneously, just as they do in real life, and builds a range and distribution of the possible results that could occur.

With risk analysis, inputs are described with probability distributions such as normal, lognormal, beta, or binomial. This is a much more detailed description of the uncertainty present in an input’s value than a simple + or − percentage variation. A probability distribution shows both the range of values possible for an input and the likelihood of occurrence of any value in the range. Simulation combines these input distributions to generate both a range of possible results from your model, and the likelihood of any result occurring.

The simple +/− change defined by a “Vary” function in a what-if analysis can be used directly in risk analysis. @RISK actually samples your “Vary” functions directly in a risk analysis.

The values sampled by @RISK from “Vary” and “VaryTable” functions, during a simulation, depend on either distribution argument entered for the function, or the default distribution setting used in TopRank. For example, the TopRank function RiskVary(100,-10,+10), using a default distribution setting of Uniform and a default range type of +/- percentage, is sampled like the @RISK distribution RiskUniform(90,110). “VaryTable” functions from TopRank are sampled as RiskDUniform functions in @RISK.
The Differences between TopRank and @RISK

TopRank and @RISK share many common features, so it's easy to think that they perform the same functions. In fact, the two programs perform different, but complementary, tasks. If you’re wondering whether to use TopRank or @RISK, the best answer is that you can benefit from using both of them.

The Similarities

Both @RISK and TopRank are add-ins for analysis of models designed in spreadsheets. By using special spreadsheet formulas, both programs explore how uncertainty affects your model, and thus the decisions you make. Also, a common user interface guarantees a smooth transition between the two products: one learning curve instead of two.

The Differences

There are three main areas where @RISK and TopRank differ:

- **Inputs**: how uncertainty is defined in your model
- **Calculations**: what happens during an analysis
- **Results**: what types of answers the analyses provide

**Inputs**

@RISK defines uncertainty in your model using probability distribution functions. These functions define all the possible values an input can have, with a corresponding probability of that value occurring. There are over 30 probability distribution functions available in @RISK.

To define uncertainty in @RISK, you need to assign a distribution function to every value that you think is uncertain. It's up to you, the user, to determine which inputs are uncertain, and which distribution function describes the uncertainty.

TopRank defines uncertainty in your model using several types of “Vary” functions. “Vary” functions are simple: they define possible values that an input can have without assigning probabilities to these values.

TopRank can automatically define variable cells in your model every time you select an output. You don't need to know which cells are uncertain or important, TopRank identifies those cells for you.

**Calculations**

@RISK runs a Monte Carlo or Latin Hypercube simulation. For each iteration (or step), every @RISK distribution in the spreadsheet model takes on a new value determined by the probability distribution function. To run a thorough analysis, @RISK needs to run hundreds or thousands of iterations.
TopRank runs a single or multi-way sensitivity analysis. During the analysis, only one cell (or a small number of cells) varies at a time according to the values defined in the “Vary” function. With TopRank, only a few iterations are needed to study a large number of uncertain cells.

For each output defined, @RISK produces a probability distribution as an analysis result. The distribution describes which values an output (such as profit) could have, as well as how probable certain outcomes are. For example, @RISK can tell you that there is a 30% chance that your company will not make a profit next quarter.

For each output defined, TopRank tells you which inputs have the largest effect on the output. The results show how much change you can expect in an output when a given input changes by a defined amount. For example, TopRank can tell you that your company’s profit is most sensitive to sales volume, and that when the sales volume is 1000 units, you will lose $1 million.

The most important difference between the two packages is that @RISK studies how the combined uncertainty of all variables affect the output. TopRank only tells you how an individual input (or a small group of inputs) affects the output. So, while TopRank is faster and easier to use, @RISK provides a more detailed, comprehensive analysis the problem. We strongly recommend using TopRank first to determine which variables are the most important. Then, you can use @RISK to run a comprehensive analysis of your problem for the best possible results.

In summary, TopRank tells you what the most important variables are in your model. The results of a TopRank what-if analysis can be used on their own to make better decisions. But for the most thorough analysis, you should use TopRank to find the most important variables in your model, and then use @RISK to define uncertainty in those variables and run a simulation. TopRank can help you guide your @RISK simulations by defining uncertainty in only the most important variables, making your simulation faster and more compact.
Palisade’s PrecisionTree add-in is a decision analysis add-in to Microsoft Excel. It enables you to define a decision tree or influence diagram directly in your spreadsheet. PrecisionTree allows you to run a complete decision analysis, without leaving the program where your data is, your spreadsheet.

**Why You Need Decision Analysis and PrecisionTree**

You might wonder whether the decisions you make are suitable for decision analysis. If you are looking for a way to structure your decisions, to make them more organized and easier to explain to others, you should definitely consider using formal decision analysis.

When faced with a complex decision, decision makers must be able to organize the problem efficiently. They have to consider all possible options by analyzing all available information. In addition, they need to present this information to others in a clear, concise format. PrecisionTree allows decision makers to do all this, and more.

But, what exactly does decision analysis allow you to do? As the decision maker, you can clarify options and rewards, describe uncertainty quantitatively, weigh multiple objectives simultaneously, and define risk preferences. And you can do all of this in an Excel spreadsheet.

**PrecisionTree Modeling Features**

As an add-in to Microsoft Excel, PrecisionTree links directly to Excel to add decision analysis capabilities. PrecisionTree provides all the necessary tools for setting up and analyzing decision trees and influence diagrams. And PrecisionTree works in a style you are familiar with: Excel-style menus and toolbars.

With PrecisionTree, there is no limit to the size of tree you can define. You can design a tree that spans multiple worksheets in an Excel workbook. PrecisionTree reduces the tree to an easy-to-understand report right in your current workbook.
PrecisionTree allows you to define decision tree nodes in Excel spreadsheets. Node types offered by PrecisionTree include:

- Chance nodes
- Decision nodes
- End nodes
- Logic nodes
- Reference nodes

Values and probabilities for nodes are placed directly in spreadsheet cells, allowing you to easily enter and edit the definition of your decision models.

PrecisionTree creates both decision trees and influence diagrams. Influence diagrams are excellent for showing the relationship between events and the general structure of a decision clearly and concisely, while decision trees outline the chronological and numerical details of the decision.

In PrecisionTree, all decision model values and probabilities are entered directly in spreadsheet cells, just like other Excel models. PrecisionTree can also link values in a decision model directly to locations you specify in a spreadsheet model. The results of that model are then used as the **payoff** for each path through the decision tree.

All calculations of **payoff** happen in real time, that is, as you edit your tree, all **payoff** and node values are automatically recalculated.

PrecisionTree's decision analyses give you straightforward reports, including statistical reports, risk profiles, and policy suggestions. Also, decision analysis can produce more qualitative results by helping you understand trade-offs, conflicts of interest, and important objectives.

All analysis results are reported directly in Excel for easy customization, printing, and saving. There is no need to learn a whole new set of formatting commands because all PrecisionTree reports can be modified like any other Excel worksheet or chart.

Have you ever wondered which variables matter most in your decision? If so, you need PrecisionTree's sensitivity analysis options. You can perform one-way and two-way sensitivity analyses, and generate tornado graphs, spider graphs, strategy region graphs, and more.

If you need more sophisticated sensitivity analyses, PrecisionTree links directly to TopRank, Palisade's sensitivity analysis add-in.
Because decision trees can expand as more possible decision options are added, PrecisionTree offers a set of features designed to help you reduce trees to a more manageable size. All nodes can be collapsed, hiding all paths that follow the node. Subtrees can be copied and pasted to other parts of the tree, saving you the repeated re-entry of the same subtree structure.

PrecisionTree contains lets you apply built-in utility functions to a decision tree. These encode your attitude toward risk and are especially useful when large amounts of money are at stake. You can even create your own utility functions.

Using @RISK with PrecisionTree

@RISK is a perfect companion to PrecisionTree. @RISK allows you to quantify the uncertainty in the values and probabilities that define your decision trees and thereby more accurately describe chance events as a continuous range of possible outcomes. Using this information, @RISK performs a Monte-Carlo simulation on your decision tree, analyzing every possible outcome and graphically illustrating the risks you face.

Using @RISK to Quantify Uncertainty

With @RISK, all uncertain values and probabilities for branches in your decision trees, and supporting spreadsheet models, can be defined with distribution functions. When a branch from a decision or chance node has an uncertain value, for example, this value can be described by an @RISK distribution function. During a normal decision analysis, the expected value of the distribution function will be used as the value for the branch. The expected value for a path in the tree will be calculated using this value.

However, when a simulation is run using @RISK, a sample will be drawn from each distribution function during each iteration of the simulation. The value of the decision tree, and its nodes, will then be recalculated using the new set of samples and the results recorded by @RISK. A range of possible values will then be displayed for the decision tree. Instead of seeing a risk profile with a discrete set of possible outcomes and probabilities, a continuous distribution of possible outcomes is generated by @RISK. You can see the chance of any result occurring.
In decision trees, chance events must be described in terms of discrete outcomes (a chance node with a finite number of outcome branches). But in real life, many uncertain events are continuous, meaning that any value between a minimum and maximum can occur.

Using @RISK with PrecisionTree, makes modeling continuous events easier, using distribution functions. Also, @RISK functions can make your decision tree smaller and easier to understand.

### Methods of Recalculation during a Simulation

Two options are available for recalculation of a decision model during a simulation performed with @RISK. The first option, Expected Values of the Model, causes @RISK to first sample all distribution functions in the model, and supporting spreadsheets each iteration, then recalculates the model using the new values to generate a new expected value. Typically, the output from the simulation is the cell containing the expected value of the model. At the end of the run an output distribution, reflecting the possible range of expected values for the model and their relative likelihood of occurrence, is generated.

The second option, Values of One Sampled Path Through the Model, causes @RISK to randomly sample a path through the model each iteration of a simulation. The branch to follow from each chance node is randomly selected, based on the branch probabilities entered. This method does not require that distribution functions be present in the model. However, if they are used, a new sample is generated each iteration and used in path value calculations. The output from the simulation is the cell containing the value of the model, such as the value of the root node of the tree. At the end of the run an output distribution reflecting the possible range of output values for the model, and their relative likelihood of occurrence, is generated.

### Using Probability Distributions in Nodes

Let’s take a look at a chance node in an oil drilling decision tree:

The results of drilling are divided into three discrete outcomes (Dry, Wet, and Soaking). But, in reality, the amount of oil found should be described with a continuous distribution. Suppose the amount of money made from drilling follows a lognormal distribution with
mean $22900 and standard deviation $50000, or the @RISK distribution RiskLognorm(22900,50000).

To use this function in the oil drilling model, you change the chance node to have only one branch, and the value of the branch is defined by the @RISK function. Here’s how the new model will look:

During an @RISK simulation, the RiskLognorm function will return random values for the payoff value of the Results node and PrecisionTree will calculate a new expected value for the tree.

But, what about the decision to Drill or Not Drill? If the expected value of the Drill node changes, the optimum decision could change from iteration to iteration. That would imply that we know the outcome of drilling before the decision is made. To avoid this situation, PrecisionTree has an option Decisions Follow Current Optimal Path to force decisions before running an @RISK simulation. Every decision node in the tree will be changed to a forced decision node, which causes each decision node to select the decision that’s optimal when the command is used. This avoids changes in a decision, due to changing a decision tree’s values and probabilities during a risk analysis.

Using @RISK to Analyze Decision Options

Suppose you want to know how much it would be worth to learn which uncertain outcome will occur before having to make a decision. In decision analysis, this is called the value of perfect information. PrecisionTree can calculate this value easily. You simply “flip” the tree so that uncertainty is resolved first and then the decisions are made.
The BigPicture add-in is the newest addition to Palisade’s DecisionTools Suite, introduced in version 7. BigPicture is a smart drawing program, developed to compete with other “mapping” software on the market. However, unlike other mapping software, BigPicture is totally integrated with Excel, and this provides several important advantages discussed below.

BigPicture is aptly named. Its main purpose is to provide a map, or diagram, of the elements of a problem, along with relationships among these elements, so that decision makers can understand and discuss the problem from a high-level view. This was the original intent of BigPicture, and it remains its primary goal, but the developers of the add-in took advantage of its integration with Excel to provide a wealth of smart features not available in other mapping software.

**BigPicture for High-Level Maps**

Suppose a company faces an important acquisition decision. There are several directions the company could go, and each involves a number of trade-offs based on monetary and non-monetary outcomes. BigPicture allows the company to “map” the decision alternatives and possible ramifications, and this map can then be the basis for a high-level discussion.

Here is a possible map to get the discussion started.
This map resides in an Excel worksheet, and it consists basically of Excel shapes, but it is much “smarter” than a collection of shapes you could draw with Excel tools only. For example, the plus signs to the right indicate that the map can be expanded in this direction. Here is one possible expansion:

The advantage of being able to expand or contract a large map should be obvious. It lets the decision makers focus on one part of the decision process at a time.
Linking to Cell Data

Each shape in a BigPicture map (the rectangles and ovals) is called a topic, and any type of information can be entered in a topic. You can type in this information, just as with regular Excel shapes, but a big advantage of BigPicture is that you can cell-reference this information. In other words, the text in a topic can be linked to values or formulas in any Excel cells, even those in a different worksheet. Here is an example:

All of the number in this map reference cell formulas in a "calculations" section of the workbook. If the data in this section change, the map will update automatically.
Free-Form Maps

BigPicture maps can be very structured, as in the above travel cost map, where everything “flows” from left to right and topics are lined up nicely. In fact, this structure can be created automatically by BigPicture, so that you don’t need to waste time lining up and resizing shapes. However, BigPicture also allows you to create “free-form” maps, where you can arrange topics and connectors to suit your needs. This is especially useful for showing the elements of a typical @RISK model, such as the following:

In this case, the color-coding was performed manually to indicate the different types of variables in the model.

If you are using such a map to explain a model to an audience, it might be nice to show the map in a step-by-step manner, explaining each step along the way. BigPicture lets you do this by creating slide shows. Each “slide” is a partial map, where you can decide exactly which topics to include, and it can be accompanied by a text box explaining that step. For example, here is the third step for this map:
Org Charts

As mentioned earlier, the original intent of BigPicture was to create maps of the type shown so far. But it evolved to include much more, specifically, data analysis. One example of this is its ability to create organization charts from tables of employee data. The table should include a row for each employee with any relevant data, including whom this employee reports to. Then BigPicture can create an org chart from this table automatically. Here is an example:

This chart not only shows the organization’s structure, but it shows calculations performed on the table data. Specifically, it shows the total salary of a supervisor and all the employees who report directly or indirectly to that supervisor.
An org chart is relevant for data with natural “parent/child” relationships. BigPicture includes similar maps, called linkes maps, for other parent/child data in a non-org chart context.

**Data Maps**

If you have used Excel pivot tables to “slice and dice” data, that is, to break down data by categories, you will love BigPicture’s data maps. They do essentially what pivot tables do, but in a more graphical format. This is a feature, maybe the feature, that definitely sets BigPicture above competing mapping software.

As with an org chart, a data map requires a data set in an Excel worksheet. Each column of this data set should correspond to a variable. Some variables will be categorical, representing categories you want to break down by, and others will be numeric. You define the categorical variables and numeric variables to be used in the map, including statistical summary measures of the numeric variable to be shown, and BigPicture does the rest. Here is an example of a company’s sales, broken down by region and product:
In this case, each topic on the right represents all products in a given type. By expanding one of these, you can see a table for the product type broken down by country:

![Hair Care/EU/Year / 2011](image)

Or broken down by product:

![Hair Care/EU/Year / 2011](image)
### Appendix F: Glossary

| **Continuous Distribution** | A continuous probability distribution is a distribution where any value in a continuum in a range is possible. |
| **Cumulative Distribution** | A cumulative distribution, or a cumulative distribution function, lists “less than or equal probabilities,” that is, its value at a point x is the probability of being less than or equal to x. |
| **Dependent Variable** | A dependent variable is one that depends in some way on the values of other independent variables in the model under consideration. In risk analysis, this is often an output variable of interest that is “driven” by one or more input variables. |
| **Deterministic Model** | A deterministic model in a model involving no uncertainty. |
| **Discrete Distribution** | A discrete distribution is a probability distribution where only a finite number of discrete values are possible. |
| **Event** | An event is a set of one or more outcomes that might result from a given action. For example, if two dice are thrown, an event of interest might be that the sum of the two dice is 7. |
| **Expected Value** | The weighted average of all possible values of a distribution, weighted by their probabilities; the same as the mean. |
| **Frequency Distribution** | A frequency distribution is a table (usually accompanied by a column bar chart, a histogram) that arranges possible outcomes into classes, or bins, and lists the number of observation in each class. |
| **Higher Moments** | Higher moments are statistics of a probability distribution. The term generally refers to the skewness and kurtosis, the third and fourth moments, respectively. The first and second moments are the mean and the standard deviation, respectively. |
| **Independent Variable** | An independent variable is one that does not depend on the values of any other variable in the model under consideration. In risk analysis, the independent variables are the uncertain input variables, each with a probability distribution. These variables “drive” the important output variables. |
**Iteration**

An iteration is one recalculation of a model during a simulation. A simulation consists of many such iterations. During each iteration, all uncertain variables are sampled once, according to their probability distributions, and the model is recalculated using these sampled values.

**Kurtosis**

Kurtosis is a measure of the shape of a distribution. It indicates how flat, or peaked, the distribution is. The higher the kurtosis value, the more peaked the distribution.

**Latin Hypercube Sampling**

Latin Hypercube is a stratified sampling technique used in simulation modeling. Latin Hypercube sampling forces sampling from all parts of a distribution, and it typically requires fewer simulation iterations than Monte Carlo sampling for the same level of accuracy.

**Mean**

The mean of a set of values is the arithmetic average, that is, the sum of all the values divided by the total number of values; same as expected value.

**Monte Carlo Sampling**

Monte Carlo sampling is the traditional method of sampling random variables in simulation modeling. Samples are chosen completely randomly across the range of the distribution. This can result in clustering, where not all parts of the distribution are adequately sampled, so it tends to be less efficient than Latin Hypercube sampling.

**Most Likely Value**

The most likely value, or mode, is the value that occurs most often in a set of values.

**Objective Risk**

Objective risk, or objective probability, refers to a probability value or distribution that is determined by “objective” evidence or accepted theory. The probabilities associated with an objective risk are known with certainty. For example, the probability of heads in a flip of a fair coin is 0.5. There is no disagreement about this value, assuming the coin is well-balanced.

**Percentile**

A percentile is a values that has a certain percentage to the left of it. For example, the 60th percentile is the value in the data set for which 60% of the values are below it and 40% are above it.

**Preference**

Preference refers to an individual's choices where many attributes of a decision or object are considered. Risk is an important consideration in personal preference.

**Probability**

Probability is a measure of how likely a value or event is to occur. It can be measured from simulation data, as frequency, by calculating the number of occurrences of the value or event, divided by the total number of occurrences. This calculation returns a value between 0 and 1, which can be converted to a percentage by multiplying by 100.
**Probability Distribution**

A probability distribution, or probability density function, is the proper statistical term for a frequency distribution constructed from an infinitely large set of values where the class size is infinitesimally small.

**Random Number Generator**

A random number generator is an algorithm for choosing random numbers, typically in the range from 0 to 1. These random numbers are equivalent to samples drawn from a uniform distribution with a minimum of 0 and a maximum of 1. Such random numbers are the basis for other routines that convert them into samples drawn from specific distribution types.

**Random Sample**

A random sample is a value that has been chosen from a probability distribution describing a random variable. Such a sample is drawn randomly according to a sampling “algorithm.” The frequency distribution constructed from a large number of random samples, drawn by such an algorithm, will closely approximate the probability distribution from which the samples were drawn.

**Range**

The range is the absolute difference between the maximum and minimum values in a set of values. The range is the simplest measure of the dispersion or “risk” of a distribution.

**@RISK**

@RISK (pronounced “at risk”) is the name of the Excel add-in for risk analysis described in this manual.

**Risk**

The term risk refers to uncertainty or variability in the outcome of some event or decision. In many cases the range of possible outcomes can include some that are perceived as a loss, or undesirable, along with others that are perceived as a gain, or desirable. The range of outcomes is often associated with levels of probability of occurrence.

**Risk Analysis**

Risk analysis is a general term used to describe any method used to study and understand the risk inherent in a situation of interest. Methods can be quantitative and/or qualitative in nature. @RISK uses a quantitative technique, generally referred to as simulation.

**Risk Averse**

Risk averse refers to a general characteristic of individuals with regard to their preference for risk. As the payoff and risk increase, the individual will be less likely to choose an action intended to achieve the higher payoff. It is generally assumed that rational individuals are risk averse, although the degree of risk averseness varies from one individual to another. There are situations or ranges of payoffs over which individuals might display the opposite behavior, namely, they will be risk takers.
**Seed**  
The seed is a number that initializes the selection of numbers by a random number generator. Given the same seed, a random number generator will generate the same series of random numbers each time a simulation is run.

**Simulation**  
Simulation is a technique whereby a model, such as an Excel worksheet, is calculated many times with different input values with the intent of getting a complete representation of all possible scenarios that might occur in an uncertain situation.

**Skewness**  
Skewness is a measure of the shape of a distribution. Skewness indicates the degree of asymmetry in a distribution. With skewed distributions, one tail is much longer than the other. A skewness of 0 indicates a symmetric distribution. Negative skewness means the distribution is skewed to the left, and positive skewness indicates a skew to the right.

**Standard Deviation**  
The standard deviation is a measure of how widely dispersed the values in a distribution are. It equals the square root of the variance.

**Stochastic**  
Stochastic is a synonym for uncertain or risky.

**Subjective Risk**  
Subjective risk, or subjective probability, is a probability value or distribution determined by an individual's best estimate based on personal knowledge, expertise, and experience. New information often causes changes in such estimates. Reasonable individuals can disagree on such estimates.

**Summary Graph**  
A Summary Graph is an output graph in @RISK that presents simulation results for a range of cells in an Excel worksheet. It is often used for a time series of values, such as cash flows, to show how the uncertainty changes through time.

**Trial**  
Trial is sometimes another term for iteration. However, in the context of optimization with RISKOptimizer, each trial represents a possible solution (a set of adjustable cell values), and for each trial, a simulation is run for a given number of iterations.

**Truncation**  
Truncation means that sampling from a given distribution is restricted to a limited range. The limits can be on both sides or one side only. For example, it is common to truncate a normal distribution on the low side at 0 to prevent negative values.

**Variable**  
A variable refers to any input or output in a risk analysis. The term derives from algebraic models, where variables are represented by symbols such as x and y. In spreadsheet models, however, variables correspond to cell values, and in risk models, these variables can include uncertainty.
**Variance**

The variance is a measure of how widely dispersed the values are in a distribution, and thus is an indication of the “risk” of the distribution. It is calculated as the average of the squared deviations about the mean. The variance is the square of the standard deviation.
Appendix G: Recommended Readings

This manual has given you a start on understanding the concepts of risk analysis and simulation. If you are interested in learning more about risk analysis, and the theory behind it, here are some relevant books and articles you might want to explore. (There might be more recent editions of some of these books. You can check online.)

**Introduction to Risk Analysis**

If you are new to risk analysis or if you would just like some more background information on the technique, the following books and articles are helpful:


Distributions and Distribution Fitting

If you are interested in finding out more about distributions and distribution fitting, you can consult any of these books:


References to Simulation and Monte Carlo Techniques

If you would like a more in depth examination of simulation, sampling techniques, and statistical theory, the following are useful:


References to Latin Hypercube Sampling Techniques

To learn more about Latin Hypercube sampling, the following sources are useful:


Examples and Case Studies Using Risk Analysis

The following references present case studies showing the use of Risk Analysis in real life situations:


*Nersesian, Roy L. @RISK Bank Credit and Financial Analysis: Palisade Corporation, 2011.

*Nersesian, Roy L. RISKOptimizer for Business Applications: Palisade Corporation, 2011.


* These titles can be purchased through Palisade Corporation. Call (800-432-7475 or 607-277-8000), fax (607-277-8001), or write to order or request further information on these and other titles relevant to risk analysis. The Palisade Technical Sales Department can also be reached by e-mail at sales@palisade.com or on the Web at http://www.palisade.com.
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