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Welcome

@RISK for Microsoft Excel

Welcome to @RISK, the revolutionary software system for the analysis of business and technical situations impacted by risk! The techniques of Risk Analysis have long been recognized as powerful tools to help decision-makers successfully manage situations subject to uncertainty. Their use has been limited because they have been expensive, cumbersome to use, and have substantial computational requirements. However, the growing use of computers in business and science has offered the promise that these techniques can be commonly available to all decision-makers.

That promise has been finally realized with @RISK (pronounced “at risk”) — a system which brings these techniques to the industry standard spreadsheet package, Microsoft Excel. With @RISK and Excel any risky situation can be modeled, from business to science and engineering. You are the best judge of what your analysis needs require, and @RISK, combined with the modeling capabilities of Excel, allows you to design a model which best satisfies those needs. Anytime you face a decision or analysis under uncertainty, you can use @RISK to improve your picture of what the future could hold.

Why You Need Risk Analysis and @RISK

Traditionally, analyses combine single “point” estimates of a model's variables to predict a single result. This is the standard Excel model — a spreadsheet with a single estimate of results. Estimates of model variables must be used because the values which actually will occur are not known with certainty. In reality, however, many things just don't turn out the way that you have planned. Maybe you were too conservative with some estimates and too optimistic with others. The combined errors in each estimate often lead to a real-life result that is significantly different from the estimated result. The decision you made based on your “expected” result might be the wrong decision, and a decision you never would have made if you had a more complete picture of all possible outcomes. Business decisions, technical decisions, scientific decisions ... all use estimates and assumptions. With @RISK, you can explicitly include the uncertainty present in your estimates to generate results that show all possible outcomes.
@RISK uses a technique called “simulation” to combine all the uncertainties you identify in your modeling situation. You no longer are forced to reduce what you know about a variable to a single number. Instead, you include all you know about the variable, including its full range of possible values and some measure of likelihood of occurrence for each possible value. @RISK uses all this information, along with your Excel model, to analyze every possible outcome. It's just as if you ran hundreds or thousands of “what-if” scenarios all at once! In effect, @RISK lets you see the full range of what could happen in your situation. It's as if you could “live” through your situation over and over again, each time under a different set of conditions, with a different set of results occurring.

All this added information sounds like it might complicate your decisions, but in fact, one of simulation's greatest strengths is its power of communication. @RISK gives you results that graphically illustrate the risks you face. This graphical presentation is easily understood by you, and easily explained to others.

So when should you use @RISK? Anytime you make an analysis in Excel that could be affected by uncertainty, you can and should use @RISK. The applications in business, science and engineering are practically unlimited and you can use your existing base of Excel models. An @RISK analysis can stand alone, or be used to supply results to other analyses. Consider the decisions and analyses you make every day! If you've ever been concerned with the impact of risk in these situations, you've just found a good use for @RISK!

**Modeling Features**

As an “add-in” to Microsoft Excel, @RISK “links” directly to Excel to add Risk Analysis capabilities. The @RISK system provides all the necessary tools for setting up, executing and viewing the results of Risk Analyses. And @RISK works in a style you are familiar with — Excel style menus and functions.

@RISK allows you to define uncertain cell values in Excel as probability distributions using functions. @RISK adds a set of new functions to the Excel function set, each of which allows you to specify a different distribution type for cell values. Distribution functions can be added to any number of cells and formulas throughout your worksheets and can include arguments which are cell references and expressions — allowing extremely sophisticated specification of uncertainty. To help you assign distributions to uncertain values, @RISK includes a graphical pop-up window where distributions can be previewed and added to formulas.
The probability distributions provided by @RISK allow the specification of nearly any type of uncertainty in cell values in your spreadsheet. A cell containing the distribution function NORMAL(10,10), for example, would return samples during a simulation drawn from a normal distribution (mean = 10, standard deviation = 10). Distribution functions are only invoked during a simulation — in normal Excel operations, they show a single cell value — just the same as Excel before @RISK. Available distribution types include:

<table>
<thead>
<tr>
<th>Beta</th>
<th>Beta-General</th>
<th>Beta-Subjective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binomial</td>
<td>Chi-Square</td>
<td>Cumulative</td>
</tr>
<tr>
<td>Discrete</td>
<td>Discrete Uniform</td>
<td>Error Function</td>
</tr>
<tr>
<td>Erlang</td>
<td>Exponential</td>
<td>Extreme Value</td>
</tr>
<tr>
<td>Gamma</td>
<td>General</td>
<td>Geometric</td>
</tr>
<tr>
<td>Histogram</td>
<td>Hypergeometric</td>
<td>Inverse Gaussian</td>
</tr>
<tr>
<td>IntUniform</td>
<td>Logistic</td>
<td>Log-Logistic</td>
</tr>
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<td>Lognormal</td>
<td>Lognormal2</td>
<td>Negative Binomial</td>
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<tr>
<td>Normal</td>
<td>Pareto</td>
<td>Pareto2</td>
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<td>Pearson V</td>
<td>Pearson VI</td>
<td>PERT</td>
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All distributions may be truncated to allow only samples within a given ranges of values within the distribution. Also, many distributions can also use alternate percentile parameters. This allows you to specify values for specific percentile locations of an input distribution as opposed to the traditional arguments used by the distribution.

@RISK has sophisticated capabilities for specifying and executing simulations of Excel models. Both Monte Carlo and Latin Hypercube sampling techniques are supported, and distributions of possible results may be generated for any cell or range of cells in your spreadsheet model. Both simulation options and the selection of model outputs are entered with Windows style menus, dialog boxes and use of the mouse.

High resolution graphics are used to present the output distributions from your @RISK simulations. Histograms, cumulative curves and summary graphs for cell ranges all lead to a powerful presentation of results. And all graphs may be displayed in Excel for further enhancement and hard copy. An essentially unlimited number of output distributions may be generated from a single simulation — allowing for the analysis of even the largest and most complex spreadsheets!
The options available for controlling and executing a simulation in @RISK are among the most powerful ever available. They include:

- Latin Hypercube or Monte Carlo sampling
- Any number of iterations per simulation
- Any number of simulations in a single analysis
- Animation of sampling and recalculation of the spreadsheet
- Seeding the random number generator
- Real time results and statistics during a simulation

@RISK graphs a probability distribution of possible results for each output cell selected in @RISK. @RISK graphics include:

- Relative frequency distributions and cumulative probability curves
- Summary graphs for multiple distributions across cell ranges (for example, a worksheet row or column)
- Statistical reports on generated distributions
- Probability of occurrence for target values in a distribution
- Export of graphics as Windows metafiles for further enhancement

Execution time is of critical importance because simulation is extremely calculation intensive. @RISK is designed for the fastest possible simulations through the use of advanced sampling techniques.
# Table of Contents

Chapter 1: Getting Started

Introduction ................................................................. 3
Installation ........................................................................ 7
Software Activation.......................................................... 9
Quick Start ....................................................................... 13

Chapter 2: An Overview of Risk Analysis

Introduction ....................................................................... 19
What Is Risk? ................................................................. 21
What Is Risk Analysis? ...................................................... 25
Developing an @RISK Model ............................................. 27
Analyzing a Model with Simulation .................................... 29
Making a Decision: Interpreting the Results ....................... 31
What Risk Analysis Can (Cannot) Do ................................. 35

Chapter 3: Upgrade Guide

Introduction ....................................................................... 39
New @RISK Toolbars, Icons and Commands ....................... 41
Building an @RISK Model ............................................... 45
Simulation Settings .......................................................... 65
Running Simulations ....................................................... 69
Reviewing Simulation Results Graphically ........................................ 71
Reports on Simulation Results ........................................................... 81
Saving Simulations ............................................................................ 87
@RISK Library .................................................................................. 89
Chapter 4: Getting to Know @RISK .................................................. 91
A Quick Overview of @RISK ............................................................. 93
Setting Up and Simulating an @RISK Model ..................................... 105
Chapter 5: @RISK Modeling Techniques ............................................ 137
Introduction ..................................................................................... 139
Modeling Interest Rates and Other Trends ....................................... 141
Projecting Known Values into the Future ....................................... 143
Modeling Uncertain or “Chance” Events ......................................... 145
Oil Wells and Insurance Claims ....................................................... 147
Adding Uncertainty Around a Fixed Trend ...................................... 149
Dependency Relationships ............................................................... 151
Sensitivity Simulation ..................................................................... 153
Simulating a New Product ............................................................... 155
Finding Value at Risk (VAR) of a Portfolio .................................... 165
Simulating the NCAA Tournament ................................................ 169
Chapter 6: Distribution Fitting ......................................................... 173
Overview ........................................................................................ 175
Define Input Data ............................................................................ 177
Select Distributions to Fit ............................................................... 181
Run the Fit ....................................................................................... 183
Interpret the Results .......................................................................... 187
Using the Results of a Fit .................................................................... 195
Chapter 7: @RISK Reference Guide ................................................... 197
Introduction ...................................................................................... 205
Reference: @RISK Icons ..................................................................... 207
Reference: @RISK Commands ............................................................. 217
Introduction ...................................................................................... 217
Model Commands .............................................................................. 219
Distribution Fitting Commands .......................................................... 275
Distribution Artist Commands ............................................................ 295
Settings Commands ......................................................................... 299
Simulation Commands ...................................................................... 317
Simulation — Advanced Analyses Commands .................................... 319
Goal Seek ......................................................................................... 321
Stress Analysis ................................................................................ 329
Advanced Sensitivity Analysis .......................................................... 343
Results Commands .......................................................................... 359
Excel Reports Command ................................................................. 385
Swap @RISK Functions Command .................................................... 387
Utilities Commands ......................................................................... 395
Saving and Opening @RISK Simulations .......................................... 403
Library Commands .......................................................................... 405
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Help Commands</td>
<td>407</td>
</tr>
<tr>
<td>Reference: @RISK Graphs</td>
<td>409</td>
</tr>
<tr>
<td>Reference: @RISK Functions</td>
<td>443</td>
</tr>
<tr>
<td>Introduction</td>
<td>443</td>
</tr>
<tr>
<td>Table of Available Functions</td>
<td>455</td>
</tr>
<tr>
<td>Reference: Distribution Functions</td>
<td>467</td>
</tr>
<tr>
<td>Reference: Distribution Property Functions</td>
<td>583</td>
</tr>
<tr>
<td>Reference: Output Functions</td>
<td>599</td>
</tr>
<tr>
<td>Reference: Statistics Functions</td>
<td>601</td>
</tr>
<tr>
<td>Reference: Six Sigma Functions</td>
<td>613</td>
</tr>
<tr>
<td>Reference: Supplemental Functions</td>
<td>625</td>
</tr>
<tr>
<td>Reference: Graphing Function</td>
<td>627</td>
</tr>
<tr>
<td>Reference: @RISK Library</td>
<td>629</td>
</tr>
<tr>
<td>Introduction</td>
<td>629</td>
</tr>
<tr>
<td>Distributions in the @RISK Library</td>
<td>631</td>
</tr>
<tr>
<td>Results in the @RISK Library</td>
<td>637</td>
</tr>
<tr>
<td>Technical Notes</td>
<td>643</td>
</tr>
<tr>
<td>Reference: @RISK for Excel Developers Kit (XDK)</td>
<td>646</td>
</tr>
<tr>
<td>Appendix A: Sampling Methods</td>
<td>647</td>
</tr>
<tr>
<td>What is Sampling?</td>
<td>647</td>
</tr>
<tr>
<td>Appendix B: Using @RISK With Other DecisionTools®</td>
<td>653</td>
</tr>
<tr>
<td>The DecisionTools Suite</td>
<td>653</td>
</tr>
<tr>
<td>Palisade’s DecisionTools Case Study</td>
<td>654</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Introduction to TopRank®</td>
<td>657</td>
</tr>
<tr>
<td>Using @RISK with TopRank</td>
<td>661</td>
</tr>
<tr>
<td>Introduction to PrecisionTree™</td>
<td>665</td>
</tr>
<tr>
<td>Using @RISK with PrecisionTree</td>
<td>669</td>
</tr>
<tr>
<td>Appendix C: Glossary</td>
<td>673</td>
</tr>
<tr>
<td>Glossary of Terms</td>
<td>673</td>
</tr>
<tr>
<td>Appendix D: Recommended Readings</td>
<td>679</td>
</tr>
<tr>
<td>Readings by Category</td>
<td>679</td>
</tr>
</tbody>
</table>
Chapter 1: Getting Started

Introduction ...........................................................................................................................................3
Checking Your Package ..................................................................................................................3
About This Version ......................................................................................................................3
Working with your Operating Environment .................................................................................4
If You Need Help ........................................................................................................................4
@RISK System Requirements .........................................................................................................6

Installation Instructions ..................................................................................................................7
General Installation Instructions .....................................................................................................7
The DecisionTools Suite .....................................................................................................................7
Setting Up the @RISK Icons or Shortcuts ......................................................................................8
Macro Security Warning Message on Startup .................................................................................8

Software Activation .........................................................................................................................9

Quick Start ..........................................................................................................................................13
On-line Tutorial ...............................................................................................................................13
Starting On Your Own .....................................................................................................................13
Quick Start with Your Own Spreadsheets ......................................................................................14
Using @RISK 5.0 Spreadsheets in @RISK 3.5 or earlier .................................................................15
Using @RISK 5.0 Spreadsheets in @RISK 4.0 ...............................................................................15
Using @RISK 5.0 Spreadsheets in @RISK 4.5 ...............................................................................15
Introduction

This introduction describes the contents of your @RISK package and shows you how to install @RISK and attach it to your copy of Microsoft Excel 2000 for Windows or higher.

Checking Your Package

Your @RISK package should contain:

*The @RISK User’s Guide (this book in .PDF format) with:
- Getting Started
- Overview of Risk Analysis and @RISK
- Upgrade Guide
- Getting to Know @RISK
- @RISK Modeling Techniques
- Distribution Fitting
- @RISK Reference Guide
- Technical Appendices

The @RISK CD-ROM including:
- @RISK Program
- @RISK Tutorial

The @RISK Licensing Agreement

If your package is not complete, please call your @RISK dealer or supplier or contact Palisade Corporation directly at (607) 277-8000.

About This Version

This version of @RISK can be used with Microsoft Excel 2000 or higher.
Working with your Operating Environment

This User’s Guide assumes that you have a general knowledge of the Windows operating system and Excel. In particular:

- You are familiar with your computer and using the mouse.
- You are familiar with terms such as icons, click, double-click, menu, window, command and object.
- You understand basic concepts such as directory structures and file naming.

If You Need Help

Technical support is provided free of charge for all registered users of @RISK with a current maintenance plan, or is available on a per incident charge. To ensure that you are a registered user of @RISK, please register online at www.palisade.com/support/register.asp.

If you contact us by telephone, please have your serial number and User’s Guide ready. We can offer better technical support if you are in front of your computer and ready to work.

Before Calling

Before contacting technical support, please review the following checklist:

- Have you referred to the on-line help?
- Have you checked this User’s Guide and reviewed the on-line multimedia tutorial?
- Have you read the README file? It contains current information on @RISK that may not be included in the manual.
- Can you duplicate the problem consistently? Can you duplicate the problem on a different computer or with a different model?
- Have you looked at our site on the World Wide Web? It can be found at http://www.palisade.com. Our Web site also contains the latest FAQ (a searchable database of tech support questions and answers) and @RISK patches in our Technical Support section. We recommend visiting our Web site regularly for all the latest information on @RISK and other Palisade software.
Palisade Corporation welcomes your questions, comments or suggestions regarding @RISK. Contact our technical support staff using any of the following methods:

- Email us at support@palisade.com
- Telephone us at (607) 277-8000 any weekday from 9:00 AM to 5:00 PM, EST. Follow the prompt to reach Technical Support
- Fax us at (607) 277-8001.
- Mail us a letter to:
  Technical Support
  Palisade Corporation
  798 Cascadilla St
  Ithaca, NY 14850
  USA

If you want to contact Palisade Europe:

- Email us at support@palisade-europe.com
- Telephone us at +44 1895 425050 (UK).
- Fax us at +44 1895 425051 (UK).
- Mail us a letter to:
  Palisade Europe
  31 The Green
  West Drayton
  Middlesex
  UB7 7PN
  United Kingdom

If you want to contact Palisade Asia-Pacific:

- Email us at support@palisade.com.au
- Telephone us at + 61 2 9252 5922 (AU).
- Fax us at + 61 2 9252 2820 (AU).
- Mail us a letter to:
  Palisade Asia-Pacific Pty Limited
  Suite 404, Level 4
  20 Loftus Street
  Sydney NSW 2000
  Australia

Regardless of how you contact us, please include the product name, exact version and serial number. The exact version can be found by selecting the Help About command on the @RISK menu in Excel.
Telephone support is not available with the student version of @RISK. If you need help, we recommend the following alternatives:

- Consult with your professor or teaching assistant.
- Log-on to http://www.palisade.com for answers to frequently asked questions.
- Contact our technical support department via e-mail or fax.

@RISK System Requirements

System requirements for @RISK 5 for Microsoft Excel for Windows include:

- Pentium PC or faster with a hard disk.
- Microsoft Windows 2000 SP4, Windows XP or higher.
- Microsoft Excel 2000 or higher.
Installation Instructions

General Installation Instructions

The Setup program copies the @RISK system files into a directory you specify on your hard disk. To run the Setup program in Windows 2000 or higher:

1) Insert the @RISK CD-ROM in your CD-ROM drive
2) Click the Start button, click Settings and then click Control Panel
3) Double-click the Add/Remove Programs icon
4) On the Install/Uninstall tab, click the Install button
5) Follow the Setup instructions on the screen

If you encounter problems while installing @RISK, verify that there is adequate space on the drive to which you’re trying to install. After you’ve freed up adequate space, try rerunning the installation.

If you wish to remove @RISK from your computer, use the Control Panel’s Add/Remove Programs utility and select the entry for @RISK.

The DecisionTools Suite

@RISK for Excel is a member of the DecisionTools Suite, a set of products for risk and decision analysis described in Appendix D: Using @RISK With Other DecisionTools. The default installation procedure of @RISK puts @RISK in a subdirectory of a main “Program Files\Palisade” directory. This is quite similar to how Excel is often installed into a subdirectory of a “Microsoft Office” directory.

One subdirectory of the Program Files\Palisade directory will be the @RISK directory (by default called RISK5). This directory contains the program files plus example models and other files necessary for @RISK to run. Another subdirectory of Program Files\Palisade is the SYSTEM directory which contains files which are needed by every program in the DecisionTools Suite, including common help files and program libraries.
Setting Up the @RISK Icons or Shortcuts

The @RISK setup program automatically creates an @RISK command in the Programs menu of the Taskbar. However, if problems are encountered during Setup, or if you wish to do this manually another time, follow these directions.

1) Click the Start button, and then point to Settings.
2) Click Taskbar, and then click the Start Menu Programs tab.
3) Click Add, and then click Browse.
4) Locate the file RISK.EXE and double click it.
5) Click Next, and then double-click the menu on which you want the program to appear.
6) Type the name “@RISK”, and then click Finish.

Macro Security Warning Message on Startup

Microsoft Office provides several security settings (under Tools>Macro>Security) to keep unwanted or malicious macros from being run in Office applications. A warning message appears each time you attempt to load a file with macros, unless you use the lowest security setting. To keep this message from appearing every time you run a Palisade add-in, Palisade digitally signs their add-in files. Thus, once you have specified Palisade Corporation as a trusted source, you can open any Palisade add-in without warning messages. To do this:

- Click Always trust macros from this source when a Security Warning dialog (such as the one below) is displayed when starting @RISK.

![Security Warning Dialog]

Creating the Shortcut in the Windows Taskbar
Software Activation

Activation is a one-time license verification process that is required in order for your @RISK software to run as a fully licensed product. An activation code is on your printed/emailed invoice and may resemble a dash separated sequence like "19a0-c7c1-15ef-1be0-4d7f-cd". If you enter your Activation code during installation, then your software is activated the first time the software is run and no further user action is required. If you wish to activate your software after installation, select the @RISK Help menu License Activation command and enter your activation code in the displayed Palisade License Activation dialog box.

1) What if my software is not activated?
If you do not enter an activation code during installation or you are installing a trial version, your software will run as a trial version with time and/or number of uses limitations and must be activated with an activation code in order to run as a fully licensed product.

2) How long can I use the product before I have to activate it?
Software that is not activated may be run for fifteen days. All of the product's features are present, but the License Activation dialog will appear each time the program is launched to remind you to activate and to indicate the time remaining. If the 15 day trial period expires, the software will require activation in order to run.
3) How do I check my activation status?

The License Activation dialog box is viewed through the @RISK Help menu License Activation command. Activated software shows a status of *Activated* and trial version software shows a status of *Not Activated*. If the software is not activated, the remaining time that the software is allowed to run is displayed.

4) How do I activate my software?

If you do not have an activation code you may obtain one by clicking the Purchase button in the License Activation dialog. An online purchase will be immediately given an activation code and an optional link to download the installer should reinstallation become necessary. To purchase by phone call the local Palisade office given in the Contacting Palisade section of this chapter.

Activation may be done over the Internet or via email:

- **Activation if you have Internet Access**

  In the Palisade License Activation dialog box, type or paste the activation code and press "Automatic via Internet". A success message should appear after a few seconds and the License Activation dialog box will reflect the software's activated status.

- **Activation if you do not have Internet Access**

  Automated activation by email requires a few steps:

  1. **Click "Manual via Email"** to display the request.xml file which you may save to disk or copy to the Windows clipboard. (It is recommended you note the location on your computer of the request.xml file.)

  2. **Copy or attach the XML file** to an email and send it to activation@palisade.com. You should receive an automatic response to the return address in your email shortly.

  3. **Save the response.xml attachment** in the response email to your hard drive.

  4. **Click on the Process button** that is now in the Palisade License Activation dialog box and navigate to the response.xml file. Select the file and click OK.

A success message should appear and the License Activation dialog will reflect the software's activated status.
5) How do I transfer my software license to another machine?

Transfer of a license, or **rehosting**, may be performed through the Palisade License Activation dialog box as a two step procedure: **deactivation** on the first machine and **activation** on the second machine. A typical use of rehosting is to transfer your copy of @RISK from your office PC to your laptop. To rehost a license from *Machine1* to *Machine2*, make sure both machines have the software installed and are connected to the Internet during the deactivation/activation rehosting.

1. On *Machine1*, click deactivate **Automatic via Internet** in the License Activation dialog. Wait for the success message.

2. On *Machine2*, click activate **Automatic via Internet**. Wait for the success message.

If the machines do not have Internet access then you may follow the similar instructions above for rehosting by the automated email process.

6) I have Internet Access but I am still unable to Activate/Deactivate automatically.

Your firewall must be set to allow TCP access to the licensing server. For single user (non network installations) this is:

Quick Start

On-line Tutorial

In the on-line tutorial, @RISK experts guide you through sample models in movie format. This tutorial is a multi-media presentation on the main features of @RISK.

The tutorial can be run by selecting the @RISK Help Menu Getting Started Tutorial command.

Starting On Your Own

If you're in a hurry, or just want to explore @RISK on your own, here's a quick way to get started.

After attaching @RISK according to the Installation instructions outlined previously in this section:

1) Click the @RISK icon in the Windows Start Programs Palisade DecisionTools group. If the Security Warning dialog is displayed, follow the instructions in the section “Setting Palisade as a Trusted Source” in this chapter.

2) Use the Excel Open command to open the example spreadsheet FINANCE.XLS. The default location for the examples is C:\PROGRAM FILES\PALISADE\RISK5\EXAMPLES\ENGLISH.

3) Click the Model window icon on the @RISK Toolbar — the one on the Toolbar with the red and blue arrow. The Outputs and Inputs list, listing the distribution functions in the FINANCE worksheet along with your output cell C10, NPV at 10%, is displayed.

4) Click the “Simulate” icon — the one with the red distribution curve. You've just started a risk analysis on NPV for the FINANCE worksheet. The Simulation analysis is underway. A graph of the output cell is displayed as the simulation runs.

For all analyses, if you want to see @RISK “animate” its operation during the simulation, click the Demo mode icon on the @RISK toolbar. @RISK then will show you how it changes your spreadsheet iteration by iteration and generates results.
Quick Start with Your Own Spreadsheets

Working through the @RISK On-Line Tutorial and reading the @RISK Reference Guide is the best method for preparing to use @RISK on your own spreadsheets. However, if you're in a hurry, or just don't want to work through the Tutorial, here is a quick step-by-step guide to using @RISK with your own spreadsheets:

1) Click the @RISK icon in the Windows Start Programs Palisade DecisionTools group.

2) If necessary, use the Excel Open command to open your spreadsheet.

3) Examine your spreadsheet and locate those cells where uncertain assumptions or inputs are located. You will substitute @RISK distribution functions for these values.

4) Enter distribution functions for the uncertain inputs which reflect the range of possible values and their likelihood of occurrence. Start with the simple distribution types — such as UNIFORM, which just requires a minimum and maximum possible value, or TRIANG which just requires a minimum, most likely and maximum possible value.

5) Once you've entered your distributions, select the spreadsheet cell or cells for which you wish to get simulation results and click the “Add Output” icon — the one with the single red arrow — on the @RISK Toolbar.

To run a simulation:

• Click the “Start Simulation” icon — the one with the red distribution curve — on the @RISK Toolbar. A simulation of your spreadsheet will be executed and results displayed.
Using @RISK 5 Spreadsheets in @RISK 3.5 or earlier

@RISK 5 spreadsheets can only be used in @RISK 3.5 or earlier when the simple forms of distribution functions are used. In the simple distribution function format only required distribution parameters can be used. No new @RISK 5 distribution property functions can be added. In addition, RiskOutput functions must be removed and outputs reselected when simulating in @RISK 3.5.

Using @RISK 5 Spreadsheets in @RISK 4.0

@RISK 5 spreadsheets can be used directly in @RISK 4.0 with the following exceptions:

- **Alternate Parameter functions**, such as RiskNormalAlt, will not work and will return an error.
- **Cumulative Descending functions**, such as RiskCumulID, will not work and will return an error.
- **Distribution property functions** specific to @RISK 5 (such as RiskUnits) will be ignored in @RISK 4.0.
- **Statistics functions** specific to @RISK 5 (such as RiskTheoMean) will return #NAME in @RISK 4.0.
- **Other new functions** specific to @RISK 5 such as RiskCompound, RiskSixSigma statistics functions, RiskConvergenceLevel and supplemental functions such as RiskStopRun will return #NAME in @RISK 4.0.

Using @RISK 5 Spreadsheets in @RISK 4.5

@RISK 5 spreadsheets can be used directly in @RISK 4.5 with the following exceptions:

- **Distribution property functions** specific to @RISK 5 (such as RiskUnits) will be ignored in @RISK 4.5. Functions that contain them, however, will sample properly.
- **Statistics functions** specific to @RISK 5 (such as RiskTheoMean) will return #NAME in @RISK 4.5.
- **Other new functions** specific to @RISK 5 such as RiskCompound, RiskSixSigma statistics functions, RiskConvergenceLevel and supplemental functions such as RiskStopRun will return #NAME in @RISK 4.5.
Using @RISK 5.5 Spreadsheets in @RISK 5.0

@RISK 5.5 spreadsheets can be used directly in @RISK 5.0 with the following exception:

- The Distribution property function RiskIsDate specific to @RISK 5.5 will return #NAME in @RISK 5.0.
Chapter 2: An Overview of Risk Analysis

Introduction..................................................................................................................19

What Is Risk? ................................................................................................................21
  Characteristics of Risk..............................................................................................21
  The Need for Risk Analysis.....................................................................................22
  Assessing and Quantifying Risk.............................................................................23
  Describing Risk with a Probability Distribution..................................................24

What Is Risk Analysis? ...............................................................................................25

Developing an @RISK Model......................................................................................27
  Variables....................................................................................................................27
  Output Variables.......................................................................................................28

Analyzing a Model with Simulation.........................................................................29
  Simulation..................................................................................................................29
  How Simulation Works ............................................................................................30
  The Alternative to Simulation..................................................................................30

Making a Decision: Interpreting the Results............................................................31
  Interpreting a Traditional Analysis..........................................................................31
  Interpreting an @RISK Analysis..............................................................................31
  Individual Preference..............................................................................................32
  The Distribution “Spread”.......................................................................................32
  Skewness...................................................................................................................34

What Risk Analysis Can (Cannot) Do.......................................................................35
Introduction

@RISK brings advanced modeling and Risk Analysis to Microsoft Excel. You might wonder if what you do qualifies as modeling and/or would be suitable for Risk Analysis. If you use data to solve problems, make forecasts, develop strategies, or make decisions, then you definitely should consider doing Risk Analysis.

Modeling is a catch-all phrase that usually means any type of activity where you are trying to create a representation of a real life situation so you can analyze it. Your representation, or model, can be used to examine the situation, and hopefully help you understand what the future might bring. If you've ever played “what-if” games with your project, by changing the values of various entries, you are well on your way to understanding the importance of uncertainty in a modeling situation.

Okay, so you do analyses and make models — what is involved in making these analyses and models, explicitly incorporating risk? The following discussion will try to answer this question, but don't worry, you don't have to be an expert in statistics or decision theory to analyze situations under risk, and you certainly don't have to be an expert to use @RISK. We can't teach you everything in a few pages, but we'll get you started. Once you begin using @RISK you'll automatically begin picking up the type of expertise that can't be learned from a book.

Another purpose of this chapter is to give you an overview of how @RISK works with your spreadsheet to perform analyses. You don't have to know how @RISK works to use it successfully, but you might find some explanations useful and interesting. This chapter discusses:

- What risk is and how it can be quantitatively assessed.
- The nature of Risk Analysis and the techniques used in @RISK.
- Running a simulation.
- Interpreting @RISK results.
- What Risk Analysis can and cannot do.
What Is Risk?

Everyone knows that risk affects the gambler about to roll the dice, the wildcatter about to drill an oil well, or the tightrope walker taking that first big step. But these simple illustrations aside, the concept of risk comes about due to our recognition of future uncertainty — our inability to know what the future will bring in response to a given action today. Risk implies that a given action has more than one possible outcome.

In this simple sense, every action is risky, from crossing the street to building a dam. The term is usually reserved, however, for situations where the range of possible outcomes to a given action is in some way significant. Common actions like crossing the street usually aren't risky, while building a dam can involve significant risk. Somewhere in between, actions pass from being nonrisky to risky. This distinction, although vague, is important — if you judge that a situation is risky, risk becomes one criterion for deciding what course of action you should pursue. At that point, some form of Risk Analysis becomes viable.

Characteristics of Risk

Risk derives from our inability to see into the future, and indicates a degree of uncertainty that is significant enough to make us notice it. This somewhat vague definition takes more shape by mentioning several important characteristics of risk.

Firstly, risk can be either objective or subjective. Flipping a coin is an objective risk because the odds are well known. Even though the outcome is uncertain, an objective risk can be described precisely based on theory, experiment, or common sense. Everyone agrees with the description of an objective risk. Describing the odds for rain next Thursday is not so clear cut, and represents a subjective risk. Given the same information, theory, computers, etc., weatherman A may think the odds of rain are 30% while weatherman B may think the odds are 65%. Neither is wrong. Describing a subjective risk is open-ended in the sense that you could always refine your assessment with new information, further study, or by giving weight to the opinion of others. Most risks are subjective, and this has important implications for anyone analyzing risk or making decisions based on a Risk Analysis.
Secondly, deciding that something is risky requires personal judgment, even for objective risks. For example, imagine flipping a coin where you win $1 for heads and lose $1 for tails. The range between $1 and -$1 would not be overly significant to most people. If the stakes were $100,000 and -$100,000 respectively, most people would find the situation to be quite risky. There would be a wealthy few, however, who would not find this range of outcome to be significant.

Thirdly, risky actions, and therefore risk, are things that we often can choose or avoid. Individuals differ in the amount of risk they willingly accept. For example, two individuals of equal net worth may react quite differently to the $100,000 coin flip bet described above — one may accept it while the other refuses it. Their personal preference for risk differs.

The Need for Risk Analysis

The first step in Risk Analysis and modeling is recognizing a need for it. Is there significant risk involved in the situation you are interested in? Here are a few examples that might help you evaluate your own situations for the presence of significant risk:

- **Risks for New Product Development and Marketing** — Will the R&D department solve the technical problems involved? Will a competitor get to market first, or with a better product? Will government regulations and approvals delay product introduction? How much impact will the proposed advertising campaign have on sales levels? Will production costs be as forecast? Will the proposed sales price have to be changed to reflect unanticipated demand levels for the product?

- **Risks for Securities Analysis and Asset Management** — How will a tentative purchase affect portfolio value? Will a new management team affect market price? Will an acquired firm add earnings as forecast? How will a market correction impact a given industry sector?

- **Risks for Operations Management and Planning** — Will a given inventory level suffice for unpredictable demand levels? Will labor costs rise significantly with upcoming union contract negotiations? How will pending environmental legislation impact production costs? How will political and market events affect overseas suppliers in terms of exchange rates, trade barriers, and delivery schedules?

- **Risks for Design and Construction of a Structure (building, bridge, dam, ...)** — Will the cost of construction materials and labor be as forecast? Will a labor strike affect the construction schedule? Will the levels of stress placed on the structure by peak loads, crowds and nature be as forecast? Will the structure ever be stressed to the point of failure?
• **Risks for Investment in Exploration for Oil and Minerals** — Will anything be found? If a deposit is found, will it be uneconomical, or a bonanza? Will the costs of developing the deposit be as forecast? Will some political event like an embargo, tax reform, or new environmental regulations drastically alter the economic viability of the project?

• **Risks for Policy Planning** — If the policy is subject to legislative approval, will it be approved? Will the level of compliance with any policy directives be complete or partial? Will the costs of implementation be as forecast? Will the level of benefits be what you projected?

### Assessing and Quantifying Risk

The first step in Risk Analysis and modeling is recognizing a need for it. Is there significant risk involved in the situation you are interested in? Here are a few examples that might help you evaluate your own situations for the presence of significant risk.

Realizing that you have a risky situation is only the first step. How do you quantify the risk you have identified for a given uncertain situation? “Quantifying risk” means determining all the possible values a risky variable could take and determining the relative likelihood of each value. Suppose your uncertain situation is the outcome from the flip of a coin. You could repeat the flip a large number of times until you had established the fact that half the time it comes up tails and half the time heads. Alternatively, you could mathematically calculate this result from a basic understanding of probability and statistics.

In most real life situations, you can't perform an “experiment” to calculate your risk the way you can at the flip of a coin. How could you calculate the probable learning curve associated with introducing new equipment? You may be able to reflect on past experiences, but once you have introduced the equipment, the uncertainty is gone. There is no mathematical formula that you can solve to get the risk associated with the possible outcomes. You have to estimate the risk using the best information you have available.

If you can calculate the risks of your situation the way you would for a coin flip, the risk is objective. This means that everyone would agree that you quantified the risk correctly. Most risk quantification, however, involves your best judgment.
There may not be complete information available about the situation, the situation may not be repeatable compared to a coin flip, or it just may be too complex to come up with an unequivocal answer. Such risk quantification is subjective, which means that someone might disagree with your evaluation.

Your subjective assessments of risk are likely to change when you get more information on the situation. If you have subjectively derived a risk assessment, you must always ask yourself whether additional information is available that would help you make a better assessment. If it is available, how hard and how expensive would it be to obtain? How much would it cause you to change the assessment you already have made? How much would these changes affect the final results of any model you are analyzing?

**Describing Risk with a Probability Distribution**

If you have quantified risk — determined outcomes and probabilities of occurrence — you can summarize this risk using a probability distribution. A probability distribution is a device for presenting the quantified risk for a variable. @RISK uses probability distributions to describe uncertain values in your Excel worksheets and to present results. There are many forms and types of probability distributions, each of which describes a range of possible values and their likelihood of occurrence. Most people have heard of a normal distribution — the traditional “bell curve”. But there is a wide variety of distribution types ranging from uniform and triangular distributions to more complex forms such as gamma and Weibull.

All distribution types use a set of arguments to specify a range of actual values and distribution of probabilities. The normal distribution, for example, uses a mean and standard deviation as its arguments. The mean defines the value around which the bell curve will be centered and the standard deviation defines the range of values around the mean. Over thirty types of distributions are available to you in @RISK for describing distributions for uncertain values in your Excel worksheets.

The @RISK Define Distribution window allows you to graphically preview distributions and assign them to uncertain values. Using its graphs, you can quickly see the range of possible values your distribution describes.
What Is Risk Analysis?

In a broad sense, Risk Analysis is any method — qualitative and/or quantitative — for assessing the impacts of risk on decision situations. Numerous techniques are used that blend both qualitative and quantitative techniques. The goal of any of these methods is to help the decision-maker choose a course of action, to enable a better understanding of the possible outcomes that could occur.

Risk Analysis in @RISK is a quantitative method that seeks to determine the outcomes of a decision situation as a probability distribution. In general, the techniques in an @RISK Risk Analysis encompass four steps:

1. **Developing a Model** — by defining your problem or situation in Excel worksheet format

2. **Identifying Uncertainty** — in variables in your Excel worksheet and specifying their possible values with probability distributions, and identifying the uncertain worksheet results you want analyzed

3. **Analyzing the Model with Simulation** — to determine the range and probabilities of all possible outcomes for the results of your worksheet

4. **Making a Decision** — based on the results provided and personal preferences

@RISK helps with the first three steps, by providing a powerful and flexible tool that works with Excel to facilitate model building and Risk Analysis. The results that @RISK generates can then be used by the decision-maker to help choose a course of action.

Fortunately, the techniques @RISK employs in a Risk Analysis are very intuitive. As a result, you won't have to accept our methodology on faith, and you won't have to shrug your shoulders and resort to calling @RISK a “black box” when your colleagues and superiors query you as to the nature of your Risk Analysis. The following discussion will give you a firm understanding of just what @RISK needs from you in the way of a model, and how an @RISK Risk Analysis proceeds.
Developing an @RISK Model

You are the “expert” at understanding the problems and situations that you would like to analyze. If you have a problem that is subject to risk, then @RISK and Excel can help you construct a complete and logical model.

A major strength of @RISK is that it allows you to work in a familiar and standard model building environment — Microsoft Excel. @RISK works with your Excel model, allowing you to conduct a Risk Analysis, but still preserves the familiar spreadsheet capabilities. You presumably know how to build spreadsheet models in Excel — @RISK now gives you the ability to easily modify these models for Risk Analysis.

Variables

Variables are the basic elements in your Excel worksheets that you have identified as being important ingredients to your analysis. If you are modeling a financial situation, your variables might be things like Sales, Costs, Revenues or Profits, whereas if you are modeling a geologic situation your variables might be things like Depth to Deposit, Thickness of Coal Seam or Porosity. Each situation has its own variables, identified by you. In a typical worksheet, a variable labels a worksheet row or column, for example:

You may know the values your variables will take in the time frame of your model — they are certain, or what statisticians call “deterministic”. Conversely, you may not know the values they will take — they are uncertain, or “stochastic”. If your variables are uncertain you will need to describe the nature of their uncertainty. This is done with probability distributions, which give both the range of values that the variable could take (minimum to maximum), and the likelihood of occurrence of each value within the range. In @RISK, uncertain variables and cell values are entered as probability distribution functions, for example:

\[
\begin{align*}
\text{RiskNormal}(100,10) \\
\text{RiskUniform}(20,30) \\
\text{RiskExpon}(A1+A2) \\
\text{RiskTriang}(A3/2.01,A4,A5)
\end{align*}
\]

These “distribution” functions can be placed in your worksheet cells and formulas just like any other Excel function.
In addition to being certain or uncertain, variables in a Risk Analysis model can be either “independent” or “dependent”. An independent variable is totally unaffected by any other variable within your model. For example, if you had a financial model evaluating the profitability of an agricultural crop, you might include an uncertain variable called Amount of Rainfall. It is reasonable to assume that other variables in your model such as Crop Price and Fertilizer Cost would have no effect on the amount of rain — Amount of Rainfall is an independent variable.

A dependent variable, in contrast, is determined in full, or in part, by one or more other variables in your model. For example, a variable called Crop Yield in the above model should be expected to depend on the independent variable Amount of Rainfall. If there's too little or too much rain, then the crop yield is low. If there's an amount of rain that is about normal, then the crop yield would be anywhere from below average to well above average. Maybe there are other variables that affect Crop Yield such as Temperature, Loss to Insects, etc.

When identifying the uncertain values in your Excel worksheet, you have to decide whether your variables are correlated. These variables would all be “correlated” with each other. The Corrmat function in @RISK is used to identify correlated variables. It is extremely important to correctly recognize correlations between variables, or your model might generate nonsensical results. For example, if you ignored the relationship between Amount of Rainfall and Crop Yield, @RISK might choose a low value for the Rainfall at the same time it picked a high value for the Crop Yield — clearly something nature wouldn't allow.

Output Variables

Any model needs both input values and output results, and a Risk Analysis model is no different. An @RISK Risk Analysis generates results on cells in your Excel worksheet. Results are probability distributions of the possible values which could occur. These results are usually the same worksheet cells that give you the results of a regular Excel analysis — profit, the “bottom line” or other such worksheet entries.
Analyzing a Model with Simulation

Once you have placed uncertain values in your worksheet cells and have identified the outputs of your analysis, you have an Excel worksheet that @RISK can analyze.

Simulation

@RISK uses simulation, sometimes called Monte Carlo simulation, to do a Risk Analysis. Simulation in this sense refers to a method whereby the distribution of possible outcomes is generated by letting a computer recalculate your worksheet over and over again, each time using different randomly selected sets of values for the probability distributions in your cell values and formulas. In effect, the computer is trying all valid combinations of the values of input variables to simulate all possible outcomes. This is just as if you ran hundreds or thousands of “what-if” analyses on your worksheet, all in one sitting.

What is meant by saying that simulation “tries all valid combinations of the values of input variables”? Suppose you have a model with only two input variables. If there is no uncertainty in these variables, you can identify a single possible value for each variable. These two single values can be combined by your worksheet formulas to calculate the results of interest — also a certain or deterministic value. For example, if the certain input variables are:

Revenues = 100
Costs = 90

then the result:

Profits = 10

would be calculated by Excel from:

Profits = 100 – 90

There is only one combination of the input variable values, because there is only one value possible for each variable.

Now consider a situation where there is uncertainty in both input variables. For example,

Revenues = 100 or 120
Costs = 90 or 80

gives two values for each input variable. In a simulation, @RISK would consider all possible combinations of these variable values to calculate possible values for the result, Profits.
There are four combinations:

**Profits = Revenues – Costs**

10 = 100 – 90  
20 = 100 – 80  
30 = 120 – 90  
40 = 120 – 80

Profits also is an uncertain variable because it is calculated from uncertain variables.

**How Simulation Works**

In @RISK, simulation uses the following two distinct operations:

- *Selecting sets of values for the probability distribution functions contained in the cells and formulas of your worksheet*
- *Recalculating the Excel worksheet using the new values*

The selection of values from probability distributions is called sampling and each calculation of the worksheet is called an iteration.

The following diagrams show how each iteration uses a set of single values sampled from distribution functions to calculate single-valued results. @RISK generates output distributions by consolidating single-valued results from all the iterations.

**The Alternative to Simulation**

There are two basic approaches to quantitative Risk Analysis. Both have the same goal — to derive a probability distribution that describes the possible outcomes of an uncertain situation — and both generate valid results. The first approach is the one just described for @RISK, namely, simulation. This approach relies on the ability of the computer to do a great deal of work very quickly — solving your worksheet problem by repeatedly using a large number of possible combinations of input variable values.

The second approach to Risk Analysis is an analytical approach. Analytical methods require that the distributions for all uncertain variables in a model be described mathematically. Then the equations for these distributions are combined mathematically to derive another equation, which describes the distribution of possible outcomes. This approach is not practical for most uses and users. It is not a simple task to describe distributions as equations, and it is even more difficult to combine distributions analytically given even moderate complexity in your model. Furthermore, the mathematical skills necessary to implement the analytical techniques are significant.
Making a Decision: Interpreting the Results

@RISK analysis results are presented in the form of probability distributions. The decision-maker must interpret these probability distributions, and make a decision based on the interpretation. How do you interpret a probability distribution?

Interpreting a Traditional Analysis

Let’s start by looking at how a decision-maker would interpret a single-valued result from a traditional analysis — an “expected” value. Most decision-makers compare the expected result to some standard or minimum acceptable value. If it’s at least as good as the standard, they find the result acceptable, but most decision makers recognize that the expected result doesn’t show the impacts of uncertainty. They have to somehow manipulate the expected result to make some allowance for risk. They might arbitrarily raise the minimum acceptable result, or they might non rigorously weigh the chances that the actual result could exceed or fall short of the expected result. At best, the analysis might be extended to include several other results — such as “worst case” and “best case” — in addition to the expected value. The decision maker then decides if the “expected” and “best case” values are good enough to outweigh the “worst case” value.

Interpreting an @RISK Analysis

In an @RISK Risk Analysis, the output probability distributions give the decision-maker a complete picture of all the possible outcomes. This is a tremendous elaboration on the “worst-expected-best” case approach mentioned above. Beyond filling in the gaps between the three values, the probability distribution also does the following:

- **Determines a “Correct” Range** — Because you have more rigorously defined the uncertainty associated with every input variable, the possible range of outcomes may be quite different from a “worst case”, “best case” range — different, and more correct.

- **Shows Probability of Occurrence** — A probability distribution shows the relative likelihood of occurrence for each possible outcome.

As a result, you no longer just compare desirable outcomes with undesirable outcomes. Instead, you can recognize that some outcomes are more likely to occur than others, and should be given more weight in your evaluation. This process is also a lot easier to understand than the traditional analysis because a probability distribution is a graph, where you can see the probabilities and get a feel for the risks involved.
Individual Preference

The results provided by an @RISK analysis must be interpreted by you as an individual. The same results given to several individuals may be interpreted differently, and lead to different courses of action. This is not a weakness in the technique, but a direct result of the fact that individuals have varying preferences with regard to possible choices, time, and risk. You might feel that the shape of the output distribution shows that the chances of an undesirable outcome far outweighs the chances of a desirable outcome. A colleague who is less risk averse might come to the opposite conclusion.

The Distribution “Spread”

Range and likelihood of occurrence are directly related to the level of risk associated with a particular event. By looking at the spread and likelihood of possible results, you can make an informed decision based on the level of risk you are willing to take. Risk averse decision makers prefer a small spread in possible results, with most of the probability associated with desirable results. But if you are a risk taker, then you will accept a greater spread or possible variation in your outcome distribution. Furthermore, a risk taker will be influenced by “bonanza” outcomes even if their likelihood of occurrence is small.

Regardless of your personal risk preferences, there are some general conclusions about riskiness that apply to all decision makers. The following probability distributions illustrate these conclusions:

*Probability distribution A represents greater risk than B despite identical shapes, because the range of A includes less desirable results — the spread relative to the mean is greater in A than B.*
Probability distribution C represents greater risk than D because the probability of occurrence is uniform across the range for C, whereas it is concentrated around 98 for D.

Probability distribution F represents greater risk than E because the range is larger and the probability of occurrence is more spread out than for E.
Skewness

A simulation output distribution can also show skewness, that is how much the distribution of possible results deviates from being symmetrical. Suppose your distribution had a large positive 'tail'. If you saw only a single number for the expected result, you might not realize the possibility of a highly positive outcome that could occur in the tail. Skewness such as this can be very important to decision makers. By presenting all the information, @RISK “opens up” a decision by showing you all possible outcomes.
What Risk Analysis Can (Cannot) Do

Quantitative analysis techniques have gained a great deal of popularity with decision makers and analysts in recent years. Unfortunately, many people have mistakenly assumed that these techniques are magic “black boxes” that unequivocally arrive at the correct answer or decision. No technique, including those used by @RISK, can make that claim. These techniques are tools that can be used to help make decisions and arrive at solutions. Like any tools, they can be used to good advantage by skilled practitioners, or they can be used to create havoc in the hands of the unskilled. In the context of Risk Analysis, quantitative tools should never be used as a replacement for personal judgment.

Finally, you should recognize that Risk Analysis cannot guarantee that the action you choose to follow — even if skillfully chosen to suit your personal preferences — is the best action viewed from the perspective of hindsight, which implies perfect information, which you never have at the time the decision is made. You can be guaranteed, however, that you have chosen the best personal strategy given the information that is available to you. That's not a bad guarantee!
Chapter 3: Upgrade Guide

Introduction .......................................................................................................................... 39

New @RISK Toolbars, Icons and Commands ................................................................. 41

Building an @RISK Model ............................................................................................. 45
  New and Enhanced @RISK Functions in Excel ...................................................... 45
  Defining Probability Distributions in your Spreadsheet .................................... 47
  Correlating Probability Distributions .................................................................... 52
  Defining Simulation Outputs in your Spreadsheet .............................................. 56
  Reviewing a Model in the @RISK Model Window ............................................. 57
  Properties for Input Distributions and Simulation Outputs ...................... 59
  Swapping @RISK Functions Out and In .............................................................. 60
  Using Data to Define Probability Distributions ................................................. 62

Simulation Settings ........................................................................................................... 65

Running Simulations ........................................................................................................ 69

Reviewing Simulation Results Graphically ..................................................................... 71
  Browse Mode ............................................................................................................... 72
  @RISK Results Summary Window .......................................................................... 73
  New @RISK 5.0 Graphs ............................................................................................... 75
  Customizing and Reporting @RISK Graphs ............................................................ 79

Reports on Simulation Results ......................................................................................... 81

Saving Simulations ............................................................................................................ 87

@RISK Library .................................................................................................................... 89
Introduction

@RISK 5 is a major upgrade to earlier versions of @RISK. @RISK 5 offers enhanced integration with Microsoft Excel to give easier access to simulation results directly in your spreadsheet. @RISK 5 is available in three versions — Standard, Professional and Industrial — to allow you to select the feature set you need.

Key features of @RISK 5 include:

- The separate Model and Results Summary Windows of @RISK 4.0 and 4.5 are now integrated into the Excel window.
- Graphs of simulation results and inputs can link directly to the cells they reference in Excel with “callout” windows.
- New “graph navigator” quickly moves through @RISK inputs and outputs in open workbooks, with graphs pointing at the cell where the input or output is located.
- Correlations between distributions are quickly defined in matrices that pop-up over Excel, and a Correlated Time Series can be added in a single button click.
- New graphics engine, designed for simulation data, provides faster graphing and real-time animation of simulation results.
- Nearly all modeling operations can be performed via drag and drop or simple clicks on the toolbar.
- New @RISK Settings toolbar in Excel provides quick access to simulation settings.
- New scatter plots and box plots provide additional insights into simulation results.
- A wider set of @RISK functions in Excel support six sigma analyses, statistics on simulation inputs and additional results processing.
- A new RiskCompound function, especially applicable to the insurance industry, combines two distributions to create a single new input, dramatically reducing the number of probability distributions required in many models and speeding analyses.
- Smart Sensitivity Analysis is performed by pre-screening inputs based on their precedence in formulas to outputs in your model.
- The @RISK Library provides a repository for sharing @RISK inputs and simulation results.
• **Function Swap** allows @RISK functions to be removed and restored from workbooks; easing the sharing of workbooks with non-@RISK users.

• **Data from a simulation may be sorted** to show key values you are interested in.

• **Iterations from a previously run simulation may be stepped through**, updating Excel with values sampled and results calculated. This is useful to investigate iterations with errors, iterations that led to certain output scenarios and similar.

• Support for versions of Microsoft Excel through Excel 2007, including the larger worksheet size of Excel 2007.
New @RISK Toolbars, Icons and Commands

@RISK 5 includes new toolbars, icons and commands that make it easy to define your simulation model directly in your spreadsheet.

@RISK Toolbar in Excel 2003 and Earlier

@RISK Ribbon Bar in Excel 2007

New icons include:

- **Define Correlation** icon pops up a correlation matrix over Excel where probability distributions can be quickly correlated.

- **Browse Results** icon turns on the new “Browse” mode of @RISK 5, where a graph of simulation results for a cell automatically pops up when you select the cell in Excel.

- Four new **Reports** icons display reports on simulation results (Detailed Statistics, Data, Sensitivity Analysis and Scenario Analysis) that are popped up directly over Excel.

- **Filter** icon allows you to enter filters to restrict the range over which statistics and graphs are calculated.

- **Swap Functions** icon swaps @RISK functions in and out of open workbooks.

- **Library** icon displays the @Risk Library where common input distributions can be defined and simulation results archived.

- **Utilities** icon includes commands such as Application Settings, where default settings for @RISK can be entered.
An @RISK Settings Toolbar is added in Excel 2003 and earlier. This allows quick access to many simulation settings. In Excel 2007, the commands of the @RISK Settings toolbar are present on the standard @RISK ribbon bar.

The icons include:

- **Simulation Settings** opens the Simulation Settings dialog box.
- **Iterations** drop-down list, where the number of iterations to run can be quickly changed from the toolbar.
- **Simulations** drop-down list, where the number of simulations to run can be quickly changed from the toolbar.
- **Random/Static Recalc** flips @RISK between returning expected or static values from distributions, to returning Monte Carlo samples in a standard Excel recalculation.
- **Show Graph, Show Results Window, Demo Mode** control what is shown on the screen during and after a simulation.
- **Live Update** controls if open windows will be updated while a simulation is running.

If you are using @RISK Industrial you will have an additional [RISKOptimizer icon](#) displayed on the @RISK Ribbon in Excel 2007. RISKOptimizer commands can be accessed from this icon directly in @RISK, as opposed to using the separate RISKOptimizer ribbon.
A new progress window is displayed during simulations. The icons allow you to run, pause or stop a simulation, as well as turn real-time updates of graphs and Excel recalcuations on and off.
The new **Application Settings** dialog sets program wide defaults for standard options (such as graph colors, descending percentiles, number of iterations, etc.) that will be used any time you run @RISK.
Building an @RISK Model

@RISK 5 (as well as prior versions of @RISK) allows you to define risk with probability distribution functions that can be added to spreadsheet formulas. @RISK also allows simulation results to be accessed directly in spreadsheet formulas through use of @RISK statistics functions.

@RISK 5 both expands the set of spreadsheet functions available for modeling. It also provides a new graphical interface for entering and editing these functions in your spreadsheet. As in prior versions of @RISK, you can type @RISK functions directly in Excel formulas or use a graphical interface to enter the functions.

New and Enhanced @RISK Functions in Excel

@RISK 5 includes both new and enhanced custom functions that can be included in Excel cells and formulas.

A new function RiskCompound, used for “frequency severity” modeling, takes two distributions to form a new input distribution. RiskCompound takes two arguments, each normally an @RISK distribution function. In a given iteration, the sample from the first distribution specifies the number of samples which will be drawn from the second distribution. Those samples from the second distribution are then summed to give the value returned by the RiskCompound function. For example, the function:

RiskCompound(RiskPoisson(5),RiskLognorm(100000,10000))

would be used in the insurance industry where the frequency or number of claims is described by RiskPoisson(5) and the severity of each claim is given by RiskLognorm(100000,10000). Here the sample value returned by RiskCompound is the total claim amount for the iteration; as given by a number of claims sampled from RiskPoisson(5), each with an amount sampled from RiskLognorm(100000,10000). Two optional arguments, Deductible and Limit, allow you to subtract a deductible from each severity sample or cap a severity sample at an upper limit.

RiskCompound can eliminate hundreds or thousands of distribution functions from existing @RISK models by encapsulating them in a single RiskCompound function. In addition, these models will run much faster.

A new set of @RISK statistics functions return a desired statistic on simulation inputs. For example, the function RiskTheoMean(A10) returns the mean of the probability distribution in the cell A10.

Existing @RISK statistics functions for simulation results (such as RiskMean) can take an optional min-max arguments to specify a percentile or actual range over which statistics should be calculated. This
allows you to calculate statistics on a small subset of collected simulation data, such as the tail of a distribution. The min-max range is entered using a \texttt{RiskTruncate} function.

A new function \texttt{RiskSensitivity} returns sensitivity analysis results directly to your spreadsheet. Using this function, the most critical inputs affecting a simulation result, and the coefficients which identify their level of impact, can be returned to spreadsheet formulas.

\textbf{Additional distribution property functions} have been added in @RISK 5. These property functions can be embedded in a distribution or output function. They are used to specify additional information about an input distribution or simulation output. For example, \texttt{RiskNormal(10,1,RiskUnits("Dollars"))} specifies that the units label used in graphs and reports for this input should be \textit{Dollars}.

\textbf{RiskMakeInput Function} specifies that the calculated value for a formula will be treated as a simulation input, in the same manner as a distribution function. This function allows the results of Excel calculations (or a combination of distribution functions) to be treated as a single “input” in a sensitivity analysis. Distributions that precede or “feed into” a RiskMakeInput function are not included in a sensitivity analysis to avoid double counting of their impacts.

The new property function \texttt{TruncateP} allows truncation of a probability distribution using percentiles instead of actual values.

\textbf{Six Sigma Statistics Functions} return a desired \textit{Six Sigma statistic} on a simulation output. For example, the function \texttt{RiskCPK(A10)} returns the CPK value for the simulation output in Cell A10. By default, these functions will use the six sigma LSL, USL and Target information entered in the \texttt{RiskSixSigma} property function for the output. However, you can also directly enter LSL, USL and Target values as optional arguments in any of the six sigma statistics functions.

@RISK can report \textit{convergence monitoring information} during a simulation via the new \texttt{RiskConvergence} function. This function allows you to specify the statistic of a specific output whose convergence you wish to monitor and convergence threshold you wish to use. The function \texttt{RiskConvergenceLevel} identifies when an output in a simulation has converged.

An additional function \texttt{RiskStopRun} can be used in conjunction with the \texttt{RiskConvergenceLevel} function to stop a simulation or to stop a simulation when a formula or function in your model evaluates to TRUE.
Defining Probability Distributions in your Spreadsheet

With @RISK 5 you can assign probability distributions functions to uncertain values in your spreadsheet model using the Define Distribution window. This window is now interactive as you can step through cells in a workbook, assigning or previewing distributions, without closing the window. The window has a callout that points to the cell you are defining distributions for. Press <Tab> to move the Define Distribution window among cells with distributions in open workbooks.

Using the Define Distribution window, you can:

- **Preview and assign probabilities to values in Excel cells and formulas.** This allows the quick, graphical assignment of distributions to any number in an Excel cell formula, plus the editing of previously entered distribution functions.

- **Automatically enter distribution functions to formulas.** All edits made via pop-up are added directly to the cell formula in Excel.

- **Edit multiple distributions in a single cell.** Clicking on any value in a formula selects it so it can be replaced by a probability distribution.
With @RISK 5’s Define Distribution window, you can interactively switch between available probability distributions and preview the probabilities they describe. While previewing distributions, you can:

- Interactively set and compare probabilities using sliding delimiters.
- Overlay multiple distributions to make comparisons.
- Change graph type and scaling using toolbars and the mouse.

New probability distributions can be added to formulas using the new Distribution Palette. Clicking on a value in a formula selects it. The value can then be replaced by a distribution type in the displayed Palette by double-clicking on the distribution’s picture.
**Entering Argument Values**

Argument values can be entered in the Distribution Argument panel or typed directly in the shown formula. This panel is displayed on the left of the graph.

**Parameter Type**

By changing the Parameter type, you can select to enter Alternate Parameters or Truncate the distribution.
**Changing Graph Type**

In the Define Distribution window (along with other graph windows), the type of the displayed graph may be changed by clicking the Graph Type icon in the lower left of the window.

**Customizing a Graph**

In the Define Distribution window (along with other graph windows), graphs may be customized with the **Graph Options** dialog. Many settings, including titles, colors, delimiters and other options may be set. In many cases (such as entering a title) you can click directly on the graph to customize it.
In the Define Distribution window, overlays may be added using a small version of the Distribution Palette. This palette, displayed below the graph, allows you to add and delete overlays.

The Distribution Argument panel at the left of the graph may be used to select cells in Excel to use as arguments to a distribution function. This is done by clicking the Excel Reference icon for the desired distribution in the Distribution Argument panel.
Correlating Probability Distributions

With @RISK 5 you can easily define correlations between probability distributions using the new **Define Correlations window**. The Define Correlations window displays a correlation matrix with the correlation coefficients between the probability distributions in the matrix.

Correlations can be added by selecting the cells in Excel that contain the input distributions you want to correlate, and then clicking the Define Correlations icon. You can also add inputs to a displayed matrix by clicking Add Inputs and selecting cells in Excel.

Once a matrix is displayed, you can enter correlation coefficients between inputs in matrix cells, copy values in from a matrix in Excel, or use scatter plots to assess and enter correlations.
A scatter plot matrix is displayed by clicking the Scatter Plots icon at the bottom left of the Define Correlations window. The scatter plots in matrix cells show how values between any two input distributions are correlated. Moving the Correlation Coefficient slider dynamically changes the correlation coefficient and scatter plot for any pair of inputs.

By dragging a scatter plot cell off the matrix you can expand the thumbnail scatter plot into a full graph window. This window will also update dynamically when the Correlation Coefficient slider is changed.
@RISK 5 allows you to place matrices anywhere in open workbooks. If you like, you can change correlation coefficients by simply typing in new values in the matrix in Excel.

All correlations entered in the Define Correlations window result in RiskCorrmat property functions being added to the correlated distribution functions in your formulas. These RiskCorrmat property functions reference the location where the displayed matrix was placed in Excel.

After a simulation, you can check the actual simulated correlations for the entered matrix by clicking on a cell in the matrix when “browsing” simulation results in your spreadsheet.
**Correlated Time Series**

A **Correlated Time Series** is created from a multi-period range that contains a set of similar distributions in each time period. You might often like to correlate each period’s distributions using the same correlation matrix. In @RISK 5 a Correlated Time Series can be created by clicking the **Correlated Time Series** icon in the Define Correlations window and selecting the time series range in Excel.

When a correlated time series is created @RISK automatically sets up a correlation matrix “instance” for each set of similar distributions in each time period.
Defining Simulation Outputs in your Spreadsheet

@RISK 5 includes enhanced tools for adding or deleting simulation outputs in your spreadsheet. Outputs may be added or removed from the pop-up dialog.

Financial Forecasting

This model demonstrates the analysis of uncertainty in a financial forecast. Imagine you are deciding whether to launch a new product line. A simplified analysis of the cash flow activity of the venture might look as shown below. Since most of the elements of the model evolve the production of cash flows, we all involve uncertainty. The values in cells are drawn from @RISK distributions and the final income in cell C9 (E3), have been modified so that @RISK outputs are displayed as bar charts and can be performed on other simulated results. By adding @RISK distributions to your financial models, you can go beyond the simplistic "deal cash flow" analysis that can lead to bad business decisions.

Cash Flow

<table>
<thead>
<tr>
<th>Year</th>
<th>2016</th>
<th>2017</th>
<th>2018</th>
<th>2019</th>
<th>2020</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales Revenue</td>
<td>$174,370.00</td>
<td>$148,370.00</td>
<td>$244,370.00</td>
<td>$321,370.00</td>
<td>$407,370.00</td>
</tr>
<tr>
<td>Costs</td>
<td>$149,780.00</td>
<td>$149,780.00</td>
<td>$149,780.00</td>
<td>$149,780.00</td>
<td>$149,780.00</td>
</tr>
<tr>
<td>Gross Margin</td>
<td>$24,590.00</td>
<td>$4,590.00</td>
<td>$954,590.00</td>
<td>$1,549,590.00</td>
<td>$2,119,590.00</td>
</tr>
<tr>
<td>Operating Expenses</td>
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<td>$104,620.00</td>
<td>$104,620.00</td>
<td>$104,620.00</td>
<td>$104,620.00</td>
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<tr>
<td>Earnings Before Tax</td>
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<td>$10,000.00</td>
<td>$10,000.00</td>
<td>$10,000.00</td>
</tr>
<tr>
<td>Tax Expense</td>
<td>$6,000.00</td>
<td>$6,000.00</td>
<td>$6,000.00</td>
<td>$6,000.00</td>
<td>$6,000.00</td>
</tr>
<tr>
<td>Income Tax</td>
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<td>$4,000.00</td>
<td>$4,000.00</td>
<td>$4,000.00</td>
<td>$4,000.00</td>
</tr>
<tr>
<td>Net Income</td>
<td>$4,000.00</td>
<td>$4,000.00</td>
<td>$4,000.00</td>
<td>$4,000.00</td>
<td>$4,000.00</td>
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</table>

Market Conditions

<table>
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<tr>
<th>Number of Competitors</th>
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<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
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<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Sheet 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Reviewing a Model in the @RISK Model Window

The @RISK Model window provides a complete table of all input probability distributions, simulation outputs and correlation matrices described in your model. This window, which pops up over Excel, replaces the separate Model window found in @RISK version 4.5 and earlier. From this list, you can:

- Edit any input distribution or output by simply typing in the table
- Quickly view thumbnail graphs of all defined inputs
- Drag and drop any thumbnail graph to expand it to a full window
- Double-click on any entry in the table to use the Graph Navigator to move through cells in your workbook that contain input distributions
- Preview and edit correlation matrices
Customizing Displayed Statistics

The Model window columns can be customized to select which statistics you want to display on the input distributions in your model. The Select Columns for Table icon at the bottom of the window displays the Columns for Table dialog.

Placing Inputs in Categories

Inputs in the Model Window are grouped by category. By default, a category is made when a group of inputs share the same row (or column) name. In addition, inputs can be placed in any category you wish.
Properties for Input Distributions and Simulation Outputs

A new Properties window allows you to quickly define property functions for input distributions and simulation outputs. This provides a wizard for entering property functions that are used in @RISK distribution functions. Any time the Function Properties icon (fx) is displayed the window can be popped open.

New property functions for input distributions include:

- **RiskUnits** — units labels for graphs and reports
- **RiskStatic** — value which 1) is returned by function during a standard Excel recalculation and 2) replaces @RISK function after @RISK functions are swapped out
- **RiskSeed** — random number generator seed for a specific input

New property functions for simulation outputs include:

- **RiskUnits** — units labels for graphs and reports
- **RiskIsDiscrete** — forces @RISK to generate graphs and statistics for the output in discrete form
- **RiskSixSigma** — specifies LSL, USL and Target values for use in six sigma statistics calculations
Swapping @RISK Functions Out and In

By clicking the new Swap Functions icon, @RISK functions in @RISK 5 can be swapped in and out of your workbooks. This makes it easy to give models to colleagues who do not have @RISK. If your model is changed when @RISK functions are swapped out, @RISK will update the locations and static values of @RISK functions when they are swapped back in.

@RISK uses a new property function called RiskStatic to help in its function swap. RiskStatic holds the value that will replace the function when it is swapped out. It also specifies the value that @RISK will return for the distribution in a standard Excel recalculation. If you enter a new distribution using the Define Distribution window, @RISK can automatically store the value that you are replacing with a distribution in a RiskStatic property function. For example; if a cell C10 has the value 1000 in it, as shown in the formula:

\[ C10: =1000 \]

Then, using the Define Distribution window, you replace this value with a Normal distribution with a mean of 990 and a standard deviation of 100. Now, the formula in Excel will be:

\[ C10: =\text{RiskNormal}(990,100,\text{RiskStatic}(1000)) \]

Note that the original cell value of 1000 has been retained in the RiskStatic property function.

If you do not use RiskStatic, @RISK can use a distribution’s expected value, median, mode or a percentile as the static value when swapping functions out.
When functions are swapped out, the @RISK toolbar is disabled and if you enter an @RISK function it will not be recognized.

The Swap options dialog allows you to specify how @RISK will operate when functions are swapped in and out. If your workbook is changed when @RISK functions are swapped out, @RISK can report to you how it will re-insert @RISK functions into your changed model. In most cases, @RISK will be able to automatically handle changes made to a workbook when functions are swapped out.

![Image of spreadsheet showing financial forecasting](image-url)
Using Data to Define Probability Distributions

Distribution fitting is now done entirely in Excel, as compared to a separate application in @RISK 4.5. The distribution fitting features of the @RISK 5 Professional and Industrial versions include:

- The fitting of sample data (continuous or discrete) and data from a density or cumulative curve.
- Ranking of fits based on Chi-Squared, Kolmogorov-Smirnov, or Anderson-Darling statistics.
- Comparison graphs, difference graphs and P-P and Q-Q plots.
- Statistics and Goodness-of-fit tests.
- A summary window with results of all fits in a single report.
- Advanced fitting control, including the ability to specify exactly how the Chi-Squared statistic is calculated using equal interval binning, equal probability binning or custom binning.
- Ability to create a custom list of predefined distributions for fitting.
- Linking of @RISK functions to fitted data so functions will be updated automatically when data changes and your model is re-simulated.

The Fit Distributions icon on the @RISK toolbar is used to fit distributions to data and to manage existing fits.
**Fit Distributions to Data Dialog**

The Fit Distributions to Data dialog allows you to select a range of data in Excel to fit and specify options to be used during fitting. You can select the type of data to be fit (such as continuous, discrete or cumulative), filter the data, specify distribution types to be fit and specify Chi-Sq binning to be used.

![Fit Distributions to Data Dialog](image)

**Fit Results Graphs**

Fit result graphs include comparison graphs, difference graphs, P-P graphs and Q-Q graphs. By clicking in the Fit Ranking list, the results for each fitted distribution is displayed.

![Fit Results Graphs](image)
Clicking **Write to Cell** places a fit result in your model as a new distribution function.

Selecting **Update and Refit at the Start of Each Simulation** causes @RISK, at the start of each simulation, to automatically refit your data when it has changed and place the new resulting distribution function in your model.

The **Distribution Artist** is used to draw freeform curves, histograms or discrete probability graphs that can be used to create @RISK distributions. This is useful for graphically assessing probabilities and then creating probability distributions from the graph.

A curve may be drawn simply by dragging the mouse through the window. Clicking **OK** places the drawn curve in your model as a new distribution function.
Simulation Settings

@RISK simulation settings have been enhanced to reflect the new design and capabilities of @RISK 5. Many of these options can also be changed from the new @RISK Settings toolbar.

The new General tab settings control the general operation of @RISK. The options under When a Simulation is Not Running, Distributions Return are displayed when <F9> is pressed and a standard Excel recalculation is performed. If Random Values (Monte Carlo) are not selected, Static Values entered in a RiskStatic property function are returned. Where no RiskStatic function is present, distribution expected value, mode, median or a selected percentile is returned.

The Random Values (Monte Carlo) or Static Values settings can be quickly changed by clicking the new Random/Static Recalc icon on the @RISK Settings toolbar.
The new View settings control what will be shown by @RISK when a simulation is running. All graphs of simulation results now popup directly over Excel and optionally can “point” at the cell in your workbook whose distribution is being displayed.

New **Automatic Results Display** settings include:

- **Show Output Graph.** In this mode, a graph of simulation results for the selected cell automatically pops up in Excel:
  - When a run starts (if real-time updating is enabled with Update Windows During Simulation Every XXX Seconds), or
  - When a simulation is over.

- **Show Results Summary Window.** Pops up the @RISK — Results Summary window when a run starts (if real-time updating is enabled with Update Windows During Simulation Every XXX Seconds), or when a simulation is over.

- **None.** No new @RISK windows are displayed at the start or end of a simulation.

![@RISK - Simulation Settings](image)
New **Options** on the View tab of the Simulation Settings dialog include:

- **Demo mode.** Demo mode is a preset view where @RISK updates the workbook each iteration to show values changing and pops up and updates a graph of the first output in your model. This mode is useful for illustrating a simulation in @RISK.

- **Update Windows During simulation Every XXX Seconds.** Turns real time updating of open @RISK windows on and off, and sets the frequency with which windows are updated. When **Automatic** is selected, @RISK selects an update frequency based on the number of iterations performed and the runtime per iteration.

The new Sampling settings control how samples will be drawn from probability distributions by @RISK when a simulation is running.

New **Random Numbers** settings include:

**Generator** — any of eight different random number generators can be selected for use when simulating, @RISK uses a new default random number generator — Mersenne Twister.
The new Convergence settings control how the convergence of simulation outputs will be monitored by @RISK when a simulation is running. Convergence testing in @RISK 5 can be controlled for individual outputs using the new RiskConvergence property function, or set globally for all outputs from a simulation in the Simulation Settings dialog.

New Convergence Options include:

- **Convergence Tolerance** — Specifies the tolerance allowed for the statistic you are testing. For example, the settings below specify that you wish to estimate the mean of each output simulated within 3% of its actual value.

- **Confidence Level** — Specifies the confidence level for your estimate. For example, the settings below specify that you want your estimate of the mean of each output simulated (within the entered tolerance) to be accurate 95% of the time.

- **Perform Tests on Simulated** — Specifies the statistics of each output that will be tested.
Running Simulations

@RISK 5 simulations include the updating of graphs and reports on top of Excel while a simulation runs. Simulations can be paused or stopped using the Progress Control window. The @RISK — Results Summary window provides a “dashboard” view of all simulation outputs with small graph thumbnails that update as a simulation runs.
Reviewing Simulation Results Graphically

Once a simulation has been run, @RISK 5 has:

- A new **Browse mode** that allows you to easily view graphs of simulation results by selecting cells in your spreadsheet.
- The **@RISK — Results Summary** window summarizes the results of your model and displays thumbnail graphs and summary statistics for your simulated output cell and input distributions.
- **New graph types** — Summary Box Plot, Tornado — Regression Mapped Values and Scatter Plots — help you review and interpret your simulation results.
- A **new graphics engine** includes extensive customization options to enhance your reports on simulation results.
Browse Mode

The Browse mode is started by clicking the Browse Results icon on the @RISK toolbar. Browse mode is automatically turned on at the end of a run if you select to popup a graph during a simulation.

In Browse mode, @RISK pops up graphs of simulation results as you click on cells in your spreadsheet, as follows:

- If the selected cell is a simulation output (or contains a simulated distribution function), @RISK will display its simulated distribution in a callout pointing at the cell.

- If the selected cell is part of a correlation matrix, a matrix of the simulated correlations between the inputs in the matrix pops up.

As you click on different cells in your workbook, their results graphs pop up. Press <Tab> to move the Graph window among output cells with simulation results in open workbooks.

From a graph window, you can easily add overlays plus create scatter plots and summary graphs by clicking the icons at the bottom of the window and selecting cells to include in the graph in Excel.

To exit the Browse mode, simply close the popup graph or click the Browse Results icon on the toolbar.
@RISK Results Summary Window

The @RISK Results Summary Window summarizes the results of your model and displays thumbnail graphs and summary statistics for your simulated output cell and input distributions. As with the Model window, you can:

- Drag and drop any thumbnail graph to expand it to a full window.
- Double-click on any entry in the table to use the Graph Navigator to move through cells in your workbook that have simulation results.
- Customize columns to select which statistics you want to display on results.
Graphs can be made in @RISK by simply dragging thumbnails off the Results or Model windows. In addition, overlays can be added to a graph by dragging a graph (or thumbnail) onto another.

Multiple graphs can be created at once by selecting multiple rows in the @RISK Results Summary Window and clicking on the Graph icon at the bottom of the window.
New @RISK 5 Graphs

Graphs of simulation results in @RISK 5 include new Summary Box Plots, Tornado Graphs and Scatter Plots to help you review and interpret your simulation results.

@RISK 5 has two types of graphs that summarize trends across a group of simulated outputs (or inputs). These are the Summary Trend and Box Plot. Each of these graphs can be made by clicking the Summary Graph icon at the bottom of a graph window and selecting the cells you want to include in the graph in Excel.
**Tornado Graphs**

Tornado graphs from a sensitivity analysis display a ranking of the input distributions which impact an output. In @RISK 5, three methods are available for displaying tornado graphs — **Regression Coefficients**, **Regression (Mapped Values)** and **Correlation Coefficients**.

Tornado graphs may be displayed by selecting a row (or rows) in the @RISK — Results Summary window, clicking **Tornado Graph** icon at the bottom of the window and one of the three Tornado graph options. Alternatively, a distribution graph for a simulated output can be changed to a tornado graph by clicking the **Tornado Graph** icon in the lower left of the graph.

@RISK 5 offers a new tornado graph type — **Regression — Mapped Values**. The values on the X-axis of this tornado graph type show the amount of change in the output due to a +1 standard deviation change in each input. For example, in the graph below, when Sales Volume 2017 increases by 8000 units (1 standard deviation), the NPV (10%) output will increase by 44,614.

@RISK 5 offers Smart Sensitivity Analysis by pre-screening inputs based on their precedence to outputs in your model. Inputs located in formulas that have no link (via your model’s formulas) to an output are removed from the sensitivity analysis, thus avoiding erroneous results.
@RISK 5 provides scatter plots to show the relationship between simulated outputs and inputs. A scatter plot is an x-y graph showing the values calculated in each iteration of the simulation for two inputs or outputs. A confidence ellipse identifies the region where, at a given confidence level, the x-y values will fall. Scatter graphs can also be standardized so that values from multiple inputs could be more easily compared on a single scatter plot.

Scatter graphs can be created by:

- Clicking the Scatter Plot icon on a displayed graph and then selecting the cell(s) in Excel whose results you want to include in the plot.
- Selecting one or more outputs or inputs in the @Risk Results Summary window and clicking the Scatter Plot icon.
- Dragging a bar (representing the input you want to show in the scatter graph) from an output’s tornado graph.
- Displaying a scatter plot matrix in the Sensitivity Analysis window (see Sensitivity Analysis Window later in this section).
- Clicking on a correlation matrix in Browse mode displays a scatter plot matrix showing the simulated correlations between the inputs correlated in the matrix.

As with other @RISK graphs, scatter plots will update in real-time when a simulation runs.
If you have defined a correlation matrix, it is often useful to check the actual simulated correlations between any pair of inputs in the matrix. To do this, simply click on a cell in the matrix when browsing simulation results. A pop-up scatter matrix appears showing scatter plots between any pair of inputs in the matrix. To show a larger graph of any of the thumbnail scatter plots in the matrix, simply drag the cell off the matrix into a new graph window.

Scatter plots, like many other @RISK graphs, may be overlaid. This shows how the values for two (or more) inputs are related to the value of an output.
Customizing and Reporting @RISK Graphs

@RISK 5 graphics use a new graphing engine designed specifically for processing simulation data. Graphs can be customized and enhanced as needed, often by simply clicking on the appropriate element in the graph. For example, to change the title of a graph, simply click on the title and type the new entry.
A displayed graph can also be customized through the **Graph Options** dialog. Customization includes colors, scaling, fonts and displayed statistics.

Displayed statistics are included in any graph that you copy and paste into your Excel workbook, or into a report in PowerPoint or Word. Simply right-click on a graph to copy it, and then paste it into your report.
Reports on Simulation Results

Once a simulation has been run, @RISK 5 has a set of reports that help explain the results of your simulation. These include Detailed Statistics, Data, Sensitivity Analysis and Scenario Analysis. In @RISK 4.5 and earlier versions, these reports were shown in the separate @RISK Results Summary window. To display any of these report windows, click the appropriate icon on the @RISK toolbar.

Report windows may be exported to Excel for use in an Excel workbook.

If the Simulation Setting Update Windows During Simulation Every XXX Seconds is selected, all report windows update as a simulation runs.

This report shows all statistics on simulated outputs and inputs, and allows the entry of target values for inputs and outputs. New to @RISK 5 is the ability to pivot this report so that statistics are reported by row instead of column.
This report shows, by iteration, all values calculated for simulated outputs, and all values sampled for input probability distributions. In addition:

- Data from a simulation may be sorted to show key values you are interested in.

Iterations from a previously run simulation may be stepped through, updating Excel with values sampled and calculated. This is useful to investigate iterations with errors, and iterations that led to certain output scenarios.
This report shows sensitivity analysis results for all outputs in your model. The reported results are ranked by the output you select. New to @RISK 5 is:

- Reporting of **Regression — Mapped Values**
- Display of a **Scatter Plot Matrix**, showing individual scatter plots for each input listed vs. each output in the report
A scatter plot is an x-y graph showing the input value sampled vs. the output value calculated in each iteration of the simulation. In the Scatter Plot Matrix, ranked sensitivity analysis results are displayed with scatter plots. To show the Scatter Plot Matrix, click the Scatter Plot icon in the lower left of the Sensitivity window.

Using Drag and Drop, a thumbnail scatter plot in the Scatter Plot Matrix can be dragged and expanded into a full graph window. In addition, overlays of scatter plots may be created by dragging additional scatter thumbnail graphs from the matrix onto an existing scatter plot.
Scenario analysis allows you to determine which input variables contribute significantly towards reaching a goal. For example, which variables contribute to exceptionally high sales? Or, which variables contribute to profits below $1,000,000?

A scatter plot in the Scenarios window is an overlay x-y scatter plot. This graph shows:

1) the input value sampled vs. the output value calculated in each iteration of the simulation,

2) overlaid with a scatter plot of the input value sampled vs. the output value calculated when the output value meets the entered scenario.

In the Scatter Plot Matrix, ranked scenario analysis results are displayed with scatter plots. To show the Scatter Plot Matrix, click the Scatter Plot icon in the lower left of the Scenarios window.
Using Drag and Drop, a thumbnail scatter plot in the Scatter Plot Matrix can be dragged and expanded into a full graph window. In addition, overlays of scatter plots may be created by dragging additional scatter thumbnail graphs from the matrix onto an existing scatter plot.

Each of the Report windows in @RISK 5 may be exported to an Excel worksheet for use in Excel. To export a report, click the Edit icon at the bottom of any Report Window and select Report in Excel.
Saving Simulations

@RISK 5 adds new options for saving simulations you have run and comparing them with other simulations. These include:

- Storing simulations in your Excel workbook
- Using the @RISK Library for storing and comparing different simulations (see the section @RISK Library)

When you want to store simulation results and graphs, @RISK 5 allows you to keep all data in your Excel workbook. This makes it easy for you to give simulations to others, without worrying about sharing a separate .RSK simulation file as was required in earlier versions of @RISK.

When a simulation is saved in your workbook, all data and graphs are stored and will be automatically opened next time you open the workbook in Excel with @RISK running.

You can also use the new **Application Settings** command in the @RISK Utilities menu to specify the default location where you wish to store your @RISK data.
@RISK Library

@RISK 5 Professional and Industrial versions include the @RISK Library, a separate database application for sharing @RISK’s input probability distributions, and comparing results from different simulations. The @RISK Library uses SQL Server to store @RISK data. Different users in an organization can access a shared @RISK Library in order to access:

- Common input probability distributions, which have been pre-defined for use in an organization’s risk models
- Simulation results from different users
Chapter 4: Getting to Know @RISK

A Quick Overview of @RISK.................................................................93
How Does Risk Analysis Work?............................................................93
How Does @Risk Link to Excel?............................................................93
Entering Distributions in Workbook Formulas.......................................95
Simulation Outputs...............................................................................96
Model Window.......................................................................................97
Using Data to Define Probability Distributions......................................98
Running a Simulation...........................................................................98
Simulation Results...............................................................................100
Advanced Analytical Capabilities.........................................................102

Setting Up and Simulating an @RISK Model.......................................105
Probability Distributions in Your Worksheet.........................................105
Correlating Input Variables..................................................................109
Fitting Distributions To Data...............................................................111
@RISK Model Window.........................................................................114
Simulation Settings.............................................................................116
Running a Simulation.........................................................................118
Browse Mode.......................................................................................121
@RISK Results Summary Window......................................................122
Detailed Statistics Window..................................................................123
Targets.................................................................................................123
Graphing Results................................................................................124
Sensitivity Analysis Results.................................................................131
Scenario Analysis Results.................................................................133
Reporting in Excel..............................................................................135
A Quick Overview of @RISK

This chapter provides a quick overview of using @RISK with Microsoft Excel. It guides you through the process of setting up an Excel model to be used with @RISK, simulating that model, and interpreting the result of your simulation.

The material in this chapter is presented on-line in the @RISK tutorial. This can be run by selecting Start / Programs / Palisade DecisionTools / Tutorials / @RISK Tutorial.

How Does Risk Analysis Work?

@RISK extends the analytical capabilities of Microsoft Excel to include risk analysis and simulation. These techniques allow you to analyze your spreadsheets for risk. Risk Analysis identifies the range of possible outcomes you can expect for a spreadsheet result and their relative likelihood of occurrence.

@RISK uses the technique of Monte Carlo simulation for risk analysis. With this technique, uncertain input values in your spreadsheet are specified as probability distributions. An input value is a value in a spreadsheet cell, or formula, which is used to generate results in your spreadsheet. In @RISK, a probability distribution which describes the range of possible values for the input is substituted for its original single fixed value. To find out more about inputs and probability distributions, see Chapter 2 of this User’s Guide: An Overview of Risk Analysis.

How Does @Risk Link to Excel?

To add risk analysis capabilities to your spreadsheet @RISK uses menus, toolbars and custom distribution functions in your spreadsheet.
@RISK Menu

An @RISK Menu is added to Excel versions 2003 and earlier, allowing you to access all commands necessary for setting up and running simulations.

@RISK Toolbars

An @RISK toolbar is added to Excel (versions 2003 and earlier) and also an @RISK ribbon bar to Excel 2007. The icons and commands on these bars allow you to quickly access most @RISK options.

@RISK Distribution Functions

In @RISK, probability distributions are entered directly into your worksheet formulas using custom distribution functions. These new functions, each of which represents a type of probability distribution (such as NORMAL or BETA), are added to your spreadsheets functions set by @RISK. When entering a distribution function you enter both the function name, such as RiskTriang — a triangular distribution — and the arguments which describe the shape and range of the distribution, such as RiskTriang (10,20,30), where 10 is the minimum value, 20 the most likely value and 30 the maximum value.

Distribution functions may be used anywhere in your spreadsheet that there is uncertainty about the value being used. The @RISK functions may be used just as you would use any normal spreadsheet functions — including them in mathematical expressions and having cell references or formulas as arguments.
Entering Distributions in Workbook Formulas

@RISK includes a pop-up Define Distribution window that allows you to easily add probability distribution functions to spreadsheet formulas. By clicking the Define Distribution icon you can display this window.

The @RISK Define Distribution window graphically displays probability distributions which can be substituted for values in a spreadsheet formula. By changing the displayed distribution you can see how various distributions would describe the range of possible values for an uncertain input in your model. The displayed statistics further show how a distribution defines an uncertain input.

The graphical display of an uncertain input is useful in showing your definition of the input to others. It displays the range of possible values for an input and the relative probability of any value in the range occurring. Working with distribution graphs you can easily incorporate assessments of uncertainty from experts into your risk analysis models.

When the Define Distribution window is displayed, press <Tab> to move the window among cells with distributions in open workbooks.
Simulation Outputs

Once distribution functions have been entered into your spreadsheet, you need to identify those cells (or ranges of cells) that you are interested in seeing simulation results for. Typically, these output cells contain the results of your spreadsheet model (such as “profit”) but they can be any cells, anywhere in your spreadsheet. To select outputs, simply highlight the cell or range of cells you want as outputs, in your worksheet, and then click the Add Output icon — the one with the red down arrow.
Model Window

The @RISK — Model window provides a complete table of all input probability distributions and simulation outputs described in your model. From this window, which pops up over Excel, you can:

- Edit any input distribution or output by simply typing in the table
- Drag and drop any thumbnail graph to expand it to a full window
- Quickly view thumbnail graphs of all defined inputs.
- Double-click on any entry in the table to use the Graph Navigator to move through cells in your workbook with input distributions

The Model window columns can be customized to select which statistics you want to display on the input distributions in your model.
Using Data to Define Probability Distributions

The @RISK Distribution Fitting toolbar (Professional and Industrial versions only) allows you to fit probability distributions to your data. Fitting is done when you have a set of collected data that you want to use as the basis for an input distribution in your spreadsheet. For example, you may have collected historical data on a product price and you might want to create a distribution of possible future prices based on this data.

If desired, the distributions which result from a fit can be assigned to an uncertain value in your spreadsheet model. In addition, if data in Excel is used in a fit, it can be “hot-linked” so that the fit will automatically update whenever your data changes and your model is re-simulated.

Running a Simulation

A simulation is run by clicking the Start Simulation icon on the @RISK toolbar or ribbon.
When a simulation is run, your spreadsheet is calculated repeatedly — each recalculation is an “iteration” — with a set of new possible values sampled from each input distribution and each iteration. With each iteration the spreadsheet is recalculated with the new set of sampled values and a new possible result is generated for your output cells.

As a simulation progresses, new possible outcomes are generated from each iteration. @RISK keeps track of these output values and displays them on a popup graph which is displayed with an output.

This graph of the distribution of possible outcomes is created by taking all the possible output values generated, analyzing them and calculating statistics on how they are distributed across their minimum-maximum range.
Simulation Results

@RISK simulation results include distributions of possible results for your outputs. In addition, @RISK generates sensitivity and scenario analysis reports which identify the input distributions most critical to your results. These results are best presented graphically. Available graphs include frequency distributions of possible output variable values, cumulative probability curves, tornado graphs which show the sensitivities of an output to different inputs, and summary graphs which summarize risk changes across a range of output cells.
The easiest way to get a report on your @RISK simulation in Excel (or Word) is to simply copy and paste a graph and the included statistics. In addition, any report window can be exported to an Excel sheet where you can access its values with formulas.

@RISK also offers a set of standard reports on simulations that summarize your results. In addition, @RISK reports generated in Excel can use pre-built template sheets that contain custom formatting, titles and logos.
Advanced Analytical Capabilities

Advanced capabilities are available in @RISK to allow sophisticated analysis of simulation data. @RISK collects simulation data by iteration for both input distributions and output variables. It analyzes this data set to determine:

- **Sensitivities**, identifying the input distributions which are 'significant' in determining output variable values, and
- **Scenarios**, or the combinations of input distributions which generate output target values.

The Sensitivity analysis — which identifies significant inputs — is carried out with two different analytical techniques — regression analysis and rank correlation calculation. The results of a sensitivity analysis can be displayed as a “tornado” type chart, with longer bars at the top representing the most significant input variables.
The Scenario analysis identifies combinations of inputs which lead to output target values. Scenario analysis attempts to identify groupings of inputs which cause certain output values. This allows simulation results to be characterized by statements such as “when Profits are high, significant inputs are, low Operating Costs, very high Sales Prices, high Sales Volumes, etc.”
Setting Up and Simulating an @RISK Model

Now that you have a quick overview as to how @RISK works, let's work through the process of setting up an @RISK model in your spreadsheet and running a simulation on it. We'll touch briefly on:

- Probability Distributions in Your Worksheet
- Correlations Between Distributions
- Running a Simulation
- Simulation Results
- Graphs of Simulation Results

Probability Distributions in Your Worksheet

As previously mentioned, uncertainty in an @RISK model is entered with distribution functions. You can choose from over thirty different functions when entering uncertainty in your spreadsheet. Each function describes a different type of probability distribution. The simplest functions are those such as `RiskTriang(min,most likely,max)` or `RiskUniform(min,max)` which take arguments specifying the minimum, most likely, or maximum possible value for the uncertain input. More complex functions take arguments specific to the distribution — such as `RiskBeta(alpha,beta)`.

For more sophisticated models, @RISK allows you to set up distribution functions which use cell references and spreadsheet formulas for function arguments. Many powerful modeling features can be created by using these types of functions. For example, you can set up a group of distribution functions across a spreadsheet row, with the mean of each function determined by the value sampled for the prior function. Mathematical expressions can also be used as arguments for distribution functions.
Distributions in the Define Distribution Window

All distribution functions can be defined and edited using the pop-up Define Distribution window. The Define Distribution window can, among other things, also be used to enter multiple distribution functions in a cell’s formula, entering names that will be used to identify an input distribution and truncate a distribution.
Argument values can be entered in the Distribution Argument panel or typed directly in the shown formula. This panel is displayed to the left of the graph.

By changing the Parameter type, you can select to enter Alternate Parameters or Truncate the distribution.
@RISK distribution functions have both required and optional arguments. The only required arguments are the numeric values which define the range and shape of the distribution. All other arguments, such as name, truncation, correlation, and others are optional, and can be entered only when needed. These optional arguments are entered using property functions using a pop-up **Input Properties** window.

All entries made in the **Define Distribution** Window are converted to distribution functions that are placed in your spreadsheet. For example, the distribution function created by the entries in the window displayed here would be:

\[ =\text{RiskNormal}(3000,1000,\text{RiskTruncate}(1000,5000)) \]

Thus, all the distribution arguments that are assigned through the Define Distribution window can also be entered directly in the distribution itself. In addition, all arguments can be entered as cell references or as formulas, as are standard Excel functions.

It often helps to first use the Define Distribution window to enter your distribution functions to better understand how to assign values to function arguments. Then, once you better understand the syntax of distribution function arguments, you can enter the arguments yourself directly in Excel, bypassing the Define Distribution window.
Correlating Input Variables

During a simulation analysis it is important to account for correlation between input variables. Correlation occurs when the sampling of two or more input distributions are related — for example, when the sampling of one input distribution returns a relatively “high” value, it may be that sampling a second input should also return a relatively high value. A good example is the case of one input named “Interest Rate” and a second input called “Housing Starts”. There may be a distribution for each of these input variables, but the sampling of them should be related to avoid nonsensical results. For example, when a high Interest Rate is sampled, Housing Starts should be sampled as relatively low. Conversely, you would expect that when Interest Rates are low, Housing Starts should be relatively high.

Correlations can be added by selecting the cells in Excel that contain the input distributions you want to correlate, and then click the **Define Correlations** icon. You can also add inputs to a displayed matrix by clicking **Add Inputs** and selecting cells in Excel.

Once a matrix is displayed, you can enter correlation coefficients between inputs in matrix cell, copy values in from a matrix in Excel, or use **scatter plots** to assess and enter correlations.
**Scatter Plots for Correlations**

A scatter plot matrix is displayed by clicking the Scatter Plots icon at the bottom left of the Define Correlations window. The scatter plots in matrix cells show how values between any two input distributions are correlated. Moving the Correlation Coefficient slider displayed with the scatter matrix dynamically changes the correlation coefficient and scatter plot for any pair of inputs.

By dragging a scatter plot cell off of the matrix you can expand the thumbnail scatter plot into a full graph window. This window will also update dynamically when the Correlation Coefficient slider is changed.

With the Define Distribution window, correlation matrices entered in it change @RISK functions in your spreadsheet model. RiskCorrmat functions are added that contain all the correlation information that was entered in your matrix. Once you see the RiskCorrmat entries that are entered, and are comfortable with their syntax, you can enter these functions yourself directly in your spreadsheet, bypassing the Define Correlations window.
Fitting Distributions To Data

The @RISK allows you to fit probability distributions to your data (Professional and Industrial versions only). Fitting is done when you have a set of collected data that you want to use as the basis for an input distribution in your spreadsheet. For example, you may have collected historical data on a product price and you might want to create a distribution of possible future prices that is based on this data.

Fitting Options

A variety of options are available for controlling the fitting process. Specific distributions can be selected to fit. In addition, input data can be in the form of sample, density or cumulative data. You can also filter your data prior to fitting.
Fit Reports

Comparison, P-P and Q-Q plots are available to help you examine the results of your fits. Delimiters on graphs allow you to quickly calculate probabilities associated with values in fitted distributions.

Placing a Fit Result in Excel

Clicking Write to Cell places a fit result in your model as a new distribution function. Selecting Update and Refit at the Start of Each Simulation causes @RISK, at the start of each simulation, to automatically refit your data when it has changed, and place the new resulting distribution function in your model.
**Fit Manager**

The Fit Manager allows you to navigate between fitted data sets in your workbook and delete previously run fits.
@RISK Model Window

To help you view your model, @RISK detects all distribution functions, outputs and correlations entered in your worksheet and lists them in the @RISK Model window. From this window, which pops up over Excel, you can:

- Edit any input distribution or output by simply typing in the table
- Drag and drop any thumbnail graph to expand it to a full window
- Quickly view thumbnail graphs of all defined inputs
- Double-click on any entry in the table to use the Graph Navigator to move through cells in your workbook with input distributions
- Edit and preview correlation matrices
The Model window columns can be customized to select which statistics you want to display on the input distributions in your model. The **Columns** icon at the bottom of the window displays the **Columns for Table** dialog.

Inputs in the **Model** Window can be grouped by category. By default, a category is made when a group of inputs share the same row (or column) name. In addition, inputs can be placed in any category you wish.
Simulation Settings

A variety of settings may be used to control the type of simulation @RISK performs. A simulation in @RISK supports nearly unlimited iterations and multiple simulations. Multiple simulations allow you to run one simulation after another on the same model. In each simulation you can change values in your spreadsheet so you can compare simulation results under different assumptions.
An @RISK Settings Toolbar is added to the Excel menu bar. This allows quick access to many simulation settings.

Icons on this toolbar include:

- **Simulation Settings** opens the Simulation Settings dialog box.
- **Iterations** drop-down list, where the number of iterations to run can be quickly changed from the toolbar.
- **Simulations** drop-down list, where the number of simulations to run can be quickly changed from the toolbar.
- **Random/Static Recalc** flips @RISK between, returning expected or static values from distributions, to returning Monte Carlo samples in a standard Excel recalculation.
- **Show Graph, Show Results Window, Demo Mode** control what is shown on the screen during and after a simulation.
- **Real Time Update** controls if open windows will be updated while a simulation is running.
Running a Simulation

A simulation in @RISK involves repetitive recalculations of your worksheet. Each recalculation is called an “iteration”. With each iteration:

- All distribution functions are sampled.
- Sampled values are returned to the cells and formulas of the worksheet.
- The worksheet is recalculated.
- Values calculated for output cells are collected from the worksheet and stored.
- Open @RISK graphs and reports are updated, if necessary

This repetitive recalculation process can run hundreds or thousands of iterations if necessary.

Clicking the Start Simulation icon starts a simulation. When a simulation is running you can watch Excel recalculate numerous times using different sampled values from distribution functions, monitor the convergence of your output distributions and watch graphs of distributions of simulation results update real-time.

Progress Window

A Progress window is displayed during simulations. The icons in this window allow you to Run, Pause or Stop a simulation, as well as turn real-time updating of graphs and Excel recalculations on and off.
@RISK shows you graphically how distributions of possible outcomes change during a simulation. Graph windows update to show calculated distributions of outcomes and their statistics. If you are starting a new simulation. For the first output cell in your model @RISK will automatically pop up a graph for the distribution.

This graph of the distribution of possible outcomes is created by taking all the possible output values generated, analyzing them and calculating statistics on how they are distributed across their minimum-maximum range.

@RISK includes a convergence monitoring capability to help evaluate the stability of the output distributions during a simulation. As more iterations are run, output distributions become more “stable” as the statistics describing each distribution change less with additional iterations. It is important to run enough iterations so that the statistics generated on your outputs are reliable. However, there comes a point when the time spent for additional iterations is essentially wasted because the statistics generated are not changing significantly.

The Convergence settings control how the convergence of simulation outputs will be monitored by @RISK when a simulation is running. Convergence testing can be controlled for individual outputs using the RiskConvergence property function or set globally for all outputs from a simulation in the Simulation Settings dialog.
@RISK monitors a set of convergence statistics on each output distribution during a simulation. During monitoring, @RISK calculates these statistics for each output at selected intervals (such as every 100 iterations) throughout the simulation.

As more iterations are run, the amount of change in the statistics becomes less and less until they reach the Convergence Tolerance and Confidence Level you enter.

If desired, @RISK can run in **Auto-Stop** mode. In this case @RISK will continue to run iterations until all outputs have converged. The number of iterations required for output distributions to converge is dependent on the model being simulated and distribution functions included in the model. More complex models with highly skewed distributions will require more iterations than simpler models.
Browse Mode

The Browse mode is started by clicking the Browse Results icon on the @RISK toolbar. Browse mode is automatically turned on at the end of a run if you select to popup a graph during a simulation.

In Browse mode, @RISK pops up graphs of simulation results as you click on cells in your spreadsheet, as follows:

- If the selected cell is a simulation output (or contains a simulated distribution function), @RISK will display its simulated distribution in a callout pointing at the cell.
- If the selected cell is part of a correlation matrix, a matrix of the simulated correlations between the inputs in the matrix pops up.

As you click on different cells in your workbook, their results graphs pop up. Press <Tab> to move the Graph window among output cells with simulation results in open workbooks.

To exit the Browse mode, simply close the popup graph or click the Browse Results icon on the toolbar.
@RISK Results Summary Window

The @RISK Results Summary Window summarizes the results of your model and displays thumbnail graphs and summary statistics for your simulated output cells and input distributions. Columns in the Results Summary Window table can be customized to select which statistics you want to display.

From the Results Summary Window, you can:

- Drag and drop any thumbnail graph to expand it to a full window
- Double-click on any entry in the table to use the Graph Navigator to move through cells in your workbook with input distributions
Detailed Statistics Window

Detailed statistics are available on simulated outputs and inputs, and target values can be entered for one or more inputs and outputs.

Targets

Target values can be calculated on simulation results. A target shows the probability of achieving a specific outcome or the value associated with any probability level. Using targets you can answer questions such as “What is the chance of a result greater than one million?” or “What is the chance of a negative outcome?”. Targets can be entered in the Detailed Statistics window, the @RISK — Results Summary window, and set directly using delimiters on graphs of simulation results.

By entering a desired target — such as 1% — for an output in the @RISK Results Summary Window and copying it across all outputs, you can quickly see the same target calculated for all simulation results.
**Graphing Results**

Simulation results are easily expressed with graphs. The Results Summary Window shows thumbnail graphs of the simulation results for all your outputs and inputs. Dragging a thumbnail graph off of the Results Summary Window allows you to expand a graph in a larger window.

A graph of the results for an output shows the range of possible outcomes and their relative likelihood of occurrence. This type of graph may be displayed in standard histogram or frequency distribution form. Distributions of possible outcomes may also be displayed in cumulative form.

Each graph created by @RISK is displayed in conjunction with the statistics for the output or input that is graphed. The type of graph displayed may be changed using the icons at the bottom of the Graph window. In addition, by clicking the right mouse button on a graph window, a pop-up menu is displayed with commands that allow the changing of a graph’s format, scaling, colors, titles and displayed statistics. Each graph may be copied to the clipboard and pasted into your spreadsheet. As graphs are transferred as Windows metafiles, they may be resized and annotated once pasted into a spreadsheet.

Using the Chart in Excel command, graphs can be drawn in Excel's native graph format. These graphs can be changed or customized just as with any Excel graph.
Many times it is useful to compare several simulated distributions graphically. This can be done using overlay graphs.

Overlays are added by clicking the **Add Overlay icon** at the bottom of a graph window, dragging one graph onto another, or by dragging a thumbnail graph from the Results Summary Window onto an open graph. Once overlays are added, delimiter statistics display probabilities for all distributions included in the overlay graph.
By dragging the delimiters displayed on a histogram or cumulative graph, target probabilities may be calculated. When delimiters are moved, calculated probabilities are shown in the delimiter bar above the graph. This is useful for graphically displaying answers to questions such as “What is the probability of a result between 1 million and 2 million occurring?” and “What is the probability of a negative result occurring?”.

Delimiters can be displayed for any number of overlays. The Graph Options dialog allows you to set the number of delimiter bars displayed.
Each distribution in an overlay graph can be formatted independently. By using the **Curves** tab options in the **Graph Options** dialog box, the color, style and pattern of each curve in the overlay graph may be set.

A **Summary graph** displays how risk changes across a range of output or input cells. You can create a summary graph for an output range, or select individual inputs or outputs to compare in a summary graph. Summary graphs have two forms — **Summary Trend** graphs and **Summary Box Plot** graphs. Each of these graphs can be made by:

- Clicking the **Summary Graph** icon at the bottom of a graph window and then selecting the cell(s) in Excel whose results you want to include in the graph.

- Selecting the rows in the @RISK Results Summary Window for the outputs, or inputs, you wish to include in the summary graph, then clicking the **Summary Graph** icon at the bottom of the window (or right-clicking in the table), and selecting **Summary Trend** or **Summary Box Plot**.

A Summary Trend graph is especially useful in displaying trends such as how risk changes across time. If, for example, a range of 10 output cells contained Profit in years 1 through 10 of a project, the Summary Trend graph for this range shows how your risk changed across the 10 year period. The narrower the band, the less the uncertainty about your Profit estimates. Conversely, the wider the band the greater the possible variance in Profit and the greater the risk.
The center line of the Summary Trend graph represents the trend in mean value across the range. The two outer bands above the mean are one standard deviation above the mean and the 95th percentile. The two outer bands below the mean are one standard deviation below the mean and the 5th percentile. The definition of these bands can be changed using the Trend tab in the Graph Options dialog box.

A **Summary Box Plot** displays a box plot for each distribution selected for inclusion in the summary graph. A box plot (or box-whisker graph) shows a box for a defined inner range of a distribution; with whisker lines showing the outer limits of the distribution. An inner line in the box marks the location of the mean, median or mode of the distribution.
**Scatter Plots**

A scatter plot is an x-y graph showing the input value sampled vs. the output value calculated in each iteration of the simulation. This graph is useful for examining in detail the relationship between an input and an output from a simulation. A confidence ellipse identifies the region where, at a given confidence level, the x-y values will fall. Scatter plots may also be standardized so that values from multiple inputs may be more easily compared on a single scatter plot.

Scatter plot windows can be created by:

- Clicking the Scatter Plot icon on a displayed graph and then selecting the cell(s) in Excel whose results you want to include in the plot.
- Selecting one or more outputs or inputs in the Results Summary window and clicking the Scatter Plot icon.
- Dragging a bar (representing the input you want to show in the scatter plot) from an output’s tornado graph.
- Displaying a scatter plot matrix in the Sensitivity Analysis report window (see the Sensitivity Analysis Window towards the end of this section).
- Clicking on a correlation matrix in Browse mode displays a scatter plot matrix showing the simulated correlations between the inputs correlated in the matrix.
Sensitivity Analysis Results

Sensitivity analysis results are displayed by clicking the Sensitivity Window icon. These results show the sensitivity of each output variable to the input distributions in your worksheet. This identifies the most "critical" inputs in your model. These are the inputs you should concentrate on most when making plans based on your model.

The data displayed in the Sensitivity window is ranked for the output selected in the Rank Inputs for Output: entry. The sensitivity of all other outputs to the ranked inputs is also shown.

Sensitivity analyses performed on the output variables and their associated inputs use either multivariate stepwise regression or a rank order correlation. The type of analysis desired is set using the Display Significant Inputs Using: entry in the Sensitivity Window.

In the regression analysis, the coefficients calculated for each input variable measure the sensitivity of the output to that particular input distribution. The overall fit of the regression analysis is measured by the reported fit or R-squared of the model. The lower the fit the less stable the reported sensitivity statistics. If the fit is too low — beneath .5 — a similar simulation with the same model could give a different ordering of input sensitivities.

The sensitivity analysis using rank correlations is based on the Spearman rank correlation coefficient calculations. With this analysis, the rank correlation coefficient is calculated between the selected output variable and the samples for each of the input distributions. The higher the correlation between the input and the output, the more significant the input is in determining the output's value.
A scatter plot is an x-y graph showing the input value sampled vs. the output value calculated for each iteration of the simulation. In the Scatter Plot Matrix, ranked sensitivity analysis results are displayed with scatter plots. To show the scatter plot matrix, click the Scatter Plot icon in the lower left of the Sensitivity window.

Using Drag and Drop, a thumbnail scatter plot in the Scatter Plot Matrix can be dragged and expanded into a full graph window. In addition, overlays of scatter plots may be created by dragging additional scatter thumbnail graphs from the matrix onto an existing scatter plot.

Sensitivity results are graphically displayed in tornado graphs. A tornado graph can be generated by right-clicking any output in the Results Summary Window or clicking the Tornado Graph icon on a graph window.
Scenario Analysis Results

The Scenarios Window icon displays the Scenario analysis results for your output variables. Up to three scenario targets may be entered for each output variable.

The Scenario analysis performed on output variable targets is based on a conditional median analysis. In performing the Scenario analysis, first @RISK subsets the iterations of the simulation into only those iterations in which the output variable achieves the entered target. It then analyzes the values sampled for each input variable in those iterations. @RISK finds the median of this “subset” of sampled values for each input and compares that with the median of the input for all iterations.

The objective of this process is to find those inputs whose subset, or conditional median, differs significantly from the overall median. If the subset median for the input variable is close to the overall median, the input variable is marked as insignificant. This is because the values sampled for the input in the iterations where the target is achieved do not differ markedly from those sampled for the input for the entire simulation. However, if the subset median for the input variable deviates significantly from the overall median (i.e. at least ½ a standard deviation) the input variable is significant. The reported scenarios show all inputs which were significant in meeting the entered target.
A scatter plot in the Scenarios window is an x-y scatter plot with an overlay. This graph shows:

3) the input value sampled vs. the output value calculated in each iteration of the simulation,

4) overlaid with a scatter plot of the input value sampled vs. the output value calculated when the output value meets the entered scenario.

In the Scatter Plot Matrix, ranked scenario analysis results are displayed with scatter plots. To show the Scatter Plot Matrix, click the Scatter Plot icon in the lower left of the Scenarios window.

Scenario analysis results are graphically displayed in tornado graphs. A tornado graph can be generated by clicking the Tornado graph icon in the Scenarios window or clicking the Scenarios icon on a graph window. This tornado graph shows the key inputs affecting the output when the output meets the entered scenario, such as when the output is above its 90th percentile.
Reporting in Excel

When you generate simulation reports and graphs in Excel, you have access to all Excel’s formatting. In addition, @RISK Reports generated in Excel can use pre-built @RISK template sheets that contain custom formatting, titles, and logos.
You can use template sheets to create your own custom simulation report. Simulation statistics and graphs are placed in a template using a set of @RISK functions added to Excel. When a statistics function or graphing function is located in a template sheet, the desired statistics and graphs are then generated at the end of a simulation in a copy of the template sheet to create your report. The original template sheet with the @RISK functions remains intact for use in generating reports from your next simulation.

Template sheets are standard Excel spreadsheets. They are identified to @RISK by having a name that starts with RiskTemplate_. These files can also contain any standard Excel formulas so custom calculations can be performed using simulation results.
Chapter 5: @RISK Modeling Techniques

Introduction..................................................................................................................139
Modeling Interest Rates and Other Trends.................................................................141
Projecting Known Values into the Future .....................................................................143
Modeling Uncertain or “Chance” Events .................................................................145
Oil Wells and Insurance Claims ..............................................................................147
Adding Uncertainty Around a Fixed Trend ...............................................................149
Dependency Relationships.........................................................................................151
Sensitivity Simulation .................................................................................................153
Simulating a New Product .........................................................................................155
Finding Value at Risk (VAR) of a Portfolio ...............................................................165
Simulating the NCAA Tournament ...........................................................................169
Introduction

The @RISK Modeling Techniques chapter will show you how to translate typical “risky” situations into @RISK models. These risky situations have been identified from real-life modeling problems that Excel users often encounter. As you use @RISK to analyze uncertainty in your Excel worksheets, look through the examples and illustrations provided in this chapter, you may just find some helpful tips or techniques which will make your @RISK models better representations of uncertain situations.

Seven @RISK techniques are presented here to illustrate common modeling situations under uncertainty. To help you understand the modeling techniques employed, example Excel worksheets and accompanying @RISK simulations are provided with your @RISK system. The simulations are even “pre-run” so you can just look at the results if you like. As you work through each modeling technique discussed, look at the corresponding worksheet and simulation, it will help you understand the @RISK concepts and techniques involved in modeling each risky situation.

The seven modeling techniques illustrated here are:

- **Modeling Interest Rates and Other Trends** — random trends over time and “random walks”.
- **Projecting Today's Known Values Into the Future** — an increasingly uncertain future or “increasing variability”.
- **Will the Flood Occur or the Competitor Enter the Market?** — modeling uncertain “chance” events.
- **Oil Wells and Insurance Claims** — modeling an uncertain number of events, each with uncertain parameters.
- **“I have to use this projection but don't trust it”** — adding uncertainty around a fixed trend using “error terms”.
- **“These values will be affected by what happens somewhere else”** — dependency relationships using variable arguments and correlations.
- **Sensitivity Simulation** — how model changes affect simulation results.
In addition to the seven models discussed here, this chapter also includes three @RISK examples from the book Financial Models Using Simulation and Optimization by Wayne Winston. These models illustrate how @RISK is applied to everyday business modeling. The full Financial Models book includes 63 examples of how @RISK and other add-ins can be applied to a wide variety of financial problems. For more information on purchasing the full Financial Models book, contact Palisade Corporation or visit www.palisade.com. 

All example spreadsheet models discussed here can be found in the default installation location C:\ PROGRAM FILES\PALISADE \RISK5\EXAMPLES.
Modeling Interest Rates and Other Trends

Projecting Trends

Example Model: RATE.XLS

Whether you are getting a mortgage or evaluating the cost of a variable rate loan, projections of future interest rates are highly uncertain. The movement of the rate of interest you are charged is often viewed as random — moving up and down erratically year after year. This movement may be completely random, or it may be a random fluctuation around an underlying known trend. In either case, modeling the random portion of any projection is an important technique in Risk Analysis.

Simulation accounts for randomness in a trend, over time, in a very powerful way — repetitively trying a different possible series of rates with each iteration of a simulation. For example, you could set up a random trend to project the interest rate over ten years. For each iteration, a new, randomly selected value is chosen for each year's interest rate, and results are calculated. By doing this, simulation includes the effects of all possible future interest rates in your results, instead of just a single, most likely, projection.

A “random trend” can be easily, and directly, included in an Excel worksheet with @RISK. And by using the Excel Copy commands, you can place a random trend anywhere in your worksheet.
**Simple Random Trends**

The simplest random trend is a distribution copied across time. The value randomly selected in one period is independent of the value selected in any other time period:

1) Enter a distribution function for the first cell of the trend
2) Copy the distribution over the range of cells

In this case, a new value will be sampled every period — a completely random trend with no correlation over time.

Maybe you don't think future rates are entirely random. Perhaps next year's rate will be influenced by this year's. In Excel terms, there will be some correlation between one cell in the range and the next. Here's a simple way to model this:

1) Enter a distribution function for the first cell in the range.
2) Enter a distribution function for the second cell in the range that uses the value sampled for the first cell as one of its arguments (such as its mean or most likely value).
3) Copy the formula of the second cell across the range. The referenced argument in the formula is a relative reference — the third cell will use the value in the second as its referenced argument and likewise for the fourth, fifth, etc.

For example:

A1: RiskNormal(100,10)
A2: RiskNormal(A1,10)
A3: RiskNormal(A2,10)
A4: RiskNormal(A3,10)

In this way there is some correlation between one cell and the next cell in the range.

**A “Random Walk” with Correlation Period to Period**

**Refining Your Random Trends**

These are just simple examples of modeling random processes over time. As you get more sophisticated, you can include your random terms in formulas which place limits or caps on the amount of change, increase the amount of possible change with time, or other such extensions or variations. Also remember that interest rates are just one application for random trends. Look at your Excel worksheets and the uncertain situations you model and you will undoubtedly see others.
Projecting Known Values into the Future

Increasing Uncertainty Over Time

Example Model: VARIABLE.XLS

You know today’s values for critical variables in your models, but what about values for these same variables in the future? Time often has a very important impact on estimates — they become less certain the further out in time your projections extend. As a consequence, results based on your single “best estimates” become more risky the further out in time they are projected.

The widening variation around the trend of best estimates illustrates this problem. @RISK lets you model the effect of time on your estimates by allowing you to easily increase the variability in a random value over time.

The range of possible values for a worksheet cell is given in your distribution function. As you move out in time — across a range of worksheet cells — the arguments of the function which specifies the range of possible values can increase. For example:

A1: RiskLognorm(10,10)
A2: RiskLognorm(10,15)
A3: RiskLognorm(10,20)
A4: RiskLognorm(10,25)
The standard deviation of the LOGNORM distribution function controls the possible variation in value. In this example, as you move out across the range of cells, the standard deviation increases.

Increasing the possible variance in value, as your projections extend further into the future, is a good “rule of thumb” to follow. By doing this your results will more accurately reflect the greater uncertainty which exists in your knowledge of the distant future.
Modeling Uncertain or “Chance” Events

Will the Flood Occur or the Competitor Enter the Market?

Example Model: DISCRETE.XLS

Uncertainty often shows up in the form of chance events which may have significant impact on your results. Either we will strike oil, or we won't. That competitor will either enter the market or he won't, but if he does, there's a 25% chance of a hailstorm that will wipe out this year's crop.

Including the possibility of these types of events in your models is an important technique in Risk Analysis. If you leave them out, the outcomes caused by these events will not be included in your results and your models will be incomplete. Using the DISCRETE function, provided by @RISK and Excel's IF function, modeling these events is easy.

The DISCRETE function is the means by which you can include probabilities for chance events in your worksheet models.

\[
\text{RiskDiscrete}([0,1],[50,50])
\]

This example models the characteristic “coin flip” — the simplest chance event. In this case, an outcome of 0 represents Heads and 1 represents Tails, and each is equally likely to occur. A more complex example illustrates four possible scenarios for annual storm damage from floods:

\[
\text{RiskDiscrete}([0,1,2,3],[20,40,30,10])
\]

In this case, outcomes valued 0 to 3 represent four possible levels of flood damage ranging from none (0) through low (1), medium (2) and high (3). The probability of occurrence of no damage is 20%, low 40%, medium 30% and high 10%.

The DISCRETE function returns a value for each iteration that indicates which event has occurred. Your worksheet model has to recognize which event has occurred and calculate different results appropriate to the event. The Excel IF function allows this. Consider the following example and Excel cell entries:

Cell C2 describes an event — the possible entry of a competitor in a given market. There is a 50-50 chance of entry. If entry occurs, your sales level will be 65; if entry does not occur, your sales level will be 100.

\[
\begin{align*}
\text{C2:} & \quad \text{RiskDiscrete}([0,1],[50,50]) \\
\text{D2:} & \quad \text{IF}(\text{C2}=1,100,65)
\end{align*}
\]
In the above example, the IF function in cell D2 will return a value of 100 if the outcome in cell C2 is 1 (no entry) and will return 65 if the outcome is 0 (entry). This simple example can be extended throughout your @RISK models. With each iteration of a simulation the DISCRETE function will return one of the possible values. Depending on the value returned, the calculations in your worksheet can change.

People used to working with single estimates in spreadsheets often substitute discrete distributions where a continuous distribution should be used. For example, they use a discrete distribution to put in three possible price levels where, in reality, price could take any value in a range.

This common mistake occurs because many people are used to manual “what-if” modeling which necessarily limits the user to a small number of discrete estimates. Use a continuous form when any value in a range is possible, and save DISCRETE for event modeling and variables that truly are discrete.

*Caution*
Oil Wells and Insurance Claims

Modeling an Uncertain Number of Events, Each with Uncertain Parameters

Example Model: CLAIMS.XLS

In real life situations, uncertainty often has two or more dimensions. The situation you face may have an uncertain number of events, each of which has an uncertain value. Think, for example, of the insurance industry. An uncertain number of claims may be filed on a new policy, and each claim filed has an uncertain dollar amount. How do you simulate the total claim amount possible? The oil industry faces a similar problem. When drilling oil wells, an uncertain number of wells will be successful, but the amount of oil discovered by each successful well is uncertain. How do you simulate the total amount of oil discovered?

Risk Analysis is very useful in modeling situations such as this. @RISK, using its RiskCompound function, can provide an easy means of performing such an analysis. It is helpful to review the example simulation, CLAIMS.XLS, while proceeding through this modeling technique.

To run the analysis:

1) A distribution is used to sample the number of events which occur in a given iteration. This is the first argument of the RiskCompound function.

2) Another distribution is used to specify the size of each event. This is the second argument to the RiskCompound function.

When the simulation runs, @RISK samples the number of events, then it draws a number of samples from the second distribution where the number of samples drawn equals the number of events. The total of the samples drawn from the second distribution is returned by the RiskCompound function. This value gives the answer desired such as total claim amount, or total amount of oil discovered.
Adding Uncertainty Around a Fixed Trend

I Have To Use This Projection But Don't Trust It

Example Model: ERROR.XLS

Often Excel modelers are given data from other sources for inclusion in their worksheets. “The economics group has provided this projection for GNP growth, so include it in your worksheet model” — could be the guideline. But how often does the future exactly follow even the best projection?

Recognizing the uncertainty inherent in projections, you still may want to stay true to the basic direction offered by the values in the trend. In this case “error terms” let you put a certain amount of variation around the values in the trend. This allows you to examine how variation in trend value will impact your results.

With @RISK, you can easily append an error term to a fixed trend you have already entered in your worksheet. Let’s say, for example, that worksheet row B contains the fixed trend in your model. An error term is just a factor you’ll multiply each worksheet cell value by. (You also could add an error term to each trend value)

Row B — GNP Growth in Percent

B1: 3.2 * RiskNormal(1,.05)  
B2: 3.5 * RiskNormal(1,.05)  
B3: 3.4 * RiskNormal(1,.05)  
B4: 4.2 * RiskNormal(1,.05)  
B5: 4.5 * RiskNormal(1,.05)  
B6: 3.5 * RiskNormal(1,.05)  
B7: 3.0 * RiskNormal(1,.05)
In this example taken from Excel, the error term for all trend values is drawn from a normal distribution with a mean of 1 and a standard deviation of .05. For each iteration of a simulation, a new error term will be sampled for each cell, and will be used to multiply the fixed trend estimate in that cell, allowing variation around the fixed estimate.

An added bonus of an error term is the expected value generated in normal Excel recalculations. Because the expected values of the error terms in the example equal one, they will not affect your normal worksheet recalculations. Thus you can leave the error terms in your formulas and only see their effects when you run your simulations. The same comment holds true if you add, instead of multiply, the error term. If you add the error term to the fixed estimate, the mean of the error term probability distribution should be zero.
Dependency Relationships

Using Variable Arguments and Correlations — These Values Will Be Affected By What Happens Someplace Else

Example Models: DEP.XLS, CORRMAT.XLS

Many times you will not know precise argument values for a distribution function in your worksheet. Often the range for a worksheet cell will depend on a value calculated, or sampled, elsewhere in your model. “If the price is low, the range for sales volume is one million to 2 million — but if the price is high, the range is only 500,000 to 750,000” is an illustration of this type of dilemma.

Two modeling techniques in @RISK helps you to resolve problems such as Variable Arguments for Distribution Functions and Correlations in Sampling.

The first technique — variable arguments for distribution functions — relies on a standard Excel capability most modelers are familiar with. The referencing of cell addresses in functions is allowed in @RISK just as in Excel. For example:

Minimum A1: RiskTriang(10,20,30)

Maximum B1: RiskNormal(80,10)

Final Price C1: RiskUniform(A1,B1)

This example shows how the range for the uniform distribution for Final Price will change with the value sampled for Minimum and Maximum. Here the range for Final Price will change with each iteration of the simulation. Final Price thus depends on the variables Minimum and Maximum.
The second modeling technique which can be used to affect sampled values based on other calculations in a worksheet is *correlations in sampling*. The @RISK function CORRMAT is used to correlate the values sampled in different distribution functions. This correlation allows you to specify a relationship between the values sampled in different worksheet cells while still maintaining a degree of uncertainty for each.

**Interest Rate**

\[ A1: \text{RiskUniform}(6,14, \text{RiskCorrmat}(D1: E2, 1)) \]

**Housing Starts:**

\[ B1: \text{RiskUniform}(100000, 200000, \text{RiskCorrmat}(D1: E2, 2)) \]

Above, the variable *Interest Rate* — described by the distribution RiskUniform(6,14) — is the distribution with which the *Housing Starts* distribution — RiskUniform(100000,200000) — should be correlated. The range D1:E2 contains a matrix of 4 cells and a single correlation coefficient, -.75. The -.75 is the coefficient which specifies how the two sampled values are correlated. Coefficients range from -1 to 1. A value of -.75 is a negative correlation — as *Interest Rate* goes up, *Housing Starts* go down.

When you are using sampled, uncertain variables in your worksheet models, it is important to recognize correlations in sampling. If you don't use methods such as the two presented here, all uncertain variables will be sampled as if they were completely independent of other variables in the model. This can cause erroneous results. Think about what could happen if *Interest Rate* and *Housing Starts* in the above example were completely independent. *Interest Rate* and *Housing Starts* would be sampled entirely independent of one another. A high *Interest Rate* and a high value for *Housing Starts* would be an entirely possible scenario during sampling. But could this happen in real life? Not in this economy.

Correlating multiple distribution functions can be accomplished using the CORRMAT function, or by selecting the cells containing the distributions and selecting the **Define Correlations** command. Either of these methods allows you to enter a matrix of correlation coefficients. @RISK then uses the coefficients to correlate the sampling of distribution functions. This is especially useful when pre-existing correlation coefficients (calculated from actual collected data) are available, and you want sampling to be governed by those coefficients. Excel can calculate correlations from existing datasets using the CORREL function. For more information on using Correlate or CORRMAT, see the example simulation CORRMAT.XLS.

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**Correlations in Sampling**

**Correlating Multiple Distributions**

**Dependency Relationships**

152
Sensitivity Simulation

How Do Changes in Model Variables Affect My Simulation Results?

Example Model: SENSIM.XLS

@RISK lets you see the impact of uncertain model parameters on your results, but what if some of the uncertain model parameters are under your control? In this case the value a variable will take is not random, but can be set by you. For example, you might need to choose between some possible prices you could charge, different possible raw materials you could use, or from a set of possible bids or bets. To properly analyze your model, you need to run a simulation at each possible value for the “user-controlled” variables and compare the results. A Sensitivity Simulation in @RISK allows you to quickly and easily do this — offering a powerful analysis technique for selecting between available alternatives.

The benefits of Sensitivity Simulation are not limited to evaluating the impacts of user-controlled variables on simulation results. A sensitivity analysis can be run on the probability distributions which describe uncertain variables in your model. You may wish to repetitively re-run a simulation, each time changing the parameters of one (or several) of the distributions in your model. After all the individual simulations are complete, you can then compare the results from each.

The key to a Sensitivity Simulation is the repetitive simulation of the same model while making selected changes to the model, each simulation. In @RISK any number of simulations can be included in a single Sensitivity Simulation. The SIMTABLE function is used to enter lists of values, which will be used in the individual simulations, into your worksheet cells and formulas. @RISK will automatically process and display the results from each of the individual simulations together, allowing easy comparison.

To run a Sensitivity Simulation:

1) Enter the lists of values you want used in each of the individual simulations into your cells and formulas using SIMTABLE. For example, possible price levels might be entered into Cell B2:

   \[ B2: \text{RiskSimtable}((100,200,300,400)) \]

   will cause simulation #1 to use a value of 100 for price, simulation #2 to use a value of 200, simulation #3 to use a value of 300 and simulation #4 to use a value of 400.
2) Set the number of simulations in the Settings dialog box and run the Sensitivity Simulation using the Start Simulation command.

Each simulation executes the same number of iterations and collects data from the same specified output ranges. However, each simulation uses a different value from the SIMTABLE functions in your worksheet.

@RISK processes Sensitivity Simulation data just as it processes data from a single simulation. Each output cell, for which data was collected, has a distribution for each simulation. By clicking the Display Simulation # icon displayed on a graph window you can compare the results of the different alternatives or “scenarios” described by each individual simulation. In addition, the Distribution Summary graph summarizes how the results for an output range change. There is a different summary graph for each output range in each simulation, and these graphs can be compared to show the differences between individual simulations. In addition, the Simulation Summary report is useful for comparing results across multiple simulations.

You also can use Sensitivity Simulation to see how different distribution functions affect your results. The values entered in the SIMTABLE function can be distribution functions. For example, you may wish to see how your results change if you alternately try TRIANG, NORMAL, or LOGNORM as the distribution type in a given cell.

Caution

It is important to distinguish between 1) controlled changes by simulation (which are modeled with the SIMTABLE function) and 2) random variation within a single simulation (which is modeled with distribution functions). SIMTABLE should not be substituted for DISCRETE when evaluating different possible random discrete events. Most modeling situations are a combination of random, uncertain variables and uncertain but “controllable” variables. Typically, the controllable variables will eventually be set to a specific value by the user, based on the comparison conducted with a Sensitivity Simulation.

Advanced Sensitivity Analysis

@RISK 5 Professional and Industrial versions include an advanced analysis tool called Advanced Sensitivity Analysis. This analysis greatly expands on the sensitivity simulation capabilities described here. For more information on Advanced Sensitivity Analysis, see the Advanced Analyses command in the Reference: @RISK Add-in Menu section of this manual.
Simulating a New Product

The Hippo Example

(Chapter 28, Financial Models Using Simulation and Optimization)

When a company develops a new product, the profitability of the product is highly uncertain. Simulation is an excellent tool to estimate the average profitability and riskiness of new products. The following example illustrates how simulation can be used to evaluate a new product.

ZooCo is thinking of marketing a new drug used to make hippos healthier. At the beginning of the current year there are 1,000,000 hippos that may use the product. Each hippo will use the drug (or a competitor’s drug) at the most once a year. The number of hippos is forecasted to grow by an average of 5% per year, and we are 95% sure that the number of hippos will grow each year by between 3% and 7%. We are not sure what the use of the drug will be during year 1, but our worst case guess is 20% use, most likely use is 40% and best case use is 70%. In later years, we feel the fraction of hippos using our drug (or a competitor’s) will remain the same, but in the year after a competitor enters, we lose 20% of our share for each competitor who enters. We will model Year 1 market use with a triangular random variable. See Figure 28.1. Basically, @RISK will generate Year 1 market use by making the likelihood of a given market use proportional to the height of the “triangle” in Figure 28.1. Thus a 40% Year 1 market use is most likely; a 30% market use occurs half as often as a Year 1 40% market use. The maximum height of the triangle is 4, because that makes the total area under the triangle equal to one. Probability of market use being in a given range is equal to area in that range under the triangle. For example, the chance of a market use being at most 40% is .5*(4)*(.4-.2) = .4 or 40%.
There are three potential entrants (in addition to ZooCo). At the beginning of each year each entrant, who has not already entered the market, has a 40% chance of entering the market. The year after a competitor enters, our market use drops by 20% for each competitor who entered. Therefore, if in Year 1 two competitors enter the market, in Year 2 our market use will be reduced by 40%. To model the number of entrants you can use the binomial random variable (in @RISK this requires us to use the =RISKBinomial function). The formula:

\[
\text{= RISKBinomial (n, p)}
\]

generates n independent binomial trials (each a success or failure) having probability of success p and keeps track of the number of successes.

We consider a “success” to be a competitor entering the market. Then the formula:

\[
\text{= RISKBinomial (2, 0.4)}
\]

will simulate the number of entrants during a year in which two competitors have yet to enter the market. Make sure that if all three entrants have entered, no more entrants may enter.

Each unit of the drug is sold for $2.20 and incurs a variable cost of $0.40. Profits are discounted by 10% (risk adjusted rate) annually.
Find a 95% CI for risk-adjusted NPV of project. For now we ignore the fixed cost of developing the drug.

Recall that risk-adjusted NPV is the expected discounted value of cash flows (discounted at risk-adjusted rate).

Our spreadsheet is in Figure 28.2 (file hippo.xls).

**Solution**

**Figure 28.2**

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Size</td>
<td>1000000</td>
<td>1050000</td>
<td>1102500</td>
<td>1157625</td>
<td>1215506.25</td>
</tr>
<tr>
<td>Use per hippo of our drug competitors (beginning of year)</td>
<td>0.433333333</td>
<td>0.329333333</td>
<td>0.27664</td>
<td>0.2545088</td>
<td>0.23418096</td>
</tr>
<tr>
<td>Entrants</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Unit Sales</td>
<td>433333333</td>
<td>345800</td>
<td>294625.7496</td>
<td>284608.4741</td>
<td></td>
</tr>
<tr>
<td>Revenues</td>
<td>$953,333</td>
<td>$760,760</td>
<td>$670,990</td>
<td>$640,177</td>
<td>$626,139</td>
</tr>
<tr>
<td>Costs</td>
<td>$173,333</td>
<td>$138,320</td>
<td>$121,998</td>
<td>$117,850</td>
<td>$113,843</td>
</tr>
<tr>
<td>Profits</td>
<td>$780,000</td>
<td>$622,440</td>
<td>$549,992</td>
<td>$522,326</td>
<td>$502,295</td>
</tr>
<tr>
<td>NPV</td>
<td>$2,316,286</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step by Step**

Step 1: In row 8 we determine the market size during each of the next five years. In B8 we enter =D3. Assuming year to year growth in market size is normally distributed, the given information tells us that the number of pigs grows from each year by a percentage which is a normal random variable having mean .05 and standard deviation of .01. This follows because 95% of the time a normal random variable is within 2 standard deviations of its mean. Therefore we may conclude $2\sigma = .02$ or $\sigma = .01$. Thus in C8 we determine the Year 2 Market Size with the formula

$$=B8*RISKNormal(1.05,0.01).$$

Essentially, this formula ensures that each year there is a 68% chance that the size of the hippo market grows by between 4% and 6%, a 95% chance that the hippo market grows by between 3% and 7%, and a 99.7% chance that the hippo market grows by between 2% and 8%. Copying this formula to D8:F8 generates the market size for Years 3-5.

Step 2: In row 9 we determine our market use/hippo for each year. Year 1 market use/hippo is computed in B9 with the formula

$$=RISKTriang(D4,D5,D6).$$

In C9:F9 we account for the fact that the year after entry, each entrant takes away 20% of our market share. Thus in C9 we compute our Year 2 Market use/hippo with the formula

$$=B9*(1-B11*$D$2).$$

Copying this formula to D9:F9 computes Years 3-5 market share.
Step 3: In Row 11 we determine the number of entrants during each year. If less than 3 competitors have entered, then each competitor who has not yet entered has a 40% chance of entering during the current year. If all three competitors have entered, then nobody can enter. In B11 we compute the number of Year 1 entrants with the formula

\[ =\text{If}(B10<3, \text{RISKBinomial}(3-B10,\$B\$5),0). \]

Copying this to C11:F11 computes Years 2-5 entrants. If we do not use the =If statement, then in a year after all 3 competitors have entered we will obtain an error message because =RISKBinomial cannot take 0 trials as the first argument.

Step 4: In Row 10 we compute the number of competitors present at the beginning of each Year by adding the number of new entrants to the number of competitors already present. In B10 we enter 0 and in C10 we enter

\[ =B10+B11. \]

Copying this formula to D10:F10 computes the number of competitors present at the beginning of each year.

Step 5: In row 12 we compute each year’s unit sales = (use/hippo)*market size by copying the formula

\[ =B8*B9 \]

from B12 to C12:F12.

Step 6: In row 13 we compute our annual revenues by copying the formula

\[ =$B$2*B12 \]


Step 7: In row 14 we compute our annual variable costs by copying the formula

\[ =$B$3*B12 \]

from B14 to C14:F14.
Step 8: In row 15 we compute our annual profits by copying the formula

\[ =B13-B14 \]

from B15 to C15:F15.

Step 9: In B17 we compute the NPV of our 5-year profits with the formula

\[ =NPV(B4,B15:F15). \]

Step 10: We now run a simulation with cell B17 (NPV) being our forecast cell. We used 500 trials. Our results follow.

\[ \begin{array}{l}
\text{Minimum} & 96890.625 \\
\text{Maximum} & 44167.89 \\
\text{Mean} & 2912372.956 \\
\text{Standard Deviation} & 633418.2967 \\
\text{Variance} & 4.01219E+11 \\
\text{Skewness} & 0.31535665 \\
\text{Kurtosis} & 2.669867004 \\
\text{NumErrs} & 0 \\
\text{Mode} & 2212183.8 \\
5\% & 1347058.879 \\
10\% & 1469903.376 \\
15\% & 1630663.376 \\
20\% & 1761009.443 \\
25\% & 1856571 \\
30\% & 1933889.883 \\
35\% & 2012303.986 \\
40\% & 2096331.542 \\
45\% & 2181681.218 \\
50\% & 2265116.9 \\
55\% & 2332834.792 \\
60\% & 2422256.516 \\
65\% & 2500136.715 \\
70\% & 2539604.196 \\
75\% & 2765056.75 \\
80\% & 2877606.315 \\
85\% & 2986681.884 \\
90\% & 3165375.941 \\
95\% & 3385483.707 \\
\end{array} \]

Our point estimate of risk adjusted NPV is the sample mean of NPV's from simulation ($2,312,372.866). To find a 95% confidence interval for the mean in a simulation use the fact that we are 95% sure, actual mean NPV is between

\[ (\text{Sample Mean of NPV}) \pm 2 * (\text{Sample Standard deviation}) / \sqrt{n} , \]

where \( n = \text{number of iterations}. \)
For example, we are 95% sure the mean NPV (or risk adjusted NPV) is between

\[2,312373 \pm 2*\frac{633418}{\sqrt{500}}\] or

$2,255,718 \text{ and } 2,369,028$.

Thus we are pretty sure risk adjusted NPV is between 2.26 and 2.37 million, as 95% of the time we are accurate within $50,000 (which is 2% of sample mean) we feel comfortable that we have run enough iterations.

The actual discounted (at 10% rate) value of cash flows has much more variability than our confidence interval for risk-adjusted NPV would indicate. To show this look at the following histogram.

**Figure 28.4**

Note: If you are going to use distribution of NPV's as a tool to compare projects, then you must discount all the company's projects at the same rate (probably obtained from CAPM). Otherwise you will be double counting risk.
Tornado Graphs and Scenarios

A natural question is what factors have the most influence on the success of the project? Does market growth matter more than the timing of the entrance of competitors? Using @RISK Tornado graphs and Scenario Analysis we can easily answer questions such as:

a. What factors appear to have the most influence on the NPV earned by the drug?

b. When NPV is in the top 10% of all possible NPV, what seems to be going on?

Here we utilize a Tornado Graph. Make sure that in “Simulation Settings” you have checked “Collect Distribution Samples”. Then click on the Tornado Graph icon on the displayed graph for NPV/B17. You have three options: A Regression Tornado Graph (see Figure 28.5), a Correlation Tornado Graph (see Figure 28.6) or a Regression Tornado graph that shows input values, instead of coefficients, at the end of the bars.

Solution – Part a

Figure 28.5
We find (ceteris paribus) from Regression Tornado Graph (obtained by selecting “Regression-Coefficients” after clicking the Tornado Graph icon that

- A one standard deviation increase in Year 1 use increases NPV by .853 standard deviations.
- A one standard deviation increase in number of Year 1 entrants decreases NPV by .371 standard deviations.
- Not much else matters.

Basically, when running a Tornado Graph @RISK runs a regression where each iteration represents an observation. The dependent variable is the output cell (NPV) and the independent variables are each “random” @RISK functions in the spreadsheet. Then the .853 coefficient for Year 1 Use is the standardized, or beta weight coefficient of Year 1 Use, in this regression.

From Tornado Correlation Graph in Figure 28.6 (obtained by a change similar to the one above, except with Correlation instead of Regression) we find:

- Year 1 Use is most highly correlated (.88) with NPV
- Next is Year 1 Entrants (-.36)
- The rest of the random cells in the spreadsheet do not matter much!
These correlations are rank correlations; for example, for all iterations the values of Year 1 Use are ranked, as are values of NPV. Then these ranks (not actual values) are correlated.

If you check “Collect Distribution Samples” under Simulation Settings you can obtain a Scenario Analysis. For a given scenario, such as all iterations where NPV is in top 10% of all iterations, the Scenario Analysis identifies random variables whose values differ significantly from their median values.\(^1\)

We find from Scenario Approach (see Figure 28.7) (click on the “Scenarios Window” icon) that in the iterations yielding the top 10% of all NPV’s the following variables differ significantly from their overall medians:

- Year 1 Use (median is .596, 1.66 sigma above average)
- Year 2 Entrants (median is 0, 1.53 sigma below average)

To change the scenario settings just click on the “Scenario=” row in the Scenario Analysis box. Figure 28.7 contains a listing of three Scenario settings (the top 25%, the bottom 25%, and the top 10% of NPV’s) along with the random variables that differ significantly from their average values when the given scenario occurs. For example, for iterations in which the NPV is in the bottom 25% of all iterations, Year 1 Market Use averaged out to 13.9%.

\(^1\) @RISK will identify any random variable whose median value in iterations satisfying the scenario condition differs by more than .5 standard deviations from the median value of the random variable in all iterations.
Finding Value at Risk (VAR) of a Portfolio

VAR
(Chapter 45, Financial Models Using Simulation and Optimization)

Anybody who owns a portfolio of investments knows there is a great deal of uncertainty about the future worth of the portfolio. Recently the concept of value at risk (VAR) has been used to help describe a portfolio's uncertainty. Simply stated, value at risk of a portfolio, at a future point in time, is usually considered to be the fifth percentile of the loss in the portfolio's value at that point in time. In short, there is considered to be only one chance in 20 that the portfolio's loss will exceed the VAR. To illustrate the idea suppose a portfolio today is worth $100. We simulate the portfolio's value one year from now, and find there is a 5% chance that the portfolio's value will be $80 or less. Then the portfolio's VAR is $20 or 20%. The following example shows how @RISK can be used to measure VAR. The example also demonstrates how buying puts can greatly reduce the risk, or hedge, a long position in a stock.

Example 45.1

Let's suppose we own one share of Dell computer on June 30, 1998. The current price is $94. From historical data (see Chapter 41) we have estimated that the growth of the price of Dell stock can be modeled as a Lognormal random variable with $\mu = 57\%$ and $\sigma = 55.7\%$. To hedge the risk involved in owning Dell we are considering buying (for $5.25) a European put on Dell with exercise price $80 and expiration date November 22, 1998. Here you will:

a) Compute the VAR on November 22, 1998 if we own Dell computer and do not buy a put.

b) Compute the VAR on November 22, 1998 if we own Dell computer and buy the put.

Solution

The key idea is to realize that in valuing the put we let the Dell price grow at the risk-free rate, but when doing a VAR calculation we should let the Dell price grow at the rate we expect it to grow. Our work is in file var.xls. See Figure 45.1.
We have created range names as indicated in Figure 45.1.

**Step 1:** In cell B11 we generate Dell's price on November 22, 1998 with the formula:

\[ S \times \exp((g-0.5\times v^2) \times d + \text{RISKNormal}(0,1) \times v \times \sqrt{d}). \]

**Step 2:** In cell B12 we compute the payments from the put at expiration with the formula:

\[ \text{If}(B11 > x, 0, x - B11). \]

**Step 3:** The percentage gain on our portfolio if we just own Dell is given by:

\[ \frac{\text{Ending Dell Price} - \text{Beginning Dell Price}}{\text{Beginning Dell Price}}. \]

In B14 we compute the percentage gain on our portfolio if we do not buy a put with the formula:

\[ \frac{(B11 - S)}{S}. \]
Step 4: The percentage gain on our portfolio if we own Dell and a put is:

\[
\text{Ending Dell Price} + \text{Cash Flows from Put} - \text{Beginning Dell Price} - \text{Put Price} = \frac{\text{Beginning Dell Price} + \text{Put Price}}{\text{Beginning Dell Price}}
\]

In cell B15 we compute the percentage gain on our portfolio if we buy the put with the formula

\[
=\frac{(B12+B11)-(S+p)}{(S+p)}.
\]

Step 5: After selecting B14 and B15 as output cells and running 1600 iterations we obtained the @RISK output in Figure 45.2.

We find our VAR if we do not buy the put to be 33.9% of our invested cash while, if we buy the put, our VAR drops to 19.4% of the invested cash. The reason for this is, of course, is that if the Dell stock drops below $80, every one dollar decrease in the value of Dell is countered by a one dollar increase in the value of the put. Also note that if we do not buy the put, Dell (despite its high growth rate) might lose up to 64% of its value.

The following histograms give the distribution of the percentage gain on our portfolio with and without the put.
From Figures 45.3 and 45.4 we see that there is a much greater chance of a big loss if we do not buy the put. Note, however, that our average return without the put is 25.4%, while our average return, with the put, is 21.1%. In effect, buying the put is a form of portfolio insurance, and we must pay for this insurance.
Simulating the NCAA Tournament

NCAA

(Chapter 62, Financial Models Using Simulation and Optimization)

The file NCAA.xls lets you play out the NCAA tournament as many times as you want. We factor in the abilities (through the SAGARIN ratings published in USA Today) of each team. Extensive data analysis has indicated that teams play on average to SAGARIN ratings and perform according to a standard deviation of 7 points about that level. For example, in 1997 SAGARIN rated NC a 94 and Fairfield a 70. Thus we would model NC's play by a RISKNormal(94,7) and Fairfield by a RISKNormal(70,7) and declare the team with the higher performance the winner. Our simulation of the 1996 NCAA tournament is in file NCAA96.xls

To begin we label the EAST teams 1-16 in the order they are listed in bracket. Then teams 17-32 are the SOUTHEAST, teams 33-48 the WEST and teams 49-64 the MIDWEST. It is important that we list things so that winner of 1 and 2 plays winner of 3 and 4.

Step by Step

Step 1: We enter the ratings, numerical codes, and team names in rows 2-4. We name the range A3:BL4 Ratings.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>UNC</td>
<td>Fairfield</td>
<td>Ind</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>94.4</td>
<td>70.3</td>
<td>85.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>East</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>94.4</td>
<td>1</td>
</tr>
</tbody>
</table>

Step 2: We model the UNC Fairfield game in A6:C7. In A7 we generate UNC's performance with the formula:

= RISKNormal(HLOOKUP(A6,Ratings,2),7).

This looks up UNC's rating and generates a performance with that mean and a standard deviation of 7.
Similarly, in C7 we generate Fairfield’s performance. In B7 we determine who wins the game with the formula

\[ =\text{If}(A7>C7, A6, C6). \]

After “playing” the Colorado-Indiana game in E6:G7 (see Figure 62.1) we play the winners of these two games in A9:C10.

**Figure 62.1**

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>85.3</td>
<td>3</td>
<td>82</td>
</tr>
</tbody>
</table>

We ensure that the entry in A9 is the winner of UNC-Fairfield Game and the entry in C9 is winner of Indiana-Colorado game. Then in Row 10 we “play” this game. See Figure 62.2.

**Figure 62.2**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>94.4</td>
<td>1</td>
<td>85.3</td>
</tr>
</tbody>
</table>

You can follow this logic down to Row 57. Here the Final Four begins! See Figure 62.3.

**Figure 62.3**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>57</td>
<td>East</td>
<td></td>
<td>West</td>
<td>Midwest</td>
<td></td>
<td>Mideast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>1</td>
<td>17</td>
<td>33</td>
<td>49</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>94.4</td>
<td>17</td>
<td>97.9</td>
<td>97.4</td>
<td>33</td>
<td>93.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>61</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>Finals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>63</td>
<td></td>
<td>17</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64</td>
<td></td>
<td>97.9</td>
<td>17</td>
<td>97.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Kansas</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In 1997 East played West and Midwest played Mideast. Each year the final four matchups will change and you will need to adjust this part of spreadsheet. In C65 we print out the winner with the formula:

\[ =\text{HLOOKUP}(C64, A1:BL2, 2). \]

This formula finds the Team Name corresponding to the code number of the winner. Hit the F9 key several times to see what happens.
We used cell C64 as our output cell and ran the tournament 5000 times. The teams having at least a 5% chance of winning were:

- UNC: 13%
- Kansas: 26%
- Kentucky: 27%
- Duke: 8%
- Minnesota: 9%

Of course, Arizona won (we gave them a .0084 chance!). That's what makes sports great!

Remarks

Remember, each year the Final Four brackets change. This will require you to rearrange the rows where the East, Midwest, Mideast and West regions are located.
Chapter 6: Distribution Fitting

Overview .................................................................................................................. 175

Define Input Data .................................................................................................. 177
  Sample Data ........................................................................................................... 177
  Density Data ........................................................................................................... 178
  Cumulative Data .................................................................................................... 179
  Filtering Your Data ............................................................................................... 179

Select Distributions to Fit .................................................................................... 181
  Continuous vs. Discrete Distributions .............................................................. 181
  Estimated Parameters vs. Predefined Distributions ....................................... 181
  Domain Limits ....................................................................................................... 182

Run the Fit ............................................................................................................... 183
  Sample Data — Maximum Likelihood Estimators (MLEs) ......................... 183
  Curve Data — The Method of Least Squares ............................................... 185

Interpret the Results .............................................................................................. 187
  Graphs ..................................................................................................................... 187
  Basic Statistics and Percentiles .......................................................................... 189
  Fit Statistics .......................................................................................................... 190
  P-Values and Critical Values .............................................................................. 192

Using the Results of a Fit ..................................................................................... 195
  Exporting Graphs and Reports .......................................................................... 195
  Using Fitted Distributions in Excel ..................................................................... 196
Overview

@RISK allows you to fit probability distributions to your data (Professional and Industrial versions only). Fitting is done when you have a set of collected data that you want to use as the basis for an input distribution in your spreadsheet. For example, you may have collected historical data on a product price and you might want to create a distribution of possible future prices that is based on this data.

To fit distributions to data using @RISK, there are five steps that you should consider:

- Define Input Data
- Specify Distributions to Fit
- Run the Fit
- Interpret the Results
- Using the Results of a Fit

Each of these steps are discussed in this chapter.
Define Input Data

@RISK allows you to analyze three kinds of data for distribution fitting: sample, density and cumulative. @RISK supports up to 100,000 data points for each of these types. The available data types are shown in the Data tab of the Fit Distributions to Data dialog.

Sample Data

Sample (or observation) data is a set of values drawn randomly from a large population. Distributions are fit to sample data to estimate the properties of that population.

Sample data is either continuous or discrete. Continuous sample data can take on any value over a continuous range, while discrete data is limited to integer values. Discrete data can be entered in two formats. In the “standard” format, you enter each data point individually. In the “counted” format, the data is entered in pairs, where the first value is the sampled value and the second is the number of samples drawn with that value.

Data requirements for sample data include:

- You must have at least five data values.
- Discrete data values must be integral.
- All sample values should fall in the range -1E+37 \(\leq x \leq +1E+37\) or be dates.
Density Data

Density data is a set of (x,y) points that describe the probability density function of a continuous distribution. Distributions are fit to density data to give the best representation of the curve points using a theoretical probability distribution.

Since all probability distribution functions must have unit area, @RISK automatically will scale your y-values so that the density curve described by your data has an area of one. Since the points you specify are isolated points on a continuum, linear interpolation between these points is used to calculate the normalization factor. In certain cases, such as fitting to data generated from a mathematical function already known to be normalized, it is undesirable to have @RISK apply its own normalization. In these cases, you may turn off this feature.

Data requirements for density data include:

♦ You must have at least three (x,y) data pairs.
♦ All x-values must be in the range $-1E+37 \leq x \leq +1E+37$ or be dates.
♦ All x-values should be distinct.
♦ All y-values must be in the range $0 \leq y \leq +1E+37$.
♦ At least one y-value must be non-zero.
Cumulative Data

Cumulative data is a set of (x,p) points that describe a continuous cumulative distribution function. The p-value associated with a given x-value is the probability of obtaining a value less than or equal to x. Distributions are fit to cumulative data to give the best representation of the curve points using a theoretical probability distribution.

In order to calculate statistics and display graphs of your cumulative data, @RISK needs to know where the input minimum and maximum are (that is, the points with p=0 and p=1). If you do not explicitly supply these points, @RISK will linearly interpolate them from your data. In general, it is recommended that you always include the p=0 and p=1 points in your data set, if possible.

Data requirements for cumulative data include:

♦ You must have at least three (x,p) data pairs.
♦ All x-values must be in the range \(-1E+37 \leq x \leq +1E+37\) or be dates.
♦ All x-values must be distinct.
♦ All p-values must be in the range \(0 \leq p \leq 1\).
♦ Increasing x-values must always correspond to increasing p-values.

Filtering Your Data

You can further refine your input data by applying an input filter. Filtering tells @RISK to ignore outliers, based on criteria you specify, without requiring you to explicitly remove them from your data set. For example, you may wish to only analyze x-values greater than zero. Or, you may wish to filter values that lie outside two standard deviations from the mean.
Select Distributions to Fit

After you define your data set, you must specify the distributions you want @RISK to attempt to fit. There are three general questions you must answer to do this.

### Continuous vs. Discrete Distributions

For sample data, you should first decide if your data is continuous or discrete. Discrete distributions always return integer values. For example, presume you have a set of data describing the number of failures in a series of 100 trial batches. You would only want to fit discrete distributions to this set because partial failures are not allowed. In contrast, continuous data can take on any value in a range. For example, presume you have a set of data describing the height, in inches, of 300 people. You would want to fit continuous distributions to this data, since heights are not restricted to integral values.

If you specify that your data is discrete, all your data values must be integers. Keep in mind, however, that the converse is not true. Just because you have all integral data values does not mean you have to fit only discrete distributions. In the previous example, the height data set may be rounded to the nearest inch, but fitting to continuous distributions is still appropriate.

@RISK does not support the fitting of discrete distributions to density and cumulative curve data.

You can specify whether your data set is continuous or discrete in the Data tab of the Fit Distributions to Data dialog.

### Estimated Parameters vs. Predefined Distributions

Generally, you will want @RISK to estimate the parameters of your distributions. However, in some cases, you may want to specify exactly what distributions to use. For example you may want to have @RISK compare two competing hypotheses and tell you which one is a better description of your data.

Predefined distributions can be set in the Distributions to Fit tab of the Fit Distributions to Data dialog.
Domain Limits

For continuous data sets (sample or curve data) you can specify how you want @RISK to treat the upper and lower limits of the distributions. For both limits there are four choices: **Fixed Bound, Bounded but Unknown, Open and Unsure.**

**Fixed Bound**

If you specify a fixed bound, you are telling @RISK that the limit of the distribution must be the value you specify. For example, if you have a data set of the times between arrivals of customers in a queue, you might want to fit distributions which have a fixed lower bound of zero, since it is impossible to have a negative time between events.

**Bounded But Unknown**

If you specify an unknown bound, you are telling @RISK that the limit of the distribution has a finite bound (that is, it does not extend to plus or minus infinity). Unlike a fixed bound, however, you do not know what the actual value of the limit is. You want @RISK to choose the value for you as it performs its fit.

**Open**

If you specify an open bound, you are telling @RISK that the limit of the distribution must extend to minus infinity (for a lower bound) or plus infinity (for an upper bound).

**Unsure**

This is the default option. It is the combination of an unknown bound and an open bound. The limits of distributions that are non-asymptotic are treated as in the unknown bound case, while asymptotic distributions are still included as in the open bound case.

Note, not all distributions functions are compatible with all the possible choices. For example, you can not specify a fixed or unknown lower bound for the Normal distribution, since it asymptotically extends to minus infinity.
Run the Fit

To start the fitting process, click the **Fit** button in the Fit Distributions to Data dialog.

For each of the distributions specified in the previous step, @RISK will try to find the set of parameters that make the closest match between the distribution function and your data set. Keep in mind, @RISK does not produce an absolute answer, but rather identifies a distribution that *most likely* produced your data. Always evaluate your @RISK results quantitatively and qualitatively, examining both the comparison graphs and statistics before using a result.

@RISK uses two methods to calculate the best distributions for your data set. For sample data, distribution parameters are estimated using Maximum Likelihood Estimators (MLEs). For density and cumulative data (collectively known as curve data), the method of least squares is used to minimize the root-mean square error between the curve points and the theoretical function.

**Sample Data — Maximum Likelihood Estimators (MLEs)**

The MLEs of a distribution are the parameters of that function that maximize the probability of obtaining the given data set.

For any density distribution \( f(x) \) with one parameter \( \alpha \), and a corresponding set of \( n \) sampled values \( X_i \), an expression called the likelihood may be defined:

\[
L = \prod_{i=1}^{n} f(X_i, \alpha)
\]

To find the MLE, simply maximize \( L \) with respect to \( \alpha \):

\[
\frac{dL}{d\alpha} = 0
\]

and solve for \( \alpha \). The method described above can be easily generalized to distributions with more than one parameter.

An exponential function with a fixed lower bound of zero has only one adjustable parameter, and its MLE is easily calculated. The distribution’s density function is:

\[
f(x) = \frac{1}{\beta} e^{-x/\beta}
\]
and the likelihood function is:

\[ L(\beta) = \prod_{i=1}^{n} \frac{1}{\beta} e^{-X_i/\beta} = \beta^{-n} \exp\left(-\frac{1}{\beta} \sum_{i=1}^{n} X_i\right) \]

To simplify matters, we can use the natural log of the likelihood function:

\[ l(\beta) = \ln L(\beta) = -n \ln(\beta) - \frac{1}{\beta} \sum_{i=1}^{n} X_i \]

To maximize the log of the likelihood, simply set its derivative with respect to \( \beta \) to zero:

\[ \frac{dl}{d\beta} = -\frac{n}{\beta} + \frac{1}{\beta^2} \sum_{i=1}^{n} X_i \]

which equals zero when:

\[ \beta = \frac{\sum_{i=1}^{n} X_i}{n} \]

Therefore, when @RISK tries to fit your data to the best Exponential function with a fixed lower bound of zero, it first finds the mean of the input data and uses it as the MLE for \( \beta \).

For some distributions, the MLE method described above does not work. For example, a 3-parameter Gamma distribution (a Gamma distribution whose lower bound is allowed to vary) cannot always be fitted using MLEs. In these cases @RISK will resort to a hybrid algorithm, which combines the standard MLE approach with a moment matching procedure.

In certain distributions, a strict MLE method produces parameters which are heavily biased for small sample sizes. For example, the MLE of the “shift” parameter of an exponential distribution, and the minimum and maximum parameters of the uniform distribution, are heavily biased for small sample sizes. Where possible, @RISK will correct for the bias.
Curve Data — The Method of Least Squares

The root-mean square error (RMSErr) between set of n curve points \((X_i, Y_i)\) and a theoretical distribution function \(f(x)\) with one parameter \(\alpha\) is:

\[
RMSErr = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (f(x_i, \alpha) - y_i)^2}
\]

The value of \(\alpha\) that minimizes this value is called the least squares fit. In a sense, this value minimizes the “distance” between the theoretical curve and the data. The formula above is easily generalized to more than one parameter.

This method is used to calculate the best distribution for both density and cumulative curve data.
Interpret the Results

Once @RISK has completed the fitting process, you should review its results. @RISK provides a powerful array of graphs, statistics, and reports to help you evaluate fits and select the best choice for your models.

@RISK ranks all the fitted distributions using one or more fit statistics. For continuous sample data, you can choose to rank fits by their chi-squared statistic, Anderson-Darling statistic, or Kolmogorov-Smirnov statistic. Each of these statistics is discussed in more detail later in this section. For discrete sample data, only the chi-squared statistic can be used. For density and cumulative curve data, the fits are ranked by their RMS Error value.

Graphs

@RISK provides four types of graphs to help you visually assess the quality of your fits.

A comparison graph superimposes the input data and fitted distribution on the same graph, allowing you to visually compare them, either as density or cumulative curves. This graph allows you to determine if the fitted distribution matches the input data in specific areas. For example, it may be important to have a good match around the mean or in the tails.
**P-P Graphs**

Probability-Probability (P-P) graphs plot the distribution of the input data ($P_i$) vs. the distribution of the result ($F(x_i)$). If the fit is “good”, the plot will be nearly linear. P-P graphs are only available for fits to sample data.

**Q-Q Graphs**

Quantile-Quantile (Q-Q) graphs plots percentile values of the input distribution ($x_i$) vs. percentile values of the result ($F^{-1}(P_i)$). If the fit is “good”, the plot will be nearly linear. Q-Q graphs are only available for fits to continuous sample data.
### Basic Statistics and Percentiles

@RISK reports basic statistics (mean, variance, mode, etc.) for each fitted distribution, which can easily be compared to the same statistics for the input data.

@RISK allows you to compare percentiles between distributions and the input data. For example, perhaps the 5th and 95th percentiles values are important to you. This can be done in two ways. First, all @RISK graphs have a set of “delimiters” which allow you to visually set different targets, or percentiles. Second, the @RISK graphs can display percentiles you select in the legend to the right of the graph.

![@RISK - Fit Results](image)

**Table:**

<table>
<thead>
<tr>
<th>Fit</th>
<th>Input</th>
<th>InvGamma</th>
<th>Logistic</th>
<th>LogLogistic</th>
<th>Normal</th>
<th>Triang</th>
<th>Weibull</th>
<th>ExValues</th>
</tr>
</thead>
<tbody>
<tr>
<td>InvGamma</td>
<td>1.8000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Logistic</td>
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<td></td>
<td></td>
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<tr>
<td>Normal</td>
<td>2.4000</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triang</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weibull</td>
<td>3.3000</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>ExValue</td>
<td>4.8000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BetaGeneral</td>
<td>5.4000</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Uniform</td>
<td>13.8000</td>
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<td></td>
</tr>
<tr>
<td>Laplace</td>
<td>42.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Percentiles

- 5%
  - Lambda: 0.019
  - Gamma: 0.021
  - Lognorm: N/A
  - Pareto: N/A
  - Pearson5: N/A
  - Pearson6: N/A

- 10%
  - Lambda: 0.031
  - Gamma: 0.061
  - Lognorm: N/A
  - Pareto: N/A
  - Pearson5: N/A
  - Pearson6: N/A

- 15%
  - Lambda: 0.056
  - Gamma: 0.122
  - Lognorm: N/A
  - Pareto: N/A
  - Pearson5: N/A
  - Pearson6: N/A

- 50%
  - Lambda: 0.081
  - Gamma: 0.138
  - Lognorm: N/A
  - Pareto: N/A
  - Pearson5: N/A
  - Pearson6: N/A

- 5%
  - Lambda: 0.160
  - Gamma: 0.273
  - Lognorm: N/A
  - Pareto: N/A
  - Pearson5: N/A
  - Pearson6: N/A

- 80%
  - Lambda: 0.233
  - Gamma: 0.253
  - Lognorm: N/A
  - Pareto: N/A
  - Pearson5: N/A
  - Pearson6: N/A

- 95%
  - Lambda: 0.285
  - Gamma: 0.321
  - Lognorm: N/A
  - Pareto: N/A
  - Pearson5: N/A
  - Pearson6: N/A

- 99%
  - Lambda: 0.342
  - Gamma: 0.376
  - Lognorm: N/A
  - Pareto: N/A
  - Pearson5: N/A
  - Pearson6: N/A

**Chi-Squared Test**

<table>
<thead>
<tr>
<th>Chi-Squared Test</th>
<th>1.8000</th>
<th>1.8000</th>
<th>1.8000</th>
<th>2.4000</th>
<th>2.4000</th>
<th>3.3000</th>
<th>4.8000</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-value</td>
<td>0.9961</td>
<td>0.9961</td>
<td>0.9961</td>
<td>0.9662</td>
<td>0.9662</td>
<td>0.9141</td>
<td>0.7787</td>
</tr>
<tr>
<td>Chi Value @ 0.100</td>
<td>0.0706</td>
<td>0.0706</td>
<td>0.0706</td>
<td>0.0706</td>
<td>0.0706</td>
<td>0.0706</td>
<td>0.0706</td>
</tr>
<tr>
<td>Chi Value @ 0.050</td>
<td>0.0706</td>
<td>0.0706</td>
<td>0.0706</td>
<td>0.0706</td>
<td>0.0706</td>
<td>0.0706</td>
<td>0.0706</td>
</tr>
</tbody>
</table>

**Note:** The image shows a screenshot of the @RISK software interface with various distributions and their parameters.
Fit Statistics

For each fit, @RISK reports one or more fit statistics. These statistics measure how well the distribution fits the input data and how confident you can be that the data was produced by the distribution function. For each of these statistics, the smaller the value, the better the fit. @RISK makes use of four different fit statistics: chi-squared, Kolmogorov-Smirnov, Anderson-Darling, and Root-Mean Squared Error.

When more than one fit statistic is available, there is no hard rule to decide which test will give you the “best” result. Each test has its strengths and weaknesses. You must decide which information is most important to you when considering which test to use.

The chi-squared statistic is the best known goodness-of-fit statistic. It can be used with both continuous and discrete sample data. To calculate the chi-squared statistic, you first must break up the x-axis domain into several “bins”. The chi-squared statistic is then defined as:

\[ \chi^2 = \sum_{i=1}^{K} \frac{(N_i - E_i)^2}{E_i} \]

where

- \( K = \) the number of bins
- \( N_i = \) the observed number of samples in the \( i^{th} \) bin
- \( E_i = \) the expected number of samples in the \( i^{th} \) bin.

A weakness of the chi-squared statistic is that there are no clear guidelines for selecting the number and location of the bins. In some situations, you can reach different conclusions from the same data depending on how you specified the bins.

Some of the arbitrariness of the bin selection can be removed by telling @RISK to use equiprobable bins. In this mode, @RISK will adjust the bin sizes based on the fitted distribution, trying to make each bin contain an equal amount of probability. For continuous distributions this is straightforward. For discrete distributions, however, @RISK will only be able to make the bins approximately equal.

@RISK allows you full control of how bins are defined for the chi-squared test. These settings are on the **Chi-Sq Binning** tab of the Fit Distributions to Data dialog.
Another fit statistic that can be used for continuous sample data is the Kolmogorov-Smirnov statistic, which is defined as

\[ D_n = \sup \left[ F_n(x) - \hat{F}(x) \right] \]

where

\[ n = \text{total number of data points} \]
\[ \hat{F}(x) = \text{the fitted cumulative distribution function} \]
\[ F_n(x) = \frac{N_x}{n} \]
\[ N_x = \text{the number of } X_i \text{'s less than } x. \]

The K-S statistic does not require binning, which makes it less arbitrary than the chi-squared statistic. A weakness of the K-S statistic is that it does not detect tail discrepancies very well.

The final fit statistic that can be used with continuous sample data is the Anderson-Darling Statistic, which is defined as:

\[ A^2_n = n \int_{-\infty}^{+\infty} \left[ F_n(x) - \hat{F}(x) \right]^2 \Psi(x) \hat{f}(x) dx \]

where

\[ n = \text{total number of data points} \]
\[ \Psi^2 = \frac{1}{\hat{F}(x)\left[1-\hat{F}(x)\right]} \]
\[ \hat{f}(x) = \text{the hypothesized density function} \]
\[ \hat{F}(x) = \text{the hypothesized cumulative distribution function} \]
\[ F_n(x) = \frac{N_x}{n} \]
\[ N_x = \text{the number of } X_i \text{'s less than } x. \]

Like the K-S statistic, the A-D statistic does not require binning. But unlike the K-S statistic, which focuses in the middle of the distribution, the A-D statistic highlights differences between the tails of the fitted distribution and input data.
For density and cumulative curve data, the only fit statistic used is the Root-Mean Squared Error. This is the same quantity that @RISK minimized to determine the distribution parameters during its fitting process. It is a measure of the “average” squared error between the input and fitted curve.

P-Values and Critical Values

The goodness-of-fit statistic reports a measure of the deviation of the fitted distribution from the input data. As mentioned earlier, the smaller the fit statistic is, the better the fit. But how small a value is needed for a “good” fit? For fits to sample data, this section explains how P-values and critical values can be used to analyze the “goodness” of a fit.

For the discussion below, suppose we have a distribution fitted to a set of $N$ sampled values, and a corresponding fit statistic, $s$.

**P-Values**

How likely is it that a new set of $N$ samples drawn from the fitted distribution would generate a fit statistic greater than or equal to $s$? This probability is referred to as the P-value and is sometimes called the “observed significance level” of the test. As the P-value decreases to zero, we are less and less confident that the fitted distribution could possibly have generated our original data set. Conversely, as the P-value approaches one, we have no basis to reject the hypothesis that the fitted distribution actually generated our data set.

**Critical Values**

Often we want to turn the same question around and specify a particular level of significance to use, usually denoted by $\alpha$. This value is the probability that we will incorrectly reject a distribution because it generated, due to statistical fluctuations, a value of $s$ that was very large. Now we want to know, given this significance level, what the largest value of $s$ is that we would accept as a valid fit. This value of $s$ is called the “critical value” of the fit statistic at the $\alpha$ level of significance. Any fit that has a value of $s$ above the critical value is rejected, while fits with values of $s$ below the critical value are accepted. Typically, critical values depend on the type of distribution fit, the particular fit statistic being used, the number of data points, and the significance level.

**Calculation Methods in @RISK**

For the chi-squared test, the P-values and critical values can be calculated by finding the appropriate points on a chi-square distribution with $k-1$ degrees of freedom (where $k$ is the number of bins). While this method is exactly correct when predefined distributions are used, it turns out to be only an approximation for distributions where @RISK estimated one or more distribution parameters. Conveniently, however, the approximation is always a conservative one. That is, the reported values for both your critical values and P-value will be slightly higher than the exact values.
More information about this can be found in Appendix D: Recommended Readings in this manual.

Most critical values and P-values for the A-D and K-S fit statistics have been found by very detailed Monte-Carlo studies (see Appendix D: Recommended Readings for references). Unfortunately, not all distributions have been analyzed in enough detail for @RISK to be able to report them. Where possible @RISK will report the appropriate P-values and critical values. Often, where an exact P-value calculation is not possible, a range is returned for the P-value, indicating that the true P-value lies between the specified upper and lower limit.
Using the Results of a Fit

Exporting Graphs and Reports

Once you have analyzed the results of your calculation, you may wish to export the results to another program. Of course, you can always copy and paste any @RISK graph or report into Excel, or another Windows application, via the clipboard. In addition, using the Chart in Excel command, @RISK allows you to create a copy of the current @RISK graph in Excel’s native chart format.
Using Fitted Distributions in Excel

Often you will want to use the result of a fit in an @RISK model. Clicking **Write to Cell** places a fit result in your model as a new distribution function.

Selecting **Update and Refit at the Start of Each Simulation** causes @RISK, at the start of each simulation, to automatically refit your data when it has changed and place the new resulting distribution function in your model.
Chapter 7: @RISK Reference Guide

Introduction..............................................................................................................205

Reference: @RISK Icons.........................................................................................207
  @RISK Ribbon Bar (Excel 2007)........................................................................207
  Main @RISK Toolbar (Excel 2003 and Earlier)..............................................211
  Settings @RISK Toolbar (Excel 2003 and Earlier) ....................................213
  Graph Window Icons .......................................................................................214

Reference: @RISK Commands...........................................................................217
  Introduction........................................................................................................217

Model Commands..................................................................................................219
  Define Distributions Command ....................................................................219
  Input Properties ...............................................................................................230
  Add Output Command ..................................................................................234
  Output Properties ..........................................................................................237
  Insert Function Command ...........................................................................241
  Define Correlations Command ....................................................................247
  Show Model Window Command ..................................................................262
  Model Window — Inputs Tab .....................................................................265
  Model Window — Outputs Tab ..................................................................273
  Model Window — Correlations Tab ..........................................................274

Distribution Fitting Commands..........................................................................275
  Fit Distributions to Data Command ............................................................275
  Data Tab — Fit Distributions to Data Command .........................................276
  Distributions to Fit Tab — Fit Distributions to Data Command ..........279
  Chi-Sq Binning Tab — Fit Distributions to Data Command ...................282
  Fit Results Window .......................................................................................285
  Fit Results — Graphs ....................................................................................288
  Write to Cell Command — Fit Results Window .........................................290
  Fit Summary Window ....................................................................................292
  Fit Manager Command ..................................................................................294
Distribution Artist Commands .......................................................... 295
  Distribution Artist Command.......................................................... 295

Settings Commands ........................................................................ 299
  Simulation Settings Command.......................................................... 299
  General Tab — Simulation Settings Command................................. 300
  View Tab — Simulation Settings Command....................................... 304
  Sampling Tab — Simulation Settings Command............................... 308
  Macros Tab — Simulation Settings Command.................................... 313
  Convergence Tab — Simulation Settings Command............................ 315

Simulation Commands .................................................................... 317
  Start Simulation Command............................................................... 317

Simulation — Advanced Analyses Commands .................................. 319

Goal Seek ........................................................................................ 321
  Goal Seek Command........................................................................ 321
  Goal Seek Dialog — Goal Seek Command....................................... 322
  Goal Seek Options Dialog — Goal Seek Command............................ 324
  Analyze — Goal Seek Command.................................................... 326

Stress Analysis .............................................................................. 329
  Stress Analysis Command............................................................... 329
  Stress Analysis Dialog — Stress Analysis Command....................... 330
  Input Definition Dialog — Stress Analysis Command..................... 332
  Stress Options Dialog — Stress Analysis Command....................... 334
  Analyze — Stress Analysis Command............................................ 336

Advanced Sensitivity Analysis ....................................................... 343
  Advanced Sensitivity Analysis Command........................................ 343
  Advanced Sensitivity Analysis Dialog — Advanced Sensitivity Analysis Command......................................................... 344
  Input Definition — Advanced Sensitivity Analysis Command......... 345
  Options — Advanced Sensitivity Analysis Command...................... 351
  Analyze — Advanced Sensitivity Analysis Command...................... 353

Results Commands ....................................................................... 359
  Browse Results Command............................................................... 359
  Results Summary Window Command.............................................. 360
  Detailed Statistics Command.......................................................... 367
  Data Command................................................................................ 370
  Sensitivities Command.................................................................... 373
  Scenarios Command........................................................................ 377
  Define Filters Command.................................................................. 382

Excel Reports Command................................................................. 385
Swap @RISK Functions Command ..........................................................387

Utilities Commands ..................................................................................395
   Application Settings Command ...............................................................395
   Windows Command ................................................................................399
   Open Simulation File Command ...............................................................400
   Clear @RISK Data Command ......................................................................401
   Unload @RISK Add-in Command ..............................................................401

Saving and Opening @RISK Simulations ..................................................403

Library Commands .....................................................................................405
   Add Results to Library ............................................................................405
   Show Library ............................................................................................405

Help Commands ..........................................................................................407
   @RISK Help ...............................................................................................407
   Online Manual ........................................................................................407
   License Activation Command .................................................................407
   About Command .......................................................................................407

Reference: @RISK Graphs .........................................................................409
   Overview ..................................................................................................409
   Histogram and Cumulative Graphs ..........................................................413
   Fitting a Distribution to a Simulated Result .............................................421
   Tornado Graphs .......................................................................................422
   Scatter Plots ............................................................................................425
   Summary Graphs ......................................................................................429
   Formatting Graphs ..................................................................................436

Reference: @RISK Functions .....................................................................443
   Introduction ...............................................................................................443
   Distribution Functions .............................................................................443
   Simulation Output Functions .................................................................451
   Simulation Statistics Functions ...............................................................452
   Graphing Function ...................................................................................453
   Supplemental Functions ..........................................................................453

Table of Available Functions .....................................................................455

Reference: Distribution Functions ............................................................467
RiskCorrel................................................................. 603
RiskData................................................................. 604
RiskKurtosis............................................................ 604
RiskMax ................................................................. 604
RiskMean ............................................................... 605
RiskMin ................................................................. 605
RiskMode ............................................................... 605
RiskPercentile, RiskPtoX, RiskPercentileD, RiskQtoX........ 606
RiskRange ............................................................. 606
RiskSensitivity ....................................................... 607
RiskSkewness ......................................................... 607
RiskStdDev ........................................................... 608
RiskTarget, RiskXtoP, RiskTargetD, RiskXtoQ ............... 608
RiskVariance ........................................................... 608
RiskTheoKurtosis .................................................... 609
RiskTheoMax ........................................................ 609
RiskTheoMean ....................................................... 609
RiskTheoMin ........................................................ 610
RiskTheoMode ....................................................... 610
RiskTheoPercentile, RiskTheoPtoX, RiskTheoPercentileD,
RiskTheoQtoX ......................................................... 611
RiskTheoRange ....................................................... 611
RiskTheoSkewness .................................................. 611
RiskTheoStdDev ..................................................... 612
RiskTheoTarget , RiskTheoXtoP, RiskTheoTargetD,
RiskTheoXtoQ ......................................................... 612
RiskTheoVariance .................................................... 612

Reference: Six Sigma Functions ................................. 613
RiskCp ................................................................. 614
RiskCpm ............................................................... 614
RiskCpk ............................................................... 615
RiskCpkLower ...................................................... 616
RiskCpkUpper ...................................................... 616
RiskDPM ............................................................. 617
RiskK ................................................................. 617
RiskLowerXBound .................................................. 618
RiskPNC .............................................................. 618
RiskPNCLower ..................................................... 619
RiskPNCUpper ..................................................... 619
RiskPPMLower ..................................................... 620
RiskPPMUpper ..................................................... 620
RiskSigmalLevel .................................................... 621
RiskUpperXBound .................................................. 622
RiskYV ............................................................... 622
RiskZlower ........................................................ 623
Chapter 7: @RISK Reference Guide

RiskZMin...........................................................................................................624
RiskZUpper.......................................................................................................624

Reference: Supplemental Functions ...............................................................625
RiskCorrectCorrmat ......................................................................................625
RiskCurrentIter ..............................................................................................626
RiskCurrentSim ..............................................................................................626
RiskStopRun ....................................................................................................626

Reference: Graphing Function ..............................................................627
RiskResultsGraph ..........................................................................................627

Reference: @RISK Library ........................................................................629
Introduction .................................................................................................629
Distributions in the @RISK Library ..............................................................631
Results in the @RISK Library ......................................................................637
Technical Notes ............................................................................................643

Reference: @RISK for Excel Developers Kit (XDK) ................................646
Introduction

This chapter describes the icons, commands, probability distribution functions and macros used to set up and execute a risk analysis using @RISK. The @RISK Reference Guide chapter is divided into six sections:

1) Reference: @RISK Icons
2) Reference: @RISK Menu Commands
3) Reference: @RISK Graphs
4) Reference: @RISK Distribution Fitting
5) Reference: @RISK Library
6) Reference: @RISK Developers Kit (XDK)
@RISK icons are used to quickly and easily perform tasks necessary to set up and run risk analyses. @RISK icons appear on the spreadsheet “toolbar” (i.e., as a custom toolbar in Excel or on a custom ribbon bar in Excel 2007) and on open graph windows. This section briefly describes each icon, outlining the functions they perform and the menu command equivalents associated with them. Note: The @RISK add-in in Excel 2003 and earlier has two available toolbars — the main toolbar and a settings toolbar that contains tools for specifying simulation settings.

If you are using @RISK Industrial you will have an additional RISKOptimizer icon displayed on the @RISK Ribbon in Excel 2007. RISKOptimizer commands can be accessed from this icon directly in @RISK, as opposed to using the separate RISKOptimizer ribbon. To use the RISKOptimizer ribbon instead, change the RISKOptimizer Add-in option to ‘Independent’ in @RISK’s Application Settings.

### @RISK Ribbon Bar (Excel 2007)

<table>
<thead>
<tr>
<th>Icon</th>
<th>Function Performed and Location</th>
</tr>
</thead>
</table>
| ![Define Distributions](image) | Add or edit probability distributions in the formula in the current cell  
  *Location: Model group, Define Distributions* |
| ![Add Output](image) | Add the current selected spreadsheet cell (or range of cells) as a simulation output  
  *Location: Model group, Add Output* |
| ![Insert Function](image) | Inserts an @RISK function in the formula of the active cell  
  *Location: Model group, Insert Function* |
| ![Define Correlations](image) | Defines correlations between probability distributions  
  *Location: Model group, Define Correlations* |
| ![Distribution Fitting](image) | Fit distributions to data  
  *Location: Model group, Distribution Fitting* |
<table>
<thead>
<tr>
<th>Icon</th>
<th>Description</th>
<th>Location</th>
</tr>
</thead>
</table>
| ![Distribution Artist](image) | Draw distribution curves  
*Location: Model group, Distribution Artist* | |
| ![Model Window](image) | Display current output cell(s) along with all distribution functions entered in the worksheet in the @RISK Model window  
*Location: Model group, Model Window* | |
| ![Iterations](image) | Sets the number of iterations to be run  
*Location: Simulation group, Iterations* | |
| ![Simulations](image) | Sets the number of simulations to be run  
*Location: Simulation group, Simulations* | |
| ![Simulation Settings](image) | View or change the simulation settings, including # of iterations, # of simulations, sampling type, standard recalc method, executed macros and other settings  
*Location: Simulation group, Simulation Settings* | |
| ![Random/Static Standard Recalc](image) | Sets the type of values (random or static) returned by @RISK distribution functions in a standard Excel recalculation  
*Location: Simulation group, Random/Static Standard Recalc* | |
| ![Automatically Show Output Graph](image) | Selects to Automatically Show Output Graph during or after simulation  
*Location: Simulation group, Automatically Show Output Graph* | |
| ![Automatically Show Results Summary Window](image) | Selects to Show Results Summary window during or after simulation  
*Location: Simulation group, Automatically Show Results Summary Window* | |
| ![Demo Mode](image) | Turns Demo mode on or off  
*Location: Simulation group, Demo Mode* | |
| ![Live Update](image) | Turns updating of open @RISK windows during simulation on and off  
*Location: Simulation group, Live Update* | |
<table>
<thead>
<tr>
<th><strong>Simulate the current worksheet(s)</strong></th>
<th>Location: Simulation group, Start Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Performs an advanced analysis</strong></td>
<td>Location: Simulation group, Advanced Analyses</td>
</tr>
<tr>
<td><strong>Runs an @RISK Goal Seek</strong></td>
<td>Location: Simulation group, Advanced Analyses, Goal Seek</td>
</tr>
<tr>
<td><strong>Runs a Stress Analysis</strong></td>
<td>Location: Simulation group, Advanced Analyses, Stress Analysis command</td>
</tr>
<tr>
<td><strong>Display an Advanced Sensitivity Analysis</strong></td>
<td>Location: Simulation group, Advanced Analyses, Advanced Sensitivity Analysis</td>
</tr>
<tr>
<td><strong>Browse results in the current worksheet(s)</strong></td>
<td>Location: Results group, Browse Results</td>
</tr>
<tr>
<td><strong>Show Results Summary Window</strong></td>
<td>Location: Results group, Summary Window</td>
</tr>
<tr>
<td><strong>Define Filters</strong></td>
<td>Location: Results group, Define Filters</td>
</tr>
<tr>
<td><strong>Display detailed statistics window</strong></td>
<td>Location: Results group, Simulation Detailed Statistics</td>
</tr>
<tr>
<td><strong>Display data window</strong></td>
<td>Location: Results group, Simulation Data</td>
</tr>
<tr>
<td><strong>Display sensitivity analysis window</strong></td>
<td>Location: Results group, Simulation Sensitivities</td>
</tr>
<tr>
<td><strong>Display scenario analysis window</strong></td>
<td>Location: Results group, Simulation Scenarios</td>
</tr>
</tbody>
</table>
| ![Excel Reports](Icons/Excel_Reports.png) | **Select Excel reports to run**  
*Location: Tools group, Excel Reports* |
| ![Swap Functions](Icons/Swap_Functions.png) | **Swap @RISK Functions in and out of open workbooks**  
*Location: Tools group, Swap Functions* |
| ![Library](Icons/Library.png) | **Add Results to or Display @RISK Library**  
*Location: Tools group, Library* |
| ![Utilities](Icons/Utilities.png) | **Open Application Settings, Show Window Panel, Open Simulation File, Clear @RISK Data, Unload @RISK Add-In**  
*Location: Tools group, Utilities* |
| ![Help](Icons/Help.png) | **Display @RISK Help**  
*Location: Tools group, Help* |
Main @RISK Toolbar (Excel 2003 and Earlier)

The following icons are shown on the main @RISK toolbar in Excel.

<table>
<thead>
<tr>
<th>Icon</th>
<th>Function Performed and Command Equivalent</th>
</tr>
</thead>
</table>
| ![Add or edit probability distributions](image) | Add or edit probability distributions in the formula in the current cell  
*Command equivalent: Model commands Define Distributions command* |
| ![Add output](image) | Add the current selected spreadsheet cell (or range of cells) as a simulation output  
*Command equivalent: Model commands Add Output command* |
| ![Insert function](image) | Inserts an @RISK function in the formula of the active cell  
*Command equivalent: Model commands Insert Function command* |
| ![Define correlations](image) | Defines correlations between probability distributions  
*Command equivalent: Model commands Define Correlations command* |
| ![Fit distributions](image) | Fit distributions to data  
*Command equivalent: Model commands Fit Distributions to Data command* |
| ![Distribution curves](image) | Draw distribution curves  
*Command equivalent: Model commands Distribution Artist command* |
| ![Display current output](image) | Display current output cell(s) along with all distribution functions entered in the worksheet in the @RISK Model window  
*Command equivalent: Model commands Model Window command* |
| ![Simulate](image) | Simulate the current worksheet(s)  
*Command equivalent: Simulation commands Start Simulation command* |
| ![Advanced analysis](image) | Performs an advanced analysis  
*Command equivalent: Simulation commands Advanced Analyses commands* |
<table>
<thead>
<tr>
<th>Icon</th>
<th>Description</th>
<th>Command equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Browse results in the current worksheet(s)" /></td>
<td>Browse results in the current worksheet(s)</td>
<td>Simulation commands Browse Results command</td>
</tr>
<tr>
<td><img src="image" alt="Display Results Summary window" /></td>
<td>Display Results Summary window</td>
<td>Results commands Results Summary Window command</td>
</tr>
<tr>
<td><img src="image" alt="Filter results" /></td>
<td>Filter results</td>
<td>Results commands Define Filters command</td>
</tr>
<tr>
<td><img src="image" alt="Display detailed statistics window" /></td>
<td>Display detailed statistics window</td>
<td>Results commands Detailed Statistics command</td>
</tr>
<tr>
<td><img src="image" alt="Display data window" /></td>
<td>Display data window</td>
<td>Results commands Data command</td>
</tr>
<tr>
<td><img src="image" alt="Display sensitivity analysis window" /></td>
<td>Display sensitivity analysis window</td>
<td>Results commands Sensitivity command</td>
</tr>
<tr>
<td><img src="image" alt="Display scenario analysis window" /></td>
<td>Display scenario analysis window</td>
<td>Results commands Scenarios command</td>
</tr>
<tr>
<td><img src="image" alt="Display reporting options" /></td>
<td>Display reporting options</td>
<td>Results commands Excel Reports command</td>
</tr>
<tr>
<td><img src="image" alt="Swap Functions" /></td>
<td>Swap Functions</td>
<td>Swap @RISK Functions command</td>
</tr>
<tr>
<td><img src="image" alt="Display @RISK Library" /></td>
<td>Display @RISK Library</td>
<td>Library commands Show @RISK Library command</td>
</tr>
<tr>
<td><img src="image" alt="Display @RISK Utilities" /></td>
<td>Display @RISK Utilities</td>
<td>Utilities commands</td>
</tr>
<tr>
<td><img src="image" alt="Display @RISK Help" /></td>
<td>Display @RISK Help</td>
<td>Help commands</td>
</tr>
</tbody>
</table>
## Settings @RISK Toolbar (Excel 2003 and Earlier)

The following icons are shown on the @RISK Settings toolbar in Excel.

<table>
<thead>
<tr>
<th>Icon</th>
<th>Function Performed and Command Equivalent</th>
</tr>
</thead>
</table>
| ![View or change the simulation settings, including # of iterations, # of simulations, sampling type, standard recalc method, executed macros and other settings](image) | View or change the simulation settings, including # of iterations, # of simulations, sampling type, standard recalc method, executed macros and other settings  
Command equivalent: Simulation command Settings command |
| ![Sets the number of iterations to be run](image) | Sets the number of iterations to be run  
Command equivalent: Settings command Simulation Settings command Number of Iterations option |
| ![Sets the number of simulations to be run](image) | Sets the number of simulations to be run  
Command equivalent: Settings command Simulation Settings command Number of Simulations option |
| ![Sets the type of values (random or static) returned by @RISK distribution functions in a standard Excel recalculation](image) | Sets the type of values (random or static) returned by @RISK distribution functions in a standard Excel recalculation  
Command equivalent: Settings command Random Standard (F9) Recalc command options |
| ![Selects to Browse Results in Spreadsheet at end of simulation and Automatically Show One Output Graph during simulation](image) | Selects to Browse Results in Spreadsheet at end of simulation and Automatically Show One Output Graph during simulation  
Command equivalent: Settings command Automatically Show Output Graph command |
| ![Selects to Show @RISK — Results Summary window during and at the end of simulation](image) | Selects to Show @RISK — Results Summary window during and at the end of simulation  
Command equivalent: Settings command Automatically Show Results Summary Window command |
| ![Turns Demo mode on or off](image) | Turns Demo mode on or off  
Command equivalent: Settings command Demo Mode command |
| ![Turns updating of open @RISK windows during simulation on and off](image) | Turns updating of open @RISK windows during simulation on and off  
Command equivalent: Settings command Live Update command |
### Graph Window Icons

The following icons are shown on the bottom of open @RISK graph windows. Depending on the type of displayed graph, some icons may not be shown.

<table>
<thead>
<tr>
<th>Icon</th>
<th>Function Performed and Command Equivalent</th>
</tr>
</thead>
</table>
| ![Icon 1] | Displays the Graph Options dialog  
*Command equivalent: Graph Options command* |
| ![Icon 2] | Copies or reports on the displayed result  
*Command equivalent: Reports Commands* |
| ![Icon 3] | Shows and sets the type of distribution graph shown  
*Command equivalent: Graph Options command Type options* |
| ![Icon 4] | Shows and sets the type of tornado graph shown  
*Command equivalent: Graph Options command Type options* |
| ![Icon 5] | Add an overlay to the displayed graph  
*Command equivalent: None* |
| ![Icon 6] | Create a scatter plot using the data from the displayed graph  
*Command equivalent: None* |
| ![Icon 7] | Shows a scenario tornado graph or edits scenarios  
*Command equivalent: None* |
| ![Icon 8] | Create a summary graph using the data from the displayed graph  
*Command equivalent: None* |
| ![Icon 9] | Add a new variable to a scatter plot or summary graph  
*Command equivalent: None* |
| ![Icon 10] | Selects a graph from a simulation# in a multi-simulation run  
*Command equivalent: None* |
| ![Icon 11] | Defines a filter for the displayed result  
*Command equivalent: Results commands Define Filters command* |
| ![Icon 12] | Fit distributions to a simulated result  
*Command equivalent: None* |
<table>
<thead>
<tr>
<th>Icon</th>
<th>Description</th>
<th>Command equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Zoom In" /></td>
<td>Zooms in on a region of a graph</td>
<td>None</td>
</tr>
<tr>
<td><img src="image" alt="Reset" /></td>
<td>Resets zoom to default scaling</td>
<td>None</td>
</tr>
<tr>
<td><img src="image" alt="Attach Graph" /></td>
<td>Changes a floating graph to a graph attached to the cell it references</td>
<td>None</td>
</tr>
</tbody>
</table>
Introduction

This section of the @RISK Reference Guide details the available @RISK commands that can be accessed through the @RISK ribbon bar in Excel 2007 or with the @RISK toolbars and menu in Excel 2003 and earlier.

An @RISK menu is added to Excel versions 2003 and earlier. All @RISK commands are accessed through the @RISK ribbon bar in Excel 2007.

Several @RISK commands are also available in a pop-up floating menu that is displayed when the right mouse button is clicked on a cell in Excel.
Model Commands

Define Distributions Command

Defines or edits probability distributions entered in the current cell formula

The Define Distributions command displays the Define Distribution pop-up window. Using this window, probability distributions can be assigned to values contained in the formula of the selected cell. This window also allows you to edit distributions already present in a cell’s formula.

The @RISK Define Distribution window graphically displays probability distributions which can be substituted for values in the formula in the current cell. By changing the displayed distribution you can see how various distributions would describe the range of possible values for an uncertain input in your model. The displayed statistics also shows how a distribution defines an uncertain input.

The graphical display of an uncertain input is useful in showing your definition of risk to others. It clearly displays the range of possible values for an input, and the relative probability of any value in the range occurring. Working with distribution graphs you can easily incorporate other individual assessments of uncertainty into your risk analysis models.
Clicking the **Define Distributions** icon displays the Define Distribution window. As you click on different cells in your spreadsheet, the Define Distribution window updates to show the formula for each cell you select. Press `<Tab>` to move the window among cells with distributions in open workbooks.

All changes and edits made are added directly to the cell’s formula when you 1) click on another cell to move the Define Distribution window to that formula or 2) click OK to close the window.

The Define Distribution window has a **Primary** curve — i.e. the one for the function entered in the cell formula — and up to ten **Overlay** curves, representing other distributions that you may wish to graphically display on top of the Primary curve. Overlays are added by clicking the **Add Overlay icon** at the bottom of the window.
The different elements of the Define Distribution window are as follows:

- **Name.** Displays the default name that @RISK has identified for the cell. By clicking the Reference Entry icon (the icon after the name), you can select an alternate cell in Excel that contains the name to use. Alternatively, simply type a name.

- **Cell Formula.** Displays the current cell formula including any @RISK distribution functions. This formula may be edited here just as in Excel. The text shown in red and underlined is the distribution that is graphed.

- **Select Distribution.** Adds the currently selected distribution in the Distribution Palette. For the shortcut to Select Distribution double-click on the distribution you wish to use from the displayed Distribution Palette.

- **Make Favorite.** Adds the currently selected distribution in the Distribution Palette to the Favorites tab in the Palette.

- **Splitter Bar.** To make the Cell Formula box larger or smaller, move the splitter bar, up and down, between the Cell Formula box and the graph. To make the Distribution Argument panel larger, move the splitter bar, left and right, between the panel and the graph.

Delimiters and Statistics are used to display underlying statistics on displayed distribution graphs:

- **Delimiters.** Delimiters allow setting of target probabilities and x-axis scaling using the mouse. Cumulative probabilities can be set directly on a distribution graph using the displayed probability delimiters. Dragging probability delimiters changes left and right x and p values, shown in the probability bar, above the graph. Dragging the delimiters, at either end of the x-axis, rescales the x-axis.

- **Statistics.** The statistics displayed for the graphed distributions, including any overlays, can be selected in the Legends tab of the Graph Options dialog. To display this dialog, click the Graph Options dialog icon in the bottom left of the window.
To assign a distribution to a specific value in the Cell Formula, simply click on it to select it (the value turns blue), then double-click the distribution you wish to use from the displayed **Distribution Palette**.

To change the distribution used in the formula, click the **Replace Distribution in Formula** button at the bottom of the window and select or double-click the distribution you want to change to from the Palette.
The small version of the Palette contains additional icons at the bottom that allow you to delete all overlays, make favorites to be shown on the Favorites tab or select a distribution you wish to use from a cell in Excel.

To add overlays to a displayed distribution graph, click the **Add Overlay icon** at the bottom of the window.
Argument values can be entered in the Distribution Argument panel, or typed directly in the shown formula. This panel is displayed to the left of the graph. Spinner buttons allow you to quickly change a parameter value. If you have Overlays, the Distribution Argument panel allows you to switch between entering arguments for the Primary curve and any of the overlays.

Options in the Distribution Argument panel include:

- **Function.** This entry selects the distribution type displayed in the graph, which can also be done by making a selection from the Distribution Palette.

- **Parameters.** This entry selects the type of arguments to be used for the distribution. This can include **Truncation Limits, Shift Factor, Date Formatting** and in many cases, **Alternate Parameters**. You can also select to display an entry for the **Static Value** to be returned for the distribution.
- Selecting **Truncation Limits** will put an entry for **Trunc. Min** and **Trunc. Max** in the Distribution Argument panel, allowing the distribution to be truncated at the values specified.

- Selecting **Shift Factor** will put an entry for **Shift** in the Distribution Argument panel. A Shift Factor shifts the domain of the distribution in which it is used by the entered shift amount.

- Selecting **Alternate Parameters** allows the entry of alternate parameters for the distribution.

- Selecting **Static Value** allows the entry of the Static Value for the distribution.

- Selecting **Date Formatting** instructs @RISK to display dates in the Distribution Argument panel and to display graphs and statistics using dates. This selection will result in a **RiskIsDate** property function being placed in your distribution.

```
Note: In the Application Settings dialog, you can specify that Truncation Limits, Shift Factor and Static Value always be displayed in the Distribution Argument panel.
```
Alternate Parameters

Alternate Parameters allow you to specify values for specific percentile locations of an input distribution as opposed to the traditional arguments used by the distribution. The percentiles to be entered are specified using the Alternate Distribution Parameters options, displayed when Alternate Parameters is selected.

With Alternate Parameters, you have the option to:

- **Specify Using Cumulative Descending Percentiles** specifies that the percentiles used for alternate parameters will be in terms of cumulative descending probabilities. Percentiles entered in this case specify the probability of a value greater than the entered argument x-value.

When making Parameter Selections, Percentile parameters may be mixed with standard parameters by clicking the appropriate radio buttons.
In the Application Settings dialog you can select the default parameters you wish to use for Alternate Parameter Distributions, or those distribution types that end in ALT (such as RiskNormalAlt). Your default parameters will be used each time you select an Alternate Parameter distribution from the Distribution Palette.
Icons in the Distribution Argument panel deletes curves, displays the Distribution Palette and allows Excel cell references to be used as argument values.

Icons in the Distribution Argument panel include:

- **Deletes the curve** whose arguments are shown in the selected region of the Distribution Argument panel.

- **Displays the Distribution Palette** for selecting a new distribution type for the selected curve.

- **Displays the Distribution Argument panel** in a mode which allows **Excel cell references** to be selected for argument values. When in this mode, simply click on the cells in Excel which contain the argument values you wish to use. Click the **Dismiss Reference entry** icon (at the top of the window) when complete.

The Distribution Argument panel may be hidden if desired. At the bottom of the window, hide or show the panel using the fifth button from the left as shown below:
Changing Graph Type

In the Define Distribution window (along with other graph windows), the type of the displayed graph may be changed by clicking the Graph Type icon in the lower left of the window.
Input Properties

@RISK distribution functions have both required and optional arguments. The only required arguments are the numeric values which define the range and shape of the distribution. All other arguments (such as name, truncation, correlation and others) are optional and can be entered only when needed. These optional arguments are entered using property functions using a pop-up Input Properties window.

Clicking the fx icon at the end of the Cell Formula text box displays the Input Properties window.

Many properties can use cell references to Excel cells. Simply click the Reference Entry icon next to the property to add a cell reference.
Distribution properties available in the Options tab of Input Properties window include:

- **Name.** The name @RISK will use for the input distribution in reports and graphs. Initially a default name determined by @RISK from row and column headings is shown. If this default name is changed, a **RiskName** property function will be added to the entered distribution function to hold the defined name.

- **Units.** The units @RISK will use for the input distribution to label the x-axis in graphs. If units are entered, a **RiskUnits** property function will be added to the entered distribution function to hold the defined units.

- **Use Static Value.** The value the distribution will 1) return in normal (not random), Excel recalculations and 2) be substituted for the input distribution when @RISK functions are swapped out. When a new input distribution is entered through the Define Distribution window, the Static Value is set to the value replaced in the formula by the distribution. If no Static value is entered, @RISK will use either the expected value, median, mode, or a percentile for the distribution in 1) normal (not random), Excel recalculations and 2) when @RISK functions are swapped out. If a static value is entered, a **RiskStatic** property function will be added to the entered distribution function to hold the defined value.
• **Date Formatting.** Specifies if the data for the input will be treated as dates in reports and graphs. The setting **Automatic** specifies that @RISK will automatically detect the date data using the format of the cell where the input is located. Selecting **Enabled** will force @RISK to always display graphs and statistics for the input using dates, regardless of the cell format. Likewise, **Disabled** will force @RISK to always generate graphs and statistics for the input in numeric format, regardless of the cell format. If Enabled or Disabled is selected, a **RiskIsDate** property function will be entered to hold the date setting.
Input Properties – Sampling Tab

Distribution properties available in the Sampling tab of Input Properties window include:

- **Separate Seed.** Sets the seed value for this input which will be used during simulation. Setting a seed value for a specific input insures that any model that uses the input distribution will have the identical stream of sampled values for the input during a simulation. This is useful when sharing input distributions between models using the @RISK Library.

- **Lock Input from Sampling.** Keeps the input from being sampled during a simulation. A locked input returns its static value (if specified) or alternatively, its expected value, or the value specified through the options under **When a Simulation is Not Running, Distributions Return** of the Simulation Settings dialog.

- **Collect Distribution Samples.** Instructs @RISK to collect samples for the input when the option **Inputs Marked with Collect** is selected in the Sampling tab of Simulation Settings dialog. If this option is chosen, only inputs marked to collect will be included in sensitivity analyses, statistics and graphs available after a simulation.
Add Output Command

Adds a cell or range of cells as a simulation output or output range

Clicking the Add Output icon adds the currently selected range of worksheet cells as a simulation output. A distribution of possible outcomes is generated for every output cell selected. These probability distributions are created by collecting the values calculated for a cell, each iteration of a simulation.

A Summary graph may be generated when a selected output range has more than one cell in it. For example, in one output range, you could select all the cells in a row in your worksheet. The output distributions from these cells would be summarized in a Summary graph. You could also see an individual probability distribution for any cell in the range.

Sensitivity and Scenario analysis results are also generated for each output cell. For more information on these analyses, see the descriptions of these analyses in the Results Summary Window section of this chapter.
When a cell is added as a simulation output, a RiskOutput function is placed in the cell. These functions allow the easy copying, pasting and moving of output cells. RiskOutput functions may also be entered in formulas, the same way you would type in any standard Excel function, bypassing the Add Output command. RiskOutput functions optionally allow you to name your simulation outputs, and add individual output cells to output ranges. A typical RiskOutput function might be:

\[ =\text{RiskOutput("Profit")} + \text{NPV}(0.1,H1:H10) \]

where the cell, prior to its selection as a simulation output, simply contained the formula:

\[ = \text{NPV}(0.1,H1:H10) \]

The added RiskOutput function selects the cell as a simulation output and gives the output the name “Profit”. For more information on RiskOutput functions, see the section: Reference: @RISK Functions.

When an output is added, you are given the opportunity to name it, or use the default name @RISK has identified. You can enter a reference to an Excel cell, containing the name, by simply clicking in the desired cell. The name (if not the @RISK default name) is added as an argument to the RiskOutput function used to identify the output cell.

At any time a name may be changed by 1) editing the name argument to the RiskOutput function, 2) re-selecting the output cell and clicking the Add Output icon again or 3) changing the name shown for the output in the Model window.
To add a new output range:

1) Highlight the range of cells in your spreadsheet that you wish to add as an output range. If multiple cells are included in the range, highlight all the cells by dragging the mouse.

2) Click the Add Output icon (the one with the single red arrow).

3) Add the name for the output range, and individual output cells in the range, in the displayed Add Output Range window. Properties for individual output cells in the range can be added by selecting the output in the table and clicking the fx icon.

![Add Output Range](image-url)
Output Properties

@RISK outputs (defined using the function RiskOutput) have optional arguments that specify properties, such as name and units, that can be entered only when needed. These optional arguments are entered using property functions through a pop-up Output Properties window.

Clicking the fx icon at the end of the Name text box displays the Output Properties window.

Many properties can use cell references to Excel cells. Simply click the Reference Entry icon next to the property to add a cell reference.
Output properties, available in the Options tab of Output Properties window, include:

- **Name.** The name @RISK will use for the output in reports and graphs. Initially a default name determined by @RISK from row and column headings is shown.

- **Units.** The units @RISK will use for the output to label the x-axis in graphs. If units are entered, a RiskUnits property function will be added to the entered distribution function, to hold the defined units.

- **Data Type.** Specifies the type of data that will be collected for the output during a simulation – **Continuous** or **Discrete.** The setting Automatic specifies that @RISK will automatically detect the type of data described by the generated data set and generate graphs and statistics for that type. Selecting Discrete will force @RISK to always generate graphs and statistics for the output in discrete format. Likewise, Continuous will force @RISK to always generate graphs and statistics for the output in discrete format. If Discrete is selected a RiskIsDiscrete property function will be entered for the output in its RiskOutput function.
• **Date Formatting.** Specifies if the data for the output will be treated as dates in reports and graphs. The setting **Automatic** specifies that @RISK will automatically detect the date data using the format of the cell where the output is located. Selecting **Enabled** will force @RISK to always display graphs and statistics for the output using dates, regardless of the cell format. Likewise, **Disabled** will force @RISK to always generate graphs and statistics for the output in numeric format, regardless of the cell format.

The settings used in monitoring convergence of an output are set on the Convergence tab. These settings include:

- **Convergence Tolerance.** Specifies the tolerance allowed for the statistic you are testing. For example, the above settings specify that you wish to estimate the mean of the output simulated within 3% of its actual value.

- **Confidence Level.** Specifies the confidence level for your estimate. For example, the above settings specify that you want your estimate of the mean of the output simulated (within the entered tolerance) to be accurate 95% of the time.

- **Perform Tests on Simulated.** Specifies the statistics of each output that will be tested.

All convergence monitoring settings are entered using the RiskConvergence property function.

Reference: @RISK Commands
The default settings for an output to be used in Six Sigma calculations are set on the Six Sigma tab. These properties include:

- **Calculate Capability Metrics for This Output**. Specifies that capability metrics will be displayed in reports and graphs for the output. These metrics will use the entered LSL, USL and Target values.

- **LSL, USL and Target**. Sets the LSL (lower specification limit), USL (upper specification limit) and Target values for the output.

- **Use Long Term Shift and Shift**. Specifies an optional shift for calculation of long-term capability metrics.

- **Upper/Lower X Bound**. The number of standard deviations to the right or the left of the mean for calculating the upper or lower X-axis values.

Entered Six Sigma settings are entered in a RiskSixSigma property function. Only outputs which contain a RiskSixSigma property function will display six sigma markers and statistics in graphs and reports. @RISK six sigma statistics functions in Excel worksheets can reference any output cell that contains a RiskSixSigma property function.

**Note:** All graphs and reports in @RISK use the LSL, USL and Target values from RiskSixSigma property functions that existed at the start of a simulation. If you change the Specification Limits for an output (and its associated RiskSixSigma property function), you need to re-run the simulation to view changed graphs and reports.
Insert Function Command

Inserts an @RISK function in the active cell

@RISK provides a variety of custom functions that can be used in Excel formulas for defining probability distributions, returning simulation statistics to Excel and performing other modeling tasks. The @RISK Insert Function command allows you to quickly insert an @RISK function into your spreadsheet model. You can also set up a list of favorite functions that can be quickly accessed. When the @RISK Insert Function command is used, the Excel Insert Function Arguments dialog is displayed where arguments to the functions can be entered.

If you use the @RISK Insert Function command to enter a distribution function, a graph of the distribution function may also be displayed. As with the Define Distribution window, you can add overlays to this graph, add input property functions or even change the type of distribution function being entered.
Three categories of @RISK functions can be entered with the Insert Function command. These include:

- **Distribution Functions**, such as RiskNormal, RiskLognorm and RiskTriang
- **Statistics Functions**, such as RiskMean, RiskTheoMode and RiskPNC
- **Other Functions**, such as RiskOutput, RiskResultsGraph and RiskConvergenceLevel

To get more information on any of the @RISK functions listed with the Insert Function command, see the Reference: @RISK Functions section of this manual.

@RISK functions that you select are listed as **Favorites** so you can quickly access them on the Insert Function menu or on the **Favorites tab of the Distribution Palette**. The Manage Favorites command displays a list of all available @RISK functions so you can select the functions you commonly use.
If the @RISK Insert Function command is used to enter a distribution function, a graph of the distribution function may also be displayed. This graph may also be displayed any time you edit an @RISK distribution using the Excel Function Arguments dialog – for example, by clicking the small Fx symbol by the Excel formula bar or by using the Excel Insert function command.

A graph of a distribution function may be displayed or hidden by clicking the Graph button in the Excel Function Arguments dialog.

If you do not want to display @RISK distribution functions graphically beside the Excel Function Arguments dialog, select the @RISK Utilities menu Application Settings command ‘Insert Function’ Graph Window option to Disabled.

Note: Graphs of RiskCompound functions cannot be displayed in the Insert Function graph window. Use the Define Distribution window to preview these functions.
A set of buttons at the bottom of the Insert Function graph window allow you to:

- Access the **Graph Options dialog** to change the scaling, titles, colors, markers and other settings for the graph.
- Create **an Excel chart** of the graph.
- Change the **type of displayed graph** (cumulative, relative frequency, etc.).
- Add **overlays** to the graph.
- Add **properties** (i.e., distribution property functions such as `RiskTruncate`) to the entered distribution function.
- Change the **type of distribution function** graphed.
To add an overlay to an Insert Function Graph, click the **Add Overlay** button at the bottom of the window and select the distribution you wish to overlay from the Distribution Palette. Once an overlay is added, you can change the function argument values in the **Distribution Argument** panel. This panel is displayed to the left of the graph. Spinner buttons allow you to quickly change a parameter value. For more information on using the Distribution Argument Panel, see the **Define Distribution command** in this chapter.

To change the distribution used in the formula from the Insert Function Graph window, click the **Distribution Palette** button at the bottom of the window and select or double-click the distribution you want to change to from the Palette. Once selected, the new distribution and arguments will be entered in the Excel formula bar and a graph of the new function will be shown.
To add input properties in the Insert Function Graph Window, click the **Input Properties** button at the bottom of the Graph Window and select the properties you wish to include. If desired, you may edit the setting for the property in the Input Properties window.

Once you click OK and a distribution property function is entered, you can click on the distribution property function in the Excel formula bar and the **Excel Function Argument window** will be displayed for the property function itself. The arguments may then be edited using the Excel Function Argument window.
Define Correlations Command

Defines correlations between probability distributions in a correlation matrix

The Define Correlations command allows the samples of input probability distributions to be correlated. When the Define Correlations icon is clicked, a matrix is displayed which includes a row and column for each probability distribution in the currently selected cells in Excel. Correlation coefficients between the probability distributions can be entered using this matrix.

Why Correlate Distributions?

Two input distributions are correlated when their samples should be “related” — that is, the value sampled for one distribution should affect the value sampled for the other. This correlation is necessary when, in reality, two input variables move to some degree in tandem. For example, imagine a model with two input distributions — Interest Rate and Housing Starts. These two inputs are related, as the value sampled for Housing Starts depends on the value sampled for Interest Rate. A high Interest Rate would tend to cause a low value for Housing Starts, and conversely a low Interest Rate would tend to cause a high value for Housing Starts. If this correlation were not accounted for in sampling, some iterations of the simulation would reflect nonsensical conditions that could not occur in reality — such as a high Interest Rate and a high value for Housing Starts.
Correlations between input distributions are entered into the displayed matrix. The rows and columns of this matrix are labeled with each of the input distributions in the currently selected cells. Any specific cell in the matrix specifies the correlation coefficient between the two input distributions identified by the row and column of the cell.

Correlation coefficients range in value between -1 and 1. A value of 0 indicates there is no correlation between the two variables — i.e., they are independent. A value of 1 is a complete positive correlation between the two variables — i.e., when the value sampled for one input is “high”, the value sampled for the second will also be “high”. A value of -1 is a complete inverse correlation between the two variables — i.e., when the value sampled for one input is “high”, the value sampled for the second will be “low”. Coefficient values in between, such as -.5 or .5, specify a partial correlation. For example, a coefficient of .5 specifies that when the value sampled for one input is “high”, the value sampled for the second value will have a tendency to, but not always, be “high”.

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Entering Correlation Coefficients
Correlations may be entered between any input distributions. A distribution may be correlated with many other input distributions. Often your correlation coefficients will be calculated from actual historical data on which you are basing the distribution functions in your model.

**Note:** There are two possible cells where the correlation between any two inputs may be entered (row of the first and column of the second, or column of the first and row of the second). You can use either cell — as you enter a coefficient value in one it is automatically entered in the second cell.

**Editing Existing Correlations**

The Define Correlations window allows you to edit existing correlation matrices and create new instances of existing matrices. If you select 1) a cell in Excel that includes a distribution that was previously correlated or 2) a cell in an existing correlation matrix, and then click the Define Correlations icon, the existing matrix will be displayed. Once displayed, you can change coefficients, add new inputs, add instances, relocate the matrix, or edit it.

**Adding Inputs to a Matrix**

Clicking the Add Inputs button in the Define Correlations window allows you to select Excel cells with @RISK distributions to add to the displayed matrix and instance. If some of the cells in a selected range do not include distributions, those cells are simply skipped.

**Deleting a Matrix**

The Delete Matrix button deletes the displayed correlation matrix. All RiskCorrmat functions will be removed from distribution functions used in the matrix and the correlation matrix displayed in Excel will be deleted.
Options in the Define Correlation window for naming and locating a matrix in Excel include:

- **Matrix Name.** Specifies the name of the matrix. This name will be used to 1) name the range where the matrix is located in Excel and 2) identify the matrix in the RiskCorrmat functions that are created for each input distribution included in the matrix. This name must be a valid Excel range name.

- **Description.** Gives a description of the correlations included in the matrix. This entry is optional.

- **Location.** Specifies the range in Excel that the matrix will occupy.

- **Add Heading Row/Column and Format.** Optionally displays a heading row and column that includes the names and cell reference for the correlated inputs and formats the matrix with colors and borders, as shown:
Matrix Instances

An instance is a new copy of an existing matrix that can be used to correlate a new set of inputs. Each instance contains the same set of correlation coefficients, however, the inputs that are correlated with each instance are different. This allows you to easily set up groups of similarly correlated variables, without repeating the entry of the same matrix. In addition, when a correlation coefficient is edited in any instance of a matrix, it is automatically changed in all instances.

Each instance of a matrix has a name. Instances can be deleted or renamed at any time.

Instance is a third optional argument to the RiskCorrmat function. This allows you to easily specify instances when entering correlation matrices and RiskCorrmat functions directly in Excel. For more information on the RiskCorrmat function and the Instance argument, see RiskCorrmat in the Reference: @RISK Functions section of this chapter.

Note: When a correlation matrix with multiple instances is created in the Define Correlations window, and entered in Excel, only the inputs for the first instance are shown in the headings of the matrix. In addition, when you display a scatter matrix of simulated correlations for the matrix after a run, only the scatter plots for the correlations in the first instance are shown.

Options for Instances include:

- **Instance**. Selects the instance that will be shown in the displayed matrix. Inputs can be added to a displayed instance by clicking the Add Inputs button.

Icons next to the Instance name allow:

- **Rename Instance**. Renames the current instance of the displayed correlation matrix.

- **Delete Instance**. Deletes the current instance of the displayed correlation matrix.

- **Add New Instance**. Adds a new instance for the displayed correlation matrix.
A **Correlated Time Series** is created from a range in Excel that contains a set of similar distributions in each row or column of range. In many cases, each row or column represents a “time period”. Often you would like to correlate each period’s distributions using the same correlation matrix but with a different instance of the matrix for each time period.

When the Create Correlated Time Series icon is clicked, you are prompted to select the block of cells in Excel that contains the distributions of the time series. You can select to have each time period represented by the distributions in a column, or row, in the range.

When a correlated time series is created, @RISK automatically sets up a correlation matrix instance for each set of similar distributions, in each row or column, in the selected range.
Columns in a correlation matrix may be reordered simply by dragging the column heading to the new desired position in the matrix.

**Rearranging Columns**

**Deleting Rows, Columns and Inputs**

Additional options shown, when you right-click on a matrix, allow you to delete rows or columns from a matrix, or remove an input from the matrix:

- **Insert Row/Column.** Inserts a new row and column in the active correlation matrix. A new column will be placed in the matrix at the cursor location, shifting existing columns to the right. A new row is also added, at the same position as the added column, shifting existing rows down.

- **Delete Selected Row/Column(s).** Deletes the selected rows and columns from the active correlation matrix.

- **Delete Inputs in Selected Row/Columns(s) from Matrix.** Removes the selected input(s) from the active correlation matrix. When inputs are deleted, only the inputs are removed — the coefficients specified in the matrix remain.
**Displaying Scatter Plots**

The **Show Scatter Plots icon** (in the bottom left of the Define Correlation window) shows a matrix of scatter plots of possible sampled values for any two inputs in the matrix, when they are correlated using the entered correlation coefficients. These scatter plots show, graphically, how the sampled values of any two inputs will be related during a simulation.

Moving the **Correlation Coefficient slider**, displayed with the scatter matrix, dynamically changes the correlation coefficient and scatter plot for any pair of inputs. If you have expanded or dragged the thumbnail scatter plot into a full graph window, that window will also update dynamically.
After a simulation, you can check the **actual simulated correlations** for the entered matrix. This is done by clicking on a cell in the matrix when “browsing” simulation results in your spreadsheet. The scatter plot matrix shows the actual correlation coefficient calculated between the samples drawn for each pair of inputs, along with the coefficient entered in the matrix before the run. If an entered matrix has multiple instances, only the scatter plots for the correlations in the first instance are shown after a run.
The Check Matrix Consistency command, displayed when you click the Check Matrix Consistency icon, verifies that the entered matrix in the active correlation window is valid. @RISK can correct any invalid matrix and generate the closest valid matrix to the entered invalid one.

An invalid matrix specifies inconsistent simultaneous relationships between three or more inputs. It is quite easy to make a correlation matrix which is invalid. A simple example is: correlate input A and B with a coefficient of +1, B and C with a coefficient of +1, and C and A with a coefficient of -1. This example is clearly illegal, but invalid matrices are not always this obvious. In general, a matrix is valid only if it is positive semi-definite. A positive semi-definite matrix has eigenvalues which are all greater than or equal to zero, and at least one eigenvalue that is greater than zero.

If @RISK determines you have an invalid matrix when the Check Matrix Consistency icon is clicked, it will give you the option of letting @RISK generate the closest valid matrix to the entered invalid one. @RISK follows these steps to modify a matrix:

1) Finds the smallest eigenvalue (E0)

2) "Shifts" the eigenvalues so that the smallest eigenvalue equals zero by adding the product of -E0 and the identity matrix (I) to the correlation matrix (C): C' = C - E0I.

3) Divides the new matrix by 1 - E0 so that the diagonal terms equal: C'' = (1/1-E0)C'

This new matrix is positive semi-definite, and therefore, valid. It is important to check the new valid matrix to ensure that its correlation coefficients accurately reflect your knowledge of the correlation between the inputs included in the matrix. Optionally you can control what coefficients are adjusted during the correction of a matrix by entering Adjustment Weights for individual coefficients.

Note: A correlation matrix entered in the Correlation window is automatically checked for consistency when the OK button is clicked, prior to entering the matrix in Excel and adding RiskCorrmat functions for each input in the matrix.
Adjustment Weights may be specified for individual coefficients in a correlation matrix. These weights control how coefficients may be adjusted when the matrix is invalid and is corrected by @RISK. An adjustment weight ranges from 0, or any change allowed, to 100, or no change allowed (if possible). You would use Adjustment Weights when you have calculated certain correlations between inputs in a matrix with certainty and do not want them modified during the adjustment process.

To enter Adjustment Weights in a Define Correlation window, select the matrix cell(s) that you wish to enter weights for and select the Enter Adjustment Weight command displayed when you right-click on the matrix or click the Check Matrix Consistency icon.

As Adjustment Weights are entered, cells in the matrix with an Adjustment Weight are colored to indicate the degree to which their coefficient is fixed.
When you place a correlation matrix in Excel (or use the Check Matrix Consistency command), @RISK will check if the entered correlation matrix is valid. If it is not, it will correct the matrix using the entered weights.

**Note:** If you enter an Adjustment Weight of 100, @RISK will make all possible efforts to keep the coefficient associated with that weight fixed. However, if no valid matrix can be generated with the fixed coefficient, it will have to be adjusted in order to create a valid matrix.

When you place a correlation matrix in Excel, its Adjustment Weights may also be placed in an Adjustment Weight matrix in Excel. This matrix has the same number of elements as the correlation matrix it is used with. Cells in this matrix hold the entered Adjustment Weight values. Any matrix cells for which no weight was entered (shown as blanks in the matrix) have a weight of 0 indicating that they may be adjusted as necessary during matrix correction. An Adjustment Weight matrix in Excel is given an Excel range name using the name of the correlation matrix it is used with plus the extension _Weights. For example, a matrix named Matrix1 could have an associated Adjustment Weight matrix with the name Matrix1_Weights.

**Note:** You do not have to place an Adjustment Weight matrix in Excel when exiting the Define Correlations window. You can just place the corrected correlation matrix in Excel and discard any entered weights if you are happy with the corrections made and do not wish to access the weights at a later time.
You may wish to view in Excel the corrected matrix that @RISK generates and uses while simulating. If @RISK detects an inconsistent correlation matrix in your model, it will correct it, using any related Adjustment Weight matrix. However, it leaves your original inconsistent matrix as you entered it in Excel. To view the corrected matrix in your spreadsheet:

1) Highlight a range with the same number of rows and columns as the original correlation matrix

2) Enter the function
   \[ \text{RiskCorrectCorrmat(CorrelationMatrixRange,AdjustmentMatrixRange)} \]

3) Press \(<\text{Ctrl}><\text{Shift}><\text{Enter}>\) at the same time to enter your formula as an array formula. Note: the AdjustmentMatrixRange is optional, and only used when you are applying adjustment weights.

For example, if the correlation matrix was in the range A1:C3, and the adjustment weight matrix was in E1:G3, you would enter:

\[ \text{RiskCorrectCorrmat(A1:C3,E1:G3)} \]

The corrected coefficients for the matrix will be returned to the range. The RiskCorrectCorrmat function will update the corrected matrix anytime you change a coefficient in the matrix or a weight in the Adjustment Weight matrix.
When you enter a correlation matrix in the Define Correlations window, and click OK, the following events take place:

1) The matrix is added to the specified location in Excel.
2) Optionally, any specified Adjustment Weights may be placed in an Adjustment Weight matrix in Excel.
3) RiskCorrmat functions are added to each of the input distribution functions that are included in the matrix. The RiskCorrmat function is added as an argument to the distribution function itself, such as:

   =RiskNormal(200000, 30000,RiskCorrmat(NewMatrix,2))

   where NewMatrix is the range name for this matrix and the 2 is the position of the distribution function in the matrix.

After the matrix and RiskCorrmat functions are added to Excel, you can change the coefficient values in your matrix (and weights in the Adjustment Weight matrix) without editing the matrix in the Define Correlations window. New inputs, however, cannot be added to the matrix displayed in Excel, unless you individually add the necessary RiskCorrmat functions in Excel. To add new inputs to a matrix, it may be easier to edit the matrix in the Define Correlations window.

Correlations between input distributions may also be entered directly in your worksheet using the RiskCorrmat function. The correlations specified, using this function, are identical to those entered from the Define Correlations window. You may also enter an Adjustment Weight matrix directly in your worksheet. If you do this, remember to specify a range name for the correlation matrix, and then use the same range name with the extension _Weights for the Adjustment Weight matrix. If it is necessary for @RISK to correct the correlation matrix at the start of a simulation, it will use the entered Adjustment Weight matrix while correcting.

For more information on using these functions to enter correlations, see the description of these functions in the Reference: @RISK Functions section of this chapter.
The correlation of input distributions in @RISK is based on the rank order correlations. The rank order correlation coefficient was developed by C. Spearman in the early 1900's. It is calculated using rankings of values, not actual values themselves, (as is the linear correlation coefficient). A values “rank” is determined by its position within the min-max range of possible values for the variable.

@RISK generates rank-correlated pairs of sampled values in a two step process. First, a set of randomly distributed “rank scores” is generated for each variable. If 100 iterations are to be run, for example, 100 scores are generated for each variable. (Rank scores are simply values of varying magnitude between a minimum and maximum. @RISK uses van der Waerden scores based on the inverse function of the normal distribution). These rank scores are then rearranged to give pairs of scores, which generate the desired rank correlation coefficient. For each iteration there is a pair of scores, with one score for each variable.

In the second step, a set of random numbers (between 0 and 1) to be used in sampling is generated for each variable. Again, if 100 iterations are to be run, 100 random numbers are generated for each variable. These random numbers are then ranked smallest to largest.

For each variable, the smallest random number is then used in the iteration with the smallest rank score; the second smallest random number is used in the iteration with the second smallest rank score, and so on. This ordering, based on ranking, continues for all random numbers, up to the point where the largest random number is used in the iteration with the largest rank score.

In @RISK this process of rearranging random numbers happens prior to simulation. It results in a set of paired random numbers, that can be used in sampling values, from the correlated distributions in each iteration of the simulation.

This method of correlation is known as a “distribution-free” approach because any distribution types may be correlated. Although the samples drawn for the two distributions are correlated, the integrity of the original distributions is maintained. The resulting samples for each distribution reflect the input distribution function from which they were drawn.
Show Model Window Command
Displays all input distributions and output cells in the @RISK — Model window

The Show Model Window command displays the @RISK — Model window. This window provides a complete table of all input probability distributions and simulation outputs described in your model. From this window, which pops up over Excel, you can:

- Edit any input distribution, or output, by simply typing in the table
- Drag and drop any thumbnail graph to expand it to a full window
- Quickly view thumbnail graphs of all defined inputs.
- Double-click on any entry in the table to use the Graph Navigator to move through cells in your workbook with input distributions
- Edit and preview correlation matrices.
The Model window is “linked” to your worksheets in Excel. As you click on an input in the table, the cells where the input and its name are located are highlighted in Excel. If you double-click on an input in the table, the graph of the input will be displayed in Excel, linked to the cell where it is located.
The commands for the Model window may be accessed by clicking the icons, displayed at the bottom of the table, or by right-clicking and selecting from the pop-up menu. Selected commands will be performed on the current selected rows in the table.

The Outputs and Inputs table displayed in the @RISK — Model window is set up automatically, when you display the window. When the window is displayed, your worksheets are scanned or re-scanned for @RISK functions.

If a name is not entered in a RiskOutput function, or in a distribution function, @RISK will automatically try to create a name. These names are created by scanning the spreadsheet around the cell where the input or output is located. To identify names, @RISK moves from the input or output cell across the row of the spreadsheet, to the left and up the column, towards the top. It moves across these ranges of the spreadsheet until it finds a label cell, or a cell without a formula in it. It then takes these row and column “headings” and combines them to create a possible name for the input or output.

How Are Variable Names Generated?
Model Window — Inputs Tab

Lists all distribution functions in open workbooks in Excel

The Inputs tab in the Model window lists all distribution functions in your model. By default, the table shows for each input:

- **Name**, or the name of the input. To change the name of the input, simply type a new name in the table, or click the Reference entry icon to select a cell in Excel where the name you wish to use is located.

- **Cell**, where the distribution is located.

- **A Thumbnail Graph**, showing a graph of the distribution. To expand a graph into a full window, simply drag the thumbnail off of the table, and it will open up in a full graph window.

- **Function**, or the actual distribution function entered in the Excel formula. You may edit this function directly in the table.

- **Min, Mean and Max**, or the range of values described by the entered input distribution.
Columns Displayed in the Model Window

The Model window columns can be customized to select which statistics you want to display on the input distributions in your model. The Select Columns for Table icon at the bottom of the window displays the Columns for Table dialog.

If you select to show Percentile values in the table, the actual percentile is entered in the rows Value at Entered Percentile.
Editable p1,x1 and p2,x2 values are columns that can be edited directly in the table. Using these columns you can enter specific target values and/or target probabilities directly in the table.
Inputs in the Model Window can be grouped by category. By default, a category is made when a group of inputs share the same row (or column) name. In addition, inputs can be placed in any category you wish. Each category of inputs can be expanded, or collapsed, by clicking on the − or + sign in the Category header.

The Arrange icon at the bottom of the Model window allows you to turn category grouping on and off, change the type of default categories used, create new categories and move inputs between categories. The property function RiskCategory is used to specify the category for an input (when it is not located in the default category identified by @RISK).
The commands on the Arrange menu include:

- **Group Inputs By Category.** This command specifies whether or not the table of inputs will be arranged by category. When Group Inputs By Category is checked, categories entered using a RiskCategory function will always be shown. Default categories will also be shown if the Default Categories command Row Heading or Column Heading option is selected.

- **Default Categories.** This command specifies how @RISK will automatically generate category names from input names. Default category names are easily created from the default input names greeted by @RISK. The section of this manual How Are Default Names Created? describes how default names are generated for an input using a Row Heading and a Column heading in your spreadsheet. The Row Heading portion of a default name is shown to the left of the “/” separator in the default name, and the Column heading portion to the right of the separator. The Default Categories options are as follows:
  - **Row Heading** specifies that names, which use a common Row Heading, will be grouped together in a category.
  - **Column Heading** specifies that names, which use a common Column Heading, will be grouped together in a category.

Default Categories can also be created from input names entered using a RiskName function, as long as a “/” separator is included, to separate text to use as row or column “headings” in the name. For example, the input:

\[ \text{=RiskNormal(100,10,RiskName("R&D Costs / 2010"))} \]

would be included in a default category named “R&D Costs”, if the Default Categories Row Heading command was checked, and would be included in a default category named “2010” if the Default Categories Column Heading command was checked.
- **Assign Input to Category Command.** This command places an input, or set of inputs, into a category. The *Input Categories* dialog allows you to create a new category, or select a previously created category, in which to place the selected inputs.

When an input is assigned to a category by you, the input category is defined in an @RISK function using the `RiskCategory` property function. For more information on this function, see the *Listing of Property Functions* in the Function Reference of this manual.
The @RISK — Model window can be copied to the clipboard or exported to Excel using the commands on the Edit menu. In addition, where appropriate, values in the table can be filled down or copied and pasted. This allows you to quickly copy an @RISK distribution function across multiple inputs, or copy editable P1 and X1 values.

Commands on the Edit menu include:

- **Copy Selection.** Copies the current selection in the table to the clipboard.
- **Paste, Fill Down.** Pastes or fills values into the current selection in the table.
- **Report in Excel.** Generates the table in a new worksheet in Excel.
Graph Menu

The Graph menu is accessed by 1), clicking the Graph icon at the bottom of the model window, or 2) right-clicking in the table. Shown commands will be performed on the selected rows in the table. This allows you to quickly make graphs of multiple input distributions in your model. Simply select the type of graph you wish to display. The command Automatic creates the graph using the default type (probability density) for input distributions.
Model Window — Outputs Tab

Lists all output cells in open workbooks in Excel

The Outputs tab in the Model window lists all outputs in your model. These are cells where RiskOutput functions are located. For each output, the table shows:

- **Name**, or the name of the output. To change the name of the output, simply type a new name in the table, or click the Reference Entry icon to select a cell in Excel, where the name you wish to use, is located.
- **Cell**, where the output is located.
- **Function**, or the actual RiskOutput function entered in the Excel formula. You may edit this function directly in the table.

The properties of each output may be entered by clicking the fx icon shown on each row. For more on properties for outputs, see the Add Output command in this chapter.
Model Window — Correlations Tab

Lists all correlation matrices in open workbooks, along with all input distributions included in them

The Correlations tab in the Model Window lists all correlation matrices in open workbooks, along with any defined correlation matrix instances for those matrices. Each input distribution contained in each matrix and instance is shown.

Inputs can be edited in the Correlations tab, just as they can be in the Inputs tab.

The correlation matrix used for any input can be edited by:

- clicking on the **Correlation Matrix** icon shown next to the Function column
- right-clicking on the input, in the Correlations tab or Inputs tab, and selecting the **Correlate Edit Matrix** command
- selecting the cell in Excel, where the input distribution (or a cell in the matrix) is located, and selecting the **Define Correlations** command

For more information on correlation, see the **Define Correlations** command in this Reference chapter.
Distribution Fitting Commands

Fit Distributions to Data Command

Fits probability distributions to data in Excel and displays the results.

The Model command **Fit Distributions to Data** (also invoked by clicking the **Fit Distributions to Data** icon) fits probability distributions to the data in a selected Excel range. This command is only available in @RISK Professional and Industrial versions.

In some cases an input distribution is selected by fitting probability distributions to a set of data. You may have a set of sample data for an input, and you wish to find the probability distribution that best describes that data. The **Fit Distributions to Data** dialog has all the commands necessary for fitting distributions to data. After fitting, the distribution may be placed in your model, as an @RISK distribution function, for use during simulations.

A distribution for a simulated result may also be used as the source of the data to be fit. To fit distributions to a simulated result, click the **Fit Distributions to Data** icon in the lower left of the graph window that displays the simulated distribution whose data you wish to use in the fit.
Data Tab — Fit Distributions to Data Command

Specifies the input data to be fitted, its type, domain, and any filtering to be applied to the data

The Data tab in the Fit Distributions to Data dialog specifies the source and type of input data entered, whether it represents a continuous or discrete distribution and whether it should be filtered in any way.

The Data Set options specify the source of the data to be fitted and its type. Options include:

- **Name**: Specifies a name for the fitted data set. This will be the name shown in the Fit Manager, and in any RiskFit functions which link a distribution function to the results from a fit.

- **Range**: Specifies a range in Excel that contains the data to be fitted.
The **Type** options specify the type of data that is to be fitted. Six different types of data can be entered:

- **Continuous Sample Data.** Specifies that the data is in the form of Sample (or observation) data, that are a set of values chosen from a population. The sample data is used to estimate the properties of that population. This data can be in a **column, row or a block of cells** in Excel.

- **Discrete Sample Data.** Specifies that the data is in the form of Sample (or observation) data that is discrete. With discrete data, the distribution described by the input data is discrete, and only integral values — and none in between — are possible. This data can be in a **column, row, or a block of cells** in Excel.

- **Discrete Sample Data (Counted Format).** Specifies that the data is in the form of Sample (or observation) data, that is discrete, and in Counted format. In this case, the input data will be in the form of $X, Count$ pairs, where $Count$ specifies the number of points that fall at value $X$. This data must be in **two columns** in Excel — with the $X$ values in the first column, and the $Count$ value in the corresponding cell, in the second column.

- **Density (X-Y) Points (Unnormalized).** Data for a Density curve are in the form of $[X, Y]$ pairs. The $Y$ value specifies the relative height (density) of the density curve at each $X$ value. Data values are used as specified. Typically, this option is used if $Y$ data is taken from a curve that has already been normalized. This data must be in **two columns** in Excel — with the $X$ values in the first column, and the $Y$ value in the corresponding cell, in the second column.

- **Density (X-Y) Points (Normalized).** Data for a Density curve are in the form of $[X, Y]$ pairs. The $Y$ value specifies the relative height (density) of the density curve at each $X$ value. Data values for the entered Density curve (in the form of $[X, Y]$ pairs) are normalized, so that the area under the density curve equals one. It is recommended that you select this option to improve fitting of density curve data. This data must be in **two columns** in Excel — with the $X$ values in the first column, and the $Y$ value in the corresponding cell, in the second column.
• **Cumulative (X-P) Points.** Data for a Cumulative Curve are in the form of \([X, p]\) pairs, where each pair has a \(X\) value, and a cumulative probability \(p\) that specifies the height (distribution) of the cumulative probability curve at the \(X\) value. A probability \(p\) represents the probability of a value occurring that is less than or equal to the corresponding \(X\) value. This data must be in **two columns** in Excel — with the \(X\) values in the first column, and the \(p\) value in the corresponding cell, in the second column.

• **Values are Dates.** This option specifies that you will be fitting date data and graphs and statistics will be displayed using dates. If @RISK detects dates in the referenced data set this option will be checked by default.

**Filter Options**

Filtering allows you to exclude unwanted values, outside an entered range, from your input data set. Filtering lets you specify outliers in your data that will be ignored during fitting. For example, you may wish to only analyze \(X\) values greater than zero. Or, you may wish to filter out tail values by only looking at data within a few standard deviations of the mean. Filtering options include:

• **None.** Specifies that the data will be fitted as entered.

• **Absolute.** Specifies a minimum \(X\)-value, a maximum \(X\)-value, or both, to define a range of valid data to be included in a fit. Values outside the entered range will be ignored. If just a minimum, or just a maximum, is entered for the range, the data will be filtered only below the entered minimum or above the entered maximum.

• **Relative.** Specifies that data outside the entered number of standard deviations from the mean will be filtered from the data set prior to fitting.
Distributions to Fit Tab — Fit Distributions to Data Command

Selects probability distributions to fit or specifies a predefined distribution to fit

The options on the Distributions to Fit tab in the Fit Distributions to Data dialog select the probability distributions to include in a fit. These options can also be used to specify predefined distributions, with preset parameter values to fit. Probability distributions, to be included in a fit, may also be selected by entering information on the lower and upper limits of the allowable distributions.

**Fitting Method** options control whether 1) a group of distribution types will be fitted or 2) a set of predefined distributions will be used. The selection for Fitting Method determines the other options that are displayed in the Distributions to Fit tab. The options available for Fitting Method include:

- **Parameter Estimation**, or find the parameters for the selected distribution types that best fit your data set.

- **Predefined Distributions**, or determine how the entered probability distributions (with preset parameter values) fit your data set.
When **Parameter Estimation** is selected as the fitting method, the following options are available in the **Distributions to Fit** tab:

- **List of Distribution Types.** Checking, or unchecking, a specific distribution type will include, or remove, that type from the fit to be performed. The list of distribution types that are displayed will change depending on the options selected for **Lower Limit** and **Upper Limit**. By default, some distribution types in the list are unchecked. This is because: 1) those types are specialized forms of an already checked distribution type (i.e., an Erlang distribution is a Gamma distribution with an integer shape parameter) and fitting them would be duplicative, or 2) they are a distribution type not normally used in fitting (such as a Student or ChiSq).

Each distribution type has different characteristics with respect to the range and limits of the data it can describe. Using the **Lower Limit** and **Upper Limit** options you can select the types of distributions to include, limit options that are set, based on your knowledge of the range of values, which could occur for the item that your input samples describe.

**Lower Limit** and **Upper Limit** options include:

- **Fixed Bound of.** Specifies a value that will fix the lower and/or upper limit of the fitted distribution to a specific value. Only specific distribution types, such as Triangular, have fixed lower and upper limits. Your entry for Fixed Bound will restrict a fit to certain types of distributions.

- **Bounded, but Unknown.** Specifies that the fitted distribution has a finite lower and/or upper limit, but you don’t know what the boundary value is.

- **Open (Extends to +/- infinity).** Specifies that the data described by the fitted distribution can possibly extend to any possible positive, or negative, value.

- **Unsure.** Specifies that you are not sure about the possible values that could occur, and thus the full range of distributions should be available for fitting.
When **Predefined Distributions** is selected as the fitting method, a set of predefined distributions are entered and only these predefined distributions will be tested during fitting.

Predefined distributions are specified using the following options:

- **Name.** Specifies the name you wish to give to a predefined distribution.
- **Function.** Specifies the predefined distribution in distribution function format.

Predefined distributions can be included, or excluded, from a fit by checking, or unchecking, their entry in the table.
Chi-Sq Binning Tab — Fit Distributions to Data Command

Defines the binning to be used in Chi-Sq goodness-of-fit tests

The Chi-Sq Binning tab in the Fit Distributions to Data dialog defines the number of bins, type of bins and custom binning to be used, for Chi-Sq goodness-of-fit tests. Bins are the groups that your input data are divided into, similar to the classes used to draw a histogram. Binning can affect the results of Chi-Sq tests and the fit results that are generated. By using the Chi-Sq Binning options you can ensure that the Chi-Sq test is using the bins that you deem appropriate. For more information on how the number of bins is used in a Chi-Sq test, see Chapter 6: Distribution Fitting.

Note: If you are unsure about the number or type of bins to use for a Chi-Square test, set “Number of Bins” to “Automatic” and set “Bin Arrangement” to “Equal Probabilities”.

![Chi-Sq Binning Tab](image)
Bin Arrangement options specify the style of the binning that will be performed or, alternatively, allow the entry of fully custom bins with user-entered minimum and maximum values. Options for Bin Arrangement include:

- **Equal Probabilities.** Specifies that bins will be made at equal probability intervals across the fitted distribution. This usually results in unequal length bins. For example, if ten bins are used, the first bin would extend from the minimum to the 10th percentile, the second from the 10th percentile to the 20th and so on. In this mode, @RISK will adjust the bin sizes, based on the fitted distribution, trying to make each bin contain an equal amount of probability. For continuous distributions this is straight-forward. For discrete distributions, however, @RISK will only be able to make the bins approximately equal.

- **Equal Intervals.** Specifies that bins will be of equal length across the input data set. Several options are available for entering equal interval bins across an input data set. Any, or all, of these options may be selected:
  
  1) **Automatic Minimum and Maximum Based on Input Data.** Specifies that the minimum and maximum of your data set will be used to calculate the minimum and maximum of equal interval bins. First and last bins, however, may be added based on the settings for *Extend First Bin* and *Extend Last Bin* options. If Automatic Minimum and Maximum Based on Input Data, is not selected, you can enter a specific Minimum and Maximum value where your bins will start and end. This allows you to enter a specific range where binning will be performed, without regard to the minimum and maximum values in your data set.

  2) **Extend First Bin from Minimum to -Infinity.** Specifies that the first bin used will stretch from the specified minimum to -Infinity. All other bins will be of equal length. In certain circumstances, this improves fitting for data sets with unknown lower bounds.

  3) **Extend Last Bin from Maximum to +Infinity.** Specifies that the last bin used will stretch from the specified maximum to +Infinity. All other bins will be of equal length. In certain circumstances, this improves fitting for data sets with unknown upper bounds.
• **Custom Bins.** There are times when you wish to have complete control over the bins that are used for Chi-Sq testing. For example, custom bins could be used when there is a natural grouping of collected sample data, and you want your Chi-Square bins to reflect that grouping. Entering custom bins allows you to enter a specific minimum-maximum range for each bin that is defined.

To enter custom bins:

1) Select **Custom** in Bin Arrangement.

2) Enter a value for the Bin Limit for each of your bins. As you enter subsequent values, the range for each bin will be automatically filled in.

**Number of Bins**

The **Number of Bins** options specifies a fixed number of bins or, alternatively, specifies that the number of bins will be automatically calculated for you.
Fit Results Window

Displays a list of fitted distributions along with graphs and statistics that describe each fit

The Fit Results window displays a list of fitted distributions and graphs that illustrate how the selected distribution fits your data, and statistics, on both the fitted distribution and the input data, and the results of the goodness-of-fit (GOF) tests on the fit.

![Fit Results Window Image]

Note: No goodness-of-fit test information is generated if the input data type is Density Points or Cumulative Points. In addition, only Comparison and Difference graphs are available for these data types.

Fit Ranking

The Fit Ranking list displays all distributions for which valid fit results were generated. These distributions are ranked, according to the goodness-of-fit test selected, with the Rank Fits icon at the top of the Fit Ranking table. Only distribution types selected, using the Distributions to Fit tab in the Fit Distributions to Data dialog, are tested when fitting.

The goodness-of-fit statistic tells you how probable it is that a given distribution function produced your data set. The goodness-of-fit statistic can be used for comparing the values to the goodness-of-fit of other distribution functions. Goodness-of-Fit information is only available when the Input Data type is Sampled Values.

Clicking on a distribution, listed in the Fit Ranking list, displays the fit results for that distribution, including graphs and statistics on the selected fit.
The **Rank By** icon selects to rank distributions according to the selected goodness-of-fit test, which measures how well the sample data fits a hypothesized probability density function. Three types of tests are available:

- **Chi Sq** or Chi-Square test. The Chi-Square test is the most common goodness-of-fit test. It can be used with sample input data, and any type of distribution function (discrete or continuous). A weakness of the Chi-Square test is that there are no clear guidelines for selecting intervals or bins. In some situations, you can reach different conclusions from the same data, depending on how you specified the bins. The bins used in the Chi-Square test can be defined using the **Fit Distributions to Data** dialog and **Define Chi-Sq Binning** tab.

- **K-S**, or Kolmogorov-Smirnov test. The Kolmogorov-Smirnov test does not depend on the number of bins, which makes it more powerful than the Chi-Square test. This test can be used with sample input data but cannot be used with discrete distribution functions. A weakness of the Kolmogorov-Smirnov test is that it does not detect tail discrepancies very well.

- **A-D**, or Anderson-Darling test. The Anderson-Darling test is very similar to the Kolmogorov-Smirnov test, but it places more emphasis on tail values. It does not depend on the number of intervals.

- **RMS Error**, or root mean squared error. If the input data type is a **Density Curve** or **Cumulative Curve** (as set using **Fit Distributions to Data** dialog **Data** tab), only the RMS Error test is used to fit distributions. For more information on the RMS Error test, see **Chapter 6: Distribution Fitting**.
To display the fit results, for different distributions in the Fitted distribution list at the same time, simply select multiple distributions in the Fit Ranking list while holding down the <Ctrl> key.
Fit Results — Graphs

When the input data type is Sampled Values, three graphs — **Comparison**, P-P and Q-Q — are available for any fit, selected by clicking in the Fitted Distribution list. If the input data type is **Density Curve** or **Cumulative Curve**, only the **Comparison** and **Difference** graphs are available.

For all graph types **delimiters** may be used to graphically set specific X-P values on the graph.

**Comparison Graph**

A Comparison Graph displays two curves — the input distribution, and the distribution created by the best fit analysis.

Two delimiters are available for a Comparison graph. These delimiters set the **Left X** and **Left P** values, along with the **Right X** and **Right P** values. Values returned by the delimiters are displayed in the probability bar above the graph.
The P-P (or Probability-Probability) graph plots the p-value of the fitted distribution vs. the p-value of the fitted result. If the fit is "good", the plot will be nearly linear.

A Q-Q (or Quantile-Quantile graph) plots the plot percentile values of the fitted distribution vs. percentile values of the input data. If the fit is "good", the plot will be nearly linear.
Write to Cell Command — Fit Results Window

Writes a fit result to an Excel cell as an @RISK distribution function

The Write to Cell button in the Fit Results window writes a fit result to an Excel cell as an @RISK distribution function.

![Write to Cell Dialog](image)

Options in the Write to Cell dialog include:

- **Select Distribution.** The distribution function to be written to Excel can be either **Best Fit Based on** (the best fitting distribution based on the selected test) or **By Name** (as specific fitted distribution in the list).

- **Link to Data.** The distribution function to be written to Excel can be automatically updated, when the input data in the referenced data range in Excel changes and a new simulation is run. If **Update and Refit at the Start of Each Simulation** is selected, a new fit will be run when @RISK starts a simulation and detects that the data has changed. Linking is done with a RiskFit property function, such as:

  \[
  \text{RiskNormal}(2.5, 1, \text{RiskFit("Price Data", "Best A-D")})
  \]

This specifies that the distribution is linked to the best fitting distribution from the Anderson-Darling test, for the data associated with the fit named “Price Data”. Currently, this distribution is a Normal distribution with a mean of 2.5 and a standard deviation of 1.
The RiskFit property function is automatically added to the function, written to Excel, when **Update and Refit at the Start of Each Simulation** is selected. If no RiskFit function is used in the distribution function for a fit result, the distribution will be “unlinked” from the data which was fitted to select it. If the data is later changed the distribution will remain as is.

For more information on the RiskFit property function, see the @RISK Property Function Reference chapter of this Manual.

- **Function to Add.** This displays the actual @RISK distribution function that will be added to Excel when the Write button is clicked.
Fit Summary Window
Displays a summary of calculated statistics and test results for all distributions fit

The Fit Summary window displays a summary of calculated statistics and test results for all distributions fit to the current data set.

The following entries are shown in the Fit Summary window:

- **Function**, or distribution and arguments for the fitted distribution. When a fit is used as an input to an @RISK model, this formula corresponds with the distribution function that will be placed in your spreadsheet.

- **Distribution Statistics (Minimum, Maximum, Mean, etc.)**. These entries display the statistics calculated for all fitted distributions and the distribution of the input data.

- **Percentiles** identify the probability of achieving a specific outcome or the value associated with any probability level.
For each of the three tests performed (Chi Sq, A-D and K-S) the Fit Summary window displays:

- **Test Value**, or the test statistic for the fitted probability distribution for each of the three tests.

- **P-Value**, or observed level of the significance of the fit. For more information on P-values, see Chapter 6: Distribution Fitting.

- **Rank**, or the rank of the fitted distribution among all distributions fit for each of the three tests. Depending on the test, the returned rank can change.

- **C. Value**, or critical values, at different significance levels for each of the three tests. For more information on critical values and their calculation, see Chapter 6: Distribution Fitting.

- **Bin statistics** for each bin, for both the input and the fitted distribution (Chi-Sq test only). These entries return the min and max of each bin, plus the probability value for the bin, for both the input and the fitted distribution. Bin sizes can be set using the Chi-Sq Binning tab in the Fit Distributions to Data dialog.
Fit Manager Command

Displays a list of fitted data sets in the current workbook for editing and deleting

The Model command Fit Manager (also invoked by clicking the Fit Distributions to Data icon) displays a list of the fitted data sets in the open workbooks.

Fitted data sets, and their settings, are saved when you save your workbook. By selecting the Fit Manager command, you can navigate between fitted data sets and delete unneeded ones.
Distribution Artist Commands

Distribution Artist Command

Displays the Distribution Artist window where a curve to be used as a probability distribution can be drawn

The Model command Distribution Artist is used to draw freeform curves that can be used to create probability distributions. This is useful for graphically assessing probabilities and then creating probability distributions from the graph. Distributions may be drawn as Probability Density (General) curves, histograms, cumulative curves or discrete distributions.

After an Artist window has been displayed using the Distribution Artist command, a curve may be drawn simply by dragging the mouse through the window.

A curve in the Distribution Artist Window may be fitted to a probability distribution by clicking the Fit Distribution to Data icon. This fits the data represented by the curve to a probability distribution. A curve in the Distribution Artist Window may also be written to a cell in Excel as a RiskGeneral, RiskHistogrm or RiskDiscrete distribution, where the actual points on the curve will be entered as arguments to the distribution.

If you select the Distribution Artist command and the active cell in Excel contains a distribution function, the Artist window will display a probability density graph of that function with points you can adjust. You can also use this capability to review previously drawn curves that you wrote to a cell in Excel as a RiskGeneral, RiskHistogrm or RiskDiscrete distribution.
The scaling and type of graph drawn in the Artist window are set using the **Distribution Artist Options** dialog. This is displayed by clicking the **Draw New Curve icon** (in the lower left of the window) or by right clicking on the graph and selecting the **Draw New Curve** command.

Distribution Artist Options include:

- **Name**. This is the default name given to the selected cell by @RISK, or the name of the distribution used to create the displayed curve as given in its RiskName property function.

- **Distribution Format**. Specifies the type of curve that will be created, where **Probability Density (General)** is a probability density curve with x-y points, **Probability Density (Histogram)** is a density curve with histogram bars, **Cumulative Ascending** is an ascending cumulative curve, **Cumulative Descending** is a descending cumulative curve, and **Discrete Probability** is a curve with discrete probabilities.

- **Date Formatting**. Specifies that dates will be used for X-axis values.

- **Minimum and Maximum**. Specifies the X-axis scaling for the drawn graph.

- **Number of Points or Bars**. Sets the number of points or bars that will be drawn as you drag across the min-max range of the graph. You can drag the points on the curve or move the bars on a histogram up and down to change a curves’ shape.

If you are drawing an ascending cumulative distribution (as specified in the Distribution Format option), you will only be able to draw a curve with ascending Y values, and vice-versa for a descending cumulative curve.

When you have completed a curve, the end-points on your curve will be automatically plotted.
Some items to note about drawing curves using the Distribution Artist:

- After drawing a curve, you may want to "drag" one of the points to a new location. Simply click the left mouse button on the point and, while holding down the button, drag the point to a new location. When you lift the button, the curve is redrawn automatically to include the new data point.

- You can move data points along the X or Y-axis (except with a histogram).

- You can drag end points outside the axes by grabbing and dragging an endpoint.

- Move a dashed vertical endline to reposition the entire curve.

- By right-clicking on the curve, you can add new points or bars as necessary.

Icons in the Distribution Artist Window include:

- **Copy.** The Copy commands copy the selected data or the graph from the Artist window to the Clipboard. **Copy Data** copies X and Y data points for markers only. **Copy Graph** places a copy of the drawn graph in the clipboard.

- **Distribution Format.** Displays the current curve in one of the other available distribution formats.

- **Draw New Curve.** Clicking the Draw New Curve icon (the third from left on the bottom of the window) erases the active curve in the Artist window and begins a new curve.

- **Fit Distributions to Data.** The Fit Distributions to Data command fits a probability distribution to the drawn curve. When a drawn curve is fitted, the X and Y values associated with the curve are fit. The results of the fit are displayed in a standard Fit Results window where each of the fitted distributions can be reviewed. All the options that can be used when fitting distributions to data in an Excel worksheet are available when fitting probability distributions to a curve drawn in the Artist window. For more information on these options, see **Chapter 6: Distribution Fitting** in this manual.
Clicking OK creates a RiskGeneral, RiskHistogrm or RiskDiscrete distribution function from the drawn curve and places it in the selected cell. A General distribution is a user-defined @RISK distribution that takes a minimum value, a maximum value and a set of \( X,P \) data points which define the distribution. These data points are taken from the X and Y values for the markers on the drawn curve. A Histogrm distribution is a user-defined @RISK distribution that takes a minimum value, a maximum value and a set of \( P \) data points which define the probabilities for the histogram. A Discrete distribution is a user-defined @RISK distribution that takes a set of \( X,P \) data points. Only the specified X values can occur.
Settings Commands

Simulation Settings Command

Changes the settings which control the simulations performed by @RISK

The Simulation Settings command affects the tasks performed during a simulation. All settings come with default values, which you may change if you wish. The simulation settings affect the type of sampling @RISK performs, the updating of the worksheet display during simulation, the values returned by Excel in a standard recalculation, seeding of the random number generator used for sampling, the status of convergence monitoring and macro execution during simulation. All simulation settings are saved when you save your workbook in Excel.

To save simulation settings, so they will be used as the default settings each time you start @RISK, use the Utilities commands Application Settings command.

The @RISK Simulation Settings toolbar is added to the Excel 2003 and earlier. The same icons are present on the @RISK ribbon bar in Excel 2007. These icons allow access to many simulation settings.

The icons in this toolbar include:

- Simulation Settings opens the Simulation Settings dialog.
- Iterations / Simulations drop-down lists, where the number of iterations to run can be quickly changed from the toolbar.
- Random/Static Recalc flips @RISK between, returning expected or static values from distributions, to returning Monte Carlo samples in a standard Excel recalculation.
- Show Graph, Show Results Window, Demo Mode control what is shown on the screen during and after a simulation.
- Real Time Update controls if open windows will be updated while a simulation is running.
General Tab — Simulation Settings Command

Allows entry of the number of iterations and simulations which will be executed, and specifies the type of values returned by @RISK distributions in normal Excel recalculations.

The Simulation Runtime options include:

- **Number of Iterations.** Allows entry, or modification, of the number of iterations which will be executed during a simulation. Any positive integer value (up to 2,147,483,647) can be entered for Number of Iterations. The default value is 100. In each iteration:
  1) All distribution functions are sampled.
  2) Sampled values are returned to the cells and formulas of the worksheet.
  3) The worksheet is recalculated.
  4) New calculated values, in the cells of the selected output ranges, are saved for use in creating output distributions.
The number of iterations performed will affect both your simulation execution time and the quality and accuracy of your results. To get a quick look at results, run 100 iterations or less. For the most accurate results you will probably need to run 300 or 500 (or more) iterations. Use the Convergence Monitoring options (described in this section) to run the amount of iterations required for accurate and stable results. The **Automatic** setting allows @RISK to determine the number of iterations to run. It is used with Convergence Monitoring to stop the simulation when all output distributions have converged. See the Convergence Tab later in this section for more information on Convergence Monitoring.

The Excel Tools menu Options Calculation command Iterations option, is used for solving worksheets which contain circular references. You may simulate worksheets which use this option as @RISK will not interfere with the solution of circular references. @RISK allows Excel to “iterate” to solve circular references each iteration of a simulation.

**Important! A single recalc with sampling, done with the When a Simulation is Not Running, Distributions Return Random Values (Monte Carlo) option on, possibly will not resolve circular references. If an @RISK distribution function is located in a cell which is recalculated during an Excel iteration, it will be resampled each iteration of the single recalc. Because of this, the When a Simulation is Not Running, Distributions Return Random Values (Monte Carlo) option should not be used for worksheets which use Excel Iterations capabilities to solve circular references.**

- **Number of Simulations.** Allows entry or modification of the number of simulations which will be executed in an @RISK simulation. You can enter any positive integer value. The default value is 1. In each iteration of each simulation:
  1) All distribution functions are sampled.
  2) SIMTABLE functions return the argument corresponding to the number of the simulations being executed.
  3) The worksheet is recalculated.
  4) New calculated values in the cells of the selected output ranges are saved for use in creating output distributions.
The number of simulations requested should be less than, or equal to, the number of arguments entered into the SIMTABLE functions. If the number of simulations is greater than the number of arguments entered into a SIMTABLE function, the SIMTABLE function will return an error value during a simulation whose number is greater than the number of arguments.

For more information on Sensitivity Simulation and using the SIMTABLE function, see Chapter 5: @RISK Modeling Techniques.

**Important! Each simulation executed, when the # Simulations is greater than one, uses the same random number generator seed value. This isolates the differences between simulations to only the changes in the values returned by SIMTABLE functions. If you wish to override this setting, select Multiple Simulations Use Different Seed Values in the Random Number Generator section of the Sampling tab prior to running multiple simulations.**

- **Multiple CPU Support.** Instructs @RISK to use all CPUs present in your computer to speed simulations.  
  Note: This option is available only for users of @RISK Industrial running Windows NT 4.0 or higher.

**Naming Simulations**

If you run multiple simulations, you can enter a name for each simulation to be run. This name will be used to label results in reports and graphs. Set the Number of Simulations to a value greater than 1, click the **Simulation Names** button and enter the name desired for each simulation.
When a Simulation is Not Running, Distributions Return options control what is displayed when <F9> is pressed and a standard Excel recalculation is performed. Options include:

- **Random Values (Monte Carlo).** In this mode, distribution functions return a random Monte Carlo sample during a regular recalculation. This setting allows your worksheet values to appear as they would during execution of a simulation with new samples drawn for distribution functions each recalculation.

- **Static Values.** In this mode, distribution functions return Static values entered in a RiskStatic property function during a regular recalculation. If a static values is not defined for a distribution function, it will return:
  - **Expected Value,** or a distribution’s expected or mean value. For discrete distributions, the setting “Corrected” Expected Value will use the discrete value in the distribution closest to the true expected value as the swap value.
  - **‘True’ Expected Value** causes the same values to be swapped as the option “Corrected” Expected Value, except in the case of discrete distribution types, such as DISCRETE, POISSON and similar distributions. For these distributions the true expected value will be used as the swap value even if the expected value could not occur for the entered distribution, i.e., it is not one of the discrete points in the distribution.
  - **Mode,** or a distribution’s mode value.
  - **Percentile,** or the entered percentile value for each distribution.

The Random (Monte Carlo) vs. Static Values setting can be quickly changed using the Random/Static icon on the @RISK Settings toolbar.
View Tab — Simulation Settings Command

Specifies what is shown on screen during and after a simulation

The View settings control what will be shown by @RISK when a simulation is running and when a simulation ends.

The Automatic Results Display options include:

- **Show Output Graph.** In this mode a graph of simulation results, for the selected cell in Excel, automatically pops up:
  - When a run starts (if real-time results is enabled with Update Windows During Simulation Every XXX Seconds), or
  - When a simulation is over.

In addition, the Browse Results mode will be enabled at the end of a run. If the selected cell is not an @RISK output or input, a graph of the first output cell in your model will be displayed.

- **Show Results Summary Window.** This option pops up the Results Summary window when at run starts (if real-time results is enabled with Update Windows During Simulation Every XXX Seconds), or when a simulation is over.
- **Demo mode** is a preset view, where @RISK updates the workbook, each iteration, to show values changing, and pops up an updated graph of the first output in your model. This mode is useful for illustrating a simulation in @RISK.

- **None.** No new @RISK windows are displayed at the start, or end, of a simulation.

Settings under **Options** on the **View** tab of the Simulation Settings dialog include:

- **Minimize Excel at Start of Simulation.** Minimizes the Excel window, and all @RISK windows, at the start of a simulation. Any window can be viewed, during the run, by clicking it in the **Task bar**.

- **Update Windows During Simulation Every XXX Seconds.** Turns real time updating of open @RISK windows on and off, and sets the frequency with which windows are updated. When **Automatic** is selected, @RISK selects an update frequency, based on the number of iterations performed, and the runtime per iteration.

- **Show Excel Recalculations** toggles, on and off, the updating of the worksheet display during a simulation. For each iteration of a simulation, all distribution functions are sampled and the spreadsheet is recalculated. **Show Excel Recalculations** allows you to either display the results of each recalculation on the screen (box checked), or suppress the display (no check). The default is off, as updating the display for new values every iteration slows down the simulation.
- **Pause on Output Errors.** Toggles the Pause on Error capability on and off. Pause on Error causes a simulation to pause, if an error value is generated in any output. When an error is generated, the Pause on Error in Outputs dialog provides a detailed listing of the outputs for which errors were generated, during a simulation, and the cells in your spreadsheet that caused the error.

The Pause on Error in Outputs dialog shows, on the left, an explorer list containing each output for which an error was generated. A cell whose formula caused an error will be showed in the field on the right when you select an output with an error in the explorer list. @RISK identifies this cell, by searching through the list of precedent cells for the output with the error, until values switch from error to a non-error value. The last precedent cell(s) returning error prior to precedent cells returning non-error values, is identified as the “error causing” cell.

You can also review the formulas and values for cells, which are precedents to the “error causing” cell, by expanding the error-causing cell in the right-hand Explorer list. This allows you to examine values which feed into the problem formula. For example, a formula might return #VALUE because of a combination of values which are referenced by the formula. Looking at precedents to the error causing formula allows you to examine these referenced values.
- **Automatically Generate Reports at End of Simulation.**
  Selects Excel reports that will be automatically generated at the end of a simulation.

For more information on these available Excel reports, see the **Excel Reports** command.
Sampling Tab — Simulation Settings Command

Specifies how samples are drawn and saved during a simulation

Random Numbers settings include:

- **Sampling Type.** Sets the type of sampling used during an @RISK simulation. Sampling types vary in how they draw samples from across the range of a distribution. Latin Hypercube sampling will accurately recreate the probability distributions specified by distribution functions in fewer iterations, when compared with Monte Carlo sampling.

  We recommend using Latin Hypercube, the default sampling type setting, unless your modeling situation specifically calls for Monte Carlo sampling. The technical details on each sampling type are presented in the Technical Appendices.

  - **Latin Hypercube.** Selects stratified sampling
  - **Monte Carlo.** Selects standard Monte Carlo sampling
**Generator**

Generator selects any of eight different random number generators for use when simulating. There are eight random number generators (RNGs) in @RISK5:

- RAN3I
- MersenneTwister
- MRG32k3a
- MWC
- KISS
- LFIB4
- SWB
- KISS_SWB

Each of the available random number generators is described here:

1) **RAN3I.** This is the RNG used in @RISK 3 & 4. It is from Numerical Recipes, and is based on a portable “subtractive” random number generator of Knuth.

2) **Mersenne Twister.** This is the default generator in @RISK 5. For more information on its characteristics, see the web page http://www.math.sci.hiroshima-u.ac.jp/~m-mat/MT/emt.html.

3) **MRG32k3a.** This is a robust generator from Pierre L’Ecuyer. For more information on its characteristics, see http://www.iro.umontreal.ca/~lecuyer/myftp/papersstreams00s.pdf.

4) **KISS.** The KISS generator, (Keep It Simple Stupid), is designed to combine the two multiply-with-carry generators in MWC with the 3-shift register SHR3 and the congruential generator CONG, using addition and exclusive-or. Period about $2^{123}$.

5) **MWC.** The MWC generator concatenates two 16-bit multiply-with-carry generators, $x(n)=36969x(n-1)+\text{carry}$, $y(n)=18000y(n-1)+\text{carry} \mod 2^{16}$, has period about $2^{60}$ and seems to pass all tests of randomness. A favorite stand-alone generator---faster than KISS, which contains it.

6) **LFIB4.** LFIB4 is defined as a lagged Fibonacci generator: $x(n)=x(n-r) \text{ op } x(n-s)$, with the $x$'s in a finite set over which there is a binary operation $\text{op}$, such as $+,-$ on integers mod $2^{32}$, $\times$ on odd such integers, exclusive-or(xor) on binary vectors.
7) **SWB.** SWB is a subtract-with-borrow generator developed to give a simple method for producing extremely long periods:

\[ x(n) = x(n-222) - x(n-237) - \text{borrow mod } 2^{32} \]

The 'borrow' is 0, or set to 1 if computing \( x(n-1) \) caused overflow in 32-bit integer arithmetic. This generator has a very long period, \( 2^{7098}(2^{480}-1) \), about \( 2^{7578} \). It seems to pass all tests of randomness, except for the Birthday Spacings test, which it fails badly, as do all lagged Fibonacci generators using +/- or xor.

8) **KISS_SWB.** KISS+SWB has period >\( 2^{7700} \) and is highly recommended. Subtract-with-borrow (SWB) has the same local behaviour as lagged Fibonacci using +/-, xor- -- the borrow merely provides a much longer period. SWB fails the birthday spacings test, as do all lagged Fibonacci and other generators that merely combine two previous values by means of =,- or xor. Those failures are for a particular case: m=512 birthdays in a year of n=\( 2^{24} \) days. There are choices of m and n for which lags >1000 will also fail the test. A reasonable precaution is to always combine a 2-lag Fibonacci or SWB generator with another kind of generator, unless the generator uses *, for which a very satisfactory sequence of odd 32-bit integers results.

(MWC, KISS, LFIB4, SWB, AND KISS+SWB) are all from George Marsaglia at Florida State University. See http://www.lns.cornell.edu/spr/1999-01/msg0014148.html for his comments.
**Seed**

**Initial Seed.** The initial seed, for the random number generator, for the simulation as a whole, can be set to either:

- **Automatic** — have @RISK randomly pick a new seed each simulation.

- **A Fixed value you enter** — have @RISK use the same seed each simulation. When you enter a fixed non-zero seed value for the random number generator, the exact same sequence of random numbers will be repeated, simulation to simulation. Random numbers are used in drawing samples from distribution functions. The same random number will always return the same sampled value, from a given distribution function. The seed value must be an integer in the range 1 to 2147483647.

Setting a fixed seed value is useful when you wish to control the simulation sampling environment. For example, you may want to simulate the same model twice, only changing the argument values for one distribution function. By setting a fixed seed, the same values will be sampled, each iteration, from all distribution functions, except the one you changed. Thus, the differences in the results between the two runs will be directly caused by changing the argument values of the single distribution function.

- **Multiple Simulations.** Specifies the seed used when @RISK performs multiple simulations. Options include:

  - **All Use Same Seed** specifies that the same seed be used simulation to simulation, when @RISK performs multiple simulations in a single run. Thus, the same stream of random numbers will be used in each simulation, allows you to isolate simulation to simulation differences to the changes introduced by RiskSimtable functions.

  - **Use Different Seed Values** instructs @RISK to use a different seed, each simulation, in a multiple simulation run.

If a **Fixed** seed is used and the **Multiple Simulations — Different Seed Values** option is selected, each simulation will use a different seed, but the same sequence of seed values will be used each time the run is re-executed. Thus, results will be reproducible run to run.
Note: The Initial Seed set, in the Sampling tab only, affects the random numbers generated for input distributions that do not have an independent seed specified using the RiskSeed property function. Input distributions which use RiskSeed always have their own reproducible stream of random numbers.

Other Sampling Options

Other settings on the Sampling tab include:

- **Collect Distribution Samples.** Specifies how @RISK will collect random samples drawn from input distribution functions during a simulation. Options include:
  - **All.** Specifies that samples will be collected for all input distribution functions.
  - **Inputs Marked with Collect.** Specifies that samples will be collected only for those input distributions which the Collect property is selected; i.e., a RiskCollect property function is entered inside the distribution. Sensitivity and Scenario analyses will only include those distributions marked with Collect.
  - **None.** Specifies that no samples will be collected during a simulation. If no samples are collected, Sensitivity and Scenario analyses will not be available as simulation results. In addition, statistics will not be provided on samples drawn for input distribution functions. Deselecting sample collection, however, does allow simulations to run faster, and will sometimes allow large simulations with many outputs to be run on memory constrained systems.

- **Smart Sensitivity Analysis.** Enables or disables Smart Sensitivity Analysis. For more information on Smart Sensitivity Analysis and situations you may wish to disable it, see the Sensitivities Command.

- **Update Statistic Functions.** Specifies when @RISK’s statistics functions (such as RiskMean, RiskSkewness, etc) will be updated during a simulation. In most cases, statistics do not need to be updated until the end of a simulation when you wish to view final simulation statistics in Excel. However, if the calculations in your model require a new statistic to be returned each iteration (for example, when a custom convergence calculation has been entered using Excel formulas), the Each Iteration should be used.
Macros Tab — Simulation Settings Command

Allows the specification of an Excel macro to be executed before, during or after a simulation

The Run an Excel Macro options allows spreadsheet macros to be executed during an @RISK simulation. Options include:

- **Before Each Simulation.** The specified macro runs before each simulation begins.

- **Before Each Iteration’s Recalc.** The specified macro runs before @RISK has placed new sampled values into the spreadsheet model, and before Excel is recalculated with these values.

- **After Each Iteration’s Recalc.** The specified macro runs after @RISK performs its sampling and worksheet recalcul, but before @RISK stores values for outputs. The AfterRecalc macro can update values in @RISK output cells, and @RISK reports and calculations use those values and not the results of the Excel recal.

- **After Each Simulation.** The specified macro runs after each simulation ends.

Reference: @RISK Commands 313
Macros can be run at any, or all, of the possible times during a simulation. This feature allows calculations which can only be performed through the use of a macro to be made during a simulation. Examples of such macro-performed calculations are optimizations, iterative “looping” calculations and calculations which require new data from external sources. In addition, a macro may include @RISK distribution functions which are sampled during the execution of the macro. The entered Macro Name should be “fully qualified”, i.e., should contain the full address (including filename) of the macro to be run.

There are no restrictions on the operations performed by the macro each iteration. The user should, however, avoid macro commands which will do things such as, close the worksheet being simulated, quit Excel, or other similar functions.

@RISK includes an object oriented programming interface (API) that allows custom applications to be built using @RISK. This programming interface is described in the help file @RISK 5 for Excel Developer Kit help, accessed for the @RISK Help menu.
Convergence Tab — Simulation Settings Command

Define settings for monitoring convergence of simulation results

The Convergence tab settings specify how @RISK will monitor convergence during a simulation. Convergence monitoring shows how statistics on output distributions change as additional iterations are run during the simulation.

As numerous iterations are executed, the output distributions generated become more “stable”. Distributions become stable because the statistics which describe them change less, as additional iterations are performed. The number of iterations required, to generate stable output distributions, varies depending on the model being simulated and the distribution functions in the model.

By monitoring convergence you can ensure that you have run a sufficient, but not excessive, number of iterations. This is especially important with complex models that take a long time to recalculate.

Convergence monitoring does add to simulation runtime. If the fastest simulation is desired for a preset number of iterations, turn convergence monitoring off to maximize speed.
Convergence testing in @RISK can also be controlled for individual outputs using the RiskConvergence property function. Convergence testing done, by any RiskConvergence functions in your worksheet, is independent of the convergence testing specified on the Convergence tab. The RiskConvergenceLevel function returns the convergence level of an output cell it references. In addition, a simulation will be halted when any RiskStopRun function has passed an argument value of TRUE, regardless of the status of the convergence testing specified on the Convergence tab.

Default Convergence Options include:

- **Convergence Tolerance** — Specifies the tolerance allowed for the statistic you are testing. For example, the above settings specify that you wish to estimate the mean of each output simulated within 3% of its actual value.

- **Confidence Level** — Specifies the confidence level for your estimate. For example, the above settings specify that you want your estimate of the mean of each output simulated (within the entered tolerance) to be accurate 95% of the time.

- **Perform Tests on Simulated** — Specifies the statistics of each output that will be tested.

If the **Number of Iterations** entry, in the Simulation Settings dialog, is set to **Auto**, @RISK will automatically stop a simulation when convergence is achieved for all entered simulation outputs.

The Results Summary window reports convergence status when a simulation is running and Convergence Monitoring is enabled. The first column in the window displays the status for each output (as a value from 1 to 99) and displays OK when an output has converged.
Simulation Commands

Start Simulation Command

Starts a simulation

Clicking the Start Simulation icon starts a simulation using the current settings.

A Progress window is displayed during simulations. The icons in this window allow you to Run, Pause or Stop a simulation, as well as turn Update Graphs/Reports Real-time and Show Excel Recalculations on and off.

The Update Display option may be toggled on and off by pressing <Num Lock> during the simulation.
Clicking the arrow button at the bottom right of the Progress window displays the **Performance Monitor**. This monitor shows additional information on the status of each CPU used during a run.

Simulation messages are also available. These messages show recommendations for increasing the speed of lengthy simulations.

All open @RISK windows will update during a simulation if the Simulation Setting **Update Windows During Simulation Every XXX Seconds** is selected. Especially useful is updating the @RISK — Results Summary window. The small thumbnail graphs, in this window, will update to show a “dashboard” summary of simulation progress.
Simulation — Advanced Analyses Commands

@RISK Professional and Industrial versions allow you to perform Advanced Analyses on your model. The Advanced Analyses include Advanced Sensitivity Analysis, Stress Analysis, and Goal Seek. These Advanced Analyses can be used to design your model, verify your model, or get many “What if?” kinds of results.

Each of the Advanced Analyses generates its own sets of reports in Excel to show the results of the analysis being run. However, each of the analyses uses standard @RISK multiple simulations to generate its results. Because of this, the @RISK — Results Summary window may also be used to review analysis results. This is useful when you wish to generate a graph of results that is not included in the Excel reports, or when you wish to review analysis data in more detail.

Simulation Settings in Advanced Analyses

Simulation settings, specified in the @RISK Simulation Settings dialog (except for # of Simulations) are those used in each of @RISK’s Advanced Analyses. As many Advanced Analyses can involve large numbers of simulations, you should review your simulation settings to insure that analysis runtimes are minimized. For example, when testing the setup of an advanced analysis you should set the Number of Iterations to a relatively low value until you have verified that your setup is correct. Then, set the Number of Iterations back to the level necessary for stable simulation results and run the full Advanced Sensitivity Analysis, Stress Analysis, or Goal Seek.
Goal Seek

Goal Seek Command

Sets up and runs an @RISK Goal Seek

Goal Seek allows you to find a specific simulated statistic for a cell (for example, the mean or standard deviation) by adjusting the value of another cell. The setup of an @RISK Goal Seek is very similar to the standard Excel’s Goal Seek. Unlike Excel’s Goal Seek, however, @RISK’s Goal Seek utilizes multiple simulations to find the adjustable cell value that achieves your results.

When you know the desired statistic value for an output, but not the input value needed to obtain that value, you can use the Goal Seek feature. An input can be any cell in your Excel workbook. An output is any cell that is an @RISK simulation output (i.e., a cell containing a RiskOutput() function). The input should be a precedent of the output cell being targeted. When goal seeking, @RISK varies the value in the input cell and runs a full simulation. This process is repeated until the desired simulation statistic for the output equals the result you want.

Goal Seek is invoked by selecting the Goal Seek command from the Advanced Analyses icon on the @RISK toolbar.

Reference: @RISK Commands
Goal Seek Dialog — Goal Seek Command

Sets the goal and changing cell for a Goal Seek

The options available in the @RISK Goal Seek dialog are as follows:

Goal options describe the goal you are trying to achieve:

- **Cell** — Identifies the cell reference for the output whose simulation statistic you are trying to set to the entered value. This cell must be an @RISK output cell. If the cell does not contain a RiskOutput() function, you will be prompted to add a RiskOutput(). Clicking the ... selection button next to the Cell entry displays a list of current outputs which you can select from:
• **Statistic** — Allows you to choose which output statistic to monitor for convergence on the goal. The list includes: Minimum, Maximum, Kurtosis, Mean, Mode, Median, 5th Percentile, 95th Percentile, Skewness, Standard Deviation, and Variance.

• **Value** — Specifies the value you want the **Statistic** for the value in **Cell** to converge on. This value is called a goal.

The **By Changing** option identifies the single cell that you want Goal Seek to change, so that the **Goal** options **Statistic** for **Cell** approximates the **Value**. The **Cell** must be dependent on the **By Changing** cell — if it is not, Goal Seek will not be able to find a solution.
Goal Seek Options Dialog — Goal Seek Command

Sets the analysis options for a Goal Seek

The Options dialog of Goal Seek allows you to set parameters that can affect the success and quality of the Goal Seek solution. The Options dialog is invoked by clicking the Options button in the Goal Seek dialog.

The Change Limits options include:

- **Minimum** — Allows you to set the minimum value to use for the By Changing Cell. Goal Seek attempts to bracket a solution by assuming that there is one between the changing cell Minimum and changing cell Maximum.

- **Maximum** — Allows you to set the maximum value to use for the By Changing Cell. Goal Seek attempts to bracket a solution by assuming that there is one between the changing cell Minimum and changing cell Maximum.
• **Comparison Accuracy** — Determines how close the actual solution must be to the target. This entry can be viewed as a range, around the desired target value, that is acceptable for the simulation statistic. Any result within this range is defined as achieving the goal.

1) **Percent of Target Value** — Specifies the accuracy as a percentage of the *To Value*.

2) **+- Actual Value** — Specifies the accuracy as the maximum difference between the goal and the value of the *Cell* statistic found by Goal Seek.

• **Maximum Number of Simulations** — Specifies how many simulations Goal Seek will attempt, while trying to meet your goal. If a solution is found, before all simulations are completed, simulation activity will halt and the Goal Seek Status dialog will be displayed.

• **Generate Complete Simulation Results for Solution** — If this is selected, after finding a solution, Goal Seek performs an additional simulation that uses the found value for the *Changing* cell. The statistics for that simulation are displayed in the @RISK — Results Summary window. This option does not actually replace the original value of the *Changing* cell with the found value in the spreadsheet. Instead, it enables you to see the effects such a replacement would have without performing it.
Analyze — Goal Seek Command

Runs a Goal Seek

Once Analyze is clicked, Goal Seek cycles through the following process until the goal statistic value is achieved, the maximum number of simulations are run, or:

1) A new value is placed in the changing input cell
2) A full simulation of all open workbooks is run using the current settings, as specified in the @RISK Simulation Settings dialog box
3) @RISK records the simulation statistic, selected in the Statistic entry for the output identified in the Cell entry. This statistic value is compared to the Value entry to see if the value achieves the goal (within the Compare Accuracy range entered)

If a solution is found within the requested accuracy, Goal Seek will display a Status dialog. This will allow you to replace the contents of the Changing Cell with the solution value. If you choose to do this, the entire cell contents will be replaced by the solution value, and any formula, or values, that were previously in that cell will be lost.

It is possible that Goal Seek will converge on a goal, but will not be able to converge within the requested accuracy. In this case Goal Seek will prompt you with its best solution.

An @RISK Goal Seek uses a two-tiered approach to converging on the target:

1) If no brackets are set using Changing Cell Minimum and Maximum, Goal Seek will attempt to bracket the target value using a geometric expansion around the original value.

2) Once a solution is bracketed, Goal Seek uses the Ridders' method of root finding. Using the Ridders' method, Goal Seek first simulates the model with the input value set to the
midpoint of the bracketed range. It then factors out that unique exponential function, which turns the residual function, into a straight line. This has some important benefits for the Goal Seek process. It insures that the tested input values never jump outside the brackets, and helps insure that Goal Seek moves toward a solution in as few cycles as possible (an important benefit when each “cycle” is a full simulation of your model).

It is possible that Goal Seek may have problems converging on a solution. Some desired solutions may just be impossible to achieve, or the model may behave so unpredictably, that the root finding algorithm cannot converge on a solution. You can help Goal Seek converge by:

• **Starting Goal Seek with a Different Value in the Changing Cell.** Because the iteration process begins with guesses around the original changing cell value, starting Goal Seek with a different value in the cell being changed may help.

• **Changing Your Brackets.** Setting the Changing Cell Minimum and Changing Cell Maximum in the Options dialog will help point Goal Seek toward a solution.

**Note:** Goal Seek is not designed to work with multiple simulation models. For RiskSimtable functions, the first value in the table will be used for all simulations.
Stress Analysis

Stress Analysis Command

Sets up and runs a Stress Analysis

Stress Analysis allows you to analyze the effects of stressing @RISK distributions. Stressing a distribution restricts samples drawn from the distribution, to values between a specified pair of percentiles. Alternatively, stressing can be done by specifying a new “stress” distribution that will be sampled, instead of the original distribution in your model. With Stress Analysis you can select a number of @RISK distributions, and run simulations while stressing those distributions jointly in one simulation, or separately in multiple simulations. By stressing the selected distributions, you can analyze scenarios without changing your model.

After completing a simulation, Stress Analysis provides you with a collection of reports and graphs that you can use to analyze the effects of stressing certain distributions on a selected model output.

Stress Analysis is invoked by clicking the Stress Analysis command, from the Advanced Analyses on the @RISK toolbar.
Stress Analysis Dialog — Stress Analysis Command

Sets the cell to monitor and lists inputs for a Stress Analysis

The Stress Analysis dialog is used to enter the cell to monitor in the Stress Analysis, along with summarizing the inputs to be included and starting the analysis.

The options in the Stress Analysis dialog are as follows:

- **Cell To Monitor** — This is a single @RISK output that you want to monitor as the specified @RISK distributions are stressed. The Cell To Monitor can be specified by entering a cell reference, clicking the desired cell, or clicking the ... button. This button displays a dialog that contains a list of all @RISK outputs in currently open Excel workbooks. Clicking the ... selection button next to the Cell to Monitor entry displays a list of current outputs which you can select from:
The **Inputs** section allows you to **Add**, **Edit**, and **Delete** the @RISK Distributions that you wish to stress. The specified distributions are maintained in a list that contains the cell range, the @RISK Name, the Current distribution, and an Analysis Name that you can edit.

- **Add and Edit** — Display the **Input Definition** dialog. This allows you to specify an @RISK distribution, or range of @RISK Distributions to be stressed. You can then select from **Low**, **High**, or **Custom** sampling ranges, or specify an alternate stress distribution or formula.

- **Delete** — Completely removes the @RISK distribution(s) that are highlighted on the list from the Stress Analysis. To temporarily exclude a distribution, or a range of distributions from the analysis without deleting them, click the check box beside its list item to remove the checkmark.
Input Definition Dialog — Stress Analysis Command

Defines inputs for a Stress Analysis

The Input Definition dialog is used to enter how a specific input to a stress analysis will be changed.

![Input Definition Dialog](image)

The options in the Input Definition dialog are as follows:

- **Type** — For Stress Analysis, only @RISK distributions can be selected as inputs, so the only option for Type is **Distributions**.

- **Reference** — Selects the distributions to stress. Distributions can be specified by typing appropriate cell references and selecting a range of cells in the worksheet, or by clicking the ... button, which will open the @RISK Distribution Functions dialog, listing all the distributions in the model.
The Variation Method options allow you to enter a range, within the selected probability distribution(s) to sample from, or enter an alternate distribution, or formula, to substitute for the selected probability distribution(s) during the analysis.

- **Stress Low Values** — Enters a low range to sample from, bounded at the minimum by the distribution minimum. The Low Range default is 0% to 5%, sampling only values below the 5th percentile of the distribution. Any desired upper percentile can be entered instead of 5%.

- **Stress High Values** — Enters a high range to sample from, bounded at the maximum by the distribution maximum. The High Range default is 95% to 100%, sampling only values above the 95th percentile. Any desired lower percentile can be entered instead of 95%.

- **Stress Custom Range of Values** — Allows you to specify any percentile range within the distribution to sample from.

- **Substitute Function or Distribution** — Allows you to enter an alternate @RISK distribution function (or any valid Excel formula) that will be substituted for the selected distribution during a Stress Analysis. You can use the Excel Function Wizard to help enter an alternate distribution, by clicking the icon to the right of the Distribution/Formula box.
Stress Options Dialog — Stress Analysis Command

Sets the analysis options for a Stress Analysis

The Options dialog is used to determine how stressing will be performed and which reports or graphs to generate. The Options dialog is displayed when the Options button in the Stress Analysis dialog is clicked.

The **Multiple Inputs** section allows you to stress all of your specified @RISK distributions during one simulation, or to run a separate simulation for each @RISK distribution.

- **Stress Each Input in its Own Simulation** — Specifies that a full simulation will be run for each stress range entered. The only change made to the model, during each simulation, will be the stressing of a single input. The number of simulations run will equal the number of stress ranges entered.

- **Stress All Inputs in a Single Simulation** — Specifies that a single simulation will be run using all stress ranges entered. The simulation results will combine the effects of all stress ranges.
The **Reports** section allows you to choose which reports and graphs you want to be generated at the end of the stress simulations. The options include a **Summary** report, **Box-Whisker Plots**, **Comparison Graphs**, **Histograms**, **Cumulative Distribution Functions** and **Quick Reports**. For more information on the reports generated by a Stress Analysis, see **Reports** in this section.

The **Place Reports in** section allows you to place your results in the active workbook, or a new workbook.

- **New Workbook** — All reports are placed in a new workbook
- **Active Workbook** — All reports are placed in the active workbook with your model
Analyze — Stress Analysis Command

Runs a Stress Analysis

Once you have selected the **Cell to Monitor**, and at least one @RISK distribution is specified to stress, you can click the **Analyze** button to run the analysis. The analysis runs one or more simulation(s) that restricts sampling of the selected @RISK distributions to the specified stress range(s), or substitutes in any alternate stress distributions, or formulas, you have entered. The results of the Stress Analysis simulations are organized in a summary sheet and several Stress Analysis graphs.

The results of the Stress Analysis are also available in the @RISK — Results Summary Window. This allows you to further analyze the results of stressing the @RISK inputs.

The Reports generated by a stress analysis include:

- Summary report
- Box-Whisker Plots
- Comparison Graphs
- Histograms
- Cumulative Distribution Functions
- Quick Reports
The **Summary** reports describes the stressed inputs, and the corresponding statistics of the monitored output: Mean, Minimum, Maximum, Mode, Standard Deviation, Variance, Kurtosis, Skewness, 5th Percentile and 95th Percentile.

![Stress Analysis Summary](image)
The **Box-Whisker Plot** gives an overall indication of the monitored output, describing the mean, median, and outlying percentiles.

The left and right of the box are indicators of the first and third quartiles. The vertical line, inside the box represents the median, and the X indicates the location of the mean. The box width represents the interquartile range (IQR). The IQR is equal to the 75th percentile data point minus the 25th percentile data point. The horizontal lines, extending from either side of the box, indicate the first data point that is less than 1.5 times the IQR below the low edge of the box, and the last data point that is less than 1.5 times the IQR above the high edge of the box. Mild outliers, shown as hollow squares, are data points between 1.5 times IQR and 3.0 times IQR outside the box. Extreme outliers, shown as solid squares, are points beyond 3.0 times IQR outside the box.
**Quick Report** A Quick Report provides a page-sized summary of the Stress Analysis as a whole. This report is designed to fit on standard sized pages.
The four Comparison Graphs compare means, standard deviations, 5th percentiles, and 95th percentiles for each of the specified @RISK inputs (or their combination) and the Baseline simulation.
**Histogram**

The Histograms are standard @RISK histograms of the monitored output for each of the stressed inputs (or their combination), and the Baseline simulation.

![Histogram Graph](image)

**Cumulative Summary**

The CDFs (Cumulative Distribution Functions) are standard @RISK cumulative ascending density graphs. There is also a Summary CDF for all of the inputs.

![CDF Graph](image)
Advanced Sensitivity Analysis

Advanced Sensitivity Analysis Command

Sets up and runs an Advanced Sensitivity Analysis

Advanced Sensitivity Analysis allows you to determine the effects of inputs on @RISK outputs. An input can be either an @RISK distribution or a cell in your Excel workbook. Advanced Sensitivity Analysis allows you to select a number of @RISK distributions, or worksheet cells, and run trial simulations while varying these inputs across a range. Advanced Sensitivity Analysis runs a full simulation at each of a set of possible values for an input, tracking the simulation results at each value. The results then show how simulation results changed as the input value was changed. As with the standard @RISK Sensitivity Analysis, the Advanced Sensitivity Analysis shows the sensitivity of an @RISK output to the specified input.

Advanced Sensitivity Analysis can be used to test the sensitivity of an @RISK output to the input distributions in a model. When testing an @RISK distribution, @RISK runs a set of simulations for the input. In each simulation, the input distribution is fixed at a different value across the min-max range of the distribution. Typically these “step” values are different percentile values for the input distribution.

Advanced Sensitivity Analysis is invoked by selecting the Advanced Sensitivity Analysis command from the Advanced Analyses icon on the @RISK toolbar.

Reference: @RISK Commands
Advanced Sensitivity Analysis Dialog — Advanced Sensitivity Analysis Command

Sets the cell to monitor and lists inputs for an Advanced Sensitivity Analysis

![Advanced Sensitivity Analysis Dialog]

The options in the Advanced Sensitivity Analysis dialog are as follows:

- **Cell To Monitor** — This is a single @RISK output that you want to monitor, as individual simulations are run, while stepping across possible input values. The Cell To Monitor can be specified by entering a cell reference, clicking the desired cell, or clicking the ... button. This button displays a dialog that contains a list of all @RISK outputs currently open in Excel workbooks.

The Inputs options allow you to **Add**, **Edit**, and **Delete** the worksheet cells and @RISK distributions that you wish to include in the analysis. The specified cells, and distributions, are maintained in a list that contains the cell range, the @RISK Name, the Current distribution, and an Analysis Name that you can edit.

- **Add and Edit** — Displays the Input Definition dialog. This allows you to specify either a single @RISK distribution or worksheet cell, or a range of @RISK distributions or worksheet cells to be analyzed.

- **Delete** — Completely removes inputs from the Advanced Sensitivity Analysis. To temporarily exclude an input, or group of inputs from the analysis without deleting them, click the check box in the appropriate line in the list to remove the check mark from that input.
Input Definition — Advanced Sensitivity Analysis Command

Defines Inputs in an Advanced Sensitivity Analysis

The Input Definition dialog allows you to enter the type of an input, its name, a base value and data which describes the possible values for the input that you wish to test in the sensitivity analysis. A full simulation will be run at each value you enter for an input. The options in the Input Definition dialog are detailed in this section.

Options in the Input Definition dialog include:

- **Type.** Type specifies the type of input you are entering (either a distribution or a worksheet cell). Inputs to an Advanced Sensitivity Analysis can be either @RISK distributions that have been entered into your worksheet formulas or worksheet cells.
• **Reference.** Reference specifies the worksheet location of your input(s). If you are selecting distribution input(s) you can click the ... button, which will open the @RISK Distributions Functions dialog, listing all the distributions in all open worksheets.

![@RISK Distribution Functions](image)

• **Name.** Name names your input(s). If you are selecting distribution inputs, the existing @RISK name for each input is shown. If you wish to use a different name for a distribution, simply change the @RISK name by adding a RiskName function to the distribution in Excel, or by editing the name in the @RISK — Model window.

If you are selecting worksheet cells as inputs, the name of a single input can be typed directly in the Name entry. When you have selected a range of inputs, the Name entry shows the names of each cell, separated by commas.

![Input Definition](image)
These names can be edited by typing in the box (keeping the comma-separated format) or by clicking the ... button, which opens the **Sensitivity Analysis Cell Names** dialog.

Cell names are defined in the Input Definition dialog only for the purposes of Advanced Sensitivity Analysis. These names are used in @RISK Results Summary Window, and in the reports generated by Advanced Sensitivity Analysis. These cell names do not, however, become part of your Excel model.

- **Base Value.** The Base Value is used to determine the sequence of values to step through for an input, and as a reference point in the Percent Change report graph. The Base Value is especially important when you wish to apply a Variation Method that is a change from base, such as +/- Percent Change from Base. By default, the Base Value is the value a distribution or cell evaluates to when Excel recalculates the worksheet, but you can change it to a different value. Note: If your distribution or cell evaluates to 0, and Base Value is set to Auto, you need to enter a non-zero base value if you use the +/- Percent Change from Base option.
Variation

The Variation options describe the type of variation you will use to select the values that will be tested for your input(s). During an analysis, inputs are “stepped” across a range of possible values and a full simulation is run at each step value. Variation defines the nature of this range — either % Change from Base, Change from Base Value, Values Across Range, Percentiles of Distribution, Table of Values, or Table from Excel Range. These different Variation approaches provide a great deal of flexibility in describing the values to be tested for an input. Depending on the Variation method you select, the entry information, for defining the actual range and step values, (as shown below in the Input Definition dialog) will change.

Each Variation method, and its associated range and value entries, is described here.

- **% Change from Base Value.** With this Variation method, the first and the last value in the sequence to step through are obtained by incrementing, or decrementing, the input’s Base Value by the percentage values specified in Min Change (%) and Max Change (%) entries. The intermediate values are at equal intervals, with the number of values to test set by # of Steps.

- **Change from Base Value.** With this Variation method, the first and the last value in the sequence to step through, are obtained by adding to the Base Value the values specified in Min Change and Max Change entries. The intermediate values are at equal intervals, with the number of values to test set by # of Steps.
• **Values Across Range.** With this Variation method, the sequence of values to step through starts at the Minimum and ends at the Maximum. The intermediate values are at equal intervals, with the number of values to test is set by # of Steps.

![Values Across Range Variation Method](image)

• **Percentiles of Distribution.** This Variation method is only used when the selected input Type is Distribution. You specify steps as percentiles of the selected @RISK distribution, and you can define up to 20 steps. During the analysis, the input will be fixed at the percentile values as calculated from the entered input distribution.

![Percentiles of Distribution Variation Method](image)

• **Table of Values.** With this Variation method, you enter the sequence of values to step through, directly in a Table in the right portion of the Input Definition dialog. The Base Value is not used as the specific values you enter are the values tested.

![Table of Values Variation Method](image)
- **Table from Excel Range.** With this Variation method, the sequence of values to step through is found in the range of worksheet cells specified in *Excel Range* entry. This range can contain any number of values; however, it is important to remember that a full simulation will be run for each value in the referenced range.

By clicking the **Add Analysis Names** button, a descriptive name can be added to each input value that will be tested in an Advanced Sensitivity Analysis. This name will be used to identify the simulation run when an input is fixed at a particular value. These names will make your reports more readable and will help identify individual simulations, when results are reviewed in the @RISK — Results Summary window.

The **Sensitivity Analysis Names** dialog allows you to enter a name for the simulation to be run at each stepped input value. The default name @RISK has created is initially shown, and you can change this as desired.
Options — Advanced Sensitivity Analysis Command

Defines analysis options for an Advanced Sensitivity Analysis

The Sensitivity Options dialog allows you to select the output statistic you wish to evaluate during the sensitivity analysis, identify the reports you want to generate, and specify the behavior of @RISK Simtable functions in the analysis.

The Sensitivity Options dialog is invoked by clicking the Options button from the main Advanced Sensitivity Analysis dialog. The selections in this dialog include:

- **Tracking Statistic** — Allows you to specify the particular statistic that you wish to monitor for the @RISK output, during each simulation. The comparison graphs and reports from the analysis will show the change in the value for this statistic, simulation to simulation.

- **Reports** — Allows you to choose which analysis reports are generated at the end of the Sensitivity run. They include Summary, Box-Whisker Plot, Input Graphs, Quick Reports, Percentile Graphs, Percent Change Graphs, and Tornado Graphs. For more information on each of these reports, see Reports in this section.
The **Place Reports** section allows you to place your results in the active workbook, or a new workbook.

- **New Workbook** — All reports are placed in a new workbook
- **Active Workbook** — All reports are placed in the active workbook with your model

If a sensitivity analysis is run on worksheets which include **RiskSimtable** functions, this option causes the values specified by these functions to be included in the analysis. If **Include Simtable Functions as Inputs to Analyze** is selected, open workbooks will be scanned for RiskSimtable functions. The Advanced Sensitivity Analysis will then step through the values specified in the RiskSimtable function arguments, running a full simulation at each value. The reports generated after the run will show the sensitivity of the output statistic to both:

1) The variation of the inputs set up in the Advanced Sensitivity Analysis dialog and

2) The variation of the values from Simtable functions.

This option is especially useful if an Advanced Sensitivity Analysis is run on an @RISK model that was set up for multiple simulations. Simtable and @RISK's multiple simulation capability are often used to analyze how simulation results change when an input value is changed, by simulation, using the Simtable function. This analysis is similar to that performed by an Advanced Sensitivity Analysis. By simply selecting the option **Include Simtable Functions as Inputs to Analyze**, and running an Advanced Sensitivity Analysis, multiple simulation models can get the benefit of all Advanced Sensitivity Analysis reports and graphs with no additional setup.

For more information on the RiskSimtable function, see the section **Reference @RISK: Functions** in this manual.
**Analyze — Advanced Sensitivity Analysis Command**

**Runs an Advanced Sensitivity Analysis**

When the Analyze button is clicked, the Advanced Sensitivity Analysis prompts the user with the number of simulations, iterations per simulation, and total number of iterations. At this point, the analysis can be canceled.

When a smaller, faster analysis, is desired, the Cancel button gives the user an opportunity to change the # of Iterations per simulation in the Simulation Settings dialog, the number of Inputs to Analyze, or the number of values in the sequence associated with each input (that is, # of Steps or table items).

When an Advanced Sensitivity Analysis is run, the following actions occur for each input in the analysis:

1) A single step value for the input is substituted for the existing cell value, or @RISK distribution, in the worksheet.

2) A full simulation of the model is run.

3) The simulation results, for the tracked output Cell to Monitor, are collected and stored.

4) This process is repeated, until a simulation has been run, for each possible step value for the input.

The results of the Sensitivity Analysis are also available in the @RISK-Results Summary Window. You can analyze them further using tools available in this window.
**Reports**

The Advanced Sensitivity Analysis reports include:

- Summary
- Box-Whisker Plot
- Input Graphs
- Quick Reports
- Percentile Graph
- Percent Change Graph
- Tornado Graph

Each of these reports is generated in Excel, either in the workbook with your model, or in a new workbook. These reports are detailed in this section.

**Summary**

The Summary report describes the values assigned to the analyzed inputs, and the corresponding statistics of the monitored output: Mean, Minimum, Maximum, Mode, Median, Standard Deviation, Variance, Kurtosis, Skewness, 5th Percentile and 95th Percentile.
The **Input Graphs** report identifies how the tracked simulation statistic changed when simulations were run at each of the selected step values for an input. These graphs include:

- **Line Graph** — Plots the value of the tracked simulation statistic for the output against the value used for the input in each simulation. There is one point on the line graph for each simulation run when the Advanced Sensitivity Analysis was stepping across the particular input.

- **Overlaid Cumulative Distribution** — Shows the cumulative distribution for the output, in each simulation run at each step value for the input. There is one cumulative distribution for each simulation run, when the Advanced Sensitivity Analysis was stepping across the particular input.

- **Box-Whisker Plots** — Give an overall indication of the output distribution, in each simulation run for the input, describing the mean, median, and outlying percentiles. There is one Box-Whisker plot for each simulation run, when the Advanced Sensitivity Analysis was stepping across the particular input. For more information on Box-Whisker graphs, see **Stress Analysis** in this manual.
Quick Reports provide single page summaries of the Advanced Sensitivity Analysis as a whole, or for a single input in an Advanced Sensitivity Analysis. These reports are designed to fit on a single page.
The Percent Change Graph plots the Cell To Monitor statistic against each of the selected inputs as a Percent Change from Base. The input value, on the X-axis, is calculated by comparing each input value tested with the entered base value for the input.

The Percentile Graph plots the Cell To Monitor statistic against percentiles of each of the @RISK distributions that were selected for analysis with step type Percentiles of Distribution. Note: Only inputs that were @RISK distributions will be displayed on this graph.
The **Tornado Graph** shows a bar for each of the inputs defined for analysis, showing the minimum and maximum values that the specified **Cell To Monitor** statistic acquires, as the values of the input vary.
Results Commands

Browse Results Command

Turns on the Browse Results mode where a graph of simulation results is displayed when a cell is selected in Excel.

The Browse Results mode allows you to see a graph of simulation results in Excel, simply by clicking on the cell of interest in your worksheet. Alternatively, press <Tab> to move the graph among output cells, with simulation results, in open workbooks.

In Browse mode, @RISK pops up graphs of simulation results as you click, or tab to cells in your spreadsheet, as follows:

- If the selected cell is a simulation output (or contains a simulated distribution function), @RISK will display a graph of its simulated distribution.
- If the selected cell is part of a correlation matrix, a scatter plot matrix of the simulated correlations between the inputs in the matrix pops up.

If the Simulation Setting, Automatic Results Display — Show Output Graph is selected, this mode will be active at the end of a run.

To exit the Browse Results mode, simply close the popup graph, or click the Browse Results icon on the toolbar.
Results Summary Window Command
Displays all simulation results including statistics and small thumbnail graphs

The @RISK Results Summary Window summarizes the results of your model and displays thumbnail graphs and summary statistics for your simulated output cell and input distributions. As with the Model window, you can:

- Drag and drop any thumbnail graph, to expand it to a full window
- Double-click on any entry in the table to use the Graph Navigator, to move through cells in your workbook with input distributions
- Customize columns to select which statistics you want to display.

Note: If a name of an input or an output is shown in red in the Results Summary window, the referenced cell for the simulated result cannot be found. This can happen if you open simulation results and do not have a workbook that was used in the simulation open, or you have deleted the cell in your workbook after the simulation was run. In this case you will still be able to drag a graph of the result off of the Results Summary window; however, you will not be able to browse to the cell and pop up a graph.
The Results Summary Window is “linked” to your worksheets in Excel. As you click on a simulated output or input in the table, the cells where the result and its name are located are highlighted in Excel. If you double-click on a thumbnail graph in the table, the graph of the simulated output, or input, will be displayed in Excel, linked to the cell where it is located.

The commands for the Results Summary Window may be accessed by clicking the icons displayed at the bottom of the table, or by right-clicking and selecting from the pop-up menu. Commands will be performed on the current selected rows in the table.
Many graphs can be made in @RISK by simply dragging thumbnails off the Results Summary Window. In addition, overlays can be added to a graph by dragging one graph (or thumbnail) onto another.
Multiple graphs can be created at once by selecting multiple rows in the Results Summary Window, and clicking Graph icon at the bottom of the window.

As you make edits to a graph, in a full window, the thumbnail graph in the Results Summary Window will update to store the changes you make. This way you can close an open graph window without losing the edits you have made. However, the Results Summary Window has only one thumbnail graph for each simulated output, or input, and you can open multiple graph windows of a single output or input. Only the edits, for the most recently changed graph, are stored.
The Results Summary Window columns can be customized to select which statistics you want to display on your results. The Columns icon, at the bottom of the window, displays the Columns for Table dialog.

If you select to show Percentile values in the table, the actual percentile is entered in the rows Value at Entered Percentile.

Note: Column selections are retained as you change them. Separate column selections can be made for the @RISK — Model and @RISK — Results Summary window.

When Convergence Monitoring is turned on via Simulation Settings, the Status column is automatically added as the first column in the Results Summary window. It displays the convergence level for each output.
Editable p1,x1 and p2,x2 values, are columns that can be edited directly in the table. Using these columns you can enter specific target values and/or target probabilities directly in the table. Use the Edit menu Fill Down command to quickly copy p or x values across multiple outputs or inputs.

The Graph menu is accessed by 1) clicking the Graph icon, at the bottom of the Results Summary Window or 2) right-clicking in the table. Selected commands will be performed on the selected rows in the table. This allows you to quickly make graphs of multiple simulation results from your model. The command Automatic creates graphs using the default type (relative frequency), for distributions of simulation results.
The Results Summary window can be copied to the clipboard, or exported to Excel, using the commands on the Copy and Report menu. In addition, where appropriate, values in the table can be filled down or copied and pasted. This allows you to quickly copy editable P1 and X1 values.

Commands on the Edit menu include:

- **Report in Excel**. Exports the table to a new worksheet in Excel.
- **Copy Selection**. Copies the current selection in the table to the clipboard.
- **Copy Grid**. Copies the entire grid (text only; no thumbnail graphs) to the clipboard.
- **Paste, Fill Down**. Pastes or fills values into the current selection in the table.
**Detailed Statistics Command**

Displays the Detailed Statistics window

Clicking the Detailed Statistics icon displays detailed statistics on simulation results, for output cells and inputs.

![Detailed Statistics Window](image)

The Detailed Statistics window displays statistics that were calculated for all output cells and sampled input distributions. In addition, percentile values (in increments of 5 perc%) are shown, along with filter information, and up to 10 target values and probabilities.

The Detailed Statistics window may be pivoted so that it displays statistics in columns and outputs and inputs, in rows. To pivot the table, click the **Pivot Table of Statistics** icon at the bottom of the window.

![Pivot Table of Statistics](image)
Targets in @RISK may be calculated for any simulation result — either a probability distribution for an output cell, or a distribution for a sampled input distribution. These targets identify the probability of achieving a specific outcome, or the value associated with any probability level. Either values or probabilities may be entered into the target entry area at the bottom (or right, if pivoted), of the Detailed Statistics window.

The target entry area is viewed by scrolling the Detailed Statistics window to the target rows, where values and probabilities can be entered. If a value is entered, @RISK calculates the probability of a value occurring that is less than, or equal to, the entered value. If the @RISK Defaults menu Display Cumulative Descending Percentiles option is selected, the reported target probability will be in terms of a probability of exceeding the entered target value.

If a probability is entered, @RISK calculates the value in the distribution whose associated cumulative probability equals the entered probability.
Once a target value, or probability, has been entered, it may be quickly copied across a range of simulation results by simply dragging the value across the range of cells you wish to enter it in. An example of this is shown above, with the 99% target entered for each of the output cells, in the Detailed Statistics window. To copy targets:

1) Enter the desired target value, or probability, in a single cell in the Detailed Statistics window target rows.

2) Highlight a range of cells, across the row adjacent to the entered value, by dragging the mouse across the range.

3) Right-click, and select the Edit menu Fill Right command, and the same target will be calculated for each of the simulation results in the highlighted range.

The Detailed Statistics window, like other @RISK reports windows, can be exported to an Excel worksheet. Click the Copy and Report icon at the bottom of the window and select Report in Excel to export the window.
Data Command

Displays the Data Window

Clicking the Data icon displays data values, calculated for output cells and sampled input distributions. A simulation generates a new set of data for each iteration. During each iteration a value is sampled for each input distribution, and a value is calculated for each output cell. The Data window displays the simulation data in a worksheet where it can be further analyzed, or exported, (using the Edit icon commands) to another application for additional analysis.

Data is displayed, by iteration, for each output cell and sampled input distribution. By moving across a row of the Data window you can see the exact combination of input samples, which led to the shown output values in any given iteration.

Data from a simulation may be sorted to show key values you are interested in. For example, you could sort to show those iterations where an error occurred. You can also sort to show, decreasing or increasing, values for any result. Optionally, you can hide filtered values or errors. Sorting can be combined with the Iteration Step option to set Excel to the values for any iteration you are interested in.
Data Sort Dialog  

The Data Sort dialog controls how the Data Window will be sorted.

![Data Sort Dialog](image)

Sort By options include:

- **Iteration Number.** Selects to show All Iterations (the default display), Iterations Where an Error Occurred, or Iterations Remaining After Iteration Filters are Applied. For more information on Iteration Filters, see the Filters command in this chapter. The option Iterations Where an Error Occurred is useful for debugging a model. First, sort to show those iterations with errors. Then, use the Iteration Step command to set Excel to the values calculated for those iterations. Then, scroll through your workbook in Excel to examine the model conditions that led to the error.

- **Specific Result.** Each column in the Data Window (representing the data for an output or input in your simulation) may be sorted individually. Use this option to show the highest, or lowest, values for a result. Selecting Hide Filtered Values For this Result or Hide Error Values for this Result, hides iterations where the selected result has an error or filtered value.
Iterations displayed in the Data Window may be “stepped” through, updating Excel with the values that were sampled, and calculated, during the simulation. This is useful to investigate iterations with errors, iterations that led to certain output scenarios.

To step through iterations:

1) **Click the Iteration Step** icon at the bottom of the Data window.

2) **Click on the row in the Data window** with the **Iteration#** whose values you wish to update Excel with. The sampled values, for all inputs for that iteration, are placed in Excel and the workbook is recalculated.

3) **Clicking on the cell in the Data window** with the value for an output, or an input, in an iteration highlights the output or input cell in Excel.

Note: If your workbook in Excel has been changed since the simulation was run, the Iteration values that were calculated in the simulation may no longer match those calculated during the Iteration Step. When this happens, the error is reported in the Title Bar of the Data window.
**Sensitivities Command**

**Displays the Sensitivity Analysis window**

Clicking the **Sensitivity Analysis** icon displays sensitivity analysis results for output cells. These results show the sensitivity of each output variable to its input variables.

The Sensitivity analysis performed on the output variables, and their associated inputs, uses either a multivariate stepwise regression analysis, or a rank order correlation analysis. The input distributions in your model are ranked by their impact on the output, whose name is selected in the drop-down list box titled **Rank Inputs for Output**. The type of data displayed in the table — **Regression (Coefficients)**, **Regression (Mapped Values)**, **Correlation (Coefficients)** or **Regression and Correlation (Coefficients)** — is selected in the drop-down list box titled **Display Significant Inputs Using**. Click the **Tornado Graph** icon to display a tornado graph for the values in the selected column.

**Note:** Clicking a column header ranks the inputs for the output in the selected column.

**Smart Sensitivity Analysis**

By default, @RISK uses a Smart Sensitivity Analysis, by pre-screening inputs based on their precedence in formulas to outputs. Inputs located in formulas that have no link (via your model’s formulas) to an output cell are removed from the sensitivity analysis, thus avoiding spurious results. In the Sensitivity Analysis window these unrelated inputs are shown with the entry **n/a**.

Smart Sensitivity Analysis is done because it is possible for simulation data to show a correlation between an input and an output when, in reality, the input has no effect on the output in your model. Without Smart Sensitivity Analysis, tornado graph bars may be displayed for those unrelated inputs.
There are isolated instances where you should disable Smart Sensitivity Analysis on the Sampling tab of the Simulation Settings dialog box to improve performance and sensitivity analysis results:

1) The Smart Sensitivity Analysis setup time for scanning precedents at the start of the simulation adds significantly to runtime of a very large model and you are not concerned that you may see sensitivity analysis results (or tornado graph bars) for inputs unrelated to outputs.

2) You use a macro or DLL that performs calculations using @RISK input values in cells that have no relationship via workbook formulas with the output. This macro or DLL then returns a result to a cell that is used in calculating the outputs value. In this case there is no relationship in workbook formulas between the output and the @RISK distributions and Smart Sensitivity Analysis should be disabled. To avoid situations like this, we recommend that you create macro functions (UDFs) that explicitly reference all used input cells in their argument lists.

In earlier versions of @RISK, Smart Sensitivity Analysis was not used. This is the equivalent to the Settings menu Simulation Settings command Smart Sensitivity Analysis Disabled option.

Two methods — Multivariate Stepwise Regression and Rank Order Correlation — are used for calculating sensitivity analysis results, as discussed here.

Regression and Correlation

Regression is simply another term for fitting data to a theoretical equation. In the case of linear regression, the input data is fit to a line. You may have heard of the “Least-Squares” method, which is a type of linear regression.

Multiple regression tries to fit multiple input data sets to a planar equation that could produce the output data set. The sensitivity values, returned by @RISK, are normalized variations of the regression coefficients.

Stepwise regression is a technique for calculating regression values with multiple input values. Other techniques exist for calculating multiple regressions, but the stepwise regression technique is preferable for large numbers of inputs, since it removes all variables that provide an insignificant contribution from the model.

The coefficients listed in the @RISK sensitivity report are normalized regression coefficients associated with each input. A regression value of 0 indicates that there is no significant relationship between the input and the output, while a regression value of 1 or -1 indicates a 1
or -1 standard deviation change in the output for a 1 standard
deviation change in the input.

The \textbf{R-squared} value, listed at the top of the column, is simply a
measurement of the percentage of variation that is explained by the
linear relationship. If this number is less than \(~ 60\%\) then the linear
regression does not sufficiently explain the relationship between the
inputs and outputs, and another method of analysis should be used.

Even if your sensitivity analysis produces a relationship with a large
value of \textbf{R-squared}, examine the results to verify that they are
reasonable. Do any of the coefficients have an unexpected magnitude
or sign?

\textbf{Mapped values} are simply a transformation of the beta coefficient for
the Regression (Coefficient) into actual values. The beta coefficient
indicates the number of standard deviations the output will change,
given a one standard deviation change in the input (assuming all
other variables are held constant).

\textbf{Correlation} is a quantitative measurement of the strength of a
relationship between two variables. The most common type of
correlation is linear correlation, which measures the linear
relationship between two variables.

The rank order correlation value returned by @RISK can vary
between \(-1\) and \(1\). A value of 0 indicates there is no correlation
between variables; they are independent. A value of 1 indicates a
complete positive correlation between the two variables; when the
input value samples “high”, the output value will sample “high”. A
value of \(-1\) indicates a complete inverse correlation between the two
variables; when the input value samples “high”, the output value will
sample “low”. Other correlation values indicate a partial correlation;
the output is affected by changes in the selected input, but may be
affected by other variables as well.

\textbf{Rank order correlation} calculates the relationship between two data
sets by comparing the rank of each value in a data set. To calculate
rank, the data is ordered from lowest to highest and assigned
numbers (the ranks) that correspond to their position in the order.

This method is preferable to linear correlation when we do not
necessarily know the probability distribution functions from which
the data were drawn. For example, if data set A was normally
distributed and data set B was lognormally distributed, rank order
correlation would produce a better representation of the relationship
between the two data sets.
So, which measurement of sensitivity should you use? In most cases, the regression analysis is the preferred measure. The statement "correlation does not imply causality" holds, as an input that is correlated with an output may have little impact on the output, even if it is correlated with it.

However, in cases where the R-squared value, reported by the Stepwise Regression is low, you can conclude that the relationship between the input and output variables is not linear. In this case, you should use the Rank-Order Correlation analysis to determine the sensitivity in your model.

If the R-squared value reported by the Stepwise Regression is high, it is easy to conclude that the relationship is linear. But, as mentioned above, you should always verify that the regression variables are reasonable. For example, @RISK might report a significant positive relationship between two variables in the regression analysis, and a significant negative correlation in the rank-order analysis. This effect is called multicollinearity.

Multicollinearity occurs when independent variables in a model are correlated to each other as well as to the output. Unfortunately, reducing the impact of multicollinearity is a complicated problem to deal with, but you may want to consider removing the variable that causes the multicollinearity from your sensitivity analysis.

Sensitivity Analysis results may be display in a Scatter Plot Matrix. A scatter plot is an x-y graph showing the input value sampled vs. the output value calculated for each iteration of the simulation. In the Scatter Plot Matrix, ranked sensitivity analysis results are displayed with scatter plots. To show the scatter plot matrix, click the Scatter Plot icon in the lower left of the Sensitivity window.

Using Drag and Drop, a thumbnail scatter plot in the Scatter Plot Matrix can be dragged, and expanded, into a full graph window. In addition, overlays of scatter plots may be created by dragging additional scatter thumbnail graphs from the matrix onto an existing scatter plot.
Scenarios Command

Displays the Scenario Analysis window

Clicking the Scenarios icon displays scenario analysis results for output cells. Up to three scenarios may be entered for each output variable. Scenarios are shown in the top row of the scenario analysis window or in the Scenarios section of the Detailed Statistics window. Targets are preceded by a > or < operator and can be specified in terms of percentiles or actual values.

What is Scenario Analysis?

Scenario analysis allows you to determine which input variables contribute significantly towards reaching a goal. For example, which variables contribute to exceptionally high sales? Or, which variables contribute to profits below $1,000,000?

@RISK allows you to define target scenarios for each output. You might be interested in the highest quartile of values in the Total Sales output, or the value less than 1 million in the Net Profits output. Enter these values directly in the Scenarios row of the @RISK Scenario Analysis window to study these situations.

When you display a scenarios window, @RISK looks at the data created by your @RISK simulation. For each output, the following steps are followed:

1) The median and standard deviation of the samples, for each input distribution for the entire simulation, are calculated.
2) A “subset” is created containing only the iterations in which the output achieves the defined target.
3) The median of each input is calculated for the subset of data.
4) For each input, the difference between the simulation median (found in step 1) and the subset median (found in step 3) is calculated and compared to the standard deviation of the input data (found in step 1). If the absolute value of the difference in medians is greater than \( \frac{1}{2} \) a standard deviation, then the input is termed “significant”; otherwise the input is ignored in the scenario analysis.

5) Each significant input found in step 4 is listed in the scenario report.

**Interpreting the Results**

From the above explanation, you know that the scenario report will list all input variables that are “significant” toward reaching a defined goal for an output variable. But what exactly does that mean?

For example, @RISK may tell you that the Retail Price input is significant when studying the highest quartile of Total Sales. So, you know that when Total Sales are high, the median Retail Price is significantly different than the median Retail Price for the whole simulation.

@RISK calculates three statistics for each significant input distribution in a scenario:

- **Actual Median of Samples in Iterations Meeting Target.**
  The median of the subset of iterations for the selected input (calculated above in step 3). You can compare this to the median of the selected output for the whole simulation (the 50% percentile reported in the statistics report).

- **Percentile Median of Samples in Iterations Meeting Target.**
  The percentile value of the subset median in the distribution generated for the whole simulation (equivalent to entering the subset median as a Target Value in the @RISK statistics report). If this value is less than 50%, the subset median is smaller than the median for the whole simulation, if it is greater than 50% the subset median is greater than the median for the whole simulation.

You might find that the subset median for Retail Price is lower than the median for the whole simulation (thus the percentile is less than 50%). This indicates that a lower Retail Price can help you reach the goal of high Total Sales.
• **Ratio Shown Median to Original Standard Deviation.** The difference between the subset median, and the median for the whole simulation, divided by the standard deviation of the input for the whole simulation. A negative number indicates that the subset median is smaller than the median for the whole simulation, a positive number indicates that the subset median is greater than the median for the whole simulation. The larger the magnitude of this ratio, the more “significant” the variable is in reaching the defined target.

Perhaps another input variable, Number of Salespeople, is significant towards reaching the target of high Total Sales, but its ratio of median to standard deviation is only half the magnitude of the ratio for the Retail Price input. You could conclude that, while the Number of Salespeople does affect your goal of high Total Sales, the Retail Price is more significant and may require more attention.

![Caution: The greatest danger in using scenario analysis is that the analysis results may be deceiving, if the subset contains a small number of data points. For example, in a simulation of 100 iterations, and a scenario target of “>90%”, the subset will contain only 10 data points!]

**Editing Scenarios**

The default scenarios may be changed by clicking the **Edit Scenarios** icon (either in a graph window or in the Scenarios window) or by double clicking on a scenario – such a >90% - that is displayed in the first row of the Scenarios Window.

![Scenarios: Net Income / 2009](image)

Three scenarios can be entered for each simulation output. Each scenario may have one or two bounds. If you enter two bounds you will be specifying a scenario that has a min-max range for the output, such as >90% and <99%. Each bound may be specified as a percentile or an actual value, such as >1000000.
If you don’t want to use a second bound, just leave it blank. This specifies that the second bound is either the minimum output value (< operator is used, such as <5%) or maximum output value (> operator is used, such as >90%).

**Note: Default scenarios settings can be entered using the Application Settings command.**

A scatter plot in the Scenarios window is an x-y scatter plot with an overlay. This graph shows:

1) the input value sampled vs. the output value calculated in each iteration of the simulation,

2) overlaid with a scatter plot of the input value sampled vs. the output value calculated when the output value meets the entered scenario.

In the **Scatter Plot Matrix**, ranked scenario analysis results are displayed with scatter plots. To show the Scatter Plot Matrix, click the **Scatter Plot** icon in the lower left of the **Scenarios** window.

**Note: You may only overlay the same input and output, under different scenarios, in a scatter graph which displays scenario analysis results.**
Scenario analysis results are graphically displayed in tornado graphs. A tornado graph can be generated by clicking the Tornado graph icon in the Scenarios window or clicking the Scenarios icon on a graph window. This tornado graph shows the key inputs affecting the output when the output meets the entered scenario, such as when the output is above its 90th percentile.
Define Filters Command

Filters values from simulation statistics calculations and graphs

Filters may be entered for each selected output cell, or sampled input probability distribution. Filters allow you to remove unwanted values from the statistics calculations and graphs generated by @RISK. Filters are entered by clicking the Filter icon on the toolbar or, alternatively, by clicking the Filter icon shown on the graph of a simulation result or in the Data Window.

A filter can be defined, for any simulation output or sampled input distribution, as listed in the Name column of the Filter Settings table. When entering a filter, a type, a values type (Percentiles or Values), a minimum allowed value, maximum allowed value or minimum-maximum range may be entered. If the Filter Minimum or Filter Maximum entry is left blank, the Filter range will be unbounded on one end — allowing a filter, with only a maximum or minimum such as “process only values, equal to or above a minimum of 0.”
Icons and options in the Filters dialog include:

- **Show Only Outputs or Inputs With Filters** — In the Filter dialog, displays only those outputs or inputs for which filters have been entered.

- **Same Filter For All Simulations** — If multiple simulations have been run, the Same Filter For All Simulations option copies the first filter entered, for an input or output, to the results for the same input or output in all other simulations.

- **Apply** — Filters are applied as soon as you click the Apply button in the Filter dialog box.

- **Clear Filters** — To remove all current filters, click the Clear Filters button to remove the filters from the currently selected rows in the table and then click Apply. To simply disable a filter, but leave the entered filter range, set the Filter Type to Off.

The available Filter Types are:

- **Standard Filter** — This type of filter is applied only to the output cell, or sampled input probability distribution, for which the filter was entered. Values below the entered minimum, or above the entered maximum are removed from the statistics, sensitivity, and scenario calculations for the result, and not included in generated graphs for the simulation result.

- **Iteration Filter** — This type of filter affects all simulation results. In processing a global iteration filter, first @RISK applies the filter to the output cell, or sampled input probability distribution, for which the filter was entered. Values below the entered minimum, or above the entered maximum, are removed from the statistics, sensitivity, and scenario calculations for the result and not included in generated graphs for the simulation result. The iterations, which satisfy the conditions of this filter for the output or input, are then “marked” and all other output cells or sampled input probability distributions are filtered to include only values generated in these iterations. This type of filter is especially useful when you want to review simulation results (for all outputs and inputs) for only those iterations which meet a specific filter condition — such as where “Profit > 0”. 

Reference: @RISK Commands
When you click the Filter icon shown on the graph of a simulation result, a quick filter dialog is displayed that allows you to set a filter for just the result displayed in the graph.

When filtering from a graph window, simply set the Type of filter and the type of values to be entered, the minimum-maximum range and click **Apply**. The graph is redisplayed (with new statistics) and the number of values used (not filtered) is shown at the bottom of the graph. As with any filter, values below the entered minimum or above the entered maximum are removed from the statistics, sensitivity and scenario calculations for the result and not included in generated graphs for the simulation result.

If you want to see the full Filter dialog listing all active Filters, click the **Show All** button.
Excel Reports Command

Selects the reports on simulation results to generate in Excel

The @RISK — Excel Reports command selects reports to be generated on the active simulation results, or the current model definition.

A variety of different pre-built simulation reports are available directly in Excel at the end of a simulation. The Quick Report is a report on simulation results designed for printing. This report contains a single page report for each output in a simulation. The other available reports, starting with Input Results Summary, contain the same information as the equivalent report in the Results Summary Window or other Report windows.

The location of your reports is set using the Utilities menu Application Settings command. Two options are available for locating reports in Excel:

- **New Workbook.** Places simulation reports in a new workbook each time reports are generated.
- **Active Workbook.** Places simulation reports in new sheets in the active workbook each time reports are generated.

For more information on these and other defaults, see the Application Settings command in this chapter.
You can use template sheets to create your own custom simulation report. Simulation statistics and graphs are placed in a template using @RISK statistics functions (such as RiskMean) or the graphing function RiskResultsGraph. When a statistics function or graphing function is located in a template sheet, the desired statistics and graphs are then generated at the end of a simulation in a copy of the template sheet when you choose the Template Sheets option in the Excel Reports dialog. The original template sheet with the @RISK functions remains intact for use in generating reports from your next simulation.

Template sheets are standard Excel sheets. They are identified to @RISK by having a name that starts with RiskTemplate_. These files can also contain any standard Excel formulas so custom calculations can be performed using simulation results.

The example file Template.XLS shown above contains a template sheet. You can review this sheet to see how to set up your own custom reports and template sheets.
Swap @RISK Functions Command

Swaps @RISK functions in and out of cell formulas

With the Swap @RISK Functions Command, @RISK functions can be swapped in and out of your workbooks. This makes it easy to give models to colleagues who do not have @RISK. If your model is changed when @RISK functions are swapped out, @RISK will update the locations and static values of @RISK functions when they are swapped back in.

@RISK uses a new property function called RiskStatic to help in its function swap. RiskStatic holds the value that will replace the function when it is swapped out. It also specifies the value that @RISK will return for the distribution in a standard Excel recalculation.

When the Swap @RISK Functions icon is clicked, you may immediately swap out functions using the current swap settings, or change the settings to be used.
When functions are swapped out, the @RISK toolbar is disabled, and if you enter an @RISK function it will not be recognized.

The Swap options dialog allows you to specify how @RISK will operate when functions are swapped in and out. If your workbook is changed, when @RISK functions are swapped out, @RISK can report to you how it will re-insert @RISK functions into your changed model. In most cases, @RISK will be able to automatically handle changes to a workbook when functions are swapped out.
**Swap Options**

Clicking the **Swap Option** icon (next to the Help icon in the Swap @RISK Functions dialog) displays the Swap Options dialog.

Swap Options are available for:

- **Swap Out** (when @RISK functions are removed)
- **Swap In** (when @RISK functions are returned to your workbook)

**Swap Out Options**

When swapping out, the primary value used for replacing an @RISK function is its **static value**. Typically this is the value in a formula in your model that was replaced by an @RISK function. It is stored in an @RISK distribution in the RiskStatic property function.

If you enter a new distribution using the Define Distribution window, @RISK can automatically store the value that you are replacing with a distribution in a RiskStatic property function. For example; if a cell C10 has the value 1000 in it, as shown in the formula:

\[ C10: =1000 \]

Then, using the Define Distribution window, you replace this value with a Normal distribution with a mean of 990 and a standard deviation of 100. Now, the formula in Excel will be:

\[ C10: =\text{RiskNormal}(990,100,\text{RiskStatic}(1000)) \]

Note that the original cell value of 1000 has been retained in the RiskStatic property function.
If a Static value is not defined (i.e., no RiskStatic function is present), a set of different values are available for replacing the @RISK functions value. These are selected in the Where RiskStatic is Not Defined, Use options, and include:

- **“Corrected” Expected Value**, or a distribution’s expected or mean value, except for discrete distributions. For discrete distributions, the setting “Corrected” Expected Value will use the discrete value in the distribution closest to the true expected value as the swap value.

- **True Expected Value**. This setting causes the same values to be swapped as the option “Corrected” Expected Value, except in the case of discrete distribution types such as DISCRETE, POISSON and similar distributions. For these distributions the true expected value will be used as the swap value, even if the expected value could not occur for the entered distribution, i.e., it is not one of the discrete points in the distribution.

- **Mode**, or a distribution’s mode value.

- **Percentile**, or the entered percentile value for each distribution.
Swap In Options

Swap In Options control how @RISK will report changes that it will make to your spreadsheet, prior to inserting distribution functions back into formulas. Spreadsheet formulas and values can be changed when @RISK functions are swapped out. When swapping in, @RISK will identify where it should re-insert @RISK functions and, if desired, show all the changes it is going to make to your formulas. You can check these changes to make sure @RISK functions are returned as you wish. In most cases, Swap In is automatic, as @RISK captures all changes to static values that were made when functions were swapped out. It also, automatically, handles moved formulas and inserted rows and columns. However, if formulas where @RISK functions were previously located were deleted when functions were swapped out, @RISK will notify you of the problem formulas prior to swapping functions back in.
Swap In options for **Prior to Restoring @RISK Functions, Preview Changes** include:

- **All.** With this option all changes to be made to the model are reported, even if a formula and swapped out value were not changed when @RISK functions were swapped out.

- **Only Where Formulas and Static Values Were Modified.** With this option only changes to be made, that include a changed static value or formula, are reported. For example, if the original @RISK distribution was:

  \[ C10: =\text{RiskNormal}(990,100,\text{RiskStatic}(1000)) \]

  After swap out, the formula would be:

  \[ C10: =1000 \]

  If the value of C10 was changed while functions were swapped out to:

  \[ C10: =2000 \]

  @RISK would swap the following function back in, updating the static value:

  \[ C10: =\text{RiskNormal}(990,100,\text{RiskStatic}(2000)) \]

  If the Swap In option **Only Where Formulas and Static Values Were Modified** was selected, @RISK would report this change prior to swapping in.

- **Only Where Formulas Were Modified.** Only changes to be made, that include a changed formula, are reported with this option. For example, if the original @RISK distribution was in a formula:

  \[ C10: =1.12+\text{RiskNormal}(990,100,\text{RiskStatic}(1000)) \]

  After swap out, the formula would be:

  \[ C10: =1.12+1000 \]

  If the formula for C10 was changed when functions were swapped out to:

  \[ C10: =1000 \]

  @RISK would swap the following formula and function back in:

  \[ C10: =\text{RiskNormal}(990,100,\text{RiskStatic}(1000)) \]
If the options **Only Where Formulas and Static Values Were Modified** or **Only Where Formulas Were Modified** were selected, @RISK would report this change prior to swapping in.

- **None.** No changes to be made to the model are reported, and @RISK automatically swaps in its recommended change.

@RISK creates a report which you can use to preview the changes that will be made to a workbook when swapping functions in. The report includes the **Original (Before Swap)**, the **Original (After Swap)**, the **Current**, and the **Recommended** formulas to be swapped back in.

If desired, you can edit the **Recommended** formula to be swapped back in, or alternatively, select one of the other displayed formulas to be used when swapping back in. By selecting the Edit icon’s **Create Report to Excel** command at the bottom of the window, you can choose to create a report in Excel of the changes made to the model.

If @RISK is running, it will automatically offer to swap in functions when a “swapped out” workbook is opened. However, this will not happen if the “swapped out” workbook is opened while @RISK’s toolbar is disabled because functions are swapped out.
Utilities Commands

Application Settings Command

Displays the Application Settings dialog where program defaults can be set

A wide variety of @RISK settings can be set at default values that will be used each time @RISK runs. These include graph color, displayed statistics, coloring of @RISK cells in Excel and others.

All @RISK windows and graphs will update when Application Settings are changed. Thus, Application Settings are an easy way to apply desired changes, across all open windows and graphs, during an @RISK session.
Many defaults are self-explanatory, and most reflect settings found in other @RISK dialogs and screens. Defaults that require more clarification include:

- **Percentiles — Ascending or Descending.** Selecting Descending as the Percentiles defaults switches all @RISK statistics reports, targets and graph x and p values, to display cumulative descending percentiles. By default, @RISK reports percentile values in terms of cumulative ascending percentiles, or the probability that a value is less than or equal to a given x value. Selecting **Descending** Percentiles causes @RISK to report cumulative descending percentiles, or the probability that a value is greater than a given x value.

  Selecting Descending Percentiles also causes @RISK to default to the entry of cumulative descending percentiles, when alternate parameter distributions are entered in the Define Distribution window. In this case, the percentage chance of a value greater than the entered value is specified.

- **Insert Static Values.** If set to True, a RiskStatic function will automatically be inserted in @RISK distributions entered using the Define Distribution window. In this case, when an existing value in a cell formula is replaced by an @RISK distribution, the value that was replaced will be included in a RiskStatic property function.

- **Smart Sensitivity Analysis.** Enables or disables Smart Sensitivity Analysis. For more information on Smart Sensitivity Analysis and situations when you may wish to disable it, see the **Sensitivities Command.**

- **Show Window List.** The @RISK Window List (displayed when the Utilities menu Windows command is selected), by default, is displayed automatically when over five @RISK windows are shown on screen. This default either suppresses the window list, always displays it, or allows it to come up automatically.
• **Cell Formatting.** If desired, you can select to apply formatting to cells in your workbook where @RISK inputs and outputs are located. You can select a color for cell font, border or background.

![Excel workbook example](image)

- **Preferred Distribution Format.** Specifies the format to be used for @RISK distribution graphs, for model inputs and simulation results. If a specific graph cannot be displayed in the preferred format, this setting is not used.

- **Number of Delimited Curves.** Sets the maximum number of delimiter bars, shown at the top of the graph, with each bar associated with a curve in the graph.

- **Marked Values.** Sets default markers that will be shown on graphs you display.

- **Number Formatting.** Sets the formatting to be used for numbers displayed on graphs and markers. **Quantities with Units** refers to reported values such as Mean and Standard Deviation that use the units of the graph. **Unitless Quantities** refers to reported statistics such as Skewness and Kurtosis that are not in the units of the values for the graph. Note: If **Currency** format is selected, it is only applied when the Excel cell for the output or input graphed is formatted as Currency.
@RISK’s Application Settings may be saved in a file RiskSettings.RSF. Once saved, this file can be used to set the Application Settings to be used in another installation of @RISK. To do this:

1) Select the **Export to File** command after clicking the second icon at the bottom of the Application Settings window.

2) Save the file **RiskSettings.RSF**.

3) Place the RiskSettings.rsf in the **RISK5** folder under the `Program Files\Palisade` folder on the system where you want to set @RISK’s Application Settings. Typically, you will place the RiskSettings.rsf in that folder after a new install of @RISK.

If the RiskSettings.rsf file is present when @RISK runs, those application settings will be used and the user will be unable to change them. (The user will still be able to change simulation settings.) The user can change application settings by removing the RiskSettings.rsf file when @RISK isn't running.

The command **Import From File** can be used to load Application Settings from a **RiskSettings.RSF** not located in the RISK5 folder. Imported settings may be changed as desired, unlike settings used from a RiskSettings.RSF file that is located in the **RISK5** folder under the `Program Files\Palisade` folder.
Windows Command

Displays the @RISK Windows List

The @RISK Windows List displays a list of all open @RISK windows, and allows activating, arranging and closing of those windows.

Double-clicking on any window in the list activates it. Any, or all windows, can be closed by clicking icons with a red Close window icon.
Open Simulation File Command

Opens Simulation Results and Graphs from a .RSK5 File

There may be times that you wish to store simulation results in external .RSK5 files as was done in earlier versions of @RISK. You might do this if your simulation was very large, and you do not want to embed that data in your workbook. If you save a .RSK5 file in the same folder, with the same name as your workbook, it will be automatically opened when you open your workbook. Otherwise, you will need to use the Utilities menu Open Simulation File command to open the .RSK5 file.

For more information on saving and opening simulation results and graphs, see the section of this manual Saving and Opening @RISK Simulations.
Clear @RISK Data Command

Clears the Selected @RISK Data from Open Workbooks

The Clear @RISK Data Command clears the selected @RISK data from open workbooks.

The following data may be cleared:

- **Simulation Results.** This clears the results of the current simulation from @RISK, as displayed in the active @RISK windows.

- **Settings.** This clears any @RISK settings and the defined Excel names associated with them. This does not clear names entered for @RISK functions; they are stored in cell formulas and not in the Defined Names list in an Excel workbook.

- **Distribution Fitting Definitions.** This clears any definitions of fitted distributions as shown in the Fit Manager.

- **Spreadsheet Functions.** This removes all @RISK functions from open workbooks, replacing them with their Static value or, if a Static value is not found, the Swap Out value as specified in the Swap Options dialog. However, this is not a Function Swap, as @RISK will not place swap information in your workbook to be used when swapping functions back in, and thus, **all model information will be gone.**

Selecting all options allow you to remove all @RISK information from open workbooks.

Unload @RISK Add-in Command

Unloads the @RISK add-in from Excel

The Unload @RISK Add-in Command unloads @RISK, closing all @RISK windows.
Saving and Opening @RISK Simulations

Opens and Saves Simulation Results and Graphs

Results from simulations (including graphs), can be stored directly in your workbook, an external .RSK5 file or, alternatively, in the @RISK Library. Using the Utilities menu Application Settings command, you can also select to have @RISK automatically or never save your simulation results, to your workbook. It is important to note that your model — including distribution functions and simulation settings — is always saved when you save your workbook. @RISK’s Excel reports placed in worksheets in Excel are also saved when their Excel workbook is saved. Save Simulation options only affect simulation results and graphs displayed in @RISK windows such as graph windows, the Data window or the Results Summary window.

If desired, @RISK will prompt you to save your simulation results whenever your workbook is saved, as shown:

The Save Options button (second from the left) selects the location to save results.

Options in the Save @RISK Results dialog include:

- **Workbook Being Saved.** This option specifies that @RISK will store all data from the simulation that has been run, including open windows and graphs, in the workbook being saved. If the Utilities menu Application Settings command specifies that @RISK will automatically save simulations to
your workbook (or the Do This Automatically checkbox is selected), @RISK data and graphs are saved, and opened automatically, when you save or open your workbook.

- **External .RSK5 File.** There may be times, however, that you wish to store simulation results in external .RSK5 files, as was done in earlier versions of @RISK. You might do this if your simulation was very large, and you do not want to embed that data in your workbook. By clicking the option button next to the filename, you can specify a file name and location for your .RSK5 file. If you save this file in the same folder with the same root name as your workbook, it will be automatically opened when you open your workbook. Otherwise, you will need to use the **Utilities menu Open Simulation File** command to open the .RSK5 file.

![Save @RISK Results to External File](image)

- **Don’t Save.** With this option selected, @RISK will not save simulation results. However, you can always re-run your simulation to view your results again, as your model — including distribution functions and simulation settings — is always saved when you save your workbook.

- **Do This Automatically.** This option specifies that you will always save your data to your workbook, or not save results. This is the same as selecting the equivalent Utilities menu Application Settings command option.
Library Commands

The Utilities Library command displays the @RISK Library. @RISK 5 Professional and Industrial versions include the @RISK Library — a separate database application for sharing @RISK’s input probability distributions, and comparing results from different simulations. The @RISK Library uses SQL Server to store @RISK data. Different users in an organization can access a shared @RISK Library in order to access:

- Common input probability distributions, which have been pre-defined for use in an organizations’ risk models
- Simulation results from different users in an organization

For more information on the @RISK Library, see the Reference: @RISK Library chapter of this manual.

Add Results to Library

Adds simulation results to the @RISK Library

Simulation results are stored in the @RISK Library by selecting the Add Result to Library command on the Library icon on the @RISK for Excel toolbar. You will then be prompted to select which outputs in the current results you wish to display in the Library.

For more information on results in the @RISK Library, see the Reference: @RISK Library chapter of this manual.

Show Library

Displays the @RISK Library

Clicking the Library icon on the @RISK toolbar displays the @RISK Library window. This allows current distributions along with stored simulation results to be reviewed.
Help Commands

@RISK Help
Opens on-line help file for @RISK
The Help menu @RISK Help command opens the main help file for @RISK. All of @RISK's features and commands are described in this file.

Online Manual
Opens online manual for @RISK
The Help menu Online Manual command opens this manual in PDF format. You must have Adobe Acrobat reader installed to view the online manual.

License Activation Command
Displays licensing information for @RISK and allows the licensing of trial versions
The Help menu License Activation command displays the License Activation dialog box, listing the version and licensing information for your copy of @RISK. Using this dialog box you can also convert a trial version of @RISK into an licensed copy.

For more information on licensing your copy of @RISK, see Chapter 1: Getting Started in this manual.

About Command
Displays version and copyright information about @RISK
The Help menu About command displays the About dialog box, listing the version and copyright information for your copy of @RISK.
Simulation inputs and results are easily expressed with graphs. Graphs are shown in many places in @RISK. For example, the Results Summary Window shows thumbnail graphs of the simulation results for all your outputs and inputs. Dragging a thumbnail graph off of the Results Summary Window allows you to graph the simulation results, for the selected output or input, in a full window. Graphs are also displayed when you click on a worksheet output or input cell in the Browse Results mode.

Overview

Graphs in @RISK come in two types of windows:

- **Floating windows**, sitting by themselves on top of Excel. These windows are permanent, until they are closed by you.

- **Callout windows**, attached to a cell. This is the type of window used in the Browse mode. Only one of these windows is open at a time, as the graph changes when a new cell is selected in Excel.

Using the icons on the graph you can detach a callout window and turn it into a floating window, or reattach a floating window to the cell it represents.

The type of graph displayed may be changed, using the icons at the bottom of the Graph window. In addition, by clicking the right mouse button on a graph window, a pop-up menu is displayed with commands that allow the changing of a graph's format, scaling, colors, titles, and other characteristics.

Each graph created by @RISK is displayed in conjunction with the statistics for the output or input that is graphed. Each graph and its statistics may be copied to the clipboard, and pasted into your spreadsheet. As graphs are transferred, as Windows metafiles, they may be resized and annotated once pasted into a spreadsheet.

Using the Chart in Excel command, graphs can be drawn in Excel's native graph format. These graphs can be changed, or customized, just as with any Excel graph.

All @RISK graph windows have a set of icons in the lower left which allows you to control the type, format, and placement of the displayed graphs. You can also use the Zoom icon to quickly zoom in on a region displayed in a graph.
@RISK graphics use a graphing engine designed specifically for processing simulation data. Graphs can be customized and enhanced as needed, often by simply clicking on the appropriated element in the graph. For example, to change the title of a graph, simply click on the title and type the new entry:
A displayed graph can also be customized through the **Graph Options** dialog. Customization includes colors, scaling, fonts and displayed statistics. The Graph Options dialog is displayed by right clicking on a graph and selecting the Graph Options command, or by clicking the Graph Options icon in the bottom left of the graph window.

The Graph Options dialog may change depending on the type of graph being customized. Graph options specific to a certain graph type are discussed in the reference section pertaining to the graph type.
When multiple simulations are run, a graph can be made for result distributions in each simulation. Often it is desirable to compare the graphs created for the same result in different simulations. This comparison shows how risk changes for the distributions by simulation.

To create a graph that compares the results for a cell in multiple simulations:

1) Run multiple simulations, by setting the **Number of Simulations** in the Simulation Settings dialog box to a value greater than one. Use the **RiskSimtable** function to change worksheet values by simulation.

2) Click the **Select Simulation # to Display icon** at the bottom of the displayed Browse window

3) Select **All Simulations** to overlay graphs for all simulations for the selected cell on the graph

To create a graph that compares the results for a different cell in multiple simulations:

4) Click the **Overlay Graph icon** at the bottom of a displayed Browse window when multiple simulations have been run

5) Select the cells in Excel whose results you want to add to the graph.

6) Select the **simulation #** for the cells you wish to overlay from the dialog.

The **Select Simulation** dialog is also available in report windows when you wish to filter the report to show only those results from a specific simulation.
Histogram and Cumulative Graphs

A histogram or cumulative graph shows the range of possible outcomes and their relative likelihood of occurrence. This type of graph may be displayed in standard histogram or frequency distribution form. Distributions of possible outcomes may also be displayed in cumulative form.
By dragging the delimiters displayed on a histogram or cumulative graph, target probabilities may be calculated. When delimiters are moved, calculated probabilities are shown in the delimiter bar above the graph. This is useful for graphically displaying answers to questions such as “What is the probability of a result between 1 million and 2 million occurring?” and “What is the probability of a negative result occurring?”.

Delimiters can be displayed for any number of overlays. The Graph Options dialog allows you to set the number of delimiter bars displayed.
Many times it is useful to compare several distributions graphically. This can be done using overlay graphs.

Overlays are added by:

- Clicking the Add Overlay icon on a displayed graph and then selecting the cell(s) in Excel whose results you want to include in the plot.
- Dragging one graph onto another, or by dragging a thumbnail graph from the Model or Results Summary Window onto an open graph. Once overlays are added, delimiter statistics display probabilities for all distributions included in the overlay graph.

Note: A shortcut to removing an overlay curve is to right-click on the colored legend, for the curve you wish to remove, and selecting the Remove Curve command.
Some times it is useful to display histogram and cumulative curve for a given output or input on the same graph. This graph type has two Y-axes, one on the left for the histogram and a secondary Y-axis on the right for the cumulative curve.

To switch a Probability Density or Relative Frequency graph to include a cumulative overlay, select the Cumulative Overlay option after clicking the Graph Type icon in a graph window.
The Graph Options dialog is displayed by right clicking on a graph and selecting the **Graph Options command**, or by clicking the **Graph Options** icon in the bottom left of the graph window. For histogram and cumulative graphs the **Graph Options — Distribution Tab** sets the type of curve displayed along with binning options.

![Graph Options - Distribution Tab](image)

Options on the **Graph Options — Distribution Tab** include:

- **Distribution Format**. Changes the format of the displayed distribution. Settings include:
  - **Automatic**. Selects Probability Density graphs
  - **Probability Density** and **Relative Frequency**. For histograms, these settings represent the unit of measure reported on the y-axis. Relative Frequency is the probability of a value in the range of a bin occurring (observations in a bin/total observations). Probability Density is the relative frequency value divided by the width of the bin, insuring that y-axis values stay constant as the number of bins is changed.
  - **Discrete Probability**. Graphs the distribution by showing the probability of each value that occurs in the minimum-maximum range. This setting applies to graphs that display discrete distributions, where a limited set of values occur.
  - **Cumulative Ascending** and **Cumulative Descending**. Displays either cumulative ascending probabilities (y-axis shows the probability of a value less than any x-axis value) or
cumulative descending probabilities (y-axis shows the probability of a value greater than any x-axis value).

- **Histogram Binning.** Specifies how @RISK will bin the data in a displayed histogram. Settings include:
  - **Minimum.** Sets the minimum value where histogram bins start. **Automatic** specifies that @RISK will start the histogram bins based on the minimum of the data graphed.
  - **Maximum.** Sets the maximum value where histogram bins end. **Automatic** specifies that @RISK will end the histogram bins based on the maximum of the data graphed.
  - **Number of Bins.** Sets the number of histogram intervals calculated across the range of a graph. The value entered must be in the range 2 to 200. The setting **Automatic** calculates the best number of bins to use, for your data is based on an internal heuristic.
  - **Overlays.** Specifies how @RISK will align bins between distributions when overlay graphs are present. Options include:
    1) **Single Histogram,** where the entire min-max range of data in all curves (including overlays) is binned, and each curve in the graph uses those bins. This allows easy comparisons of bins between curves
    2) **Single Histogram with Adjusted Limits,** which is the same as the Single Histogram option except at the endpoints of each curve. Smaller or larger bins are used at the endpoints to ensure that each curve does not go below its minimum data value, or above its maximum.
    3) **Independent Histograms,** where each curve uses independent binning based on its own minimum — maximum data values.
    4) **Automatic** selects between Single Histogram with Adjusted Limits and Independent Histograms, depending on the overlap of the data between curves. Curves with sufficient data overlap will use Single Histogram with Adjusted Limits.
**Graph Options – Delimiters Tab**

For histogram and cumulative graphs, the Graph Options – Delimiters Tab specifies how delimiters will be displayed with the graph.

When delimiters are moved, calculated probabilities are shown in the delimiter bar above the graph. Delimiters can be shown for any or all of the curves in a graph.
For histogram and cumulative graphs, the **Graph Options — Markers Tab** specifies how markers will be displayed with the graph. Markers annotate key values on a graph.

When markers are displayed, they are included in graphs when you copy them into a report.
Fitting a Distribution to a Simulated Result

Clicking the Fit Distributions to Data icon in the lower left of a graph window fits probability distributions to the data for simulated result. All the options that can be used when fitting distributions to data in an Excel worksheet are available when fitting probability distributions to a simulated result. For more information on these options, see Chapter 6: Distribution Fitting in this manual.
Tornado Graphs

Tornado graphs from a sensitivity analysis display a ranking of the input distributions which impact an output. Inputs that have the largest impact on the distribution of the output will have the longest bars in the graph. In @RISK, three methods are available for displaying tornado graphs — regression coefficients, regression (mapped values), and correlation coefficients.

Tornado graphs for an output may be displayed by selecting a row, or rows, in the @RISK — Results Summary window, clicking Tornado Graph icon at the bottom of the window and selecting one of the three Tornado graph options. Alternatively, a distribution graph for a simulated output can be changed to a tornado graph by clicking the Tornado Graph icon, in the bottom left of the graph, and selecting a tornado graph.

@RISK has three types of tornado graphs — Regression Coefficients, Correlation Coefficients and Regression — Mapped Values. To learn more about how the values displayed on each type of tornado graph are calculated, see the Sensitivities Command section of the Reference: @RISK Commands chapter.

For tornado graphs showing Regression Coefficients and Correlation Coefficients, the length of the bar shown for each input distribution is based on the coefficient value calculated for the input. The values shown on each bar of the tornado graph are the coefficient value.

An additional tornado graph is available for scenario analysis results. This tornado graph can be generated by clicking the Tornado graph icon in the Scenarios window or clicking the Scenarios icon on a graph window. This graph shows the key inputs affecting the output when the output meets the entered scenario, such as when the output is above its 90th percentile.
For tornado graphs showing **Regression — Mapped Values**, the length of the bar, shown for each input distribution, is the amount of change in the output due to a +1 standard deviation change in the input. The values shown on each bar of the tornado graph are the output value associated with +1 standard deviation change in the input. Thus, when the input changes by +1 standard deviation, the output will change by the X-axis value associated with the length of the bar.

The maximum number of bars that can be displayed in a tornado graph is 16. If you want to display tornado graphs with fewer bars, use the **Maximum Number of Bars** setting in the Graph Options dialog. To set a default maximum number of bars, use the **Application Settings dialog Tornado Maximum # Bars** setting.

**Note:** If your tornado graph has many bars, there may not be enough room to display labels for each bar. In that case, simply drag a corner of the graph to increase its size, which will allow more bar labels to be shown.
Scenario analysis results are graphically displayed in tornado graphs. A tornado graph can be generated by clicking the Tornado graph icon in the Scenarios window or clicking the Scenarios icon on a graph window. This tornado graph shows the key inputs affecting the output when the output meets the entered scenario, such as when the output is above its 90th percentile.
Scatter Plots

@RISK provides scatter plots to show the relationship between a simulated output and the samples from an input distribution. Scatter graphs can be created by:

- Clicking the **Scatter Plot** icon on a displayed graph and then selecting the cell(s) in Excel whose results you want to include in the plot
- Selecting one or more outputs, or inputs, in the Results Summary window and clicking the **Scatter Plot** icon
- Dragging a bar (representing the input you want to show in the scatter) from an output’s **tornado graph**
- Displaying a scatter plot matrix in the Sensitivity Analysis window *(see the Sensitivities Command in this chapter)*
- Clicking on a **correlation matrix in Browse mode** displays a scatter plot matrix showing the simulated correlations between the inputs correlated in the matrix

As with other @RISK graphs, scatter plots will update in real-time when a simulation runs.

A scatter plot is an x-y graph showing the input value sampled vs. the output value calculated in each iteration of the simulation. A confidence ellipse identifies the region where, at a given confidence level, the x-y values will fall. Scatter graphs may also be standardized so that values from multiple inputs may be more easily compared on a single scatter plot.

**Note:** Scatter graphs are always displayed in floating, not callout windows.
Scatter graphs, like many other @RISK graphs, may be overlaid. This shows how the values for two (or more) inputs are related to the value of an output.

Multiple outputs may also be included in a scatter plot overlay. This is useful in examining how the same input affects different simulation outputs.

In the above scatter plot, the input has a large affect on the output Net Income/ 2010 but no impact on the output Net Income/ 2011.

**Note:** Overlays may be added to a scatter plot by clicking the Add icon (with a plus sign) shown at the bottom of the graph window.
For scatter plots, the **Graph Options — Scatter Tab** specifies whether values displayed in a scatter plot will be standardized, and the settings for confidence ellipses.

Options on the **Graph Options — Scatter Tab** include:

- **Standardization.** Selects whether values displayed in a scatter plot will be standardized. When values are standardized, values are displayed in terms of standard deviation change from the mean, instead of actual values. Standardization is especially useful when overlaying scatter plots from different input distributions. This allows a common scale between the inputs, making comparisons of impacts on outputs easier. **Y Values** standardization standardizes input values, while **X Values** standardization standardizes output values.

- **Confidence Ellipses (Assuming Underlying Bivariate Normal).** A confidence ellipse is generated by fitting the best bivariate normal distribution to the x-y dataset represented on the scatter plot. The region shown by the ellipse is where, given the entered confidence level, a sample from that bivariate normal will fall. Thus, if the **Confidence Level** is 99%, there is a 99% certainty that a sample from the best fitting bivariate normal distribution will fall within the displayed ellipse.
Scatter plots have both X and Y axis delimiters that can be used to shown the % of the total graph points that fall in each of the delimited quadrant of the graph. If you have overlays in your scatter plot, the % value for each displayed plot are color coded.

As with distribution graphs, the number of plots in an overlay graph for which percentages are reported can be set in on the Delimiters tab of the Graph Options dialog.

If you zoom in on a region of the scatter plot, the % value shown in each quadrant represents the % of the total graph points that are in the visible quadrant (where total graph points = the total # of points in the original “non-zoomed” graph).

**Note:** Grabbing the crossing point of the X and Y axis delimiters allows you to adjust both delimiters at the same time.
Summary Graphs

@RISK has two types of graphs that summarize trends across a group of simulated outputs (or inputs). These are the Summary Trend graph and Box Plot. Each of these graphs can be made by:

- Clicking the Summary Graph icon at the bottom of a graph window and then selecting the cell(s) in Excel whose results you want to include in the plot.

- Selecting the rows in the @RISK Results Summary Window for the outputs, or inputs, you wish to include in the summary graph, then clicking the Summary Graph icon at the bottom of the window (or right-clicking in the table), and selecting Summary Trend or Summary Box Plot.

For an output range, you can also click on the Range Name header and select Summary Graph.

Summary Trend and Summary Box-Plot graphs can be interchanged for a shown summary graph. To change the type of graph shown, simply click the appropriate icon in the bottom left of the graph window and select the new graph type.

Note: Elements may be added to a summary graph by clicking the Add icon (with a plus sign) shown at the bottom of the graph window.
**Summary Trend**

A Summary Trend graph summarizes changes in multiple probability distributions or an output range. The Summary graph takes five parameters from each selected distribution — the mean, two upper and two lower band values — and graphs the changes in the five parameters across the output range. The upper band values default to +1 standard deviation and the 95th percentile of each distribution, while the two lower band values default to -1 standard deviation and the 5th percentile of each distribution. These may be changed by using the Trend Tab options in the Graph Options dialog box.

The Summary graph is especially useful in displaying changes in risk over time. For example, an output range may be an entire worksheet row — such as Profit by Year. The Summary graph would then show the trends in the distributions for profit year to year. The wider the band around the mean, the larger the variability in possible results.

When generating a Summary graph, @RISK calculates the mean and the four band values (such as 5th and 95th percentile) for each cell in the output range graphed. These points are graphed with hi-lo lines. Patterns in between the points for each cell are then added. The mean, and two band values for these added points, are calculated by interpolation.
The Graph Options — Trend Tab specifies the values displayed in each band of the Summary Trend graph, and the colors for those bands.

Options on the Graph Options — Trend Tab include:

- **Statistics.** Selects the values displayed for the Center Line, the Inner Band and the Outer Band of the Summary Trend graph. Settings include:
  - Center Line — selects Mean, Median or Mode
  - Inner Band, Outer Band — selects the range each band will describe. The inner band must always be “narrower” than the outer band — that is, you must pick a set of statistics that include a larger range of the distribution for the outer band vs. the inner band.

- **Formatting.** Selects the color, and shading, used for each of the three bands in the Summary Trend graph.
Summary Box-Plot

A Summary Box-Plot displays a box plot for each distribution selected for inclusion in the summary graph. A box plot (or box-whisker graph) shows a box for a defined inner range of a distribution, with whisker lines showing the outer limits of the distribution. An inner line in the box marks the location of the mean, median, or mode of the distribution.
The **Graph Options — Box-Whisker Tab** specifies the values used for the Center Line, Box, and Whiskers, in each box of the Summary Box Plot graph and the colors for the boxes.

Options on the **Graph Options — Box-Whisker Tab** include:

- **Statistics.** Selects the values displayed for the Center Line, the Box and the Whiskers of the Box-Plot. Settings include:
  - **Center Line** — selects Mean, Median or Mode
  - **Box** — selects the range each box will describe. The range for the box must always be “narrower” than the whiskers — that is, you must pick a set of statistics that include a larger range of the distribution for the whiskers vs. the box.
  - **Whiskers** — selects the end points for the whiskers.

- **Formatting.** Selects the color and shading used for the box.
When multiple simulations are run, a summary graph can be made for sets of result distributions in each simulation. Often it is desirable to compare the summary graphs, created for the same distributions, in different simulations. This comparison shows how the trend in expected value and risk changes for the distributions by simulation.

To create a summary graph that compares the results for a range of cells in multiple simulations:

1) Run multiple simulations, by setting the **Number of Simulations** in the Simulation Settings dialog box to a value greater than one. Use the **RiskSimtable** function to change worksheet values by simulation.

2) Click the **Summary Graph icon** at the bottom of the displayed Browse window for the first cell to be added to the Summary Graph.

3) Select the cells in Excel whose results you want to add to the graph.

4) Select **All Simulations** from the dialog.
To create a summary graph that compares the results for a single cell across multiple simulations, follow the previous steps but in Step 3 only select a single cell in Excel to include in the summary graph. The displayed graph shows the five parameters from the cell’s distribution (the mean, two upper and two lower band values) in each simulation. This summarizes how the distribution for the cell changed by simulation.

Summary graphs of multiple simulations can also be made by selecting the rows in the @RISK Results Summary Window for the outputs or inputs (by simulation) you wish to include in the summary graph. Then click the Summary Graph icon at the bottom of the window (or right-click in the table), and select Summary Trend or Summary Box Plot.
Formatting Graphs

@RISK graphics use a graphing engine designed specifically for processing simulation data. Graphs can be customized and enhanced as needed; titles, legends, colors, scaling and other settings can all be controlled through the selections in the Graph Options dialog. The Graph Options dialog is displayed by right clicking on a graph, and selecting the Graph Options command, or by clicking the Graph Options icon in the bottom left of the graph window.

The available options on tabs in the Graph Options dialog are described here. **Note – not all options are available for all graph types, and available options may change by graph type.**

**Graph Options – Title Tab**

The options on the **Graph Options – Title Tab** specify the titles that will be displayed on the graph. An entry for main graph title and description are available. If you do not enter a title, @RISK will automatically assign one for you based on the name(s) of the output or input cells being graphed.
The options on the **Graph Options — X and Y Axis Tabs** specify the scaling and axis titles that will be used in the graph. A **Scale Factor** (such as thousands or millions) can be applied to entered axis minimum and maximum values and number of axis ticks may be changed. Axis scaling may also be changed directly on the graph by dragging the limits of an axis to a new minimum or maximum position. The **Graph Options — X Axis Tab** for a distribution graph is shown here.

![Graph Options](image)

**Note:** Depending on the type of graph in use, the options displayed on the X and Y axis tabs may be different; as different scaling options are available for different types of graphs (summary, distribution, scatter, etc.).
**Graph Options – Curves Tab**

The options on the **Graph Options — Curves Tab** specify the color, style and value interpolation for each curve in the graph. The definition of a “curve” changes depending on the type of graph. For example, in a histogram or cumulative graph, a curve is associated with the primary graph and each overlay. In a scatter graph, a curve is associated with each X-Y data set shown on the graph. Clicking on a curve in the **Curves**: list displays the available options for that curve.
The options on the **Graph Options — Legend Tab** specify the statistics that will be displayed with the graph.

A number of statistics may be displayed for each curve in a graph. The available statistics change depending on the type of graph displayed. These statistics are copied with the graph when it is pasted into a report. They also update as a simulation runs. To change the statistics displayed with a graph:

1) Uncheck **Automatic** to allow customization of the displayed statistics

2) Check the **Statistics** desired

3) Click **Redefine** to change the percentile values that will be reported, if desired

To remove the statistics from a graph:

- Change the **Style** option to **Simple Legend**.

To remove legend and statistics from a graph:

- Change **Show** option to **Never**.
Graph Options — Other Tab

The options on the **Graph Options — Other Tab** specify other available settings for a displayed graph. These include the **Basic Color Scheme** used and the **formatting of numbers and dates** displayed in the graph.

Numbers displayed on a graph can be formatted to show the level of precision desired using the **Number Formats** options shown on the **Other** tab. The available numbers for formatting changes depending on the type of graph displayed.

Dates displayed on a graph can be formatted to show the level of precision desired using the **Date Formats** options shown on the **Other** tab. The available dates for formatting changes depending on the type of graph displayed.

For distribution graphs, **Statistics (Unitless)** refers to reported statistics such as Skewness and Kurtosis that are not in the units of the values for the graph. **Statistics (with Units)** refers to reported statistics such as Mean and Standard Deviation that use the units of the graph.
Often graphs may be formatted by simply clicking on the appropriated element in the graph. For example, to change the title of a graph, simply click on the title and type the new entry.

Items that can be formatted directly on a graph include:

- **Titles** — simply click on the title in the graph and type in the new entry
- **X-Axis Scaling** — select the end line of the axis and move it to rescale the graph
- **Delete an Overlay** — right-click on the colored legend of the curve you wish to delete and select Remove Curve

In addition, the menu displayed when you right-click on a graph allows quick access to formatting items associated with the location of your click.
Introduction

@RISK includes custom functions that can be included in Excel cells and formulas. These functions are used for:

1) **Defining probability distributions** (@RISK distribution functions and distribution property functions).

2) **Defining simulation outputs** (RiskOutput function)

3) ** Returning simulation results to your spreadsheet** (@RISK statistics and graphing functions)

This reference chapter describes each of these types of @RISK functions, and gives details about both the required and optional arguments for each function.

Distribution Functions

Probability distribution functions are used for adding uncertainty — in the form of probability distributions — to the cells and equations in your Excel worksheet. For example, you could enter RiskUniform(10,20) to a cell in your worksheet. This specifies that the values for the cell will be generated by a uniform distribution with a minimum of 10 and a maximum of 20. This range of values replaces the single “fixed” value required by Excel.

Distribution functions are used by @RISK, during a simulation, for sampling sets of possible values. Each iteration of a simulation uses a new set of values sampled from each distribution function in your worksheet. These values are then used in recalculating your worksheet and generating a new set of possible results.

As with Excel functions, distribution functions contain two elements, a function name and argument values which are enclosed in parentheses. A typical distribution function is:

\[ \text{RiskNormal}(100,10) \]
A different distribution function is used for each type of probability distribution. The type of distribution which will be sampled is given by the name of the function. The parameters which specify the distribution are given by the arguments of the function.

The number and type of arguments required for a distribution function vary by function. In some cases, such as with:

\textbf{RiskNormal(mean,standard deviation)}

a fixed number of arguments are specified each time you use the function. For others, such as DISCRETE, you specify the number of arguments you desire, based on your situation. For example, a DISCRETE function may specify two or three outcomes, or possibly more, as needed.

Like Excel functions, distribution functions may have arguments which are references to cells or expressions. For example:

\textbf{RiskTriang(B1,B2\times 1.5,B3)}

In this case the cell value would be specified by a triangular distribution with a minimum value taken from cell B1, a most likely value calculated by taking the value for cell B2 and multiplying it by 1.5, and a maximum value taken from cell B3.

Distribution functions also may be used in cell formulas, just as are Excel functions. For example, a cell formula could read:

\textbf{B2: 100+RiskUniform(10,20)+(1.5\times RiskNormal(A1,A2))}

All standard Excel editing commands are available to you when entering distribution functions. However, you will need to have @RISK loaded for the distribution functions to be sampled by Excel.

To enter probability distribution functions:

- Examine your worksheet and identify those cells which you think have uncertain values

Look for those cells where the actual values which occur could vary from those shown in the worksheet. At first, identify those important variables whose cell values may have the largest variation in value. As your Risk Analysis gets more refined, you can further expand your use of distribution functions throughout the worksheet.

- Select distribution functions for the cells you have identified. In Excel, use the Insert menu Function command to enter the selected functions into formulas.
You have over thirty types of distributions to choose from when selecting a distribution function. Unless you know specifically how uncertain values are distributed, it is a good idea to start with some of the simpler distribution types — uniform, triangular, or normal. As a starting point, if possible, specify the current cell value as the mean, or most likely, value of the distribution function. The range of the function you are using then reflects the possible variation around the mean or most likely value.

The simple distribution functions can be very powerful as you can describe uncertainty with only a few values or arguments. For example:

- **RiskUniform** (*Minimum, Maximum*) uses only two values to describe the full range of the distribution and assign probabilities for all the values in the range.

- **RiskTriang** (*Minimum, Most Likely, Maximum*) uses three easily identifiable values to describe a complete distribution.

As your models become more complex, you probably will want to choose from more complex distribution types in order to meet your specific modeling needs. Use the listings in this Reference section to guide you in selecting and comparing distribution types.

A graph of the distribution is often helpful in selecting and specifying distribution functions. You can use the @RISK Define Distributions window to display distribution graphs and add distribution functions to cell formulas. To do this, select the cell where you wish to add a distribution function and click the Define Distribution icon, or the @RISK add-in menu Model Define Distributions command. The online file also contains graphic depictions of different functions at selected argument values. For more information on the Define Distribution window, see the Model commands : Define Distributions Command in the @RISK Commands section in this manual.

It often helps to first use the Define Distribution window to enter your distribution functions to better understand how to assign values to function arguments. Then, once you better understand the syntax of distribution function arguments, you can enter the arguments yourself directly in Excel, bypassing the Define Distribution window.

**Defining Distributions Graphically**
RISK (Professional and Industrial versions only) allows you to fit probability distributions to your data. The distributions which result from a fit are then available as input distributions that can be added to your spreadsheet model. For more information on distribution fitting see the **Fit Distributions to Data command** in this manual.

Optional arguments to distribution functions can be entered using **Distribution Property** functions. These optional arguments are used to name an input distribution for reporting and graphing, truncate the sampling of a distribution, correlate the sampling of a distribution with other distributions, and keep a distribution from being sampled during a simulation. These arguments are not required, but can be added as needed.

Optional arguments specified using @RISK distribution property functions are embedded inside of a distribution function. Distribution Property functions are entered, just as are standard Excel functions, and can include cell references and mathematical expressions as arguments.

For example, the following function truncates the entered normal distribution to a range with a minimum value of 0 and a maximum value of 20:

\[
=\text{RiskNormal}(10,5,\text{RiskTruncate}(0,20))
\]

No samples will be drawn outside this minimum-maximum range.

Supplemental functions such as **RiskTNormal**, **RiskTExpon** and **RiskTLognorm** were used in versions of @RISK prior to 4.0 to truncate distributions such as normal, exponential and lognormal. These distribution functions can still be used in newer versions of @RISK; however, their functionality has been replaced by the **RiskTruncate** distribution property function, a more flexible implementation which can be used with any probability distribution. Graphs of these older functions are not displayed in the Define Distribution window; however they will be shown in the Model window and can be used in simulations.

Many distribution functions can be entered by specifying percentile values for the distribution you want. For example, you may want to enter a distribution that is normal in shape and has a 10th percentile of 20 and a 90th percentile of 50. These percentiles may be the only values you know about this normal distribution — the actual mean and standard deviation, required by the traditional normal distribution, are unknown.
Alternate parameters may be used instead of (or in conjunction with) the standard arguments for the distribution. When entering percentile arguments, the **Alt** form of the distribution function is used, such as **RiskNormalAlt** or **RiskGammaAlt**.

Each parameter to an alternate parameter distribution function requires a pair of arguments in the function. Each pair of arguments specifies:

1) The **type of parameter** being entered
2) The **value for the parameter**.

Each argument in a pair is entered directly in the Alt function, such as **RiskNormalAlt**(arg1type, arg1value, arg2type, arg2value). For example:

- **RiskNormalAlt**(5%, 67.10, 95%, 132.89) — specifies a normal distribution with the 5th percentile at the value of 67.10 and the 95th percentile at the value of 132.89.

Alternate parameters may be percentiles or standard distribution arguments. If a **type of parameter** argument is a label in quotes (such as “**mu**”), the parameter specified is the standard distribution argument that has the entered name. This allows percentiles to be mixed with standard distribution arguments, such as:

- **RiskNormalAlt**("**mu**", 100, 95%, 132.89) — specifies a normal distribution with a mean of 100 and the 95th percentile at the value of 132.89.

The allowable names for the standard arguments of each distribution can be found in the heading for each function in this chapter, in the Excel Function Wizard in the @RISK Distrib (Alt Param) category, or by using the Define Distribution window.

**Types of Alternate Parameters**

- Percentiles
- Standard distribution arguments
- Labels in quotes
- Values between 0 and 1 or 0% to 100%

**Note:** You can specify Alternate Parameters under the Parameters option for a specific distribution in the Define Distribution window. If your parameters include a standard argument and you click OK, @RISK will write the appropriate name for the standard argument in quotes in the function in the formula bar of the Define Distribution window.

If a **type of parameter** argument is a value between 0 and 1 (or 0% to 100%), the parameter specified is the entered percentile for the distribution.
Some distributions will have an additional **location** parameter when they are specified using alternate parameters. This parameter is typically available for distributions that do not have a location value specified in one of their standard arguments. **Location is equivalent to the minimum or 0 perc% value of the distribution.** For example, the Gamma distribution does not have a location value specified through its standard arguments, and thus a location parameter is available. The normal distribution, on the other hand, does have a location parameter in its standard arguments — the mean or mu — and thus does not have a separate location parameter when it is entered using alternate parameters. The purpose of this “extra” parameter is to allow you to specify percentiles for shifted distributions (e.g. a three parameter Gamma with a location of 10 and two percentiles).

During a simulation @RISK calculates the appropriate distribution whose percentile values equal those alternate parameter values entered, and then samples that distribution. Just like all @RISK functions, the entered arguments may be references to other cells or formulas, just as with any Excel function; and argument values may change iteration to iteration during a simulation.

Alternate percentile parameters to probability distributions may be specified in terms of cumulative descending percentiles, as well as the standard cumulative ascending percentiles. Each of the Alt forms for probability distribution functions (such as `RiskNormalAlt`) has a corresponding AltD form (such as `RiskNormalAltD`). If the AltD form is used, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.

If you select the @RISK Utilities menu Application Settings command Descending Percentiles option, all @RISK reports will show cumulative descending percentile values. In addition, when you select the Alternate Parameters option in the Define Distribution window to enter distributions using alternate parameters, cumulative descending percentiles will automatically be shown and AltD forms of probability distribution functions will be entered.

In addition to cumulative descending percentiles for alternate parameters distributions, the @RISK cumulative probability distribution (`RiskCumul`) may also be specified using cumulative descending percentiles. To do this, use the function `RiskCumulD`. 
The guidelines for entering Excel functions presented in the relevant User’s Guide are also applicable to entering @RISK functions. However, some additional guidelines specific to @RISK functions are:

- Where integer arguments are required by a distribution function, any non-integer argument values will be truncated to integers.

- Distribution functions with varying numbers of arguments (such as HISTOGRM, DISCRETE, and CUMUL) require that arguments of the same type be entered as arrays. Arrays in Excel are denoted by either enclosing the values of the array in {} brackets or using a reference to a contiguous range of cells — such as A1:C1. If a function takes a varying number of value/probability pairs, the values will be one array and the probabilities another. The first value in the value array is matched with the first probability in the probability array and so on.

@RISK supports the entry of dates in distribution functions and the display of graphs and statistics using dates. A RiskIsDate(TRUE) property function instructs @RISK to display graphs and statistics using dates. @RISK will also display dates in the Distribution Argument panel in the Define Distribution window when date formatting is enabled. You can specify that date formatting should be used for a distribution by selecting Date Formatting in the Parameters window of the Distribution Argument panel, or enabling Date Formatting in the Input Properties dialog. Any of these selections will result in a RiskIsDate property function being placed in your distribution.

Typically date arguments to @RISK distribution functions are entered with references to cells where the desired dates are entered. For example:

=RiskTriang(A1,B1,C1,RiskIsDate(TRUE))


Date arguments entered directly in @RISK distribution functions need to be entered using an Excel function that converts a date to a value. Several Excel functions are available to do this. For example, the function for a triangular distribution with a minimum value of 10/1/2009, a most likely value of 1/1/2010 and a maximum value of 10/10/2010, is entered as:

=RiskTriang(DATEVALUE("10/1/2009"),DATEVALUE("1/1/2010"),DATEVALUE("10/10/2010"),RiskIsDate(TRUE))
Here the Excel DATEVALUE function is used to convert the entered dates to values. The function:

\[ \text{=RiskTriang(DATE(2009,10,4)+TIME(2,27,13),DATE(2009,12,29)+TIME(2,25,4),DATE(2010,10,10)+TIME(11,46,30),RiskIsDate(TRUE))} \]

uses the Excel DATE and TIME functions to convert the entered dates and times to values. The advantage of this approach is that the entered dates and times will convert properly if the workbook is moved to a system with different dd/mm/yy formatting.

Not all arguments for all functions can be logically specified with dates. For example, functions such as RiskNormal(mean, stdDev) support a mean entered as a date but not a standard deviation. The Distribution Argument panel in the Define Distribution window shows the type of data (dates or numeric) that can be entered for each distribution type when date formatting is enabled.

Some @RISK functions have optional arguments, or arguments that may be used but are not required. The RiskOutput function, for example, has only optional arguments. You can use it with 0, 1 or 3 arguments, depending on what information you wish to define about the output cell where the function is used. You can:

1) Just identify the cell as an output, letting @RISK automatically generate a name for you (i.e., =RiskOutput()).

2) Give the output a name you select (i.e., =RiskOutput("Profit 1999")).

3) Give the output a name you select and identify it as part of an output range (i.e., =RiskOutput("Profit 1999", "Profit By Year",1)).

Any of these forms of the RiskOutput function are allowed because all of its arguments are optional.

When an @RISK function has optional arguments you can add the optional arguments you choose and ignore the rest. You must, however, include all required arguments. For example, for the RiskNormal function, two arguments, mean and standard deviation, are required. All of the arguments which can be added to the RiskNormal function via distribution property functions are optional and can be entered in any order you like.
In Excel, you may not list cell references or names in arrays as you would list constants. For example, you could not use \{A1,B1,C1\} to represent the array containing the values in cells A1, B1, and C1. Instead, you must use the cell range reference A1:C1 or enter the values of those cells directly in the arrays as constants — for example, \{10,20,30\}.

- Distribution functions with fixed numbers of arguments will return an error value if an insufficient number of arguments is entered, and will ignore extra arguments if too many are entered.

- Distribution functions will return an error value if arguments are of the wrong type (number, array or text).

This section briefly describes each probability distribution function available and the arguments required for each. In addition, the online help file describes the technical characteristics of each probability distribution function. The appendices include formulas for density, distribution, mean, mode, distribution parameters and graphs of the probability distributions generated using typical argument values.

**Simulation Output Functions**

Output cells are defined using RiskOutput functions. These functions allow the easy copying, pasting and moving of output cells. RiskOutput functions are automatically added when the standard @RISK Add Output icon is pressed. RiskOutput functions optionally allow you to name your simulation outputs, and add output cells to output ranges. A typical RiskOutput function might be:

\[=\text{RiskOutput("Profit")}+\text{NPV}(.1,H1:H10)\]

where the cell, prior to its selection as a simulation output, simply contained the formula

\[=\text{NPV}(.1,H1:H10)\]

The added RiskOutput function selects the cell as a simulation output and gives the output the name “Profit”.

**Important Note on Excel Arrays**

More Information
Simulation Statistics Functions

@RISK statistics functions return a desired statistic on simulation results or an input distribution. For example, the function RiskMean(A10) returns the mean of the simulated distribution for the cell A10. These functions are updated real-time as a simulation is running.

@RISK statistics functions include all standard statistics plus percentiles and targets (for example, =RiskPercentile(A10,.99) returns the 99th percentile of the simulated distribution). @RISK statistics functions can be used the way you would use any standard Excel function.

@RISK statistics functions that return a desired statistic on a simulation input distribution have the identifier Theo in the function name. For example, the function RiskTheoMean(A10) returns the mean of the probability distribution in the cell A10. If multiple distribution functions are present in the formula for a cell referenced in a RiskTheo statistics function, @RISK returns the desired statistic on the last calculated function in the formula. For example, in the formula in A10:

=RiskNormal(10,1)+RiskTriang(1,2,3)

The function RiskTheoMean(A10) returns the mean of RiskTriang(1,2,3).

In a different formula in A10:

=RiskNormal(10,RiskTriang(1,2,3))

The function RiskTheoMean(A10) returns the mean of RiskNormal(10,RiskTriang(1,2,3)), since the function RiskTriang(1,2,3) is nested inside the RiskNormal function.

@RISK statistics functions can include a RiskTruncate or a RiskTruncateP property function. This will cause the statistic to be calculated on the min-max range specified by the truncation limits.

Note: The values returned from @RISK statistics functions only reflect the range set using a RiskTruncate or a RiskTruncateP property function entered in the statistics function itself. Filters set for simulation results and shown in @RISK graphs and reports do not impact the values returned by @RISK statistics functions.
Statistics functions may also reference a simulation output or input by name. This allows them to be included in templates which are used to generate pre-formatted reports in Excel on simulation results. For example, the function =RiskMean("Profit") would return the mean of the simulated distribution for the output cell named Profit defined in a model.

**Note:** A cell reference entered in a statistics function does not have to be a simulation output identified with a RiskOutput function.

### Graphing Function

A special @RISK function RiskResultsGraph will automatically place a graph of simulation results, wherever it is used, in a spreadsheet. For example, =RiskResultsGraph(A10) would place a graph of the simulated distribution for A10 directly in your spreadsheet at the function's location at the end of a simulation. Additional optional arguments to RiskResultsGraph allow you to select the type of graph you want to create, its format, scaling and other options.

### Supplemental Functions

Additional functions such as RiskCurrentIter, RiskCurrentSim, and RiskStopSimulation are provided for use in the development of macro-based applications using @RISK. These functions return the current iteration and current simulation, respectively, of an executing simulation, or stop a simulation.
## Table of Available Functions

This table lists the custom functions that are added to Excel by @RISK.

<table>
<thead>
<tr>
<th>Distribution Function</th>
<th>Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskBeta</strong>(alpha1,alpha2)</td>
<td>beta distribution with shape parameters alpha1 and alpha2</td>
</tr>
<tr>
<td><strong>RiskBetaGeneral</strong>( alpha1, alpha2,minimum, maximum)</td>
<td>beta distribution with defined minimum, maximum and shape parameters alpha1 and alpha2</td>
</tr>
<tr>
<td><strong>RiskBetaGeneralAlt</strong>(arg1type, arg1value, arg2type,arg2value, arg3type, arg3value, arg4type, arg4value)</td>
<td>beta distribution with four parameters named arg1type to arg4type which can be either a percentile between 0 and 1 or “alpha1”, “alpha2”, “min” or “max”</td>
</tr>
<tr>
<td><strong>RiskBetaSubj</strong>(minimum, most likely, mean, maximum)</td>
<td>beta distribution with defined minimum, maximum, most likely and mean</td>
</tr>
<tr>
<td><strong>RiskBinomial</strong>(n,p)</td>
<td>binomial distribution with n draws and p probability of success on each draw</td>
</tr>
<tr>
<td><strong>RiskChiSq</strong>(v)</td>
<td>Chi-Square distribution with v degrees of freedom</td>
</tr>
<tr>
<td><strong>RiskCompound</strong>(dist#1 or value or cellref,dist#2,deductible,limit)</td>
<td>the sum of a number of samples from dist#2 where the number of samples drawn from dist#2 is given by the value sampled from dist#1 or by value. Optionally, deductible is subtracted from each dist#2 sample and if (dist#2 sample-deductible) exceeds limit dist#2 sample is set equal to limit.</td>
</tr>
<tr>
<td><strong>RiskCumul</strong>(minimum,maximum, {X1,X2,...,Xn},{p1,p2,...,pn})</td>
<td>cumulative distribution with n points between minimum and maximum with cumulative ascending probability p at each point</td>
</tr>
<tr>
<td><strong>RiskCumulD</strong>(minimum,maximum, {X1,X2,...,Xn},{p1,p2,...,pn})</td>
<td>cumulative distribution with n points between minimum and maximum with cumulative descending probability p at each point</td>
</tr>
<tr>
<td><strong>RiskDiscrete</strong>((X1,X2,...,Xn), (p1,p2,...,pn))</td>
<td>discrete distribution with n possible outcomes with the value X and probability weight p for each outcome</td>
</tr>
<tr>
<td><strong>RiskDuniform</strong>((X1,X2,...Xn))</td>
<td>discrete uniform distribution with n outcomes valued at X1 through Xn</td>
</tr>
<tr>
<td><strong>RiskErf</strong>(h)</td>
<td>error function distribution with variance parameter h</td>
</tr>
<tr>
<td><strong>RiskErlang</strong>(m,beta)</td>
<td>m-erlang distribution with integral shape parameter m and scale parameter beta</td>
</tr>
<tr>
<td><strong>RiskExpon</strong>(beta)</td>
<td>exponential distribution with decay constant beta</td>
</tr>
</tbody>
</table>

Reference: @RISK Functions
<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskExponAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>exponential distribution with two parameters named arg1type and arg2type which can be either a percentile between 0 and 1 or “beta” or “loc”</td>
</tr>
<tr>
<td>RiskExtvalue(alpha, beta)</td>
<td>extreme value (or Gumbel) distribution with location parameter alpha and scale parameter beta</td>
</tr>
<tr>
<td>RiskExtvalueAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>extreme value (or Gumbel) distribution with two parameters named arg1type and arg2type which can be either a percentile between 0 and 1 or “alpha” or “beta”</td>
</tr>
<tr>
<td>RiskGamma(alpha, beta)</td>
<td>gamma distribution with shape parameter alpha and scale parameter beta</td>
</tr>
<tr>
<td>RiskGammaAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>gamma distribution with three parameters named arg1type, arg2type and arg3type which can be either a percentile between 0 and 1 or “alpha”, “beta” or “loc”</td>
</tr>
<tr>
<td>RiskGeneral(minimum, maximum, {X1, X2, ..., Xn}, {p1, p2, ..., pn})</td>
<td>general density function for a probability distribution ranging between minimum and maximum with n (x,p) pairs with value X and probability weight p for each point</td>
</tr>
<tr>
<td>RiskGeometric(p)</td>
<td>geometric distribution with probability p</td>
</tr>
<tr>
<td>RiskHistogrm(minimum, maximum, {p1, p2, ..., pn})</td>
<td>histogram distribution with n classes between minimum and maximum with probability weight p for each class</td>
</tr>
<tr>
<td>RiskHypergeo(n, D, M)</td>
<td>hypergeometric distribution with sample size n, D number of items and M population size</td>
</tr>
<tr>
<td>RiskIntUniform(minimum, maximum)</td>
<td>uniform distribution which returns integer values only between minimum and maximum</td>
</tr>
<tr>
<td>RiskInvGauss(mu, lambda)</td>
<td>inverse gaussian (or Wald) distribution with mean mu and shape parameter lambda</td>
</tr>
<tr>
<td>RiskInvGaussAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>inverse gaussian (or Wald) distribution with three parameters named arg1type, arg2type and arg3type which can be either a percentile between 0 and 1 or “mu”, “lambda” or “loc”</td>
</tr>
<tr>
<td>RiskJohnsonSB(alpha1, alpha2, a, b)</td>
<td>Johnson “system bounded” distribution with the entered alpha1, alpha2, a and b values</td>
</tr>
<tr>
<td>RiskJohnsonSU(alpha1, alpha2, gamma, beta)</td>
<td>Johnson “system unbounded” distribution with the entered alpha1, alpha2, gamma and beta values</td>
</tr>
<tr>
<td>RiskJohnsonMoments(mean, standardDeviation, skewness, kurtosis)</td>
<td>a distribution from the Johnson family of distributions (normal, lognormal, JohnsonSB, and JohnsonSU) that has as its moments the entered mean, standardDeviation, skewness, and kurtosis parameters</td>
</tr>
<tr>
<td>RiskLogistic(alpha, beta)</td>
<td>logistic distribution with location parameter alpha and scale parameter beta</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>RiskLogisticAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>logistic distribution with two parameters named arg1type and arg2type which can be either a percentile between 0 and 1 or “alpha” or “beta”</td>
</tr>
<tr>
<td>RiskLoglogistic(gamma, beta, alpha)</td>
<td>log-logistic distribution with location parameter gamma, scale parameter beta and shape parameter alpha</td>
</tr>
<tr>
<td>RiskLoglogisticAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>log-logistic distribution with three parameters named arg1type, arg2type and arg3type which can be either a percentile between 0 and 1 or “gamma”, “beta” or “alpha”</td>
</tr>
<tr>
<td>RiskLognorm(mean, standard deviation)</td>
<td>lognormal distribution with specified mean and standard deviation</td>
</tr>
<tr>
<td>RiskLognormAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>lognormal distribution with three parameters named arg1type, arg2type and arg3type which can be either a percentile between 0 and 1 or “mu”, “sigma” or “loc”</td>
</tr>
<tr>
<td>RiskLognorm2(mean, standard deviation)</td>
<td>lognormal distribution generated from the “log” of a normal distribution with specified mean and standard deviation</td>
</tr>
<tr>
<td>RiskMakeInput(formula)</td>
<td>specifies that the calculated value for formula will be treated as a simulation input, just as is a distribution function</td>
</tr>
<tr>
<td>RiskNegbin(s, p)</td>
<td>negative binomial distribution with s successes and p probability of success on each trial</td>
</tr>
<tr>
<td>RiskNormal(mean, standard deviation)</td>
<td>normal distribution with given mean and standard deviation</td>
</tr>
<tr>
<td>RiskNormalAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>normal distribution with two parameters named arg1type and arg2type which can be either a percentile between 0 and 1 or “mu” or “sigma”</td>
</tr>
<tr>
<td>RiskPareto(theta, alpha)</td>
<td>pareto distribution</td>
</tr>
<tr>
<td>RiskParetoAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>pareto distribution with two parameters named arg1type and arg2type which can be either a percentile between 0 and 1 or “theta” or “alpha”</td>
</tr>
<tr>
<td>RiskPareto2(b, q)</td>
<td>pareto distribution</td>
</tr>
<tr>
<td>RiskPareto2Alt(arg1type, arg1value, arg2type, arg2value)</td>
<td>pareto distribution with two parameters named arg1type and arg2type which can be either a percentile between 0 and 1 or “b” or “q”</td>
</tr>
<tr>
<td>RiskPearson5(alpha, beta)</td>
<td>Pearson type V (or inverse gamma) distribution with shape parameter alpha and scale parameter beta</td>
</tr>
<tr>
<td>RiskPearson5Alt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>Pearson type V (or inverse gamma) distribution with three parameters named arg1type, arg2type and arg3type which can be either a percentile between 0 and 1 or “alpha”, “beta” or “loc”</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>--------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>RiskPearson6(beta, alpha1, alpha2)</td>
<td>Pearson type VI distribution with scale parameter beta and shape parameters alpha1 and alpha2</td>
</tr>
<tr>
<td>RiskPert(minimum, most likely, maximum)</td>
<td>Pert distribution with specified minimum, most likely and maximum values</td>
</tr>
<tr>
<td>RiskPertAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>Pert distribution with three parameters named arg1type, arg2type and arg3type which can be either a percentile between 0 and 1 or “min”, “max” or “m. likely”</td>
</tr>
<tr>
<td>RiskPoisson(lamda)</td>
<td>Poisson distribution</td>
</tr>
<tr>
<td>RiskRayleigh(beta)</td>
<td>Rayleigh distribution with scale parameter beta</td>
</tr>
<tr>
<td>RiskRayleighAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>Rayleigh distribution with two parameters named arg1type and arg2type which can be either a percentile between 0 and 1 or “beta” or “loc”</td>
</tr>
<tr>
<td>RiskResample(sampMethod, {X1, X2, ...Xn})</td>
<td>Samples using sampMethod from a data set with n possible outcomes with an equal probability of each outcome occurring.</td>
</tr>
<tr>
<td>RiskSimtable({X1, X2, ...Xn})</td>
<td>Lists values to be used in each of a series of simulations</td>
</tr>
<tr>
<td>RiskSplice(dist#1 or cellref, dist#2 or cellref, splice point)</td>
<td>Specifies a distribution created by splicing dist#1 to dist#2 at the x-value given by splice point.</td>
</tr>
<tr>
<td>RiskStudent(nu)</td>
<td>Student's t distribution with nu degrees of freedom</td>
</tr>
<tr>
<td>RiskTriang(minimum, most likely, maximum)</td>
<td>Triangular distribution with defined minimum, most likely and maximum values</td>
</tr>
<tr>
<td>RiskTriangAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>Triangular distribution with three parameters named arg1type, arg2type and arg3type which can be either a percentile between 0 and 1 or “min”, “max” or “m. likely”</td>
</tr>
<tr>
<td>RiskTrigen(bottom, most likely, top, bottom perc., top perc.)</td>
<td>Triangular distribution with three points representing value at bottom percentile, most likely value and value at top percentile.</td>
</tr>
<tr>
<td>RiskUniform(minimum, maximum)</td>
<td>Uniform distribution between minimum and maximum</td>
</tr>
<tr>
<td>RiskUniformAlt(arg1type, arg1value, arg2type, arg2value)</td>
<td>Uniform distribution with two parameters named arg#type which can be either a percentile between 0 and 1 or “min” or “max”</td>
</tr>
<tr>
<td>RiskWeibull(alpha, beta)</td>
<td>Weibull distribution with shape parameter alpha and scale parameter beta</td>
</tr>
<tr>
<td>RiskWeibullAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)</td>
<td>Weibull distribution with three parameters named arg1type, arg2type and arg3type which can be either a percentile between 0 and 1 or “alpha”, “beta” or “loc”</td>
</tr>
<tr>
<td>Distribution Property Function</td>
<td>Specifies</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>RiskCategory(categoryName)</td>
<td>Names the category to be used in when displaying an input distribution.</td>
</tr>
<tr>
<td>RiskCollect()</td>
<td>Causes samples to be collected during a simulation for the distribution in which the Collect function is included (when simulation settings specify Collect Distribution Samples for Inputs Marked with Collect)</td>
</tr>
<tr>
<td>RiskConvergence(tolerance, toleranceType, confidenceLevel, useMean, useStdDev, usePercentile, percentile)</td>
<td>Specifies convergence monitoring information for an output.</td>
</tr>
<tr>
<td>RiskCormat(matrix cell range, position, instance)</td>
<td>Identifies a matrix of rank correlation coefficients and a position in the matrix for the distribution in which the Corrmat function is included. Instance specifies the instance of the matrix at matrix cell range that will be used for correlating this distribution.</td>
</tr>
<tr>
<td>RiskDepC(ID,coefficient)</td>
<td>Identifies dependent variable in correlated sampling pair with rank correlation coefficient and ID identifier string</td>
</tr>
<tr>
<td>RiskFit(ProjID,FitID,selected fit result)</td>
<td>Links a data set identified by ProjID and FitID and its fit results to the input distribution so the input can be updated when data changes</td>
</tr>
<tr>
<td>RiskIndepC(ID)</td>
<td>Identifies independent distribution in rank correlated sampling pair — ID is the identifier string</td>
</tr>
<tr>
<td>RiskIsDate(TRUE)</td>
<td>Specifies that the values for the input or output should be displayed as date values in graphs and reports</td>
</tr>
<tr>
<td>RiskIsDiscrete(TRUE)</td>
<td>Specifies that an output should be treated as a discrete distribution when displaying graphs of simulation results and calculating statistics</td>
</tr>
<tr>
<td>RiskLibrary(position, ID)</td>
<td>Specifies that a distribution is linked to a distribution in an @RISK Library with the entered position and ID</td>
</tr>
<tr>
<td>RiskLock()</td>
<td>Blocks sampling of the distribution in which the Lock function is included</td>
</tr>
<tr>
<td>RiskName(input name)</td>
<td>Input name for the distribution in which the Name function is included</td>
</tr>
<tr>
<td>RiskSeed(random number generator type, seed value)</td>
<td>Specifies that an input will use its own random number generator of the entered type and it will be seeded with seed value</td>
</tr>
<tr>
<td>Function</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>RiskShift(shift)</td>
<td>Shifts the domain of the distribution in which the Shift function is included by shift value</td>
</tr>
<tr>
<td>RiskSixSigma(LSL, USL, target, Long Term Shift, Number of Standard Deviations)</td>
<td>Specifies the lower specification limit, upper specification limit, target value, long term shift, and the number of standard deviations for six sigma calculations for an output</td>
</tr>
<tr>
<td>RiskStatic(static value)</td>
<td>Defines the static value 1) returned by a distribution function during a standard Excel recalculation and 2) that replaces the @RISK function after @RISK functions are swapped out</td>
</tr>
<tr>
<td>RiskTruncate(minimum, maximum)</td>
<td>Minimum-maximum range allowable for samples drawn for the distribution in which the Truncate function is included</td>
</tr>
<tr>
<td>RiskTruncateP(perc% minimum, perc% maximum)</td>
<td>Minimum-maximum range allowable (defined with percentiles) for samples drawn for the distribution in which the TruncateP function is included</td>
</tr>
<tr>
<td>RiskUnits(units)</td>
<td>Names the units to be used in labeling an input distribution or output</td>
</tr>
</tbody>
</table>

**Output Function**

<table>
<thead>
<tr>
<th>Function</th>
<th>Specifies</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskOutput(name, output range name, position in range)</td>
<td>Simulation output cell with name, output range name to which the output belongs, and the position in range (Note: all arguments to this function are optional)</td>
</tr>
<tr>
<td>Statistics Function</td>
<td>Returns</td>
</tr>
<tr>
<td>-----------------------------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>RiskConvergenceLevel</strong>(cellref or output name, Sim#)</td>
<td>Returns the level of convergence (0 to 100) for an output in Sim#. TRUE is returned on convergence.</td>
</tr>
<tr>
<td><strong>RiskCorrel</strong>(cellref1 or output/input1 name, cellref2 or output/input2 name, correlationType, Sim#)</td>
<td>Returns the correlation coefficient using correlationType for the data for the simulated distributions for cellref1 or output/input name1 and cellref2 or output/input name2 in Sim#. correlationType is either Pearson or Spearman Rank correlation.</td>
</tr>
<tr>
<td><strong>RiskKurtosis</strong>(cellref or output/input name, Sim#)</td>
<td>Kurtosis of the simulated distribution for the entered cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskMax</strong>(cellref or output/input name, Sim#)</td>
<td>Maximum value of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskMean</strong>(cellref or output/input name, Sim#)</td>
<td>Mean of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskMin</strong>(cellref or output/input name, Sim#)</td>
<td>Minimum value of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskMode</strong>(cellref or output/input name, Sim#)</td>
<td>Mode of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskPercentile</strong>(cellref or output/input name, perc%, Sim#)</td>
<td>Percentile perc% of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskPercentileD</strong>(cellref or output/input name, perc%, Sim#)</td>
<td>Percentile perc% of the simulated distribution for cellref or output/input name in Sim# (perc% is a cumulative descending percentile)</td>
</tr>
<tr>
<td><strong>RiskQtoX</strong>(cellref or output/input name, perc%, Sim#)</td>
<td>Percentile perc% of the simulated distribution for cellref or output/input name in Sim# (perc% is a cumulative descending percentile)</td>
</tr>
<tr>
<td><strong>RiskRange</strong>(cellref or output/input name, Sim#)</td>
<td>Range of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskSensitivity</strong>(cellref or output name, Sim#, rank, analysisType, returnValueType)</td>
<td>Returns the sensitivity analysis information of the simulated distribution for cellref or output name</td>
</tr>
<tr>
<td><strong>RiskSkewness</strong>(cellref or output/input name, Sim#)</td>
<td>Skewness of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskStdDev</strong>(cellref or output/input name, Sim#)</td>
<td>Standard deviation of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskTarget</strong>(cellref or output/input name, target value, Sim#)</td>
<td>Ascending cumulative probability of target value in the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td><strong>RiskXtoP</strong>(cellref or output/input name, target value, Sim#)</td>
<td>Ascending cumulative probability of target value in the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td>Function Name</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>RiskTargetD</td>
<td>Descending cumulative probability of target value in the simulated distribution for cellref, or output/input name in Sim#</td>
</tr>
<tr>
<td>RiskXtoQ</td>
<td>Descending cumulative probability of target value in the simulated distribution for cellref, or output/input name in Sim#</td>
</tr>
<tr>
<td>RiskVariance</td>
<td>Variance of the simulated distribution for cellref or output/input name in Sim#</td>
</tr>
<tr>
<td>RiskTheoKurtosis</td>
<td>Kurtosis of the distribution for the entered cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoMax</td>
<td>Maximum value of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoMean</td>
<td>Mean of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoMin</td>
<td>Minimum value of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoMode</td>
<td>Mode of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoPtoX</td>
<td>Percentile perc% of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoQtoX</td>
<td>Percentile perc% of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoRange</td>
<td>Range of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoSkewness</td>
<td>Skewness of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoStdDev</td>
<td>Standard deviation of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoXtoP</td>
<td>Ascending cumulative probability of target value in the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoXtoQ</td>
<td>Descending cumulative probability of target value in the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>RiskTheoVariance</td>
<td>Variance of the distribution for cellref or distribution function</td>
</tr>
<tr>
<td>Six Sigma Statistics Function</td>
<td>Returns</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td><strong>RiskCp</strong>&lt;br&gt;(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations))</td>
<td>Calculates the Process Capability for cellref or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskCPM</strong>&lt;br&gt;(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations))</td>
<td>Calculates the Taguchi capability index for cellref or output name in Sim# optionally using the LSL, USL and LongTerm Shift in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskCpk</strong>&lt;br&gt;(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations))</td>
<td>Calculates the Process Capability Index for cellref or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskCpkLower</strong>&lt;br&gt;(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations))</td>
<td>Calculates the one-sided capability index based on the Lower Specification limit for cellref or output name in Sim# optionally using the LSL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskCpkUpper</strong>&lt;br&gt;(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations))</td>
<td>Calculates the one-sided capability index based on the Upper Specification limit for cellref or output name in Sim# optionally using the USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskDPM</strong>&lt;br&gt;(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations))</td>
<td>Calculates the defective parts per million for cellref or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskK</strong>&lt;br&gt;(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations))</td>
<td>This function calculates a measure of process center for cellref or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskLowerXBound</strong>&lt;br&gt;(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, Long Term Shift, Number of Standard Deviations))</td>
<td>Returns the lower X-value for a given number of standard deviations from the mean for cellref or output name in Sim #, optionally using the Number of Standard Deviations in the RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskPNC</strong>&lt;br&gt;(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations))</td>
<td>Calculates the total probability of defect outside the lower and upper specification limits for cellref or output name in Sim# optionally using the LSL, USL and LongTerm Shift in the included RiskSixSigma property function.</td>
</tr>
<tr>
<td><strong>RiskPNCLower</strong>&lt;br&gt;(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations))</td>
<td>Calculates the probability of defect outside the lower specification limits for cellref or output name in Sim# optionally using the LSL and LongTerm Shift in the included RiskSixSigma property function.</td>
</tr>
<tr>
<td>Function Name</td>
<td>Description</td>
</tr>
<tr>
<td>----------------------</td>
<td>---------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>RiskPNCUpper</strong></td>
<td>Calculates the probability of defect outside the upper specification limits for cellref or output name in Sim# optionally using the USL and LongTerm Shift in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskPPMLower</strong></td>
<td>Calculates the number of defects below the lower specification limit for cellref or output name in Sim# optionally using the LSL and LongTerm Shift in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskPPMUpper</strong></td>
<td>Calculates the number of defects above the upper specification limit for cellref or output name in Sim# optionally using the USL and LongTerm Shift in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskSigmaLevel</strong></td>
<td>Calculates the Process Sigma level for cellref or output name in Sim# optionally using the USL and LSL and Long Term Shift in the included RiskSixSigma property function. (Note: This function assumes that the output is normally distributed and centered within the specification limits.)</td>
</tr>
<tr>
<td><strong>RiskUpperXBond</strong></td>
<td>Returns the upper X-value for a given number of standard deviations from the mean for cellref or output name in Sim#, optionally using the Number of Standard Deviations in the RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskYV</strong></td>
<td>Calculates the yield or the percentage of the process that is free of defects for cellref or output name in Sim# optionally using the LSL, USL and Long Term Shift in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskZlower</strong></td>
<td>Calculates how many standard deviations the Lower Specification Limit is from the mean for cellref or output name in Sim# optionally using the LSL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskZMin</strong></td>
<td>Calculates the minimum of Z-Lower and Z-Upper for cellref or output name in Sim# optionally using the USL and LSL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>RiskZUpper</strong></td>
<td>Calculates how many standard deviations the Upper Specification Limit is from the mean for cellref or output name in Sim# optionally using the USL in the included RiskSixSigma property function</td>
</tr>
<tr>
<td><strong>Supplemental Functions</strong></td>
<td><strong>Returns</strong></td>
</tr>
<tr>
<td>------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>RiskCorrectCorrmat</strong></td>
<td>Returns the corrected correlation matrix for the matrix located in correlationMatrixRange using the adjustment weight matrix located in adjustmentWeightsMatrixRange.</td>
</tr>
<tr>
<td><strong>RiskCurrentIter()</strong></td>
<td>Returns the current iteration number of an executing simulation.</td>
</tr>
<tr>
<td><strong>RiskCurrentSim()</strong></td>
<td>Returns the current simulation number of an executing simulation.</td>
</tr>
<tr>
<td><strong>RiskStopRun</strong></td>
<td>Stops a simulation when the value of cellRef returns TRUE, or the entered formula evaluates to TRUE.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Graphing Function</strong></th>
<th><strong>Returns</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskResultsGraph</strong></td>
<td>Adds a graph of simulation results to a worksheet.</td>
</tr>
</tbody>
</table>
Reference: Distribution Functions

Distribution functions are listed here with their required arguments. Optional arguments may be added to these required arguments using the @RISK Distribution Property functions listed in the next section.
**RiskBeta**

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskBeta(alpha1,alpha2)</strong> specifies a beta distribution using the shape parameters <em>alpha1</em> and <em>alpha2</em>. These two arguments generate a beta distribution with a minimum value of 0 and a maximum value of 1. The Beta distribution is often used as a starting point to derive other distributions (such as the BetaGeneral, PERT and BetaSubjective). It is intimately related to the Binomial distribution, representing the distribution for the uncertainty of the probability of a Binomial process based on a certain number of observations of that process.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskBeta(1,2)</strong> specifies a beta distribution using the shape parameters 1 and 2. <strong>RiskBeta(C12,C13)</strong> specifies a beta distribution using the shape parameter <em>alpha1</em> taken from cell C12 and a shape parameter <em>alpha2</em> taken from cell C13.</td>
</tr>
</tbody>
</table>
| Guidelines | *alpha1* continuous shape parameter *alpha1* > 0
*alpha2* continuous shape parameter *alpha2* > 0 |
| Domain | 0 ≤ x ≤ 1 continuous |
| Density and Cumulative Distribution Functions | \[ f(x) = \frac{x^{\alpha_1 - 1}(1 - x)^{\alpha_2 - 1}}{B(\alpha_1, \alpha_2)} \]
\[
F(x) = \frac{B_x(\alpha_1, \alpha_2)}{B(\alpha_1, \alpha_2)} \equiv I_x(\alpha_1, \alpha_2)
\]
where B is the Beta Function and Bx is the Incomplete Beta Function. |
| Mean | \[ \frac{\alpha_1}{\alpha_1 + \alpha_2} \]
| Variance | \[ \frac{\alpha_1 \alpha_2}{(\alpha_1 + \alpha_2)^2(\alpha_1 + \alpha_2 + 1)} \]
| Skewness | \[ 2 \frac{\alpha_2 - \alpha_1}{\alpha_1 + \alpha_2 + 2} \sqrt{\frac{\alpha_1 + \alpha_2 + 1}{\alpha_1 \alpha_2}} \]
<p>| Kurtosis | [ 3 \frac{(\alpha_1 + \alpha_2 + 1)(2(\alpha_1 + \alpha_2)^2 + \alpha_1 \alpha_2(\alpha_1 + \alpha_2 - 6))}{\alpha_1 \alpha_2(\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3)} ] |</p>
<table>
<thead>
<tr>
<th>Mode</th>
<th>( \frac{\alpha_1 - 1}{\alpha_1 + \alpha_2 - 2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha_1 &gt; 1, \alpha_2 &gt; 1 )</td>
</tr>
<tr>
<td>0</td>
<td>( \alpha_1 &lt; 1, \alpha_2 \geq 1 ) or ( \alpha_1 = 1, \alpha_2 &gt; 1 )</td>
</tr>
<tr>
<td>1</td>
<td>( \alpha_1 \geq 1, \alpha_2 &lt; 1 ) or ( \alpha_1 &gt; 1, \alpha_2 = 1 )</td>
</tr>
</tbody>
</table>

**Examples**

![CDF - Beta(2,3)](image1)

![PDF - Beta(2,3)](image2)
RiskBetaGeneral

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskBetaGeneral}(\alpha_1, \alpha_2, \text{minimum}, \text{maximum})$ specifies a beta distribution with the defined minimum and maximum using the shape parameters $\alpha_1$ and $\alpha_2$. The BetaGeneral is directly derived from the Beta distribution by scaling the [0,1] range of the Beta distribution with the use of a minimum and maximum value to define the range. The PERT distribution can be derived as a special case of the BetaGeneral distribution.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskBetaGeneral}(1,2,0,100)$ specifies a beta distribution using the shape parameters 1 and 2 and a minimum value of 0 and a maximum value of 100. $\text{RiskBetaGeneral}(C12,C13,D12,D13)$ specifies a beta distribution using the shape parameter $\alpha_1$ taken from cell C12 and a shape parameter $\alpha_2$ taken from cell C13 and a minimum value from D12 and a maximum value of from D13.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>$\alpha_1$ continuous shape parameter $\alpha_1 &gt; 0$ $\alpha_2$ continuous shape parameter $\alpha_2 &gt; 0$ min continuous boundary parameter min &lt; max max continuous boundary parameter</td>
</tr>
<tr>
<td>Domain</td>
<td>$\text{min} \leq x \leq \text{max}$ continuous</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>$f(x) = \frac{(x - \text{min})^{\alpha_1 - 1}(\text{max} - x)^{\alpha_2 - 1}}{B(\alpha_1, \alpha_2)(\text{max} - \text{min})^{\alpha_1 + \alpha_2 - 1}}$ $F(x) = \frac{B_z(\alpha_1, \alpha_2)}{B(\alpha_1, \alpha_2)} \equiv I_z(\alpha_1, \alpha_2)$ with $z = \frac{x - \text{min}}{\text{max} - \text{min}}$ where $B$ is the Beta Function and $B_z$ is the Incomplete Beta Function.</td>
</tr>
<tr>
<td>Mean</td>
<td>$\text{min} + \frac{\alpha_1}{\alpha_1 + \alpha_2}(\text{max} - \text{min})$</td>
</tr>
<tr>
<td>Variance</td>
<td>$\frac{\alpha_1 \alpha_2}{(\alpha_1 + \alpha_2)^2(\alpha_1 + \alpha_2 + 1)}(\text{max} - \text{min})^2$</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>[2 \frac{\alpha_2 - \alpha_1}{\alpha_1 + \alpha_2 + 2} \sqrt{\frac{\alpha_1 + \alpha_2 + 1}{\alpha_1 \alpha_2}}]</td>
</tr>
<tr>
<td>--------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>[3 \frac{(\alpha_1 + \alpha_2 + 1)(2(\alpha_1 + \alpha_2)^2 + \alpha_1 \alpha_2 (\alpha_1 + \alpha_2 - 6))}{\alpha_1 \alpha_2 (\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3)}]</td>
</tr>
<tr>
<td><strong>Mode</strong></td>
<td>[\text{min} + \frac{\alpha_1 - 1}{\alpha_1 + \alpha_2 - 2} (\text{max} - \text{min}) \quad \alpha_1 &gt; 1, \quad \alpha_2 &gt; 1]</td>
</tr>
<tr>
<td></td>
<td>[\text{min} \quad \alpha_1 &lt; 1, \quad \alpha_2 \geq 1 \quad \text{or} \quad \alpha_1 = 1, \quad \alpha_2 &gt; 1]</td>
</tr>
<tr>
<td></td>
<td>[\text{max} \quad \alpha_1 \geq 1, \quad \alpha_2 &lt; 1 \quad \text{or} \quad \alpha_1 &gt; 1, \quad \alpha_2 = 1]</td>
</tr>
</tbody>
</table>

**Examples**

PDF - BetaGeneral(2,3,0,5)
**RiskBetaGeneralAlt, RiskBetaGeneralAltD**

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskBetaGeneralAlt}(\text{arg1type, arg1value, arg2type, arg2value, arg3type, arg3value, arg4type, arg4value}) ) specifies a beta distribution with four arguments of the type ( \text{arg1type} ) to ( \text{arg4type} ). These arguments can be either a percentile between 0 and 1 or ( \alpha_1, \alpha_2, \text{min or max} ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskBetaGeneralAlt(&quot;min&quot;,0,10%,1,50%,20,&quot;max&quot;,50)) specifies a beta distribution with a minimum value of 0 and a maximum value of 50, a 10\text{th} percentile of 1 and a 50\text{th} percentile of 20.}</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Both ( \alpha_1 ) and ( \alpha_2 ) must be greater than zero and ( \text{max} &gt; \text{min} ). With ( \text{RiskBetaGeneralAltD} ), any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
**RiskBetaSubj**

<table>
<thead>
<tr>
<th>Description</th>
<th>(\text{RiskBetaSubj}(\text{minimum, most likely, mean, maximum})) specifies a beta distribution with a minimum and maximum value as specified. The shape parameters are calculated from the defined most likely value and mean. The BetaSubjective distribution is rather like a BetaGeneral distribution in the sense that the range of the underlying Beta distribution has been scaled. However, its parameterisation allows it to be used in cases where one wishes not only to use a minimum-most likely-maximum parameter set (as for the PERT distribution) but also to use the mean of the distribution as one of its parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>(\text{RiskBetaSubj}(0,1,2,10)) specifies a beta distribution with a minimum of 0, a maximum of 10, a most likely value of 1 and a mean of 2. (\text{RiskBetaSubj}(A1,A2,A3,A4)) specifies a beta distribution with a minimum value taken from cell A1, a maximum value taken from cell A4, a most likely value taken from cell A2 and a mean value taken from cell A3.</td>
</tr>
</tbody>
</table>
| Definitions | \[
\text{mid} \equiv \frac{\text{min} + \text{max}}{2} \\
\alpha_1 \equiv 2 \frac{(\text{mean} - \text{min})(\text{mid} - \text{m.likely})}{(\text{mean} - \text{m.likely})(\text{max} - \text{min})} \\
\alpha_2 \equiv \alpha_1 \frac{\text{max} - \text{mean}}{\text{mean} - \text{min}}
\]
| Parameters | min \quad \text{continuous boundary parameter}  \\
| | \quad \text{min} < \text{max}  \\
| | m.likely \quad \text{continuous parameter}  \\
| | \quad \text{min} < \text{m.likely} < \text{max}  \\
| | mean \quad \text{continuous parameter}  \\
| | \quad \text{min} < \text{mean} < \text{max}  \\
| | max \quad \text{continuous boundary parameter}  \\
| | \quad \text{mean} > \text{mid} \quad \text{if m.likely} > \text{mean}  \\
| | \quad \text{mean} < \text{mid} \quad \text{if m.likely} < \text{mean}  \\
| | \quad \text{mean} = \text{mid} \quad \text{if m.likely} = \text{mean}  \\
| Domain | \text{min} \leq x \leq \text{max} \quad \text{continuous} |
Density and Cumulative Distribution Functions

\[
f(x) = \frac{(x - \text{min})^{\alpha_1 - 1} (\text{max} - x)^{\alpha_2 - 1}}{\text{B}(\alpha_1, \alpha_2)(\text{max} - \text{min})^{\alpha_1 + \alpha_2 - 1}}
\]

\[
F(x) = \frac{\text{B}_z(\alpha_1, \alpha_2)}{\text{B}(\alpha_1, \alpha_2)} \equiv \text{I}_z(\alpha_1, \alpha_2) \quad \text{with} \quad z \equiv \frac{x - \text{min}}{\text{max} - \text{min}}
\]

where \(B\) is the Beta Function and \(B_z\) is the Incomplete Beta Function.

Mean

\(\text{mean}\)

Variance

\[
\frac{(\text{mean} - \text{min})(\text{max} - \text{mean})(\text{mean} - \text{m.likely})}{2 \cdot \text{mid} + \text{mean} - 3 \cdot \text{m.likely}}
\]

Skewness

\[
\frac{2(\text{mid} - \text{mean})}{[\text{mean} + \text{mid} - 2 \cdot \text{m.likely}]^{1/2}} \sqrt{\frac{(\text{mean} - \text{m.likely})(2 \cdot \text{mid} + \text{mean} - 3 \cdot \text{m.likely})}{(\text{mean} - \text{min})(\text{max} - \text{mean})}}
\]

Kurtosis

\[
\frac{3(\alpha_1 + \alpha_2 + 1)(2(\alpha_1 + \alpha_2)^2 + \alpha_1 \alpha_2 (\alpha_1 + \alpha_2 - 6))}{\alpha_1 \alpha_2 (2(\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3))}
\]

Mode

\(\text{m.likely}\)
Examples

CDF - BetaSubj(0,1,2,5)

PDF - BetaSubj(0,1,2,5)
## RiskBinomial

| Description | \( \text{RiskBinomial}(n, p) \) specifies a binomial distribution with \( n \) number of trials and \( p \) probability of success on each trial. The number of trials is often referred to as the number of draws or samples made. The binomial distribution is a discrete distribution returning only integer values greater than or equal to zero. This distribution corresponds to the number of events that occur in a trial of a set of independent events of equal probability. For example, \( \text{RiskBinomial}(10, 20\%) \) would represent the number of discoveries of oil from a portfolio of 10 prospects, where each prospect has a 20% chance of having oil. The most important modelling application is when \( n=1 \), so that there are two possible outcomes (0 or 1), where the 1 has a specified probability \( p \), and the 0 has probability 1-\( p \). With \( p=0.5 \), it is equivalent to tossing a fair coin. For other values of \( p \), the distribution can be used to model event risk i.e. the occurrence or not of an event, and to transform registers of risks into simulation models in order to aggregate the risks. |
| Examples | \( \text{RiskBinomial}(5, .25) \) specifies a binomial distribution generated from 5 trials or “draws” with a 25% probability of success on each draw. \( \text{RiskBinomial}(C10*3,B10) \) specifies a binomial distribution generated from the trials or “draws” given by the value in cell C10 times 3. The probability of success on each draw is given in cell B10. |
| Guidelines | The number of trials \( n \) must be a positive integer greater than zero and less than or equal to 32,767. Probability \( p \) must be greater than or equal to zero and less than or equal to 1. |
| Parameters | \( n \) discrete “count” parameter \( n > 0 \) * \( p \) continuous “success” probability \( 0 < p < 1 \) * \( *n = 0, p = 0 \) and \( p = 1 \) are supported for modeling convenience, but give degenerate distributions. |
| Domain | \( 0 \leq x \leq n \) discrete integers |
| Mass and Cumulative Distribution Functions | \( f(x) = \binom{n}{x} p^x (1-p)^{n-x} \) \( F(x) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i} \) |
| Mean | \( np \) |
| Variance | \( np(1-p) \) |

Reference: @RISK Functions
<table>
<thead>
<tr>
<th>Skewness</th>
<th>( \frac{(1 - 2p)}{\sqrt{np(1 - p)}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td>( 3 - \frac{6}{n} + \frac{1}{np(1 - p)} )</td>
</tr>
<tr>
<td>Mode</td>
<td>(bimodal) ( p(n+1)-1 ) and ( p(n+1) ) if ( p(n+1) ) is integral</td>
</tr>
<tr>
<td></td>
<td>(unimodal) largest integer less than ( p(n+1) ) otherwise</td>
</tr>
</tbody>
</table>

**Examples**

**PMF - Binomial(8, .4)**

**CDF - Binomial(8, .4)**
# RiskChiSq

**Description**  
*RiskChiSq*(\(v\)) specifies a Chi-Square distribution with \(v\) degrees of freedom.

**Examples**  
*RiskChiSq*(5) generates a Chi-Square distribution with 5 degrees of freedom.  
*RiskChiSq*(A7) generates a Chi-Square distribution with the degrees of freedom parameter taken from cell A7.

**Guidelines**  
Number of degrees of freedom \(v\) must be a positive integer.

**Parameters**  
\(ν\)  
Discrete shape parameter  
\(ν > 0\)

**Domain**  
\(0 \leq x < +\infty\)  
Continuous

**Density and Cumulative Distribution Functions**

\[
f(x) = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} e^{-x/2} x^{(\nu/2)-1}
\]

\[
F(x) = \frac{\Gamma_{x/2}(\nu/2)}{\Gamma(\nu/2)}
\]

where \(\Gamma\) is the Gamma Function, and \(\Gamma_x\) is the Incomplete Gamma Function.

**Mean**  
\(ν\)

**Variance**  
\(2ν\)

**Skewness**  
\[
\sqrt{\frac{8}{\nu}}
\]

**Kurtosis**  
\[
3 + \frac{12}{\nu}
\]

**Mode**  
\(ν-2\) if \(ν \geq 2\)

0 if \(ν = 1\)

---

Reference: @RISK Functions 479
Examples

PDF - ChiSq(5)

CDF - ChiSq(5)
### RiskCompound

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCompound(dist#1 or value or cellref, dist#2 or cellref, deductible, limit) returns the sum of a number of samples from dist#2 where the number of samples drawn from dist#2 is given by the value sampled from dist#1 or value. Typically, dist#1 is the frequency distribution and dist#2 is the severity distribution. Optionally, deductible is subtracted from each dist#2 sample and if (dist#2 sample - deductible) exceeds limit, dist#2 sample is set equal to limit. RiskCompound is evaluated each iteration of a simulation. The first argument's value is calculated using a sample drawn from dist#1 or a value taken from cellRef. Then, a number of samples, equaling the first argument's value, are drawn from dist#2 and summed. This sum is the return value for the RiskCompound function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskCompound(RiskPoisson(5), RiskLognorm(10000, 1000)) sums a number of samples drawn from RiskLognorm(10000, 1000) where the number of samples to be summed is given by the value sampled from RiskPoisson(5).</td>
</tr>
<tr>
<td>Guidelines</td>
<td>dist#1, but not dist#2, may be correlated. RiskCompound itself may not be correlated. deductible and limit are optional arguments. If (dist#2 sample - deductible) exceeds limit, the sample for dist#2 is set equal to limit. dist#1, dist#2, and RiskCompound itself may include property functions; except RiskCorrmat as noted above. Input distribution functions dist#1 or dist#2, along with any distribution functions in cells referenced in the RiskCompound function, are not displayed in sensitivity analysis results for outputs affected by the RiskCompound function. The RiskCompound function itself, however, includes sensitivity analysis results. Those results include the effects of dist#1, dist#2, and any distribution functions in cells referenced in a RiskCompound function. dist#2 may be a reference to a cellRef that contains a distribution function or a formula. If a formula is entered, this formula will be recalculated each time a severity value is needed. For example, the severity formula for cell A10 and compound function in A11 could be entered as follows: A10: =RiskLognorm(10000, 1000)/(1.1^RiskWeibull(2, 1)) A11:= RiskCompound(RiskPoisson(5), A10) In this case, the “sample” for the severity distribution would be generated by evaluating the formula in A10. Each iteration this formula would be evaluated the number of times specified by the sample drawn from the frequency distribution. Note: the formula entered needs to be &lt;256 characters; if more complex calculations are needed, a user defined function (UDF) may be entered as the formula to be evaluated. In addition, all @RISK distributions to be sampled in the severity calculation need to be entered in the cell’s formula (for example, in the formula for cell A10 above) and not referenced in other cells.</td>
</tr>
</tbody>
</table>
It is important to note that a single distribution of simulation results is not available for the severity distribution or severity calculation after a run. No entry is made for the severity distribution in the Results Summary window and a Browse window graph cannot be displayed for the severity distribution. This is because the severity distribution can be sampled any number of times during a single iteration, vs. one time for all other input distributions.

### RiskCumul

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskCumul}(\text{minimum}, \text{maximum}, {X_1, X_2, \ldots, X_n}, {p_1, p_2, \ldots, p_n}) ) specifies a cumulative distribution with ( n ) points. The range of the cumulative curve is set by the ( \text{minimum} ) and ( \text{maximum} ) arguments. Each point on the cumulative curve has a value ( X ) and a probability ( p ). Points on the cumulative curve are specified with increasing value and increasing probability. Any number of points may be specified for the curve.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskCumul}(0, 10, {1, 5, 9}, {0.1, 0.7, 0.9}) ) specifies a cumulative curve with 3 data points and a range of 0 to 10. The first point on the curve is 1 with a cumulative probability of 0.1 (10% of the distribution values are less than or equal to 1, 90% are greater). The second point on the curve is 5 with a cumulative probability 0.7 (70% of the distribution values are less than or equal to 5, 30% are greater). The third point on the curve is 9 with a cumulative probability of 0.9 (90% of the distribution values are less than or equal to 9, 10% are greater). ( \text{RiskCumul}(100, 200, A1:C1, A2:C2) ) specifies a cumulative distribution with 3 data points and a range of 100 to 200. Row 1 of the worksheet — A1 through C1 — holds the values of each data point while row 2 — A2 through C2 — holds the cumulative probability at each of the 3 points in the distribution. In Excel braces are not required when cell ranges are used as entries to the function.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>The points on the curve must be specified in order of increasing value (( X_1 &lt; X_2 &lt; X_3, \ldots, &lt; X_n )). The cumulative probabilities ( p ) for points on the curve must be specified in order of increasing probability (( p_1 &lt; p_2 &lt; p_3, \ldots, &lt; p_n )). The cumulative probabilities ( p ) for points on the curve must be greater than or equal to 0 and less than or equal to 1. ( \text{minimum} ) must be less than ( \text{maximum} ). ( \text{maximum} ) must be less than ( X_1 ) and ( \text{minimum} ) must be greater than ( X_n ).</td>
</tr>
</tbody>
</table>
| Parameters | \( \text{min} \) \hspace{1cm} \text{continuous parameter} \hspace{1cm} \text{min} < \text{max} \)

\( \text{max} \) \hspace{1cm} \text{continuous parameter} 

\( \{X\} = \{x_1, x_2, \ldots, x_n\} \) \hspace{1cm} \text{array of continuous parameters} \hspace{1cm} \text{min} \leq x_i \leq \text{max} |
\( \{p\} = \{p_1, p_2, \ldots, p_N\} \) array of continuous parameters

\( 0 \leq p_i \leq 1 \)

<table>
<thead>
<tr>
<th>Domain</th>
<th>( \min \leq x \leq \max ) continuous</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Density and Cumulative Distribution Functions</th>
<th>( f(x) = \frac{p_{i+1} - p_i}{x_{i+1} - x_i} ) for ( x_i \leq x &lt; x_{i+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( F(x) = p_i + \left(p_{i+1} - p_i\right) \left(\frac{x - x_i}{x_{i+1} - x_i}\right) ) for ( x_i \leq x \leq x_{i+1} )</td>
</tr>
</tbody>
</table>

With the assumptions:
- The arrays are ordered from left to right
- The \( i \) index runs from 0 to \( N+1 \), with two extra elements:
  - \( x_0 = \min \), \( p_0 = 0 \)
  - \( x_{N+1} = \max \), \( p_{N+1} = 1 \)

<table>
<thead>
<tr>
<th>Mean</th>
<th>\textit{No Closed Form}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>\textit{No Closed Form}</td>
</tr>
<tr>
<td>Skewness</td>
<td>\textit{No Closed Form}</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>\textit{No Closed Form}</td>
</tr>
<tr>
<td>Mode</td>
<td>\textit{No Closed Form}</td>
</tr>
</tbody>
</table>
Examples

CDF - Cumul(0,5,{1,2,3,4},{.2,.3,.7,.8})

PDF - Cumul(0,5,{1,2,3,4},{.2,.3,.7,.8})
**RiskCumulD**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCumulD(minimum, maximum, {X1, X2, ..., Xn}, {p1, p2, ..., pn}) specifies a cumulative distribution with ( n ) points. The range of the cumulative curve is set by the minimum and maximum arguments. Each point on the cumulative curve has a value ( X ) and a probability ( p ). Points on the cumulative curve are specified with increasing value and decreasing probability. Probabilities entered are cumulative descending probabilities, or the probability of a value greater than the entered ( X ) value. Any number of points may be specified for the curve.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskCumulD(0, 10, {1, 5, 9}, {.9, .3, .1}) specifies a cumulative curve with 3 data points and a range of 0 to 10. The first point on the curve is 1 with a cumulative descending probability of .9 (10% of the distribution values are less than or equal to 1, 90% are greater). The second point on the curve is 5 with a cumulative descending probability .3 (70% of the distribution values are less than or equal to 5, 30% are greater). The third point on the curve is 9 with a cumulative descending probability of .1 (90% of the distribution values are less than or equal to 9, 10% are greater). RiskCumulD(100, 200, A1:C1, A2:C2) specifies a cumulative distribution with 3 data points and a range of 100 to 200. Row 1 of the worksheet — A1 through C1 — holds the values of each data point while row 2 — A2 through C2 — holds the cumulative probability at each of the 3 points in the distribution. In Excel, braces are not required when cell ranges are used as entries to the function.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>The points on the curve must be specified in order of increasing value (( X1 &lt; X2 &lt; X3, ... &lt; Xn )). The cumulative probabilities ( p ) for points on the curve must be specified in order of decreasing cumulative descending probabilities (( p1 &gt;= p2 &gt;= p3, ... &gt;= pn )). The cumulative descending probabilities ( p ) for points on the curve must be greater than or equal to 0 and less than or equal to 1. minimum must be less than maximum. minimum must be less than ( X1 ) and maximum must be greater than ( Xn ).</td>
</tr>
<tr>
<td>Parameters</td>
<td>( \text{min} ) continuous parameter ( \text{min} &lt; \text{max} )</td>
</tr>
<tr>
<td></td>
<td>( \text{max} ) continuous parameter</td>
</tr>
<tr>
<td></td>
<td>( {x} = {x_1, x_2, ..., x_n} ) array of continuous parameters ( \text{min} \leq x_i \leq \text{max} )</td>
</tr>
<tr>
<td></td>
<td>( {p} = {p_1, p_2, ..., p_n} ) array of continuous parameters ( 0 \leq p_i \leq 1 )</td>
</tr>
<tr>
<td>Domain</td>
<td>( \text{min} \leq x \leq \text{max} ) continuous</td>
</tr>
</tbody>
</table>
Density and Cumulative Distribution Functions

\[ f(x) = \frac{p_i - p_{i+1}}{x_{i+1} - x_i} \quad \text{for } x_i \leq x < x_{i+1} \]

\[ F(x) = 1 - p_i + (p_i - p_{i+1}) \left( \frac{x - x_i}{x_{i+1} - x_i} \right) \quad \text{for } x_i \leq x \leq x_{i+1} \]

With the assumptions:
The arrays are ordered from left to right
The \(i\) index runs from 0 to \(N+1\), with two extra elements:
\(x_0 = \min\), \(p_0 = 1\) and \(x_{N+1} = \max\), \(p_{N+1} = 0\).

Mean

*No Closed Form*

Variance

*No Closed Form*

Skewness

*No Closed Form*

Kurtosis

*No Closed Form*

Mode

*No Closed Form*

Examples

CDF - CumulD(0,5,{1,2,3,4},{.8,.7,.3,.2})
PDF - CumulD(0,5,\{1,2,3,4\},\{.8,.7,.3,.2\})
**RiskDiscrete**

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskDiscrete}({X_1, X_2, \ldots, X_n}, {p_1, p_2, \ldots, p_n}) ) specifies a discrete distribution with a number of outcomes equaling ( n ). Any number of outcomes may be entered. Each outcome has a value ( X ) and a weight ( p ) which specifies the outcome's probability of occurrence. As with the RiskHistogrm function, weights may sum to any value — they are normalized to probabilities by @RISK. This is a user-defined distribution in which the user specifies all possible outcomes and their probabilities. It can be used where it is believed there are several discrete outcomes (e.g. worst case, expected case, best case), to replicate some other discrete distributions (such as the Binomial distribution), and to model discrete scenarios.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskDiscrete}({0, .5}, {1, 1}) ) specifies a discrete distribution with 2 outcomes valued 0 and .5. Each outcome has an equal probability of occurrence as the weight for each is 1. The probability of 0 occurring is 50% (1/2) and the probability of .5 occurring is 50% (1/2). ( \text{RiskDiscrete}(\text{A1:C1}, \text{A2:C2}) ) specifies a discrete distribution with three outcomes. The first row of the worksheet — A1 through C1 — holds the values of each outcome while row 2 — A2 through C2 — holds the probability “weight” of each occurring.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Weight values ( p ) must be greater than, or equal to, zero and the sum of all weights must be greater than zero.</td>
</tr>
<tr>
<td>Parameters</td>
<td>( {x} = {x_1, x_2, \ldots, x_N} ) array of continuous parameters</td>
</tr>
<tr>
<td></td>
<td>( {p} = {p_1, p_2, \ldots, p_N} ) array of continuous parameters</td>
</tr>
<tr>
<td>Domain</td>
<td>( x \in {x} ) discrete</td>
</tr>
<tr>
<td>Mass and Cumulative Distribution Functions</td>
<td>( f(x) = p_i ) for ( x = x_i )</td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td></td>
<td>( f(x) = 0 ) for ( x \not\in {x} )</td>
</tr>
<tr>
<td></td>
<td>( F(x) = 0 ) for ( x &lt; x_1 )</td>
</tr>
<tr>
<td></td>
<td>( F(x) = \sum_{i=1}^{s} p_i ) for ( x_s \leq x &lt; x_{s+1}, s &lt; N )</td>
</tr>
<tr>
<td></td>
<td>( F(x) = 1 ) for ( x \geq x_N )</td>
</tr>
</tbody>
</table>

With the assumptions:
- The arrays are ordered from left to right
- The \( p \) array is normalized to 1.

<table>
<thead>
<tr>
<th>Mean</th>
<th>( \sum_{i=1}^{N} x_i p_i \equiv \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>( \sum_{i=1}^{N} (x_i - \mu)^2 p_i \equiv V )</td>
</tr>
<tr>
<td>Skewness</td>
<td>( \frac{1}{V^{3/2}} \sum_{i=1}^{N} (x_i - \mu)^3 p_i )</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>( \frac{1}{V^2} \sum_{i=1}^{N} (x_i - \mu)^4 p_i )</td>
</tr>
<tr>
<td>Mode</td>
<td>The ( x )-value corresponding to the highest ( p )-value.</td>
</tr>
</tbody>
</table>
Examples

CDF - Discrete({1,2,3,4},{2,1,2,1})

PMF - Discrete({1,2,3,4},{2,1,2,1})
# RiskDUniform

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskDUniform}{X_1, X_2, \ldots, X_n}$ specifies a discrete uniform distribution with $n$ possible outcomes with an equal probability of each outcome occurring. The value for each possible outcome is given by the $X$ value entered for the outcome. Each value is equally likely to occur. To generate a discrete uniform distribution where every integer in a range is a possible outcome, use the RiskIntUniform function.</th>
</tr>
</thead>
</table>
| Examples | $\text{RiskDUniform}\{1, 2.1, 4.45, 99\}$ specifies a discrete uniform distribution with 4 possible outcomes. The possible outcomes have the values 1, 2.1, 4.45 and 99.  
$\text{RiskDUniform}(A1:A5)$ specifies a discrete uniform distribution with 5 possible outcomes. The possible outcomes have the values taken from cells A1 through A5. |
| Guidelines | None. |
| Parameters | $\{x\} = \{x_1, x_2, \ldots, x_N\}$ array of continuous parameters |
| Domain | $x \in \{x\}$ discrete |
| Mass and Cumulative Distribution Functions | $f(x) = \frac{1}{N}$ for $x \in \{x\}$  
$f(x) = 0$ for $x \not\in \{x\}$  
$F(x) = 0$ for $x < x_1$  
$F(x) = \frac{i}{N}$ for $x_i \leq x < x_{i+1}$  
$F(x) = 1$ for $x \geq x_N$  
assuming the $\{x\}$ array is ordered. |
| Mean | $\frac{1}{N} \sum_{i=1}^{N} x_i \equiv \mu$ |
| **Variance** | \[ \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2 \equiv V \] |
| **Skewness** | \[ \frac{1}{NV^{3/2}} \sum_{i=1}^{N} (x_i - \mu)^3 \] |
| **Kurtosis** | \[ \frac{1}{NV^2} \sum_{i=1}^{N} (x_i - \mu)^4 \] |
| **Mode** | Not uniquely defined |
| **Examples** | CDF - DUniform({1,5,8,11,12}) |
PMF - DUniform({1,5,8,11,12})
## RiskErf

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskErf}(h) ) specifies an error function with a variance parameter ( h ). The error function distribution is derived from a normal distribution.</th>
</tr>
</thead>
</table>
| Examples | \( \text{RiskErf}(5) \) generates an error function with a variance parameter 5. 
\( \text{RiskErf}(A7) \) generates an error function with a variance parameter taken from cell A7. |
| Guidelines | Variance parameter \( h \) must be greater than 0. |
| Parameters | \( h \) continuous inverse scale parameter \( h > 0 \) |
| Domain | \( -\infty < x < +\infty \) continuous |
| Density and Cumulative Distribution Functions | \( f(x) = \frac{h}{\sqrt{\pi}} e^{-hx^2} \)

\[
F(x) = \Phi(\sqrt{2hx}) = \frac{1 + \text{erf}(hx)}{2}
\]

where \( \Phi \) is called the Laplace-Gauss Integral and erf is the Error Function. |
| Mean | 0 |
| Variance | \( \frac{1}{2h^2} \) |
| Skewness | 0 |
| Kurtosis | 3 |
| Mode | 0 |
Examples

**CDF - Erf(1)**

---

**PDF - Erf(1)**
## RiskErlang

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskErlang}(m,\beta) ) generates an ( m )-erlang distribution with the specified ( m ) and ( \beta ) values. ( m ) is an integer argument for a gamma distribution and ( \beta ) is a scale parameter.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskErlang}(5,10) ) specifies an ( m )-erlang distribution with an ( m ) value of 5 and a scale parameter of 10. ( \text{RiskErlang}(A1,A2/6.76) ) specifies an ( m )-erlang distribution with an ( m ) value taken from cell A1 and a scale parameter equaling the value in cell A2 divided by 6.76.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>( m ) must be a positive integer. ( \beta ) must be greater than zero.</td>
</tr>
<tr>
<td>Parameters</td>
<td>( m ) integral shape parameter ( m &gt; 0 ) ( \beta ) continuous scale parameter ( \beta &gt; 0 )</td>
</tr>
<tr>
<td>Domain</td>
<td>( 0 \leq x &lt; +\infty ) continuous</td>
</tr>
</tbody>
</table>
| Density and Cumulative Distribution Functions | \[
    f(x) = \frac{1}{\beta (m-1)!} \left( \frac{x}{\beta} \right)^{m-1} e^{-x/\beta} \\
    F(x) = \frac{\Gamma_x/\beta}{\Gamma(m)} = 1 - e^{-x/\beta} \sum_{i=0}^{m-1} \frac{(x/\beta)^i}{i!}
\]
where \( \Gamma \) is the Gamma Function and \( \Gamma_x \) is the Incomplete Gamma Function. |
| Mean | \( m\beta \) |
| Variance | \( m\beta^2 \) |
| Skewness | \( \frac{2}{\sqrt{m}} \) |
| Kurtosis | \( \frac{3 + \frac{6}{m}}{} \) |
| Mode | \( \beta(m-1) \) |
Examples

CDF - Erlang(2,1)

PDF - Erlang(2,1)
## RiskExpon

| Description | \( \text{RiskExpon}(\text{beta}) \) specifies an exponential distribution with the entered \text{beta} value. The mean of the distribution equals \text{beta}. \\
| This distribution is the continuous time equivalent to the Geometric distribution.
| It represents the waiting time for the first occurrence of a process which is continuous in time and of constant intensity. It could be used in similar applications to the Geometric distribution (e.g. queuing, maintenance and breakdown modelling), although suffers in some practical applications from the assumption of constant intensity. |
| **Examples** | \( \text{RiskExpon}(5) \) specifies an exponential distribution with a beta value of 5. \\
| \( \text{RiskExpon}(\text{A1}) \) specifies an exponential distribution with a beta value taken from cell A1. |
| **Guidelines** | \text{beta} must be greater than zero. |
| **Parameters** | \( \beta \) continuous scale parameter \( \beta > 0 \) |
| **Domain** | \( 0 \leq x < +\infty \) continuous |
| **Density and Cumulative Distribution Functions** | \( f(x) = \frac{e^{-x/\beta}}{\beta} \) |
| \( F(x) = 1 - e^{-x/\beta} \) |
| **Mean** | \( \beta \) |
| **Variance** | \( \beta^2 \) |
| **Skewness** | 2 |
| **Kurtosis** | 9 |
| **Mode** | 0 |
Examples

CDF - Expon(1)

PDF - Expon(1)
### RiskExponAlt, RiskExponAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskExponAlt(arg1type, arg1value, arg2type, arg2value) specifies an exponential distribution with two arguments of the type arg1type and arg2type. arg1type and arg2type can be either a percentile between 0 and 1 or beta or loc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskExponAlt(&quot;beta&quot;,1,95%,10) specifies an exponential distribution with a beta value of 1 and a 95% percentile of 10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>beta must be greater than zero. With RiskExponAltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>

### RiskExtValue

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskExtValue(alpha, beta) specifies an extreme value distribution with location parameter alpha and shape parameter beta.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskExtvalue(1,2) specifies an extreme value distribution with an alpha value of 1 and a beta value of 2. RiskExtvalue(A1,B1) specifies an extreme value distribution with an alpha value taken from cell A1 and a beta value of taken from cell B1.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>beta must be greater than zero.</td>
</tr>
<tr>
<td>Parameters</td>
<td>alpha continuous location parameter</td>
</tr>
<tr>
<td>beta continuous scale parameter</td>
<td></td>
</tr>
<tr>
<td>beta &gt; 0</td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td>$-\infty &lt; x &lt; +\infty$</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>$f(x) = \frac{1}{b} \left( \frac{1}{e^z + \exp(-z)} \right)$</td>
</tr>
<tr>
<td>$F(x) = \frac{1}{\exp(-z)}$ where $z = \frac{(x - a)}{b}$</td>
<td></td>
</tr>
<tr>
<td>Where a= alpha, b= beta</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>$a - b\Gamma'(1) \approx a + .577b$</td>
</tr>
<tr>
<td>where $\Gamma'(x)$ is the derivative of the Gamma Function.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>$\frac{\pi^2 b^2}{6}$</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>$\frac{12\sqrt{6}}{\pi^3} \zeta(3) \approx 1.139547$</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>5.4</td>
</tr>
<tr>
<td><strong>Mode</strong></td>
<td>a</td>
</tr>
</tbody>
</table>

**Examples**

![PDF - ExtValue(0,1)](image)

![CDF - ExtValue(0,1)](image)
**RiskExtValueAlt, RiskExtValueAltD**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskExtValueAlt(arg1type, arg1value, arg2type, arg2value) specifies an extreme value distribution with two arguments of the type arg1type and arg2type. These arguments can be either a percentile between 0 and 1 or alpha or beta.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskExtValueAlt(5%,10,95%,100) specifies an extreme value distribution with a 5th percentile of 10 and a 95th percentile of 100.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>beta must be greater than zero. With RiskExtValueAltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
### RiskGamma

**Description**

*RiskGamma*(alpha,beta) specifies a gamma distribution using the shape parameter *alpha* and the scale parameter *beta*.

The Gamma distribution is the continuous time equivalent of the Negative Binomial i.e. it represents the distribution of inter-arrival times for several events from a Poisson process. It can also be used to represent the distribution of possible values for the intensity of a Poisson process, when observations of the process have been made.

**Examples**

*RiskGamma*(1,1) specifies a gamma distribution where the shape parameter has a value of 1 and the scale parameter has a value of 1.

*RiskGamma*(C12,C13) specifies a gamma distribution, where the shape parameter has a value taken from cell C12, and the scale parameter has a value taken from cell C13.

**Guidelines**

Both *alpha* and *beta* must be greater than zero.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>continuous shape parameter</td>
<td>α &gt; 0</td>
</tr>
<tr>
<td>β</td>
<td>continuous scale parameter</td>
<td>β &gt; 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domain</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; x &lt; +∞</td>
<td>continuous</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density and Cumulative Distribution Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ f(x) = \frac{1}{\beta \Gamma(\alpha)} \left( \frac{x}{\beta} \right)^{\alpha-1} e^{-x/\beta} ]</td>
</tr>
<tr>
<td>[ F(x) = \frac{\Gamma_{x/\beta}(\alpha)}{\Gamma(\alpha)} ]</td>
</tr>
</tbody>
</table>

where \( \Gamma \) is the Gamma Function and \( \Gamma_x \) is the Incomplete Gamma Function.

<table>
<thead>
<tr>
<th>Mean</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>βα</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>β²α</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Skewness</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{\sqrt{\alpha}} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Kurtosis</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3 + \frac{6}{\alpha} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta(\alpha - 1) )</td>
<td></td>
<td>if ( \alpha \geq 1 )</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>if ( \alpha &lt; 1 )</td>
</tr>
</tbody>
</table>
**RiskGammaAlt, RiskGammaAltD**

<table>
<thead>
<tr>
<th>Description</th>
<th>\textbf{RiskGammaAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)} specifies a gamma distribution with three arguments of the type \textit{arg1type} to \textit{arg3type}. These arguments can be either a \textit{percentile} between 0 and 1 or \textit{alpha}, \textit{beta} or \textit{loc}.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>\textbf{RiskGammaAlt(&quot;alpha&quot;,1,&quot;beta&quot;,5,95%,10)} specifies a gamma distribution where the shape parameter has a value of 1, the scale parameter has a value of 5 and the 95th percentile has a value of 10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Both \textit{alpha} and \textit{beta} must be greater than zero. With \textbf{RiskGammaAltD}, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
### RiskGeneral

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskGeneral}(\text{minimum}, \text{maximum}, {X_1, X_2, \ldots, X_n}, {p_1, p_2, \ldots, p_n}) ) generates a generalized probability distribution based on a density curve created using the specified ((X, p)) pairs. Each pair has a value (X) and a probability weight (p) which specifies the relative height of the probability curve at that (X) value. The weights (p) are normalized by @RISK in determining the actual probabilities used in sampling.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskGeneral}(0, 10, {2, 5, 7, 9}, {1, 2, 3, 1}) ) specifies a general probability distribution density function with four points. The distribution ranges from 0 to 10 with four points — 2, 5, 7, 9 — specified on the curve. The height of the curve at 2 is 1, at 5 is 2, at 7 is 3 and at 9 is 1. The curve intersects the (X)-axis at 0 and 10. ( \text{RiskGeneral}(100, 200, A1:C1, A2:C2) ) specifies a general probability distribution with three data points and a range of 100 to 200. The first row of the worksheet — A1 through C1 — holds the (X) value of each data point while row 2 — A2 through C2 — holds the (p) value at each of the three points in the distribution. Note that braces are not required when cell ranges are used as array entries to the function.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Probability weights (p) must be greater than, or equal to, zero. The sum of all weights must be greater than zero. (X) values must be entered in increasing order and must fall within the minimum-maximum range of the distribution. (\text{minimum}) must be less than (\text{maximum}).</td>
</tr>
<tr>
<td>Parameters</td>
<td>( \text{min} ) \hspace{1cm} \text{continuous parameter} \hspace{1cm} \text{min} &lt; \text{max} |</td>
</tr>
<tr>
<td></td>
<td>( \text{max} ) \hspace{1cm} \text{continuous parameter}</td>
</tr>
<tr>
<td></td>
<td>( {x} = {x_1, x_2, \ldots, x_N} ) \hspace{1cm} \text{array of continuous parameters} \hspace{1cm} \text{min} \leq x_i \leq \text{max}</td>
</tr>
<tr>
<td></td>
<td>( {p} = {p_1, p_2, \ldots, p_n} ) \hspace{1cm} \text{array of continuous parameters} \hspace{1cm} p_i \geq 0</td>
</tr>
<tr>
<td>Domain</td>
<td>( \text{min} \leq x \leq \text{max} ) \hspace{1cm} \text{continuous}</td>
</tr>
</tbody>
</table>
Density and Cumulative Distribution Functions

\[
f(x) = p_i + \left[ \frac{x - x_i}{x_{i+1} - x_i} \right] (p_{i+1} - p_i)
\]

for \( x_i \leq x \leq x_{i+1} \)

\[
F(x) = F(x_i) + (x - x_i) \left[ p_i + \frac{(p_{i+1} - p_i)(x - x_i)}{2(x_{i+1} - x_i)} \right]
\]

for \( x_i \leq x \leq x_{i+1} \)

With the assumptions:
The arrays are ordered from left to right
The \( \{p\} \) array has been normalized to give the general distribution unit area.
The \( i \) index runs from 0 to \( N+1 \), with two extra elements:
\( x_0 \equiv \min, p_0 \equiv 0 \) and \( x_{N+1} \equiv \max, p_{N+1} \equiv 0 \).

<table>
<thead>
<tr>
<th>Mean</th>
<th>No Closed Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Skewness</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>No Closed Form</td>
</tr>
<tr>
<td>Mode</td>
<td>No Closed Form</td>
</tr>
</tbody>
</table>
CDF - General(0,5,{1,2,3,4},{2,1,2,1})

PDF - General(0,5,{1,2,3,4},{2,1,2,1})
### RiskGeomet

| Description | **RiskGeomet**(\(p\)) generates a geometric distribution with the probability \(p\). The value returned represents the number of failures prior to a success on a series of independent trials. There is a \(p\) probability of success on each trial. The geometric distribution is a discrete distribution returning only integer values greater than or equal to zero.

This distribution corresponds to the uncertainty about the number of trials of a Binomial distribution that would be required for an event of given probability to occur for the first time. Examples would include the distribution for the number of times a coin is tossed before a head is produced, or the number of sequential bets that one needs to make on roulette before the selected number occurs. The distribution can also be used in basic maintenance modelling, for example to represent the number of months before a car breaks down. However, since the distribution requires a constant probability of breakdown per trial, other models are often used i.e. where the probability of breakdown increases with age. |
| Examples | **RiskGeomet**(0.25) specifies a geometric distribution with a 25% probability of success on each trial. **RiskGeomet**(A18) specifies a geometric distribution with a probability of success on each trial taken from cell A18. |
| Guidelines | Probability \(p\) must be greater than zero and less than, or equal to, one. |
| Parameters | \(p\) continuous “success” probability \(0 < p \leq 1\) |
| Domain | \(0 \leq x < +\infty\) discrete integers |
| Mass and Cumulative Distribution Functions | \(f(x) = p(1 - p)^x\) \(F(x) = 1 - (1 - p)^{x+1}\) |
| Mean | \(\frac{1}{p} - 1\) |
| Variance | \(\frac{1 - p}{p^2}\) |
| Skewness | \(\frac{(2 - p)}{\sqrt{1 - p}}\) for \(p < 1\) Not Defined for \(p = 1\) |
### Kurtosis

\[
9 + \frac{p^2}{1-p} \quad \text{for } p < 1
\]

Not Defined \quad \text{for } p = 1

<table>
<thead>
<tr>
<th>Mode</th>
<th>0</th>
</tr>
</thead>
</table>

**Examples**

CDF - Geomet(.5)

PMF - Geomet(.5)
### RiskHistogram

**Description**  
RiskHistogram\((\text{minimum}, \text{maximum}, \{p_1, p_2, \ldots, p_n\})\) specifies a user-defined histogram distribution with a range defined by the specified minimum and maximum values. This range is divided into \(n\) classes. Each class has a weight \(p\) reflecting the probability of occurrence of a value within the class. These weights may be any values — the only important factor is the weight of one class relative to the others. This means that the sum of all the weights need not equal 100%. @RISK normalizes the class probabilities for you. Normalizing is done by summing all specified weights and dividing each weight by this sum.

**Examples**  
RiskHistogram\((10, 20, \{1, 2, 3, 2, 1\})\) specifies a histogram with a minimum value of 10 and a maximum value of 20. This range is divided into 5 equal length classes as there are 5 probability values. The probability weights for the five classes are the arguments 1, 2, 3, 2 and 1. The actual probabilities which would correspond with these weights are 11.1% (1/9), 22.2% (2/9), 33.3% (3/9), 22.2% (2/9) and 11.1% (1/9). Division by 9 normalizes these values so that their sum now equals 100%.

RiskHistogram\((A1, A2, B1:B3)\) specifies a histogram with a minimum value taken from cell A1 and a maximum value taken from cell A2. This range is divided into 3 equal length classes as there are 3 probability values. The probability weights are taken from cells B1 through B3.

**Guidelines**  
Weight values \(p\) must be greater than, or equal to, zero, and the sum of all weights must be greater than zero.

**Parameters**  
- \(\text{min}\) \hspace{1cm} \text{continuous parameter}\hspace{1cm} \text{min} < \text{max} *
- \(\text{max}\) \hspace{1cm} \text{continuous parameter}
- \(\{p\} = \{p_1, p_2, \ldots, p_n\}\) \hspace{1cm} \text{array of continuous parameters}\hspace{1cm} \text{\(p_i \geq 0\)}

\* \(\text{min} = \text{max}\) is supported for modelling convenience, but yields a degenerate distribution.

**Domain**  
\(\text{min} \leq x \leq \text{max}\) \hspace{1cm} \text{continuous}
<table>
<thead>
<tr>
<th>Density and Cumulative Distribution Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = p_i ) for ( x_i \leq x &lt; x_{i+1} )</td>
</tr>
<tr>
<td>( F(x) = F(x_i) + p_i \left( \frac{x - x_i}{x_{i+1} - x_i} \right) ) for ( x_i \leq x \leq x_{i+1} )</td>
</tr>
<tr>
<td>( x_i = \min + i \left( \frac{\max - \min}{N} \right) )</td>
</tr>
<tr>
<td>where the ( {p} ) array has been normalized to give the histogram unit area.</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
</tr>
<tr>
<td><strong>Variance</strong></td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
</tr>
<tr>
<td><strong>Mode</strong></td>
</tr>
</tbody>
</table>
Examples

CDF - Histogram(0,5,{6,5,3,4,5})

PDF - Histogram(0,5,{6,5,3,4,5})
RiskHypergeo

**Description**

RiskHypergeo(n,D,M) specifies a hypergeometric distribution with sample size n, number of items of a certain type equaling D and population size M. The hypergeometric distribution is a discrete distribution returning only non-negative integer values.

**Examples**

RiskHypergeo(50,10,1000) returns a hypergeometric distribution generated using a sample size of 50, 10 items of the relevant type and a population size of 1000.

RiskHypergeo(A6,A7,A8) returns a hypergeometric distribution generated using a sample size taken from cell A6, a number of items taken from cell A7 and a population size taken from cell A8.

**Guidelines**

All arguments — n, D and M — must be positive integer values. The value for sample size n must be less than or equal to the population size M. The value for number of items D must be less than or equal to the population size M.

**Parameters**

- **n** the number of draws integer
  - 0 ≤ n ≤ M
- **D** the number of "tagged" items integer
  - 0 ≤ D ≤ M
- **M** the total number of items integer
  - M ≥ 0

**Domain**

max(0,n+D-M) ≤ x ≤ min(n,D) discrete integers

**Mass and Cumulative Distribution Functions**

\[
f(x) = \binom{D}{x} \frac{M-D}{n-x} \binom{M}{n} \]

\[
F(x) = \sum_{i=1}^{x} \binom{D}{x} \frac{M-D}{n-x} \binom{M}{n}
\]

**Mean**

\[
\begin{align*}
\text{Mean} & = \frac{nD}{M} & \text{for } M > 0 \\
& = 0 & \text{for } M = 0
\end{align*}
\]
| **Variance** | \[ \frac{nD}{M^2} \left( \frac{(M - D)(M - n)}{(M - 1)} \right) \] for \( M > 1 \)  
0 for \( M = 1 \) |
| **Skewness** | \( \frac{(M - 2D)(M - 2n)}{M - 2} \sqrt{\frac{M - 1}{nD(M - D)(M - n)}} \) for \( M > 2, M > D > 0, M > n > 0 \)  
Not Defined otherwise |
| **Kurtosis** | \[ \frac{M^2(M - 1)}{n(M - 2)(M - 3)(M - n)} \left( \frac{M(M + 1) - 6n(M - n)}{D(M - D)} + \frac{3n(M - n)(M + 6)}{M^2} - 6 \right) \] for \( M > 3, M > D > 0, M > n > 0 \)  
Not Defined otherwise |
| **Mode** | (bimodal) \( x_m \) and \( x_m - 1 \) if \( x_m \) is integral  
(unimodal) biggest integer less than \( x_m \) otherwise  
\( x_m = \frac{(n + 1)(D + 1)}{M + 2} \) |

Reference: @RISK Functions
Examples

CDF - HyperGeo(6,5,10)

PMF - HyperGeo(6,5,10)
**RiskIntUniform**

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskIntUniform}(\text{minimum},\text{maximum}) ) specifies a uniform probability distribution with the entered minimum and maximum values. Only integer values across the range of the uniform distribution can occur, and each has an equal likelihood of occurrence.</th>
</tr>
</thead>
</table>
| Examples | \( \text{RiskIntUniform}(10,20) \) specifies a uniform distribution with a minimum value of 10 and a maximum value of 20.  
\( \text{RiskIntUniform}(A1+90,B1) \) specifies a uniform distribution with a minimum value equaling the value in cell A1 plus 90 and a maximum value taken from cell B1. |
| Guidelines | minimum must be less than maximum. |
| Parameters | min discrete boundary parameter min < max  
max discrete boundary probability |
| Domain | \( \text{min} \leq x \leq \text{max} \) discrete integers |
| Mass and Cumulative Distribution Functions | \( f(x) = \frac{1}{\text{max} - \text{min} + 1} \)  
\( F(x) = \frac{x - \text{min} + 1}{\text{max} - \text{min} + 1} \) |
| Mean | \( \frac{\text{min} + \text{max}}{2} \) |
| Variance | \( \frac{\Delta(\Delta + 2)}{12} \)  
where \( \Delta = (\text{max} - \text{min}) \) |
| Skewness | 0 |
| Kurtosis | \( \left( \frac{9}{5} \right) \cdot \left( \frac{n^2 - 7/3}{n^2 - 1} \right) \)  
where \( n = (\text{max} - \text{min} + 1) \) |
| Mode | Not uniquely defined |

Reference: @RISK Functions
Examples

CDF - IntUniform(0,8)

PMF - IntUniform(0,8)
## RiskInvgauss

<table>
<thead>
<tr>
<th>Description</th>
<th>$RiskInvgauss(mu, lambda)$ specifies an inverse gaussian distribution with mean $mu$ and shape parameter $lambda$.</th>
</tr>
</thead>
</table>
| Examples    | $RiskInvgauss(5,2)$ returns an inverse gaussian distribution with a $mu$ value of 5 and a $lambda$ value of 2.  
$RiskInvgauss(B5,B6)$ returns an inverse gaussian distribution with a $mu$ value taken from cell B5 and a $lambda$ value taken from cell B6. |
| Guidelines  | $mu$ must be greater than zero.  
$lambda$ must be greater than zero. |
| Parameters  | $\mu$ continuous parameter $\mu > 0$  
$\lambda$ continuous parameter $\lambda > 0$ |
| Domain      | $x > 0$ continuous |
| Density and Cumulative Distribution Functions |

\[
f(x) = \sqrt{\frac{\lambda}{2\pi x^3}} e^{-\left[\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right]}
\]

\[
F(x) = \Phi\left[\frac{\lambda}{\sqrt{\frac{x}{\mu}}} (x - 1)\right] + e^{2\lambda/\mu} \Phi\left[-\frac{\lambda}{\sqrt{\frac{x}{\mu}}} (x + 1)\right]
\]

where $\Phi(z)$ is the cumulative distribution function of a Normal$(0,1)$, also called the Laplace-Gauss Integral.

<table>
<thead>
<tr>
<th>Mean</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>$\frac{\mu^3}{\lambda}$</td>
</tr>
<tr>
<td>Skewness</td>
<td>$3\sqrt{\frac{\mu}{\lambda}}$</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>$3 + 15\frac{\mu}{\lambda}$</td>
</tr>
</tbody>
</table>

Reference: @RISK Functions
<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu = \sqrt{1 + \frac{9\mu^2}{4\lambda^2} - \frac{3\mu}{2\lambda}}$</td>
</tr>
</tbody>
</table>

**Examples**

**PDF - InvGauss(1,2)**

**CDF - InvGauss(1,2)**
### RiskInvgaussAlt, RiskInvgaussAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskInvgaussAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value)} ) specifies an inverse gaussian distribution with three arguments of the type ( \text{arg1type} ) to ( \text{arg3type} ). These arguments can be either a \text{percentile} between 0 and 1 or ( \text{mu, lambda or loc} ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskInvgaussAlt(&quot;mu&quot;,10,5%,1,95%,25)} ) returns an inverse gaussian distribution with a ( \text{mu} ) value of 1, a 5(^{th} ) percentile of 1 and a 95(^{th} ) percentile of 25.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>( \text{mu} ) must be greater than zero. ( \text{lambda} ) must be greater than zero. With ( \text{RiskInvgaussAltD} ), any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
### RiskJohnsonMoments

#### Description

RiskJohnsonMoments\((mean, standardDeviation, skewness, kurtosis)\)

chooses one of four distributions functions (all members of the so-called Johnson system) that matches the specified mean, standard deviation, skewness, and kurtosis. This resulting distribution is either a JohnsonSU, JohnsonSB, lognormal, or normal distribution.

#### Examples

RiskJohnsonMoments\((10, 20, 4, 41)\) returns a distribution from the Johnson family that has a mean value of 10, a standard deviation value of 20, a skewness value of 4 and a kurtosis value of 41.

RiskJohnsonMoments\((A6, A7, A8, A9)\) returns a distribution from the Johnson family that has a mean value taken from cell A6, a standard deviation value taken from cell A7, a skewness value taken from cell A8 and a kurtosis value taken from cell A9.

#### Guidelines

- standard deviation must be a positive value.
- kurtosis must be greater than 1.

#### Parameters

- \(\mu\) continuous location parameter
- \(\sigma\) continuous scale parameter \(\sigma > 0\)
- \(s\) continuous shape parameter
- \(k\) continuous shape parameter \(k > 1\)
- \(k - s^2 \geq 1\)

#### Domain

\(-\infty < x < +\infty\) continuous

#### Density and Cumulative Distribution Functions

See entries for each Johnson system distribution

#### Mean

\(\mu\)

#### Variance

\(\sigma^2\)

#### Skewness

\(s\)

#### Kurtosis

\(k\)

#### Mode

No Closed Form
### Examples

<table>
<thead>
<tr>
<th>PDF - JohnsonMoments(0,1,1,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
</tr>
<tr>
<td>0.45</td>
</tr>
<tr>
<td>0.40</td>
</tr>
<tr>
<td>0.35</td>
</tr>
<tr>
<td>0.30</td>
</tr>
<tr>
<td>0.25</td>
</tr>
<tr>
<td>0.20</td>
</tr>
<tr>
<td>0.15</td>
</tr>
<tr>
<td>0.10</td>
</tr>
<tr>
<td>0.05</td>
</tr>
<tr>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CDF - JohnsonMoments(0,1,1,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
</tr>
<tr>
<td>0.80</td>
</tr>
<tr>
<td>0.60</td>
</tr>
<tr>
<td>0.40</td>
</tr>
<tr>
<td>0.20</td>
</tr>
<tr>
<td>0.00</td>
</tr>
</tbody>
</table>
### RiskJohnsonSB

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskJohnsonSB</strong>(alpha1, alpha2, a, b) specifies a Johnson “system bounded” distribution with the entered alpha1, alpha2, a and b values.</th>
</tr>
</thead>
</table>
| Examples    | **RiskJohnsonSB**(10,20,1,2) returns a JohnsonSB distribution generated using an alpha1 value of 10, an alpha2 value of 20, an a value of 1 and a b value of 2.  
**RiskJohnsonSB**(A6,A7,A8,A9) returns a JohnsonSB distribution generated using an alpha1 value taken from cell A6, an alpha2 value taken from cell A7, an a value taken from cell A8 and a b value taken from cell A9. |
| Guidelines  | b must be greater than a |
| Parameters  | alpha1  | continuous shape parameter |
|             | alpha2  | continuous shape parameter     |
|             | a       | continuous boundary parameter |
|             | b       | continuous boundary parameter |
| Domain      | a ≤ x ≤ b | continuous |

#### Density and Cumulative Distribution Functions

\[
f(x) = \frac{\alpha_2 (b - a)}{\sqrt{2\pi(x - a)(b - x)}} \times e^{-\frac{1}{2} \left[ \alpha_1 + \alpha_2 \ln\left(\frac{x - a}{b - x}\right) \right]}
\]

\[
F(x) = \Phi\left[ \alpha_1 + \alpha_2 \ln\left(\frac{x - a}{b - x}\right) \right]
\]

where \(\Phi\) is the cumulative distribution function of a standard Normal(0,1).

<table>
<thead>
<tr>
<th>Mean</th>
<th>Closed Form exists but is extremely complicated.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>Closed Form exists but is extremely complicated.</td>
</tr>
<tr>
<td>Skewness</td>
<td>Closed Form exists but is extremely complicated.</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>Closed Form exists but is extremely complicated.</td>
</tr>
<tr>
<td>Mode</td>
<td>No Closed Form.</td>
</tr>
</tbody>
</table>
Examples

PDF - JohnsonSB(2,2,0,1)

CDF - JohnsonSB(2,2,0,1)
**RiskJohnsonSU**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskJohnsonSU((alpha_1, alpha_2, gamma, beta)) specifies a Johnson &quot;system unbounded&quot; distribution with the entered (alpha_1, alpha_2, gamma) and (beta) values.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Examples</strong></td>
<td>RiskJohnsonSU(10,20,1,2) returns a JohnsonSU distribution generated using an (alpha) value of 10, an (alpha_2) value of 20, a (gamma) value of 1 and a (beta) value of 2. RiskJohnsonSU(A6,A7,A8,A9) returns a JohnsonSU distribution generated using an (alpha) value taken from cell A6, an (alpha_2) value taken from cell A7, a (gamma) value taken from cell A8 and a (beta) value taken from cell A9.</td>
</tr>
<tr>
<td><strong>Guidelines</strong></td>
<td>(alpha_2) must be a positive value. (beta) must be a positive value.</td>
</tr>
<tr>
<td><strong>Parameters</strong></td>
<td>(alpha_1) continuous shape parameter (alpha_2) continuous shape parameter (gamma) continuous location parameter (beta) continuous scale parameter (alpha_2 &gt; 0) (beta &gt; 0)</td>
</tr>
<tr>
<td><strong>Domain</strong></td>
<td>(-\infty &lt; x &lt; +\infty) continuous</td>
</tr>
<tr>
<td><strong>Definitions</strong></td>
<td>(\theta = \exp\left(\frac{1}{\alpha_2}\right)^2) (r = \frac{\alpha_1}{\alpha_2})</td>
</tr>
<tr>
<td><strong>Density and Cumulative Distribution Functions</strong></td>
<td>(f(x) = \frac{\alpha_2}{\beta \sqrt{2\pi (1 + z^2)}} \times e^{-\frac{1}{2}\left[\alpha_1 + \alpha_2 \sinh^{-1}(z)\right]^2}) (F(x) = \Phi\left(\alpha_1 + \alpha_2 \sinh^{-1}(z)\right)) Where (z = \frac{(x - \gamma)}{\beta}) and (\Phi) is the cumulative distribution function of a standard Normal(0,1).</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>(\gamma - \beta \sqrt{\theta} \sinh(r))</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>(\frac{\beta^2}{2} \left((\theta - 1)(\theta \cosh(2r) + 1)\right))</td>
</tr>
</tbody>
</table>
### Skewness

\[-\frac{1}{4} \sqrt{\theta} (\theta - 1)^2 \left[ \theta (\theta + 2) \sinh(3r) + 3 \sinh(r) \right] \]

\[= \left[ \frac{1}{2} (\theta - 1) (\theta \cosh(2r) + 1) \right]^3 \]

### Kurtosis

\[\frac{1}{8} (\theta - 1)^2 \left[ \theta^2 \left( \theta^4 + 2\theta^3 + 3\theta^2 - 3 \right) \cosh(4r) + 4\theta^2 (\theta + 2) \cosh(2r) + 3(2\theta + 1) \right] \]

\[= \left[ \frac{1}{2} (\theta - 1) (\theta \cosh(2r) + 1) \right]^2 \]

### Mode

*No Closed Form.*

### Examples

![PDF - JohnsonSU(2, 2, 0, 1)](image_url)
## RiskLogistic

<table>
<thead>
<tr>
<th>Description</th>
<th>\textit{RiskLogistic}(alpha,beta) specifies a logistic distribution with the entered \textit{alpha} and \textit{beta} values.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guidelines</td>
<td>\textit{beta} must be a positive value.</td>
</tr>
<tr>
<td>Parameters</td>
<td>$\alpha$ \hspace{1cm} \text{continuous location parameter} \hspace{1cm} $\beta$ \hspace{1cm} \text{continuous scale parameter} $\beta &gt; 0$</td>
</tr>
<tr>
<td>Domain</td>
<td>$-\infty &lt; x &lt; +\infty$ \hspace{1cm} \text{continuous}</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>\begin{align*} f(x) &amp;= \frac{\text{sech}^2 \left( \frac{1}{2} \left( \frac{x - \alpha}{\beta} \right) \right)}{4\beta} \ F(x) &amp;= \frac{1 + \text{tanh} \left( \frac{1}{2} \left( \frac{x - \alpha}{\beta} \right) \right)}{2} \end{align*} \text{where “sech” is the Hyperbolic Secant Function and “tanh” is the Hyperbolic Tangent Function.}</td>
</tr>
<tr>
<td>Mean</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Variance</td>
<td>$\frac{\pi^2\beta^2}{3}$</td>
</tr>
<tr>
<td>Skewness</td>
<td>0</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.2</td>
</tr>
<tr>
<td>Mode</td>
<td>$\alpha$</td>
</tr>
</tbody>
</table>

Reference: @RISK Functions
Examples

PDF - Logistic(0,1)

CDF - Logistic(0,1)
**RiskLogisticAlt, RiskLogisticAltD**

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskLogisticAlt}(\text{arg1type, arg1value, arg2type, arg2value}) ) specifies a logistic distribution with two arguments of the type \text{arg1type} and \text{arg2type}. These arguments can be either a \text{percentile} between 0 and 1 or \text{alpha} or \text{beta}.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskLogisticAlt}(5%, 1, 95%, 100) ) returns a logistic distribution with a 5(^{th}) percentile of 1 and a 95(^{th}) percentile of 100.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>\text{beta} must be a positive value. With \text{RiskLogisticAltD}, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
### RiskLogLogistic

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskLoglogistic}(\gamma, \beta, \alpha) ) specifies a log-logistic distribution with location parameter ( \gamma ) and shape parameter ( \alpha ) and scale parameter ( \beta ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskLoglogistic}(-5, 2, 3) ) returns a log-logistic distribution generated using a ( \gamma ) value of -5, a ( \beta ) value of 2, and an ( \alpha ) value of 3. ( \text{RiskLoglogistic}(A1, A2, A3) ) returns a log-logistic distribution generated using a ( \gamma ) value taken from cell A1, a ( \beta ) value taken from cell A2, and an ( \alpha ) value taken from cell A3.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>( \alpha ) must be greater than zero. ( \beta ) must be greater than zero.</td>
</tr>
<tr>
<td>Parameters</td>
<td>( \gamma ) continuous location parameter ( \beta ) continuous scale parameter ( \beta &gt; 0 ) ( \alpha ) continuous shape parameter ( \alpha &gt; 0 )</td>
</tr>
<tr>
<td>Definitions</td>
<td>( \theta \equiv \frac{\pi}{\alpha} )</td>
</tr>
<tr>
<td>Domain</td>
<td>( \gamma \leq x &lt; +\infty ) continuous</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>( f(x) = \frac{\alpha}{\beta} \frac{t^{\alpha-1}}{(1 + t^\alpha)^2} ) ( F(x) = \frac{1}{1 + \left(\frac{1}{t}\right)^\alpha} ) with ( t \equiv \frac{x - \gamma}{\beta} )</td>
</tr>
<tr>
<td>Mean</td>
<td>( \beta \theta \csc(\theta) + \gamma ) for ( \alpha &gt; 1 )</td>
</tr>
<tr>
<td>Variance</td>
<td>( \beta^2 \theta \left[ 2\csc(2\theta) - \theta \csc^2(\theta) \right] ) for ( \alpha &gt; 2 )</td>
</tr>
<tr>
<td>Skewness</td>
<td>( \frac{3\csc(3\theta) - 6\theta \csc(2\theta) \csc(\theta) + 2\theta^2 \csc^3(\theta)}{\sqrt{\theta \left[ 2\csc(2\theta) - \theta \csc^2(\theta) \right]^3}} ) for ( \alpha &gt; 3 )</td>
</tr>
</tbody>
</table>
| **Kurtosis** | \[
\frac{4 \csc(4\theta) - 12\theta \csc(3\theta) \csc(\theta) + 12\theta^2 \csc(2\theta) \csc^2(\theta) - 3\theta^3 \csc^4(\theta)}{\theta \left[2\csc(2\theta) - \theta \csc^2(\theta)\right]^2}
\] for \(\alpha > 4\) |
| **Mode** | \[
\gamma + \beta \left[\frac{\alpha - 1}{\alpha + 1}\right]^{\frac{1}{\alpha}}
\] for \(\alpha > 1\)  
\[
\gamma
\] for \(\alpha \leq 1\) |

**Examples**

**PDF - LogLogistic(0,1,5)**

**CDF - LogLogistic(0,1,5)**

Reference: @RISK Functions
RiskLogLogisticAlt, RiskLogLogisticAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskLogLogisticAlt}(\text{arg1type}, \text{arg1value}, \text{arg2type}, \text{arg2value}, \text{arg3type}, \text{arg3value}) ) specifies a log-logistic distribution with three arguments of the type ( \text{arg1type} ) to ( \text{arg3type} ). These arguments can be either a percentile between 0 and 1 or ( \text{gamma}, \text{beta} ) or ( \text{alpha} ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskLogLogisticAlt}(&quot;\text{gamma&quot;}, 5, &quot;\text{beta&quot;}, 2, 90%, 10) ) returns a log-logistic distribution generated using a gamma value of 5, a beta value of 2, and a 90(^{th}) percentile of 10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>( \text{alpha} ) must be greater than zero. ( \text{beta} ) must be greater than zero. With ( \text{RiskLogLogisticAltD} ), any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
### RiskLognorm

**Description**

RiskLognorm(mean,standard deviation) specifies a lognormal distribution with the entered mean and standard deviation. The arguments for this form of the lognormal distribution specify the actual mean and standard deviation of the generated lognormal probability distribution.

Like the Normal distribution, the Lognormal has two parameters (µ,σ) corresponding to the mean and standard deviation. Just as the Normal distribution results from adding many random processes, so the Lognormal arises by multiplying many random processes. From a technical perspective, this is a direct extension of the previous results because the logarithm of the product of random numbers is equal to the sum of the logarithms. In practice it is often used as a representation of the future value of an asset whose value in percentage terms changes in a random and independent fashion. It is often used in the oil industry as a model of reserves following geological studies whose results are uncertain. The distribution has a number of desirable properties of real world processes. These include that it is skewed, and that it has a positive and unbounded range i.e. it ranges from 0 to infinity. Another useful property is that when σ is small compared to µ, the skew is small and the distribution approaches a Normal distribution; so any Normal distribution can be approximated by a Lognormal by using the same standard deviation, but increasing the mean (so that the ratio σ / µ is small), and then shifting the distribution by adding a constant amount so that the means match.

**Examples**

RiskLognorm(10,20) specifies a lognormal distribution with a mean of 10 and a standard deviation of 20.

RiskLognorm(C10*3.14,B10) specifies a lognormal distribution with a mean equaling the value in cell C10 times 3.14 and a standard deviation equaling the value in cell B10.

**Guidelines**

The mean and standard deviation must be greater than 0.

**Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>µ</td>
<td>continuous parameter</td>
<td>µ &gt; 0</td>
</tr>
<tr>
<td>σ</td>
<td>continuous parameter</td>
<td>σ &gt; 0</td>
</tr>
</tbody>
</table>

**Domain**

0 ≤ x < +∞  continuous
Density and Cumulative Distribution Functions

\[ f(x) = \frac{1}{x \sqrt{2\pi \sigma'}} e^{-\frac{1}{2} \left( \frac{\ln x - \mu'}{\sigma'} \right)^2} \]

\[ F(x) = \Phi\left( \frac{\ln x - \mu'}{\sigma'} \right) \]

\[ \mu' \equiv \ln \left( \frac{\mu^2}{\sqrt{\sigma^2 + \mu^2}} \right) \quad \text{and} \quad \sigma' \equiv \sqrt{\ln \left( 1 + \left( \frac{\sigma}{\mu} \right)^2 \right)} \]

where \( \Phi(z) \) is the cumulative distribution function of a Normal(0,1) also called the Laplace-Gauss Integral.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>( \mu )</td>
</tr>
<tr>
<td>Variance</td>
<td>( \sigma^2 )</td>
</tr>
<tr>
<td>Skewness</td>
<td>( \left( \frac{\sigma}{\mu} \right)^3 ) + 3 ( \frac{\sigma}{\mu} )</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>( \omega^4 + 2\omega^3 + 3\omega^2 - 3 ) with ( \omega \equiv 1 + \left( \frac{\sigma}{\mu} \right)^2 )</td>
</tr>
<tr>
<td>Mode</td>
<td>( \frac{\mu^4}{(\sigma^2 + \mu^2)^{3/2}} )</td>
</tr>
</tbody>
</table>
Examples

PDF - Lognorm(1,1)

CDF - Lognorm(1,1)
## RiskLognormAlt, RiskLognormAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskLognormAlt(arg1type, arg1value, arg2type,arg2value, arg3type,arg3value) specifies a lognormal distribution function with three arguments of the type arg1type to arg3type. These arguments can be either a percentile between 0 and 1 or mu, sigma or loc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskLognormAlt(&quot;mu&quot;,2,&quot;sigma&quot;,5,95%,30) specifies a lognormal distribution with a mean of 2 and a standard deviation of 5 and a 95\textsuperscript{th} percentile of 30.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>mu and sigma must be greater than 0. With RiskLognormAltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
### RiskLognorm2

| Description | RiskLognorm2*(mean of corresponding normal dist., std. dev. of normal)*
specifies a lognormal distribution, where the entered mean and standard deviation, equals the mean and standard deviation of the corresponding normal distribution. The arguments entered are the mean and standard deviation of the normal distribution, for which an exponential of the values in the distribution was taken to generate the desired lognormal.

| Examples | RiskLognorm2*(10, .5)* specifies a lognormal distribution generated by taking the exponential of the values from a normal distribution with a *mean* of 10 and a *standard deviation* of .5.

RiskLognorm2*(C10*3.14, B10)* specifies a lognormal distribution generated by taking the exponential of the values from a normal distribution with a *mean* equaling the value in cell C10 times 3.14 and a *standard deviation* equaling the value in cell B10.

| Guidelines | The *standard deviation* must be greater than 0.

| Parameters | µ continuous parameter
σ continuous parameter
\[ \sigma > 0 \]

| Domain | \[0 \leq x < +\infty\] continuous

| Density and Cumulative Distribution Functions |

\[
f(x) = \frac{1}{x \sqrt{2\pi\sigma}} e^{-\frac{1}{2} \left[ \ln x - \mu \right]^2/\sigma^2}
\]

\[
F(x) = \Phi \left( \frac{\ln x - \mu}{\sigma} \right)
\]

where \( \Phi(z) \) is the cumulative distribution function of a Normal(0,1), also called the Laplace-Gauss Integral

| Mean | \[
\mu + \frac{\sigma^2}{e} \frac{\sigma}{2}
\]

| Variance | \[
e^{2\mu} \omega (\omega - 1)
\]

with \( \omega \equiv e^{\sigma^2} \)

| Skewness | \[
(\omega + 2)\sqrt{\omega - 1}
\]

with \( \omega \equiv e^{\sigma^2} \)

Reference: @RISK Functions
<table>
<thead>
<tr>
<th>Kurtosis</th>
<th>$\omega^4 + 2\omega^3 + 3\omega^2 - 3$ with $\omega \equiv e^{\sigma^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>$e^{\mu - \sigma^2}$</td>
</tr>
<tr>
<td><strong>Examples</strong></td>
<td><img src="image" alt="CDF - Lognorm2(0,1)" /> <img src="image" alt="PDF - Lognorm2(0,1)" /></td>
</tr>
</tbody>
</table>
## RiskMakeInput

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskMakeInput(formula) specifies that the calculated value for formula will be treated as a simulation input, the same way as a distribution function. This function allows the results of Excel calculations (or a combination of distribution functions) to be treated as a single “input” in a sensitivity analysis.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskMakeInput (RiskNormal(10,1)+RiskTriang(1,2,3)+A5) specifies that the sum of the samples from the distributions RiskNormal(10,1) and RiskTriang(1,2,3) plus the value of cell A5 will be treated as a simulation input by @RISK. An entry for the distribution for this formula will be shown in the Inputs tab of the Results Summary window, and it will be used in sensitivity analyses for outputs which it affects.</td>
</tr>
</tbody>
</table>
| Guidelines | Distributions that precede, or “feed into”, a RiskMakeInput function are not included in a sensitivity analysis of the outputs they impact to avoid double counting of their impacts. Their impact is included in the sensitivity analysis through the RiskMakeInput function.  
The RiskMakeInput function does not have to be a precedent of an output to be included in its sensitivity analysis – only the distributions that precede the RiskMakeInput do. For example, you can add a single RiskMakeInput function that averages a set of distributions. Each distribution in the averaged set might be precedents of an output. They will be replaced in the sensitivity analysis for that output by the RiskMakeInput function.  
The following distribution property functions may be included in a RiskMakeInput function: RiskName, RiskCollect, RiskCategory, RiskStatic, RiskUnits, RiskSixSigma, RiskConvergence, RiskIsDiscrete, and RiskIsDate.  
RiskMakeInput functions may not be correlated using RiskCorrmat, since they are not sampled the same way as standard distribution functions are.  
No graph is available of a RiskMakeInput function, prior to simulation, in the Define Distribution or Model windows.  
RiskTheo functions cannot be used with inputs specified using RiskMakeInput. |
### RiskNegbin

| Description | $\text{RiskNegbin}(s, p)$ specifies a negative binomial distribution with $s$ number of successes and $p$ probability of success on each trial. The negative binomial distribution is a discrete distribution returning only integer values greater than, or equal to, zero.  
This distribution represents the number of failures before several successes of a binomial distribution have occurred, so that $\text{NegBin}(1, p) = \text{Geomet}(p)$. It is sometimes used in models of quality control and production testing, breakdown and maintenance modelling. |
|---|---|
| Examples | $\text{RiskNegbin}(5, .25)$ specifies a negative binomial distribution with 5 successes with a 25% probability of success on each trial.  
$\text{RiskNegbin}(A6, A7)$ specifies a negative binomial distribution with the number of successes taken from cell A6 and a probability of success taken from cell A7. |
| Guidelines | Number of successes $s$ must be a positive integer less than or equal to 32,767.  
Probability $p$ must be greater than zero and less than, or equal to, one. |
| Parameters | $S$ the number of successes  
discrete parameter $s \geq 0$  
$p$ probability of a single success  
continuous parameter $0 < p \leq 1$ |
| Domain | $0 \leq x < +\infty$ discrete integers |
| Mass and Cumulative Distribution Functions | $f(x) = \binom{s + x - 1}{x} p^s (1 - p)^x$  
$F(x) = p^s \sum_{i=0}^{x} \binom{s + i - 1}{i} (1 - p)^i$  
Where $\binom{ }{ }$ is the Binomial Coefficient. |
| Mean | $\frac{s(1 - p)}{p}$ |
| Variance | $\frac{s(1 - p)}{p^2}$ |
| Skewness | $\frac{2 - p}{\sqrt{s(1 - p)}}$ for $s > 0$, $p < 1$ |
Kurtosis

\[ 3 + \frac{6}{s} + \frac{p^2}{s(1-p)} \]

for \( s > 0 \), \( p < 1 \)

Mode

(bimodal) \( z \) and \( z + 1 \) \( \text{integer } z > 0 \)

(unimodal) \( 0 \) \( z < 0 \)

(unimodal) smallest integer greater than \( z \) otherwise

Mode

\[ z = \frac{s(1-p)-1}{p} \]

where

Examples

PDF - NegBin(3, 0.6)

CDF - NegBin(3, 0.6)
RiskNormal

| Description | RiskNormal(mean,standard deviation) specifies a normal distribution with the entered mean and standard deviation. This is the traditional “bell shaped” curve applicable to distributions of outcomes in many data sets.

The Normal distribution is a symmetric continuous distribution which is unbounded on both sides, and described by two parameters (µ and σ i.e. its mean and standard deviation). The use of the Normal distribution can often be justified with reference to a mathematical result called the Central Limit Theorem. This loosely states that if many independent distributions are added together, then the resulting distribution is approximately Normal. The distribution therefore often arises in the real world as the compound effect of more detailed (non-observed) random processes. Such a result applies independently of the shape of the initial distributions being added.

The distribution can be used to represent the uncertainty of a model’s input whenever it is believed that the input is itself the result of many other similar random processes acting together in an additive manner (but where it may be unnecessary, inefficient, or impractical to model theses detailed driving factors individually). Examples could include the total number of goals scored in the a soccer season, the amount of oil in the world, assuming that there are many reservoirs of approximately equal size, but each with an uncertain amount of oil. Where the mean is much larger than the standard deviation (e.g. 4 times or more) then a negative sampled value of the distribution would occur only rarely (so that the number of goals would not be sampled negatively in most practical cases). More generally, the output of many models is approximately normally distributed, because many models have an output which results from adding many other uncertain processes. An example might be the distribution of discounted cash flow in a long-term time series models, which consists of summing the discounted cash flows from the individual years.

| Examples | RiskNormal(10,2) specifies a normal distribution with a mean of 10 and a standard deviation of 2.

RiskNormal(SQRT(C101),B10) specifies a normal distribution with a mean equaling the square root of the value in cell C101 and a standard deviation taken from cell B10.

| Guidelines | The standard deviation must be greater than 0.

| Parameters | µ  continuous location parameter

σ  continuous scale parameter  σ > 0 *

*σ = 0 is supported for modeling convenience, but gives a degenerate distribution with x = µ.

| Domain | -∞ < x < +∞  continuous |
Density and Cumulative Distribution Functions

\[
f(x) = \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2}
\]

\[
F(x) \equiv \Phi \left( \frac{x - \mu}{\sigma} \right) = \frac{1}{2} \left[ \text{erf} \left( \frac{x - \mu}{\sqrt{2\sigma}} \right) + 1 \right]
\]

where \( \Phi \) is called the Laplace-Gauss Integral and erf is the Error Function.

<table>
<thead>
<tr>
<th><strong>Mean</strong></th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variance</strong></td>
<td>( \sigma^2 )</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>0</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>3</td>
</tr>
<tr>
<td><strong>Mode</strong></td>
<td>( \mu )</td>
</tr>
</tbody>
</table>

Tables:

<table>
<thead>
<tr>
<th>x</th>
<th>PDF - Normal(0,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0.00</td>
</tr>
<tr>
<td>-2</td>
<td>0.05</td>
</tr>
<tr>
<td>-1</td>
<td>0.10</td>
</tr>
<tr>
<td>0</td>
<td>0.15</td>
</tr>
<tr>
<td>1</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>0.35</td>
</tr>
<tr>
<td>5</td>
<td>0.40</td>
</tr>
<tr>
<td>6</td>
<td>0.45</td>
</tr>
<tr>
<td>7</td>
<td>0.50</td>
</tr>
<tr>
<td>8</td>
<td>0.55</td>
</tr>
<tr>
<td>9</td>
<td>0.60</td>
</tr>
<tr>
<td>10</td>
<td>0.65</td>
</tr>
<tr>
<td>11</td>
<td>0.70</td>
</tr>
<tr>
<td>12</td>
<td>0.75</td>
</tr>
<tr>
<td>13</td>
<td>0.80</td>
</tr>
<tr>
<td>14</td>
<td>0.85</td>
</tr>
<tr>
<td>15</td>
<td>0.90</td>
</tr>
<tr>
<td>16</td>
<td>0.95</td>
</tr>
<tr>
<td>17</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Reference: @RISK Functions
CDF - Normal(0,1)
**RiskNormalAlt, RiskNormalAltD**

<table>
<thead>
<tr>
<th>Description</th>
<th><em>RiskNormalAlt</em>(arg1type, arg1value, arg2type, arg2value) specifies a normal distribution with two arguments of the type <em>arg1type</em> and <em>arg2type</em>. These arguments can be either a <em>percentile</em> between 0 and 1 or <em>mu</em> or <em>sigma</em>.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><em>RiskNormalAlt</em>(5%, 1, 95%, 10) specifies a normal distribution with a 5(^{\text{th}}) percentile of 1 and a 95(^{\text{th}}) percentile of 10.</td>
</tr>
</tbody>
</table>
| Guidelines | *sigma* must be greater than 0.  
With *RiskNormalAltD*, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value. |
## RiskPareto

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskPareto}(\theta, \alpha) ) specifies a pareto distribution with the entered ( \theta ) and ( \alpha ) values.</th>
</tr>
</thead>
</table>
| Examples | \( \text{RiskPareto}(5,5) \) specifies a pareto distribution with a \( \theta \) value of 5 and an \( \alpha \) value of 5.  
\( \text{RiskPareto}(A10,A11+A12) \) specifies a pareto distribution with a \( \theta \) value taken from cell A10 and an \( \alpha \) value given by the result of the expression \( A11+A12 \). |
| Guidelines | \( \theta \) must be greater than 0.  
\( \alpha \) must be greater than 0. |
| Parameters | \( \theta \) continuous shape parameter \( \theta > 0 \)  
\( \alpha \) continuous scale parameter \( \alpha > 0 \) |
| Domain | \( \alpha \leq x < +\infty \) continuous |
| Density and Cumulative Distribution Functions | \( f(x) = \frac{\theta a^\theta}{x^{\theta+1}} \)  
\( F(x) = 1 - \left( \frac{a}{x} \right)^\theta \)  
where \( a = \alpha \) |
<p>| Mean | ( \frac{a\theta}{\theta - 1} ) for ( \theta &gt; 1 ) |
| Variance | ( \frac{\theta a^2}{(\theta - 1)^2(\theta - 2)} ) for ( \theta &gt; 2 ) |
| Skewness | ( 2 \frac{\theta + 1}{\theta - 3} \sqrt{\frac{\theta - 2}{\theta}} ) for ( \theta &gt; 3 ) |
| Kurtosis | ( \frac{3(\theta - 2)(3\theta^2 + \theta + 2)}{\theta(\theta - 3)(\theta - 4)} ) for ( \theta &gt; 4 ) |</p>
<table>
<thead>
<tr>
<th>Mode</th>
<th>$alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Examples</strong></td>
<td></td>
</tr>
</tbody>
</table>

**PDF - Pareto(2,1)**

![PDF - Pareto(2,1)](image1)

**CDF - Pareto(2,1)**

![CDF - Pareto(2,1)](image2)
### RiskParetoAlt, RiskParetoAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskParetoAlt}(\text{arg1type}, \text{arg1value}, \text{arg2type}, \text{arg2value}) ) specifies a pareto distribution with two arguments of the type ( \text{arg1type} ) and ( \text{arg2type} ). These arguments can be either a percentile between 0 and 1 or ( \theta ) or ( \alpha ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskParetoAlt}(5%, 1, 95%, 4) ) specifies a pareto distribution with a 5(^{th}) percentile of 1 and a 95(^{th}) percentile of 4.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>( \theta ) must be greater than 0. ( \alpha ) must be greater than 0. With ( \text{RiskParetoAltD} ), any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
# RiskPareto2

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskPareto2}(b,q) ) specifies a pareto distribution with the entered ( b ) and ( q ) values.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskPareto2}(5,5) ) specifies a pareto distribution with a ( b ) value of 5 and a ( q ) value of 5.</td>
</tr>
<tr>
<td></td>
<td>( \text{RiskPareto2}(A10,A11+A12) ) specifies a pareto distribution with a ( b ) value taken from cell A10 and a ( q ) value given by the result of the expression A11+A12.</td>
</tr>
</tbody>
</table>
| Guidelines | \( b \) must be greater than 0.  
\( q \) must be greater than 0. |
| Parameters | \( b \) continuous scale parameter \( b > 0 \)  
\( q \) continuous shape parameter \( q > 0 \) |
| Domain      | \( 0 \leq x < +\infty \) continuous |
| Density and Cumulative Distribution Functions | \( f(x) = \frac{qb^q}{(x+b)^{q+1}} \)  
\( F(x) = 1 - \frac{b^q}{(x+b)^q} \) |
| Mean        | \( \frac{b}{q-1} \) for \( q > 1 \) |
| Variance    | \( \frac{b^2q}{(q-1)^2(q-2)} \) for \( q > 2 \) |
| Skewness    | \( 2 \left[ \frac{q+1}{q-3} \right] \sqrt{\frac{q-2}{q}} \) for \( q > 3 \) |
| Kurtosis    | \( \frac{3(q-2)(3q^2 + q + 2)}{q(q-3)(q-4)} \) for \( q > 4 \) |
| Mode        | 0 |

Reference: @RISK Functions
Examples

PDF - Pareto2(3, 3)

CDF - Pareto2(3, 3)
## Description

$\text{RiskPareto2Alt}(\text{arg1type}, \text{arg1value}, \text{arg2type}, \text{arg2value})$ specifies a pareto distribution with two arguments of the type $\text{arg1type}$ and $\text{arg2type}$. These arguments can be either a percentile between 0 and 1 or $b$ or $q$.

## Examples

$\text{RiskPareto2Alt}(5\%, .05, 95\%, 5)$ specifies a pareto distribution with a 5th percentile of .05 and a 95th percentile of 5.

## Guidelines

- $b$ must be greater than 0.
- $q$ must be greater than 0.

With $\text{RiskPareto2AltD}$, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.
# RiskPearson5

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskPearson5(alpha,beta) specifies a Pearson type V distribution with shape parameter alpha and scale parameter beta.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskPearson5(1,1) specifies a Pearson type V distribution where the shape parameter has a value of 1 and the scale parameter has a value of 1. RiskPearson5(C12,C13) specifies a Pearson type V distribution where the shape parameter has a value taken from cell C12 and the scale parameter has a value taken from cell C13.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>alpha must be greater than zero. beta must be greater than zero.</td>
</tr>
<tr>
<td>Parameters</td>
<td>α is a continuous shape parameter, α &gt; 0. β is a continuous scale parameter, β &gt; 0.</td>
</tr>
<tr>
<td>Domain</td>
<td>0 ≤ x &lt; +∞, continuous</td>
</tr>
</tbody>
</table>

## Density and Cumulative Distribution Functions

\[
f(x) = \frac{1}{\beta \Gamma(\alpha)} \cdot \frac{e^{-\beta/x}}{(x/\beta)^{\alpha+1}}
\]

F(x) Has No Closed Form

## Mean

\[
\frac{\beta}{\alpha - 1}
\]

for \( \alpha > 1 \)

## Variance

\[
\frac{\beta^2}{(\alpha - 1)^2 (\alpha - 2)}
\]

for \( \alpha > 2 \)

## Skewness

\[
\frac{4\sqrt{\alpha - 2}}{\alpha - 3}
\]

for \( \alpha > 3 \)

## Kurtosis

\[
\frac{3(\alpha + 5)(\alpha - 2)}{(\alpha - 3)(\alpha - 4)}
\]

for \( \alpha > 4 \)

## Mode

\[
\frac{\beta}{\alpha + 1}
\]
Examples

PDF - Pearson5(3,1)

CDF - Pearson5(3,1)
**RiskPearson5Alt, RiskPearson5AltD**

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{RiskPearson5Alt}(\text{arg1type, arg1value, arg2type, arg2value, arg3type, arg3value})$ specifies a Pearson type V distribution with three arguments of the type $\text{arg1type}$ to $\text{arg3type}$. These arguments can be either a percentile between 0 and 1 or $\text{alpha}$, $\text{beta}$ or $\text{loc}$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskPearson5Alt(&quot;alpha&quot;, 2, &quot;beta&quot;, 5, 95%, 30) specifies a Pearson type V distribution with a $\text{alpha}$ of 2 and a $\text{beta}$ of 5 and a 95(^{\text{th}}) percentile of 30.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{alpha}$ must be greater than zero.</td>
</tr>
<tr>
<td>$\text{beta}$ must be greater than zero.</td>
</tr>
<tr>
<td>With $\text{RiskPearson5AltD}$, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
<tr>
<td>Description</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Examples</td>
</tr>
</tbody>
</table>
| Guidelines | *alpha1* must be greater than zero. 
*alpha2* must be greater than zero. 
beta must be greater than zero. |
| Parameters | *α*<sub>1</sub> continuous shape parameter  
*α*<sub>2</sub> continuous shape parameter  
*β* continuous scale parameter |
| Domain | 0 ≤ x < +∞  continuous |
| Density and Cumulative Distribution Functions | 
\[ f(x) = \frac{1}{\beta B(\alpha_1, \alpha_2)} \times \left(\frac{x}{\beta}\right)^{\alpha_1-1} \left(1 + \frac{x}{\beta}\right)^{-\alpha_1-\alpha_2} \]
F(x) Has No Closed Form. 
where B is the Beta Function. |
| Mean | \( \frac{\beta \alpha_1}{\alpha_2 - 1} \) for \( \alpha_2 > 1 \) |
| Variance | \( \frac{\beta^2 \alpha_1 (\alpha_1 + \alpha_2 - 1)}{(\alpha_2 - 1)^2 (\alpha_2 - 2)} \) for \( \alpha_2 > 2 \) |
| Skewness | \( 2 \sqrt{\frac{\alpha_2 - 2}{\alpha_1 (\alpha_1 + \alpha_2 - 1)}} \left[ \frac{2\alpha_1 + \alpha_2 - 1}{\alpha_2 - 3} \right] \) for \( \alpha_2 > 3 \) |

Reference: @RISK Functions
<table>
<thead>
<tr>
<th>Phase</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td>[ \frac{3(\alpha_2 - 2)}{(\alpha_2 - 3)(\alpha_2 - 4)} \left( \frac{2(\alpha_2 - 1)^2}{\alpha_1(\alpha_1 + \alpha_2 - 1)} + (\alpha_2 + 5) \right) ] for ( \alpha_2 &gt; 4 ]</td>
</tr>
<tr>
<td>Mode</td>
<td>[ \frac{\beta(\alpha_1 - 1)}{\alpha_2 + 1} ] for ( \alpha_1 &gt; 1 ]</td>
</tr>
</tbody>
</table>

0 otherwise

**Examples**

PDF - Pearson6(3,3,1)

CDF - Pearson6(3,3,1)
**RiskPert**

**Description**

RiskPert\((\text{minimum, most likely, maximum})\) specifies a PERT distribution (as special form of the beta distribution) with a minimum and maximum value as specified. The shape parameter is calculated from the defined most likely value.

The PERT distribution (meaning Program Evaluation and Review Technique) is rather like a Triangular distribution, in that it has the same set of three parameters. Technically it is a special case of a scaled Beta (or BetaGeneral) distribution. In this sense it can be used as a pragmatic and readily understandable distribution. It can generally be considered as superior to the Triangular distribution when the parameters result in a skewed distribution, as the smooth shape of the curve places less emphasis in the direction of skew. As for the Triangular distribution, the PERT distribution is bounded on both sides, and hence may not be adequate for some modelling purposes where it is desired to capture tail or extreme events.

**Examples**

RiskPert\((0,2,10)\) specifies a beta distribution with a minimum of 0, a maximum of 10, and a most likely value of 2.

RiskPert\((\text{A1,A2,A3})\) specifies a PERT distribution with a minimum value taken from cell A1, a maximum value taken from cell A3, and a most likely value taken from cell A2.

**Guidelines**

*minimum* must be less than *maximum*.

*most likely* must be greater than *minimum* and less than *maximum*.

**Definitions**

\[
\mu = \frac{\text{min} + 4 \cdot \text{m.likely} + \text{max}}{6} \\
\alpha_1 = 6 \left[ \frac{\mu - \text{min}}{\text{max} - \text{min}} \right] \\
\alpha_2 = 6 \left[ \frac{\text{max} - \mu}{\text{max} - \text{min}} \right]
\]

**Parameters**

- **min**: continuous boundary parameter
  - \(\text{min} < \text{max}\)

- **m.likely**: continuous parameter
  - \(\text{min} < \text{m.likely} < \text{max}\)

- **max**: continuous boundary parameter

**Domain**

\(\text{min} \leq x \leq \text{max}\) continuous
### Density and Cumulative Distribution Functions

The density function is given by:

\[
f(x) = \frac{(x - \min)^{\alpha_1 - 1}(\max - x)^{\alpha_2 - 1}}{B(\alpha_1, \alpha_2)(\max - \min)^{\alpha_1 + \alpha_2 - 1}}
\]

The cumulative distribution function is:

\[
F(x) = \frac{B_z(\alpha_1, \alpha_2)}{B(\alpha_1, \alpha_2)} = I_z(\alpha_1, \alpha_2)
\]

where \( z = \frac{x - \min}{\max - \min} \) with \( B \) is the *Beta Function* and \( B_z \) is the *Incomplete Beta Function*.

### Mean

\[
\mu \equiv \frac{\min + 4 \cdot \text{m.likely} + \max}{6}
\]

### Variance

\[
\frac{(\mu - \min)(\max - \mu)}{7}
\]

### Skewness

\[
\frac{\min + \max - 2\mu}{4} \sqrt{\frac{7}{(\mu - \min)(\max - \mu)}}
\]

### Kurtosis

\[
3 \left( \frac{\alpha_1 + \alpha_2 + 1}{\alpha_1 \alpha_2 (\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3)} \right) \left( \frac{2(\alpha_1 + \alpha_2)^2 + \alpha_1 \alpha_2 (\alpha_1 + \alpha_2 - 6)}{\alpha_1 \alpha_2 (\alpha_1 + \alpha_2 + 2)(\alpha_1 + \alpha_2 + 3)} \right)
\]

### Mode

\( \text{m.likely} \)

### Examples

- **PDF - Pert(0,1,3)**

![PDF - Pert(0,1,3)](image-url)
RiskPertAlt, RiskPertAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskPertAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value) specifies a PERT distribution with three arguments of the type arg1type to arg3type. These arguments can be either a percentile between 0 and 1 or min, m. likely or max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskPertAlt(&quot;min&quot;, 2,&quot;m. likely&quot;, 5,95%,30) specifies a PERT distribution with a minimum of 2 and a most likely value of 5 and a 95th percentile of 30.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>min must be less than or equal to the m. likely value. m. likely must be less than or equal to the max value. min must be less than the max value. With RiskPertAltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
**RiskPoisson**

<table>
<thead>
<tr>
<th>Description</th>
<th><em>RiskPoisson</em>(lambda) specifies a poisson distribution with the specified lambda value. The argument lambda is also the same as the mean of the poisson distribution. The poisson distribution is a discrete distribution returning only integer values greater than or equal to zero. The Poisson distribution is a model for the number of events that occur in a given time period where the intensity of the process is constant (it can equally be applied to processes over other domains e.g. spatial). The distribution can be thought of as an extension of the Binomial distribution (which has a discrete domain). It is often used in insurance modeling and financial markets as a distribution of the number of events (e.g. earthquakes, fires, stock market crashes) that might occur in a given period.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guidelines</td>
<td>lambda must be greater than zero.</td>
</tr>
<tr>
<td>Parameters</td>
<td>lambda mean number of successes continuous lambda &gt; 0 *</td>
</tr>
<tr>
<td>Domain</td>
<td>0 ≤ x &lt; +∞ discrete integers</td>
</tr>
<tr>
<td>Mass and Cumulative Distribution Functions</td>
<td>f(x) = ( \frac{\lambda^x e^{-\lambda}}{x!} ) F(x) = ( e^{-\lambda} \sum_{n=0}^{x} \frac{\lambda^n}{n!} )</td>
</tr>
<tr>
<td>Mean</td>
<td>lambda</td>
</tr>
<tr>
<td>Variance</td>
<td>lambda</td>
</tr>
<tr>
<td>Skewness</td>
<td>( \frac{1}{\sqrt{\lambda}} )</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3 + ( \frac{1}{\lambda} )</td>
</tr>
<tr>
<td>Mode</td>
<td>(bimodal) lambda and lambda-1 (bimodal) if lambda is an integer (unimodal) largest integer less than lambda otherwise</td>
</tr>
</tbody>
</table>
### Examples

**CDF - Poisson(3)**

![CDF Graph]

**PMF - Poisson(3)**

![PMF Graph]
## RiskRayleigh

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskRayleigh( (\text{beta}) ) specifies a rayleigh distribution with mode ( \text{beta} ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskRayleigh(3) specified a rayleigh distribution with a mode of 3. RiskRayleigh(C7) specifies a rayleigh distribution with a mode taken from the value in cell C7.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>( \text{beta} ) must be greater than zero.</td>
</tr>
<tr>
<td>Parameters</td>
<td>( \text{beta} ) continuous scale parameter ( \text{beta} &gt; 0 )</td>
</tr>
<tr>
<td>Domain</td>
<td>( 0 \leq x &lt; +\infty ) continuous</td>
</tr>
</tbody>
</table>
| Density and Cumulative Distribution Functions | \[
\begin{align*}
  f(x) &= \frac{x}{b^2} e^{-\left(\frac{x}{b}\right)^2} \\
  F(x) &= 1 - e^{-\left(\frac{x}{b}\right)^2}
\end{align*}
\]
Where \( b = \text{beta} \) |
| Mean        | \( b \sqrt{\frac{\pi}{2}} \)                                                                    |
| Variance    | \( b^2 \left( 2 - \frac{\pi}{2} \right) \)                                                      |
| Skewness    | \( \frac{2(\pi - 3)\sqrt{\pi}}{(4 - \pi)^{3/2}} \approx 0.6311 \)                               |
| Kurtosis    | \( \frac{32 - 3\pi^2}{(4 - \pi)^2} \approx 3.2451 \)                                           |
| Mode        | \( b \)                                                                                         |
Examples

PDF - Rayleigh(1)

CDF - Rayleigh(1)
<table>
<thead>
<tr>
<th>RiskRayleighAlt, RiskRayleighAltD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description</strong></td>
</tr>
<tr>
<td>\textit{RiskRayleighAlt}(arg1type, arg1value, arg2type, arg2value) specifies a rayleigh distribution with two arguments of the type arg1type and arg2type. These arguments can be either a percentile between 0 and 1 or beta or loc.</td>
</tr>
<tr>
<td><strong>Examples</strong></td>
</tr>
<tr>
<td>\textit{RiskRayleighAlt}(5%,1.95%,10) specifies a normal distribution with a 5th percentile of 1 and a 95th percentile of 10.</td>
</tr>
<tr>
<td><strong>Guidelines</strong></td>
</tr>
<tr>
<td>\textit{beta} must be greater than zero.</td>
</tr>
<tr>
<td>With \textit{RiskRayleighAltD}, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RiskResample</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description</strong></td>
</tr>
<tr>
<td>\textit{RiskResample}(sampMethod, {X1,X2,\ldots,Xn}) samples from a data set with n possible outcomes with an equal probability of each outcome occurring. The value for each possible outcome is given by the X value entered for the outcome. Each value is equally likely to occur. \textit{@RISK} will sample from the X values using \textit{sampMethod}. Available \textit{sampMethods} are order, random with replacement and random without replacement.</td>
</tr>
<tr>
<td><strong>Examples</strong></td>
</tr>
<tr>
<td>\textit{RiskResample}(2,{1,2.1,4.45,99}) specifies a data set with 4 possible outcomes. The possible outcomes have the values 1, 2.1, 4.45 and 99. Samples will be randomly drawn with replacement from these 4 values.</td>
</tr>
<tr>
<td>\textit{RiskResample}(1,A1:A500) specifies a data set with 500 possible values. The possible values are taken from cells A1 through A500. In a simulation values will be sampled in order from this range.</td>
</tr>
<tr>
<td><strong>Guidelines</strong></td>
</tr>
<tr>
<td>\textit{sampMethod} can be 1-order, 2-random with replacement or 3-random without replacement.</td>
</tr>
<tr>
<td>A RiskLibrary property function can be included with a Resample function to link the X data to a simulation output that is stored in the \textit{@RISK} Library. A RiskLibrary property function causes \textit{@RISK} to update the Resample X data with the current data stored for the simulation output at the start of each simulation. Thus, if a new version of the simulation containing the output has been saved in the \textit{@RISK} Library, \textit{@RISK} will automatically update the RiskResample function with the new data for that output prior to simulating.</td>
</tr>
<tr>
<td>When using the random with or without replacement methods, you should also consider the sampling type defined under Simulation Settings. When the number of possible values and the number of iterations are the same, all values will be used in both the random with replacement and random without replacement methods when the sampling type is set to Latin Hypercube, since each stratification will be covered exactly once. Using the sampling type set to Monte Carlo will result in duplicate values when the RiskResample sampling method is random with replacement.</td>
</tr>
</tbody>
</table>
## RiskSimtable

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskSimtable(val1, val2,..., valn) specifies a list of values which will be used sequentially in individual simulations executed during a Sensitivity Simulation. In a Sensitivity Simulation, the number of simulations, set using the Iterations Simulations command, is greater than one. In a single simulation, or a normal recalculation, RiskSimtable returns the first value in the list. Any number of RiskSimtable functions may be included in a single worksheet. As with other functions, the arguments of RiskSimtable may include distribution functions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskSimtable({10, 20, 30, 40}) specifies four values to be used in each of four simulations. In Simulation #1 the SIMTABLE function will return 10, Simulation #2 the value 20 and so on. RiskSimtable(A1:A3) specifies a list of three values for three simulations. In Simulation #1 the value from cell A1 will be returned. In Simulation #2 the value from cell A2 will be returned. In Simulation #3 the value from cell A3 will be returned.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Any number of arguments may be entered. The number of simulations executed must be less than, or equal to, the number of arguments. If the number of arguments is less than the number of an executing simulation, ERR will be returned by the function for that simulation.</td>
</tr>
</tbody>
</table>
**RiskSplice**

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskSplice}(\text{dist#1 or cellref,dist#2 or cellref,splice point})$ specifies a distribution created by splicing $\text{dist#1}$ to $\text{dist#2}$ at the x-value given by $\text{splice point}$. Samples below the $\text{splice point}$ are drawn from $\text{dist#1}$ and above from $\text{dist#2}$. The resulting distribution is treated as a single input distribution in a simulation and may be correlated.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskSplice}(\text{RiskNormal}(1,1),\text{RiskPareto}(1,1),2)$ splices a normal distribution with a mean of 1 and a standard deviation of 1 to a pareto distribution with a $\theta=1$ and $\alpha=1$ at the splice point of 2.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>$\text{dist#1}$ and $\text{dist#2}$ may not be correlated. The $\text{RiskSplice}$ itself may be correlated. $\text{dist#1}$, $\text{dist#2}$, and $\text{RiskSplice}$ itself may include property functions, except $\text{RiskCorrmat}$ as noted above. $\text{dist#1}$ and $\text{dist#2}$ may be a reference to a cell that contains a distribution function. The two pieces of the distribution will be re-weighted since the total area under the (spliced) curve still has to equal 1. Thus the probability density of any given x value in the resulting spliced distribution will probably be different from what it was in the original distribution.</td>
</tr>
</tbody>
</table>
**RiskStudent**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskStudent(nu) specified a student's t distribution with nu degrees of freedom.</th>
</tr>
</thead>
</table>

**Examples**

- RiskStudent(10) specifies a student's t distribution with 10 degrees of freedom.
- RiskStudent(J2) specifies a student's t distribution with the degrees of freedom taken from the value in cell J2.

**Guidelines**

nu must be a positive integer.

**Parameters**

<table>
<thead>
<tr>
<th>ν</th>
<th>the degrees of freedom</th>
<th>integer</th>
<th>ν &gt; 0</th>
</tr>
</thead>
</table>

**Domain**

-∞ < x < +∞ continuous

**Density and Cumulative Distribution Functions**

\[
f(x) = \frac{1}{\sqrt{\pi \nu}} \frac{\Gamma \left( \frac{\nu + 1}{2} \right)}{\Gamma \left( \frac{\nu}{2} \right)} \left[ \frac{\nu}{\nu + x^2} \right]^{\frac{\nu + 1}{2}}
\]

\[
F(x) = \frac{1}{2} \left[ 1 + I_s \left( \frac{1}{2}, \frac{\nu}{2}, \frac{x^2}{\nu + x^2} \right) \right]
\]

where \( \Gamma \) is the Gamma Function and \( I_s \) is the Incomplete Beta Function.

**Mean**

0 for \( \nu > 1^* \)

*even though the mean is not defined for \( \nu = 1 \), the distribution is still symmetrical about 0.

**Variance**

\[ \frac{\nu}{\nu - 2} \] for \( \nu > 2 \)

**Skewness**

0 for \( \nu > 3^* \)

*even though the skewness is not defined for \( \nu \leq 3 \), the distribution is still symmetric about 0.

**Kurtosis**

\[ 3 \left( \frac{\nu - 2}{\nu - 4} \right) \] for \( \nu > 4 \)

**Mode**

0
Examples

CDF - Student(3)

PDF - Student(3)
**RiskTriang**

| Description | **RiskTriang**(minimum,most likely,maximum) specifies a triangular distribution with three points — a minimum, most likely, and maximum. The direction of the "skew" of the triangular distribution is set by the size of the most likely value relative to the minimum and the maximum.

This distribution is perhaps the most readily understandable and pragmatic distribution for basic risk models. It has a number of desirable properties, including a simple set of parameters including the use of a modal value i.e. a most likely case. There are two main disadvantages of a Triangular distribution. First, when the parameters result in a skewed distribution, then there may be an over-emphasis of the outcomes in the direction of the skew. Second, the distribution is bounded on both sides, whereas many real-life processes are bounded on one side but unbounded on the other. |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskTriang</strong>(100,200,300) specifies a triangular distribution with a minimum value of 100, a most likely value of 200 and a maximum value of 300. <strong>RiskTriang</strong>(A10/90,B10,500) specifies a triangular distribution with a minimum value equaling the value in cell A10 divided by 90, a most likely value taken from cell B10 and a maximum value of 500.</td>
</tr>
</tbody>
</table>
| Guidelines | minimum must be less than, or equal to, most likely.
most likely must be less than, or equal to, maximum.
minimum must be less than maximum. |
| Parameters | min continuous boundary parameter
min < max *
m.likely continuous mode parameter
min ≤ m.likely ≤ max
max continuous boundary parameter

*min = max is supported for modeling convenience, but gives a degenerate distribution. |
| Domain | min ≤ x ≤ max continuous |
# Density and Cumulative Distribution Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$f(x) = \frac{2(x - \text{min})}{(\text{m.likely} - \text{min})(\text{max} - \text{min})}$ for $\text{min} \leq x \leq \text{m.likely}$</td>
</tr>
<tr>
<td>Density</td>
<td>$f(x) = \frac{2(\text{max} - x)}{(\text{max} - \text{m.likely})(\text{max} - \text{min})}$ for $\text{m.likely} \leq x \leq \text{max}$</td>
</tr>
<tr>
<td>Cumulative Distribution</td>
<td>$F(x) = \frac{(x - \text{min})^2}{(\text{m.likely} - \text{min})(\text{max} - \text{min})}$ for $\text{min} \leq x \leq \text{m.likely}$</td>
</tr>
<tr>
<td>Cumulative Distribution</td>
<td>$F(x) = 1 - \frac{(\text{max} - x)^2}{(\text{max} - \text{m.likely})(\text{max} - \text{min})}$ for $\text{m.likely} \leq x \leq \text{max}$</td>
</tr>
</tbody>
</table>

### Mean
\[
\text{Mean} = \frac{\text{min} + \text{m.likely} + \text{max}}{3}
\]

### Variance
\[
\text{Variance} = \frac{\text{min}^2 + \text{m.likely}^2 + \text{max}^2 - (\text{max})(\text{m.likely}) - (\text{m.likely})(\text{min}) - (\text{max})(\text{min})}{18}
\]

### Skewness
\[
\text{Skewness} = \frac{2\sqrt{2} \left(f \left(\frac{f^2 - 9}{f^2 + 3}^{3/2}\right) - 1\right)}{5}
\]
where
\[
f = \frac{2(\text{m.likely} - \text{min})}{\text{max} - \text{min}}
\]

### Kurtosis
2.4

### Mode
\text{m.likely}
Examples

PDF - Triang(0,3,5)

CDF - Triang(0,3,5)
## RiskTriangAlt, RiskTriangAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskTriangAlt}(\text{arg1type, arg1value, arg2type, arg2value, arg3type, arg3value}) ) specifies a triangular distribution with three arguments of the type ( \text{arg1type} ) to ( \text{arg3type} ). These arguments can be either a \textit{percentile} between 0 and 1 or \textit{min}, \textit{m. likely} or \textit{max}.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskTriangAlt}(&quot;\text{min}&quot;,2,&quot;\text{m. likely}&quot;,5,95%,30) ) specifies a triangular distribution with a \textit{minimum} of 2 and a most likely value of 5 and a 95\textsuperscript{th} percentile of 30.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>\textit{min} must be less than, or equal to, the \textit{m. likely} value. \textit{m. likely} must be less than, or equal to, the \textit{max} value. \textit{min} must be less than the \textit{max} value. With ( \text{RiskTriangAltD} ), any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>

## RiskTrigen

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskTrigen}(\text{bottom value, most likely value, top value, bottom perc., top perc.}) ) specifies a triangular distribution with three points — one at the most likely value and two at the specified bottom and top percentiles. The bottom percentile and top percentile are values between 0 and 100. Each percentile value gives the percentage of the total area under the triangle that falls to the left of the entered point. Use of the RiskTrigen function avoids the problem of the minimum and maximum values not actually being possible occurrences in the standard RiskTriang function. This is because in the RiskTriang function these are the points where the distribution intersects the X-axis, or points of zero probability.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskTrigen}(100,200,300,10,90) ) specifies a triangular distribution with a 10\textsuperscript{th} percentile value of 100, a most likely value of 200 and a 90\textsuperscript{th} percentile value of 300. ( \text{RiskTrigen}(A10/90,B10,500,30,70) ) specifies a triangular distribution with a 30\textsuperscript{th} percentile value equaling the value in cell A10 divided by 90, a most likely value taken from cell B10 and a 70\textsuperscript{th} percentile value of 500.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>\textit{bottom value} must be less than, or equal to, \textit{most likely value}. \textit{most likely value} must be less than, or equal to, \textit{top value}. \textit{bottom perc.} must be less than \textit{top perc.}.</td>
</tr>
</tbody>
</table>
### RiskUniform

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskUniform(minimum, maximum) specifies a uniform probability distribution with the entered minimum and maximum values. Every value across the range of the uniform distribution has an equal likelihood of occurrence. This distribution is sometimes referred to as a “no knowledge” distribution. Processes which might be considered to follow a uniform continuous distribution include the position of a particular air molecule in a room, or the point on a car tyre where the next puncture will occur. In many uncertain situations, there is in fact a base or modal value, where the relative likelihood of other outcomes decreases as one moves away from this base value. For this reason there are only a few real life cases where this distribution genuinely captures all the knowledge that one has about a situation. The distribution is nevertheless extremely important, not least because it is often used by random number algorithms as a first step to generate samples from other distributions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskUniform(10, 20) specifies a uniform distribution with a minimum value of 10 and a maximum value of 20. RiskUniform(A1+90,B1) specifies a uniform distribution with a minimum value equaling the value in cell A1 plus 90 and a maximum value taken from cell B1.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>minimum must be less than maximum.</td>
</tr>
<tr>
<td>Parameters</td>
<td>min continuous boundary parameter min &lt; max *</td>
</tr>
<tr>
<td></td>
<td>max continuous boundary parameter</td>
</tr>
<tr>
<td></td>
<td>* min = max is supported for modeling convenience, but gives a degenerate distribution.</td>
</tr>
<tr>
<td>Domain</td>
<td>min ≤ x ≤ max continuous</td>
</tr>
<tr>
<td>Density and Cumulative Distribution Functions</td>
<td>f(x) = (\frac{1}{\text{max} - \text{min}})</td>
</tr>
<tr>
<td></td>
<td>F(x) = (\frac{x - \text{min}}{\text{max} - \text{min}})</td>
</tr>
<tr>
<td>Mean</td>
<td>(\frac{\text{max} + \text{min}}{2})</td>
</tr>
<tr>
<td>Variance</td>
<td>(\frac{(\text{max} - \text{min})^2}{12})</td>
</tr>
<tr>
<td>Skewness</td>
<td>0</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.8</td>
</tr>
<tr>
<td>Mode</td>
<td>Not uniquely defined</td>
</tr>
</tbody>
</table>

Reference: @RISK Functions
Examples

**PDF - Uniform(0,1)**

**CDF - Uniform(0,1)**
## RiskUniformAlt, RiskUniformAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskUniformAlt}(\text{arg1type, arg1value, arg2type, arg2value}) ) specifies a uniform distribution with two arguments of the type ( \text{arg1type} ) and ( \text{arg2type} ). These arguments can be either a \text{percentile} between 0 and 1 or \text{min} or \text{max}.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskUniformAlt}(5%, 1, 95%, 10) ) specifies a uniform distribution with a 5\text{th} percentile of 1 and a 95\text{th} percentile of 10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>\text{min} must be less than \text{max}. With \text{RiskUniformAltD}, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
### RiskWeibull

<p>| Description | RiskWeibull(alpha,beta) generates a Weibull distribution with the shape parameter alpha and a scale parameter beta. The Weibull distribution is a continuous distribution whose shape and scale vary greatly depending on the argument values entered. This distribution is often used as a distribution of time to first occurrence for other continuous time processes, where it is desired to have a non-constant intensity of occurrence. This distribution is flexible enough to allow an implicit assumption of constant, increasing or decreasing intensity, according to the choice of its parameter α (α&lt;1, =1, or &gt;1 represent processes of increasing, constant, and decreasing intensity respectively; a constant intensity process is the same as an Exponential distribution). For example in maintenance or lifetime modelling, one may choose to use α&lt;1 to represent that the older something is, the more likely it is to fail. |
| Examples | RiskWeibull(10,20) generates a Weibull distribution with a shape parameter 10 and a scale parameter 20. RiskWeibull(D1,D2) generates a Weibull distribution with a shape parameter taken from cell D1 and a scale parameter taken from cell D2. |
| Guidelines | Both shape parameter alpha and scale parameter beta must be greater than zero. |
| Parameters | α  continuous shape parameter  α &gt; 0 |
| β  continuous scale parameter  β &gt; 0 |
| Domain | 0 ≤ x &lt; +∞  continuous |
| Density and Cumulative Distribution Functions | f(x) = (αx)^α−1 e^{−(x/β)α}  F(x) = 1 − e^{−(x/β)α} |
| Mean | βΓ(1+1/α)  where Γ is the Gamma Function. |
| Variance | β^2 [Γ(1+2/α) − Γ^2(1+1/α)]  where Γ is the Gamma Function. |</p>
<table>
<thead>
<tr>
<th>Skewness</th>
</tr>
</thead>
</table>
| \[
\frac{\Gamma\left(1 + \frac{3}{\alpha}\right) - 3\Gamma\left(1 + \frac{2}{\alpha}\right)\Gamma\left(1 + \frac{1}{\alpha}\right) + 2\Gamma^3\left(1 + \frac{1}{\alpha}\right)}{\left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \Gamma^2\left(1 + \frac{1}{\alpha}\right)\right]^{3/2}}
\] |

where \(\Gamma\) is the Gamma Function.

<table>
<thead>
<tr>
<th>Kurtosis</th>
</tr>
</thead>
</table>
| \[
\frac{\Gamma\left(1 + \frac{4}{\alpha}\right) - 4\Gamma\left(1 + \frac{3}{\alpha}\right)\Gamma\left(1 + \frac{1}{\alpha}\right) + 6\Gamma\left(1 + \frac{2}{\alpha}\right)\Gamma^2\left(1 + \frac{1}{\alpha}\right) - 3\Gamma^4\left(1 + \frac{1}{\alpha}\right)}{\left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \Gamma^2\left(1 + \frac{1}{\alpha}\right)\right]^2}
\] |

where \(\Gamma\) is the Gamma Function.

<table>
<thead>
<tr>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>[\beta\left(1 - \frac{1}{\alpha}\right)^{1/\alpha}]</td>
</tr>
</tbody>
</table>

for \(\alpha > 1\)

\[0\]                                               

for \(\alpha \leq 1\)
Examples

PDF - Weibull(2, 1)

CDF - Weibull(2, 1)
## RiskWeibullAlt, RiskWeibullAltD

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskWeibullAlt(arg1type, arg1value, arg2type, arg2value, arg3type, arg3value) specifies a Weibull distribution with three arguments of the type arg1type to arg3type. These arguments can be either a percentile between 0 and 1 or alpha, beta or loc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskWeibullAlt(&quot;alpha&quot;,1,&quot;beta&quot;,1,95%,3) specifies a Weibull distribution with a alpha of 1 and a beta of 1 and a 95th percentile of 3.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Both shape parameter alpha and scale parameter beta must be greater than zero. With RiskWeibullAltD, any entered percentile values are cumulative descending percentiles, where the percentile specifies the chance of a value greater or equal to the entered value.</td>
</tr>
</tbody>
</table>
Reference: Distribution Property Functions

The following functions are used to add optional arguments to distribution functions. The arguments added by these functions are not required, but can be added as needed.

Optional arguments are specified using @RISK distribution property functions that are embedded inside of a distribution function.

<table>
<thead>
<tr>
<th>RiskCategory</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description</strong></td>
</tr>
<tr>
<td><em>RiskCategory</em>(category name) names the category to be used when displaying an input distribution. This name defines the grouping in which an input will appear in the @RISK Model Window Inputs list and in any reports which include simulation results for the input.</td>
</tr>
<tr>
<td><strong>Examples</strong></td>
</tr>
<tr>
<td><em>RiskTriang</em>(10,20,30,RiskCategory(&quot;Prices&quot;)) places the probability distribution <em>RiskTriang</em>(10,20,30) in the category “Prices”.</td>
</tr>
<tr>
<td><strong>Guidelines</strong></td>
</tr>
<tr>
<td>The category name specified must be entered in quotes. Any valid cell references can be used to define a category name.</td>
</tr>
</tbody>
</table>
## RiskCollect

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCollect() identifies specific distribution functions whose samples are collected during a simulation and whose:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• statistics are displayed</td>
</tr>
<tr>
<td></td>
<td>• data points are available</td>
</tr>
<tr>
<td></td>
<td>• sensitivities and scenario values are calculated</td>
</tr>
</tbody>
</table>

When RiskCollect is used and **Inputs Marked With Collect** is selected for **Collect Distribution Samples** in the Simulation Settings dialog, only functions identified by RiskCollect are displayed in the Results Summary Window Explorer list.

Earlier versions of @RISK had the RiskCollect function entered by placing it in the cell formula, immediately preceding the distribution function for which samples will be collected, e.g.:

\[ =\text{RiskCollect()} + \text{RiskNormal(10,10)} \]

RiskCollect is typically used when a large number of distribution functions are present in a simulated worksheet, but sensitivities and scenario analyses are desired on only a pre-identified subset of important distributions. It can also be used to bypass Windows memory constraints, that keep sensitivities and scenario analyses from being performed on all functions in a large simulation.

| Examples | RiskNormal(10,2,RiskCollect()) collects samples from the probability distribution RiskNormal(10,2). |
| Guidelines | The “Inputs Marked With Collect” box in Simulation Settings must be selected for COLLECT functions to take effect. |
## RiskConvergence

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskConvergence(tolerance, toleranceType, confidenceLevel, useMean, useStdDev, usePercentile, percentile) specifies convergence monitoring information for a specific output. tolerance is the +/- tolerance amount desired, toleranceType specifies the type of tolerance value entered (1 for +/- actuals, 2 for +/- percentage or relative), confidenceLevel specifies the confidence level for your estimate, useMean, useStdDev, usePercentile is set to TRUE to select the monitoring statistic desired, and percentile enters the percentile to monitor when usePercentile is set to TRUE. RiskConvergence returns FALSE if the output has not converged, and TRUE when it has.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskOutput(,,,RiskConvergence(3%,2,95%,TRUE)) specifies a +/- 3% tolerance with a 95% confidence level, where the monitored statistic is the mean</td>
</tr>
<tr>
<td>Guidelines</td>
<td>This property function overrides any default convergence monitoring specified in the Simulation Settings dialog. The RiskConvergence property function is only available for simulation outputs.</td>
</tr>
</tbody>
</table>
RiskCorrmat

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
</table>
| **RiskCorrmat**(*matrix cell range*, *position*, *instance*) identifies a distribution function belonging to a set of correlated distribution functions. The function is used to specify multivariate correlation. RiskCorrmat identifies 1) a matrix of rank correlation coefficients and 2) the location in the matrix of the coefficients used in correlating the distribution function which follows the RiskCorrmat function.

Correlated distribution functions typically are defined using the @RISK Define Correlations command; however, the same type of correlation may be directly entered in your spreadsheet using the RiskCorrmat function.

The matrix identified by the *matrix cell range* is a matrix of rank correlation coefficients. Each element (or cell) in the matrix contains a correlation coefficient. The number of distribution functions correlated by the matrix equals the number of rows or columns in the matrix. The argument *position* specifies the column (or row) in the matrix to use in correlating the distribution function which follows the RiskCorrmat function. The coefficients located in the column (or row), identified by *position*, are used in correlating the identified distribution function with each of the other correlated distribution functions represented by the matrix. The value in any given cell in the matrix gives the correlation coefficient between 1) the distribution function whose RiskCorrmat *position* equals the column coordinate of the cell and 2) the distribution function whose RiskCorrmat *position* equals the row coordinate of the cell. *Positions* (and coordinates) range from 1 to N, where N is the number of columns or rows in the matrix.

The *instance* argument is optional and is used when multiple groups of correlated inputs use the same matrix of correlation coefficients. *Instance* is an integer or string argument, and all inputs in a correlated group of inputs share the same instance value or string. String arguments used to specify *instance* need to be in quotes.

The RiskCorrmat function generates correlated sets of random numbers to be used in sampling each of the correlated distribution functions. The sample matrix of rank correlation coefficients, calculated on the correlated set of random numbers, approximates as closely as possible the target correlation coefficient matrix which was entered in the worksheet.

Correlated sets of random numbers specified by the RiskCorrmat function are generated when the first RiskCorrmat function is called during a simulation. This is usually during the first iteration of the simulation. This may cause a delay while values are sorted and correlated. The length of the delay is proportional to the number of iterations and the number of correlated variables.

The method used for generating multiple rank correlated distribution functions is based on the method used for DEPC and INDEPC functions. For more information on this, see the section **Understanding Rank Order Correlation Coefficient Values** under the DEPC function in this section.
Entry of CORRMAT functions outside of a distribution function (in the form `RiskCorrmat+distribution function`) as done in earlier versions of @RISK, is still supported. However, these functions will be moved inside the distribution function they are correlating whenever the formula or correlated distribution is edited in the @RISK Model window.

### Examples

`RiskNormal(10,10, RiskCorrmat(C10:G14,1,"Matrix 1"))` specifies that the sampling of the distribution `Normal(10,10)` will be controlled by the first column of the 5 by 5 matrix of correlation coefficient values located in the cell range C10:G14. There are five correlated distributions represented by the matrix, as there are five columns in the matrix. The coefficients used in correlating `Normal(10,10)` with the other four correlated distributions are found in row 1 of the matrix. This distribution — `Normal(10,10)` — will be correlated with the other distributions that include the instance `Matrix 1` in their embedded `RiskCorrmat` functions.

### Guidelines

Multiple matrices of correlation coefficients may be used in a single worksheet.

The sample matrix of correlation coefficients (calculated on the correlated random numbers generated by @RISK) approximates as closely as possible the target correlation coefficient matrix located in the *matrix cell range*. It is possible that the target coefficients are inconsistent and approximation cannot be performed. @RISK will notify the user if this occurs.

Any blank cells, or labels in the *matrix cell range*, specify a correlation coefficient of zero.

*Position* may be a value between 1 and N, where N is the number of columns in the matrix.

The *matrix cell range* must be square, that is, with an equal number of rows and columns.

By default @RISK uses the correlation coefficients in the *matrix cell range* on a row wise basis. Because of this, only the top 'half' of the matrix — the top right of the matrix when it is split on the diagonal — needs to be entered.

Correlation coefficients must be less than or equal to 1 and greater than or equal to -1. Coefficients on the diagonal of the matrix must equal 1.

An *Adjustment Weight matrix* may be defined in Excel to control how coefficients are adjusted if an entered correlation matrix is inconsistent. This matrix is 1) given an Excel range name using the name of the correlation matrix it is used with plus the extension `_Weights` and 2) has the same number of elements as the related correlation matrix. Adjustment Weight matrix cells take values from 0 to 100 (a blank cell is 0). A weight of 0 indicates that the coefficient in the related correlation matrix may be adjusted as necessary during matrix correction and 100 indicates that the corresponding coefficient is fixed. Values in between these two extremes allow a proportional amount of change in the related coefficient.
RiskDepC

**Description**

RiskDepC(ID,coefficient) designates a dependent variable in a correlated sampling pair. The ID is the string used to identify the independent variable being correlated with. The string must be set in quotes. This is the same ID used in the RiskIndepC function for the independent variable. The coefficient entered is the rank-order correlation coefficient, which describes the relationship of the values sampled for the distributions identified by the RiskDepC and RiskIndepC. The RiskDepC function is used with the distribution function which specifies the possible values for the dependent variable.

**Understanding Rank-Order Correlation Coefficient Values**

The rank-order correlation coefficient was developed by C. Spearman in the early 1900’s. It is calculated using rankings of values, not actual values themselves (as is the linear correlation coefficient). A value’s “rank” is determined by its position within the min-max range of possible values for the variable.

The coefficient is a value between -1 and 1 which represents the desired degree of correlation between the two variables during sampling. Positive coefficient values indicate a positive relationship between the two variables — when the value sampled for one is high, the value sampled for the second will also tend to be high. Negative coefficient values indicate an inverse relationship between the two variables — when the value sampled for one is high, the value sampled for the second will tend to be low.

@RISK generates rank-correlated pairs of sampled values in a two step process. First, a set of randomly distributed “rank scores” are generated for each variable. If 100 iterations are to be run, for example, 100 scores are generated for each variable. (Rank scores are simply values of varying magnitude between a minimum and maximum. @RISK uses van der Waerden scores based on the inverse function of the normal distribution). These rank scores are then rearranged to give pairs of scores which generate the desired rank correlation coefficient. For each iteration there is a pair of scores, with one score for each variable.

In the second step, a set of random numbers (between 0 and 1) to be used in sampling is generated for each variable. Again, if 100 iterations are to be run, 100 random numbers are generated for each variable. These random numbers are then ranked smallest to largest. For each variable, the smallest random number is then used in the iteration with the smallest rank score, the second smallest random number is used in the iteration with the second smallest rank score and so on. This ordering based on ranking continues for all random numbers, up to the point where the largest random number is used in the iteration with the largest rank score.
In @RISK this process of rearranging random numbers happens prior to simulation. It results in a set of pairs of random numbers that can be used in sampling values from the correlated distributions in each iteration of the simulation.

This method of correlation is known as a “distribution-free” approach, because any types of distributions may be correlated. Although the samples drawn for the two distributions are correlated, the integrity of the original distributions are maintained. The resulting samples for each distribution reflect the input distribution function from which they were drawn.

Earlier versions of @RISK had the RiskDepC function entered by placing it in the cell formula immediately preceding the distribution function which will be correlated, e.g.:

```
=RiskDepC("Price 1",.9)+RiskNormal(10,10)
```

This form of function entry is still supported. However, these functions will be moved inside the distribution function they are correlating whenever the formula or correlated distribution is edited in the @RISK Model window.

The correlation coefficient generated by using RiskDepC and RiskIndepC is approximate. The generated coefficient will more closely approximate the desired coefficient, as increasing numbers of iterations are executed. There may be a delay at the start of a simulation when distributions are correlated when RiskDepC and RiskIndepC are used. The length of the delay is proportional to the number of RiskDepC functions in the worksheet, and the number of iterations to be performed. See the @RISK Modeling Techniques chapter for a detailed example of dependency relationships.

### Examples

RiskNormal(100,10, RiskDepC("Price",.5)) specifies that the sampling of the distribution RiskNormal(100,10) will be correlated with the sampling of the distribution identified with the function RiskIndepC("Price"). The sampling of RiskNormal(100,10) will be positively correlated with the sampling of the distribution identified with the function RiskIndepC("Price") as the coefficient is greater than 0.

### Guidelines

coefficient must be a value greater than or equal to -1 and less than or equal to 1. ID must be the same string of characters used to identify the independent variable in the RiskIndepC function. ID may be a reference to cell that contains an identifier string.
### RiskFit

| Description | $\text{RiskFit}(\text{fit name}, \text{selected fit result})$ links a data set, and its fit results, to the input distribution the RiskFit function is used in. The \textit{fit name} in quotes is the name of the fit given when the data was fit using the Fit Distributions to Data command. The \textit{selected fit result} in quotes is a string used to identify the type of fit result to select. The RiskFit function is used to link an input to the fit results for a data set, so that when the data is changed the input distribution selected from the fit can be updated. The \textit{selected fit result} can be any of the following entries:

- **Best Chi-Sq**, indicating the best fitting distribution from the Chi-Sq test should be used
- **Best A-D**, indicating the best fitting distribution from the Anderson-Darling test should be used
- **Best K-S**, indicating the best fitting distribution from the Kolmogorov-Smirnov test should be used.
- **Best RMS Err**, indicating the best fitting distribution from the RMS Error test should be used
- A \textit{distribution name}, such as “Normal”, indicating that the best-fitting distribution of the entered type should be used.

### What Happens When Data Changes When RiskFit is Used

The RiskFit function “hot-links” a distribution function to a data set and the fit of that data set. The data used in a fit is in a range in Excel. When the fitted data changes and a simulation is started, the following actions take place:

- @RISK re-runs the fit using the current settings on the fit tab where the fit was originally run
- The distribution function that includes the RiskFit function that references the fit is changed to reflect the new fit results. The changed function replaces the original one in Excel. If, for example, the distribution function’s RiskFit argument specified “Best Chi-sq” for \textit{selected fit result}, the new best-fitting distribution based on the Chi-Sq test would replace the original one. This new function would also include the same RiskFit function as the original one.

### Examples

| Examples | $\text{RiskNormal}(2.5, 1, \text{RiskFit("Price Data", "Best A-D")})$ specifies that the best fitting distribution from the Anderson-Darling test for the fitted data associated with the fit named Price Data is a Normal distribution with a mean of 2.5 and a standard deviation of 1. |

### Guidelines

None.
### RiskIndepC

<table>
<thead>
<tr>
<th>Description</th>
<th></th>
</tr>
</thead>
</table>
| **RiskIndepC**(*ID*) designates an independent variable in a rank correlated sampling pair. The *ID* is a string used to identify the independent variable. The RiskIndepC function is used with the distribution function, which specifies the possible values for the independent variable. RiskIndepC is just an identifier. Earlier versions of @RISK had the RiskIndepC function entered by placing it in the cell formula immediately preceding the distribution function which will then be correlated, e.g.:  

=RiskIndepC("Price 1")+RiskNormal(10,10)  
This form of function entry is still supported. However, these functions will be moved inside the distribution function they are correlating whenever the formula or correlated distribution is edited in the @RISK Model window.  |

<table>
<thead>
<tr>
<th>Examples</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskNormal(10,10, RiskIndepC(&quot;Price&quot;)) identifies the function NORMAL(10,10) as the independent variable “Price”. This function will be used as the independent variable anytime a DEPC function with the ID string “Price” is used.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
<th></th>
</tr>
</thead>
</table>
| *ID* must be the same string of characters used to identify the dependent variable in the DEPC function. *ID* must be the same string of characters used to identify the independent variable in the INDEPC function. *ID* may be a reference to cell that contains an identifier string.  
A maximum of 64 individual INDEPC functions may be used in a single worksheet. Any number of DEPC functions may be dependent on those INDEPC functions.  
See the @RISK Modeling Techniques chapter for a detailed example of dependency relationships.  |

### RiskIsDiscrete

<table>
<thead>
<tr>
<th>Description</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskIsDiscrete</strong>(TRUE) specifies that the output for which it is entered should be treated as a discrete distribution, when displaying graphs of simulation results and calculating statistics. If RiskIsDiscrete is not entered, @RISK will attempt to detect when an output represents a distribution of discrete values.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskOutput(,,,RiskIsDiscrete(TRUE))+NPV(.1,C1:C10) specifies that the output distribution of NPV will be a discrete distribution</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>None.</td>
<td></td>
</tr>
</tbody>
</table>
### RiskIsDate

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskIsDate</strong>(TRUE or FALSE) specifies whether the input or output for which it is entered should be treated as a distribution of dates, when displaying graphs of simulation results and calculating statistics. If RiskIsDate is not entered, @RISK will use the formatting of the cell where the input or output is located in Excel to decide to display simulation results as dates. When RiskIsDate(TRUE) is entered for an input distribution, argument values will be displayed as dates in the Define Distribution window.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskOutput</strong>,,<strong>RiskIsDate</strong>(TRUE)) specifies that the output distribution will be displayed using dates, regardless of the cell formatting in Excel. <strong>RiskTriang</strong>(DATE(2009,10,4),DATE(2009,12,29),DATE(2010,10,10),RiskIsDate(TRUE)) specifies a triangular distribution with a minimum value of 10/4/2009, a most likely value of 12/29/2009 and a maximum value of 10/10/2010.</td>
</tr>
<tr>
<td>Guidelines</td>
<td><strong>RiskIsDate</strong>(FALSE) causes @RISK to display graphs and reports for the input or output in values, not dates, even if the cell where the function is located in Excel has date formatting.</td>
</tr>
</tbody>
</table>

### RiskLibrary

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskLibrary</strong>(position,ID) specifies that the distribution, for which it is entered, is linked to a distribution in an @RISK Library with the entered position and ID. Each time a simulation is run the distribution function will update with the current definition of the distribution in an @RISK Library with the entered ID.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskNormal</strong>(5000,1000,<strong>RiskName</strong>(&quot;Sales Volume / 2010&quot;)),<strong>RiskLibrary</strong>(2,&quot;LV6W59J5&quot;),<strong>RiskStatic</strong>(0.46)) specifies that the entered distribution is taken from the @RISK Library with the position 2 and the ID LV6W59J5. The current definition of this library distribution is RiskNormal(10,10, <strong>RiskName</strong>(&quot;Sales Volume / 2010&quot;)) however, this will change when the distribution in the library changes.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>A RiskStatic value is not updated from the @RISK Library, as it is unique to the model where the library distribution is used.</td>
</tr>
</tbody>
</table>
## RiskLock

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskLock() keeps a distribution from being sampled in a simulation. Locking an input distribution causes it to return the value set in using the Simulation Settings standard recalc options.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskNormal(10,2,RiskLock()) stops samples from being drawn from the probability distribution RiskNormal(10,2).</td>
</tr>
<tr>
<td>Guidelines</td>
<td>The optional argument Lock_Mode is used internally by @RISK, but is not available to users in the @RISK Define Distribution window.</td>
</tr>
</tbody>
</table>

## RiskName

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskName(input name) names the input distribution in which the function is used as an argument. This name will appear in both the @RISK Model Window Outputs and Inputs list, and in any reports and graphs which include simulation results for the input.</th>
</tr>
</thead>
</table>
| Examples    | RiskTriang(10,20,30,RiskName("Price")) gives the name Price to the input described by the probability distribution RiskTriang(10,20,30).  
RiskTriang(10,20,30,RiskName(A10)) gives the name contained in the cell A10 to the input described by the probability distribution RiskTriang(10,20,30). |
| Guidelines  | input name must be entered in quotes.  
Any valid cell references can be used to define a name. |
### RiskSeed

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskSeed}(\text{random number generator type, seed value})$ specifies that an input will use its own random number generator of the specified type and it will be seeded with $\text{seed value}$. Seeding an individual input is useful when the same distribution is shared between models using the @RISK Library and a reproducible set of samples is wanted for the input in every model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskBeta}(10,2,\text{RiskSeed}(1,100))$ the input $\text{RiskBeta}(10,2)$ will use its own Mersenne Twister random number generator, and it will use the seed 100.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>Input distributions that use RiskSeed always have their own reproducible stream of random numbers. The Initial Seed, set in the Simulation Settings Sampling tab, only affects the random numbers generated for input distributions that do not have an independent seed specified using the RiskSeed property function. The $\text{random number generator type}$ is specified as value between 1 and 8, where 1=MersenneTwister, 2=MRG32k3a, 3=MWC, 4=KISS, 5=LFIB4, 6=SWB, 7=KISS_SWB, 8=RAN3I. For more information on the available random number generators, see the Simulation Settings command. $\text{Seed value}$ is an integer between 1-2,147,483,647. RiskSeed has no effect when an input is correlated.</td>
</tr>
</tbody>
</table>

### RiskShift

<table>
<thead>
<tr>
<th>Description</th>
<th>$\text{RiskShift}(\text{shift amount})$ shifts the domain of the distribution in which it is used by the entered $\text{shift amount}$. This function is automatically entered when a fit result includes a shift factor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>$\text{RiskBeta}(10,2,\text{RiskShift}(100))$ shifts the domain of the distribution $\text{RiskBeta}(10,2)$ by 100.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskSixSigma

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskSixSigma</strong>(LSL, USL, Target, Long Term Shift, Number of Standard Deviations) specifies the lower specification limit, upper specification limit, target value, long term shift, and the number of standard deviations for the six sigma calculations for an output. These values are used in calculating six sigma statistics displayed in the Results window and on graphs for the output.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskOutput</strong>(A10,,,,<strong>RiskSixSigma</strong>(.88,.95,.915,1.5,6)) specifies an LSL of .88, a USL of .95, target value of .915, long term shift of 1.5, and a number of standard deviations of 6 for the output located in Cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>By default, @RISK six sigma statistics functions in Excel will use the entered LSL, USL, Target, Long Term Shift, and Number of Standard Deviations entered in the RiskSixSigma property function for an output (when the statistics function references the output). These values can be overridden by entering the LSL, USL, Target, Long Term Shift, and Number of Standard Deviations directly in the statistics function. LSL, USL, Target, Long Term Shift, and Number of Standard Deviations entered in the RiskSixSigma property function for an output, are read at the start of a simulation. If you change the property function values you need to re-run the simulation, to update six sigma statistics displayed in the Results window and on graphs for the output.</td>
</tr>
</tbody>
</table>

### RiskStatic

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskStatic</strong>(static value) defines the static value 1) returned by a distribution function during a standard Excel recalculation and 2) that replaces the @RISK function after @RISK functions are swapped out.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td><strong>RiskBeta</strong>(10,2,<strong>RiskStatic</strong>(9.5)) specifies that the static value for the distribution function <strong>RiskBeta</strong>(10,2) will be 9.5.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
## RiskTruncate

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTruncate(minimum,maximum) truncates the input distribution in which the function is used as an argument. Truncating a distribution restricts samples drawn from the distribution to values within the entered minimum-maximum range. Truncated forms of specific distributions available in earlier versions of @RISK (such as RiskTnormal and RiskTlognorm) are still supported.</th>
</tr>
</thead>
</table>
| Examples | RiskTriang(10,20,30,RiskTruncate(13,27)) restricts the samples drawn from the probability distribution RiskTriang(10,20,30) to a minimum possible value of 13 and a maximum possible value of 27.  
RiskTriang(10,20,30,RiskTruncate(D11,D12)) restricts the samples drawn from the probability distribution RiskTriang(10,20,30) to a minimum possible value of taken from cell D11 and a maximum possible value of taken from cell D12. |
| Guidelines | minimum must be less than or equal to maximum.  
To enter a distribution that is truncated on only one side, leave the argument for the unbounded side blank, such as RiskNormal(10,1,RiskTruncate(5,)). This would set the minimum equal to 5, but leave the maximum unbounded. |

## RiskTruncateP

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTruncateP(perc% minimum, perc% maximum) truncates the input distribution in which the function is used as an argument. Truncating a distribution restricts samples drawn from the distribution to values within the entered minimum-maximum range. Truncated forms of specific distributions available in earlier versions of @RISK (such as RiskTnormal and RiskTlognorm) are still supported.</th>
</tr>
</thead>
</table>
| Examples | RiskTriang(10,20,30,RiskTruncate(.01,.99)) restricts the samples drawn from the probability distribution RiskTriang(10,20,30) to a minimum possible value of the 1st percentile of the distribution and a maximum possible value of 99th percentile of the distribution.  
RiskTriang(10,20,30,RiskTruncate(D11,D12)) restricts the samples drawn from the probability distribution RiskTriang(10,20,30) to a minimum possible percentile value of taken from cell D11 and a maximum possible percentile value of taken from cell D12. |
| Guidelines | Perc% minimum must be less than or equal to perc% maximum.  
Perc% minimum and perc% maximum must be in the range 0<=perc%<=1.  
Distribution functions which contain a RiskTruncateP property function cannot be displayed in the Define Distribution window  
As with RiskTruncate, to enter a distribution that is truncated on only one side, leave the argument for the unbounded side blank. |
## RiskUnits

| Description | \(\text{RiskUnits}(\text{units})\) names the units to be used in labeling an input distribution or output. This name will appear in both the @RISK Model Window Outputs and Inputs list, and in any reports and graphs which include simulation results for the input or output. |
| Examples | \(\text{RiskTriang}(10,20,30,\text{RiskUnits}("Dollars"))\) gives the name Dollars to the units described by the probability distribution \(\text{RiskTriang}(10,20,30)\).  
\(\text{RiskTriang}(10,20,30, \text{RiskUnits}(A10))\) gives the name contained in the cell A10 to the units described by the probability distribution \(\text{RiskTriang}(10,20,30)\). |
| Guidelines | \(\text{units}\) must be entered in quotes. Any valid cell references can be used to define a \(\text{units}\) name.  
If \(\text{RiskUnits}\) is used as a property function for a \(\text{RiskOutput}\) function, it needs to come after the three possible arguments for \(\text{RiskOutput}\). Thus, if you are using \(\text{RiskOutput}\) with no name, range name or position arguments, you need to enter \(\text{RiskOutput},,,\text{RiskUnits}("MyUnits")\). |
Reference: Output Functions

Output cells are defined using RiskOutput functions. These functions allow the easy copying, pasting, and moving of output cells. RiskOutput functions are automatically added when the standard @RISK Add Output icon is pressed. RiskOutput functions optionally allow you to name your simulation outputs, and add individual output cells to output ranges.

The property functions RiskUnits, RiskConvergence, RiskSixSigma, and RiskIsDiscrete may be used with RiskOutput functions.

### RiskOutput

<table>
<thead>
<tr>
<th>Description</th>
<th>The function RiskOutput is used to identify output cells you have selected in your spreadsheet. This function takes up to three arguments as shown here:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>=RiskOutput(&quot;output cell name&quot;, &quot;output range name&quot;, element# in range)</td>
</tr>
<tr>
<td></td>
<td>These arguments are optional, as a simple =RiskOutput() is sufficient for entering a single element output range where @RISK creates the name of the output for you. RiskOutput used with a single argument such as:</td>
</tr>
<tr>
<td></td>
<td>=RiskOutput (&quot;output cell name&quot;)</td>
</tr>
<tr>
<td></td>
<td>specifies for a single element output range where the name is entered by you.</td>
</tr>
<tr>
<td></td>
<td>When a multiple element output range is identified, the form: =RiskOutput (&quot;output cell name&quot;, &quot;output range name&quot;, position# in range) is used. However, the output cell name entry can be omitted if you wish to have @RISK automatically generate a name for each output cell in the range.</td>
</tr>
<tr>
<td></td>
<td>RiskOutput functions are automatically generated for you when you select outputs using the @RISK Add Output icon. However, like any other @RISK function, RiskOutput may be typed directly in the cell which you wish to reference as a simulation output.</td>
</tr>
<tr>
<td></td>
<td>A RiskOutput function is entered by adding it to the cell formula, which is already present in the cell that is to be a simulation output. For example, a cell containing the formula:</td>
</tr>
<tr>
<td></td>
<td>=NPV(.1,G1…G10)</td>
</tr>
<tr>
<td></td>
<td>would become</td>
</tr>
<tr>
<td></td>
<td>=RiskOutput()+NPV(.1,G1…G10)</td>
</tr>
<tr>
<td></td>
<td>when the cell is selected as an output.</td>
</tr>
<tr>
<td>Examples</td>
<td>–RiskOutput(&quot;Profit 1999&quot;, &quot;Annual Profit&quot;, 1) + NPV(.1,G1...G10) identifies the cell where the RiskOutput function is located as a simulation output and gives it the name Profit 1999, and makes it the first cell in a multiple cell output range named Annual Profit.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>If names are entered directly in the RiskOutput function, the entered output cell name and output range name must be enclosed in quotes. Names may also be included by referencing cells with labels in them. Position# must be a positive integer &gt;=1. Any property functions need to follow the first three arguments of the RiskOutput function. Thus, if you add a RiskUnits property function to a default RiskOutput function, you would need to enter =RiskOutput(,,RiskUnits(&quot;MyUnits&quot;)) If you are using RiskOutput with a property function such as RiskSixSigma, the Property Functions section of this Reference section describes the arguments to the property function in use. If the @RISK Insert Function command is being used to enter RiskOutput in Six Sigma format, simply click in the formula bar on the displayed RiskSixSigma property function to enter its arguments or to view help on the RiskSixSigma property function.</td>
</tr>
</tbody>
</table>
Reference: Statistics Functions

Statistics Functions return a desired statistic on simulation results for 1) a specified cell or 2) a simulation output or input. These functions are updated in real-time as a simulation is running. Statistics functions, located in template sheets used for creating custom reports on simulation results, are only updated when a simulation is completed.

If a cell reference is entered as the first argument, the cell does not have to be a simulation output identified with a RiskOutput function.

If a name is entered instead of cellref, @RISK first checks for an output with the entered name. If none exists, @RISK looks for an input probability distribution with the entered name and, if one is found, returns the appropriate statistic for the samples drawn for that input. It is up to the user to insure that unique names are given to outputs and inputs referenced in statistics functions.

The Sim# argument entered selects the simulation for which a statistic will be returned when multiple simulations are run. This argument is optional and can be omitted for single simulation runs.

Statistics functions which calculate a statistic on a distribution for a simulation result can include a RiskTruncate or a RiskTruncateP property function. This will cause the statistic to be calculated on the min-max range specified by the truncation limits. For example, if you want to calculate statistics on a percentile range of a distribution, use RiskTruncateP, such as:

\[ \text{RiskMean}(A1, \text{RiskTruncateP}(0.9,1)) \]

In this case, the mean of the data in the top 10% for cell A1 will be returned from the RiskMean function.

@RISK’s statistics functions can be updated either 1) at the end of a simulation or 2) with each iteration during a simulation. In most cases, statistics do not need to be updated until the end of a simulation when you wish to view final simulation statistics in Excel. However, if the calculations in your model require a new statistic to be returned each iteration (for example, when a custom convergence calculation has been entered using Excel formulas), the Each Iteration should be used. Use the Update Statistic Functions option on the Sampling tab of the Simulation Settings dialog to control this.

Note: The default setting for updating statistics functions in @RISK 5.5 and later releases is End of Simulation.
**RiskConvergenceLevel**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskConvergenceLevel(cellref or output name, Sim#) returns the convergence level (0 to 100) for cellref or output name. TRUE is returned on convergence.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskConvergenceLevel(A10) returns the convergence level for cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>A RiskConvergence property function needs to be entered for cellref or output name, or convergence monitoring needs to be enabled in the Simulation Settings dialog for this function to return a convergence level.</td>
</tr>
</tbody>
</table>

**RiskCorrel**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCorrel(cellref1 or output/input name1, cellref2 or output/input name2, correlationType, Sim#) returns the correlation coefficient using correlationType for the data for the simulated distributions for cellref1 or output/input name1 and cellref2 or output/input name2 in simulation Sim#. correlationType is either Pearson or Spearman Rank correlation.</th>
</tr>
</thead>
</table>
| Examples    | RiskCorrel(A10,A11,1) returns the Pearson correlation coefficient for the simulation data collected for the output or input in A10 and the output or input in A11.  
**RiskCorrel ("Profit","Sales",2) returns the Spearman Rank correlation coefficient for the simulation data collected for the output or input named “Profit” and the output or input named “Sales”.** |
| Guidelines  | correlationType is 1 for Pearson correlation or 2 for Spearman Rank correlation.  
All iterations that contain ERR or are filtered in either cellref1 or output/input name1 and cellref2 or output/input name2 are removed, and the correlation coefficient is calculated based on the remaining data.  
If you wish to calculate correlations for a subset of the data collected for the simulated distributions, you need to enter a RiskTruncate or RiskTruncateP property function for each distribution whose data you wish to truncate. The first entered RiskTruncate function is used for the data for cellref1 or output/input name1 and the second RiskTruncate function is used for the data for cellref2 or output/input name2. |
## RiskData

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskData</strong>(cellref or output/input name,iteration#,Sim#) returns the data point(s) of the simulated distribution for cellref in the specified iteration# and simulation#. RiskData can optionally be entered as an array formula, where iteration# is the iteration to be returned in the first cell in the array formula range. The data points for each subsequent iteration will be filled into cells in the range where the array formula is entered.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskData</strong>(A10,1) returns the data point of the simulated distribution for cell A10 in iteration #1 of a simulation.</td>
</tr>
<tr>
<td><strong>RiskData</strong>(&quot;Profit&quot;,100,2) returns the data point of the simulated distribution for the output cell named &quot;Profit&quot; in the current model for the 100th iteration of the second simulation, executed when multiple simulations are run.</td>
</tr>
</tbody>
</table>

| Guidelines | None. |

## RiskKurtosis

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskKurtosis</strong>(cellref or output/input name,Sim#) returns the kurtosis of the simulated distribution for cellref. Use the RiskTruncate property function to optionally specify a range of the simulated distribution over which to calculate the statistic.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskKurtosis</strong>(A10) returns the kurtosis of the simulated distribution for cell A10.</td>
</tr>
<tr>
<td><strong>RiskKurtosis</strong>(&quot;Profit&quot;,2) returns the kurtosis of the simulated distribution for the output cell named &quot;Profit&quot; in the current model for the second simulation, executed when multiple simulations are run.</td>
</tr>
</tbody>
</table>

| Guidelines | None. |

## RiskMax

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskMax</strong>(cellref or output/input name,Sim#) returns the maximum value of the simulated distribution for cellref. Use the RiskTruncate property function to optionally specify a range of the simulated distribution over which to calculate the statistic.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskMax</strong>(A10) returns the maximum of the simulated distribution for cell A10.</td>
</tr>
<tr>
<td><strong>RiskMax</strong>(&quot;profit&quot;) returns the maximum of the simulated distribution for the output cell in the current model named “profit”.</td>
</tr>
</tbody>
</table>

| Guidelines | None. |
## RiskMean

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskMean(cellref or output/input name,Sim#) returns the mean value of the simulated distribution for cellref. Use the RiskTruncate property function to optionally specify a range of the simulated distribution, over which to calculate the statistic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskMean(A10) returns the mean of the simulated distribution for cell A10. RiskMean(&quot;Price&quot;) returns the mean of the simulated distribution for the output cell named “Price”.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

## RiskMin

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskMin(cellref or output/input name,Sim#) returns the minimum value of the simulated distribution for cellref. Use the RiskTruncate property function to optionally specify a range of the simulated distribution, over which to calculate the statistic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskMin(A10) returns the minimum of the simulated distribution for cell A10. RiskMin(&quot;Sales&quot;) returns the minimum value of the simulated distribution for the output cell in the current model named “Sales”.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

## RiskMode

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskMode(cellref or output/input name,Sim#) returns the mode of the simulated distribution for cellref. Use the RiskTruncate property function to optionally specify a range of the simulated distribution, over which to calculate the statistic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskMode(A10) returns the mode of the simulated distribution for cell A10. RiskMode(&quot;Sales&quot;) returns the mode of the simulated distribution for the output cell in the current model named “Sales”.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
RiskPercentile, RiskPtoX, RiskPercentileD, RiskQtoX

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskPercentile(cellref or output/input name, percentile, Sim#) or RiskPtoX(cellref or output/input name, percentile, Sim#) returns the value of the entered percentile of the simulated distribution for cellref. Use the RiskTruncate property function to optionally specify a range of the simulated distribution, over which to calculate the statistic.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskPercentile(C10,.99) returns the 99th percentile of the simulated distribution for cell C10.</td>
</tr>
<tr>
<td>RiskPercentile(C10,A10) returns the percentile value from cell A10 of the simulated distribution for cell C10.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>The entered percentile must be a value &gt;=0 and &lt;=1. RiskPercentileD and RiskQtoX take a cumulative descending percentile value. RiskPercentile and RiskPtoX (along with RiskPercentileD and RiskQtoX) are simply alternate names for the same function.</td>
</tr>
</tbody>
</table>

RiskRange

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskRange(cellref or output/input name,Sim#) returns the minimum-maximum range of the simulated distribution for cellref. Use the RiskTruncate property function to optionally specify a range of the simulated distribution, over which to calculate the statistic.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskRange(A10) returns the range of the simulated distribution for cell A10.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskSensitivity

| Description | \( \text{RiskSensitivity}(\text{cellref or output name}, \text{Sim#}, \text{rank}, \text{analysisType}, \text{returnValueTyp}) \) returns the sensitivity analysis information of the simulated distribution for \( \text{cellref or output name} \). The \text{rank} argument specifies the rank in the sensitivity analysis for the input whose results are desired, where 1 is the top ranking, or most important, input. The \text{analysisType} argument selects the type of analysis desired; 1 for regression, 2 for regression — mapped values and 3 for correlation. The \text{returnValueTyp} selects the type of data to be returned: 1 for input name/cell reference/distribution function, 2 for the sensitivity coefficient or value and 3 for equation coefficient (regression only). |
| Examples | \( \text{RiskSensitivity}(A10,1,1,1,1) \) returns a description of the top ranking input for a regression sensitivity analysis performed on the simulation results for cell A10. |
| Guidelines | None. |

### RiskSkewness

| Description | \( \text{RiskSkewness}(\text{cellref or output/input name}, \text{Sim#}) \) returns the skewness of the simulated distribution for \( \text{cellref} \). Use the RiskTruncate property function to optionally specify a range of the simulated distribution, over which to calculate the statistic. |
| Examples | \( \text{RiskSkewness}(A10) \) returns the skewness of the simulated distribution for cell A10. |
| Guidelines | None. |
### RiskStdDev

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskStdDev(cellref or output/input name, Sim#) returns the standard deviation of the simulated distribution for cellref. Use the RiskTruncate property function to optionally specify a range of the simulated distribution, over which to calculate the statistic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskStdDev(A10) returns the standard deviation of the simulated distribution for cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskTarget, RiskXtoP, RiskTargetD, RiskXtoQ

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTarget(cellref or output/input name, targetValue, Sim#) or RiskXtoP(cellref or output/input name, targetValue, Sim#) returns the cumulative probability for targetValue, in the simulated distribution for cellref. The cumulative probability returned is the probability of a value &lt;= targetValue occurring. Use the RiskTruncate property function to optionally specify a range of the simulated distribution, over which to calculate the statistic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTarget(C10,100000) returns the cumulative probability of the value 100000 as calculated using the simulated distribution for cell C10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>targetValue can be any value. RiskTargetD and RiskXtoQ return a cumulative descending probability RiskTarget and RiskXtoP (along with RiskTargetD and RiskXtoQ) are simply alternate names for the same function.</td>
</tr>
</tbody>
</table>

### RiskVariance

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskVariance(cellref or output/input name, Sim#) returns the variance of the simulated distribution for cellref. Use the RiskTruncate property function to optionally specify a range of the simulated distribution, over which to calculate the statistic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskVariance(A10) returns the variance of the simulated distribution for cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskTheoKurtosis

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoKurtosis(cellref or distribution function) returns the kurtosis of the distribution function in the formula in cellref or the entered distribution function.</th>
</tr>
</thead>
</table>
| Examples    | RiskTheoKurtosis(A10) returns the kurtosis of the distribution function in cell A10.  
 RiskTheoKurtosis(RiskNormal(10,1)) returns the kurtosis of the distribution RiskNormal(10,1). |
| Guidelines  | None.                                                                                                                                                                                                                                                                 |
### RiskTheoMin

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskTheoMin}(\text{cellref or distribution function}) ) returns the minimum value of the last distribution function in the formula in ( \text{cellref} ), or the entered \textit{distribution function}.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskTheoMin}(A10) ) returns the minimum of the distribution function in cell A10.  ( \text{RiskTheoMin}(\text{RiskNormal}(10,1)) ) returns the minimum of the distribution RiskNormal((10,1)).</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskTheoMode

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskTheoMode}(\text{cellref or distribution function}) ) returns the mode of the last distribution function in the formula in ( \text{cellref} ), or the entered \textit{distribution function}.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskTheoMode}(A10) ) returns the mode of the distribution function in cell A10.  ( \text{RiskTheoMode}(\text{RiskNormal}(10,1)) ) returns the mode of the distribution RiskNormal((10,1)).</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
### RiskTheoPercentile, RiskTheoPtoX, RiskTheoPercentileD, RiskTheoQtoX

**Description**

- **RiskTheoPercentile**(cellref or distribution function, percentile) or **RiskTheoPtoX**(cellref or distribution function, percentile) returns the value of the entered percentile of the last distribution function in the formula in cellref, or the entered distribution function.

**Examples**

- RiskTheoPtoX(C10,.99) returns the 99th percentile of the distribution in cell C10.
- RiskTheoPtoX(C10,A10) returns the percentile value from cell A10 of the distribution in cell C10.

**Guidelines**

- percentile must be a value >=0 and <=1.
- RiskTheoQtoX is equivalent to RiskTheoPtoX (and RiskTheoPercentile is equivalent to RiskTheoPercentileD) except that percentile is entered as a cumulative descending value.
- RiskTheoPercentile and RiskTheoPtoX (along with RiskTheoPercentileD and RiskTheoQtoX) are simply alternate names for the same function.

### RiskTheoRange

**Description**

- **RiskTheoRange**(cellref or distribution function) returns the minimum-maximum range of the last distribution function in the formula in cellref, or the entered distribution function.

**Examples**

- RiskTheoRange(A10) returns the range of the distribution function in cell A10.

**Guidelines**

- None.

### RiskTheoSkewness

**Description**

- **RiskTheoSkewness**(cellref or distribution function) returns the skewness of the last distribution function in the formula in cellref, or the entered distribution function.

**Examples**

- RiskTheoSkewness(A10) returns the skewness of the distribution function in cell A10.

**Guidelines**

- None.
### RiskTheoStdDev

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoStdDev(cellref or distribution function) returns the standard deviation of the last distribution function in the formula in cellref, or the entered distribution function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTheoStdDev(A10) returns the standard deviation of the distribution function in cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

### RiskTheoTarget, RiskTheoXtoP, RiskTheoTargetD, RiskTheoXtoQ

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoTarget(cellref or distribution function, targetValue) or RiskTheoXtoP(cellref or distribution function, targetValue) returns the cumulative probability for targetValue in the last distribution function in the formula in cellref, or the entered distribution function. The cumulative probability returned is the probability of a value &lt;= targetValue occurring.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTheoXtoP(C10,100000) returns the cumulative probability of the value 100000 as calculated using the distribution in cell C10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>targetValue can be any value. RiskTheoTargetD and RiskTheoXtoQ return a cumulative descending probability RiskTheoTarget and RiskTheoXtoP (along with RiskTheoTargetD and RiskTheoXtoQ) are simply alternate names for the same function.</td>
</tr>
</tbody>
</table>

### RiskTheoVariance

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskTheoVariance(cellref or distribution function) returns the variance of the last distribution function in the formula in cellref, or the entered distribution function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskTheoVariance(A10) returns the variance of the distribution function in cell A10.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
Six Sigma Functions return a desired Six Sigma statistic on simulation results for 1) a specified cell or 2) a simulation output. These functions are updated real-time as a simulation is running. Statistics functions located in template sheets used for creating custom reports on simulation results are only updated when a simulation is completed.

If a cell reference is entered as the first argument, the cell does not have to be a simulation output identified with a RiskOutput function.

If a name is entered instead of cellref, @RISK first checks for an output with the entered name, and the reads its RiskSixSigma property function settings. It is up to the user to insure that unique names are given to outputs referenced in statistics functions.

The Sim# argument entered selects the simulation for which a statistic will be returned when multiple simulations are run. This argument is optional and can be omitted for single simulation runs.

For all Six Sigma statistics functions, an optional RiskSixSigma property function can be entered directly in the function. Doing this causes @RISK to override any Six Sigma settings specified in the RiskSixSigma property function entered in the simulation output referenced by the statistics function. This allows you to calculate Six Sigma statistics at differing LSL, USL, Target, Long Term Shift and Number of Standard Deviations values for the same output.

When an optional RiskSixSigma property function is entered directly in a Six Sigma statistics function, different arguments from the property function are used depending on the calculation being performed.

For more information on using @RISK with Six Sigma, see the separate guide Using @RISK with Six Sigma that was installed with your copy of @RISK.
### RiskCp

| Description | RiskCp(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the Process Capability for cellref or output name in Sim#, optionally using the LSL and USL in the included RiskSixSigma property function. This function will calculate the quality level of the specified output and what it is potentially capable of producing. |
| Examples | Examples | RiskCP(A10) returns the Process Capability for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10. RiskCP(A10, ,RiskSixSigma(100, 120, 110, 0, 6)) returns the Process Capability for the output cell A10, using an USL of 120 and a LSL of 100. |
| Guidelines | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |

### RiskCpm

| Description | RiskCpm(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) returns the Taguchi capability index for cellref or output name in Sim#, optionally using the USL, LSL, and the Target in the RiskSixSigma property function. This function is essentially the same as the Cpk but incorporates the target value which in some cases may or may not be within the specification limits. |
| Examples | Examples | RiskCpm(A10) returns the Taguchi capability index for cell A10. RiskCpm(A10, ,RiskSixSigma(100, 120, 110, 0, 6)) returns the Taguchi capability index for cell A10, using an USL of 120, LSL of 100, and a Target of 110. |
| Guidelines | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |
# RiskCpk

<table>
<thead>
<tr>
<th>Description</th>
<th><strong>RiskCpk</strong>(<em>cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations]</em>) calculates the Process Capability Index for <em>cellref</em> or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function. This function is similar to the Cp but takes into account an adjustment of the Cp for the effect of an off-centered distribution. As a formula, Cpk = either (USL-Mean) / (3 x sigma) or (Mean-LSL) / (3 x sigma) whichever is the smaller.</th>
</tr>
</thead>
</table>
| Examples | **RiskCpk(A10)** returns the Process Capability Index for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.  
**RiskCpk(A10, ,RiskSixSigma(100,120,110,1.5,6))** returns the Process Capability Index for the output cell A10, using an LSL of 100 and a USL of 120. |
| Guidelines | A RiskSixSigma property function needs to be entered for *cellref or output name*, or a RiskSixSigma property function needs to be included |
### RiskCpkLower

<table>
<thead>
<tr>
<th>Description</th>
<th>[\text{RiskCpkLower}(\text{cellref or output name, Sim#}, \text{RiskSixSigma}(\text{LSL, USL, Target, LongTerm Shift, Number of Standard Deviations}))] calculates the one-sided capability index based on the Lower Specification limit for cellref or output name in Sim# optionally using the LSL in the RiskSixSigma property function.</th>
</tr>
</thead>
</table>
| Examples | RiskCpkLower(A10) returns the one-sided capability index based on the Lower Specification limit for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.  
RiskCpkLower(A10, \text{RiskSixSigma}(100,120,110,1.5,6)) returns the one-sided capability index for the output cell A10, using an LSL of 100. |
| Guidelines | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |

### RiskCpkUpper

<table>
<thead>
<tr>
<th>Description</th>
<th>[\text{RiskCpkUpper}(\text{cellref or output name, Sim#}, \text{RiskSixSigma}(\text{LSL, USL, Target, LongTerm Shift, Number of Standard Deviations}))] calculates the one-sided capability index based on the Upper Specification limit for cellref or output name in Sim# optionally using the USL in the included RiskSixSigma property function.</th>
</tr>
</thead>
</table>
| Examples | RiskCpkUpper(A10) returns the one-sided capability index based on the Upper Specification limit for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.  
RiskCpkUpper(A10, \text{RiskSixSigma}(100,120,110,1.5,6)) returns the Process Capability Index for the output cell A10, using an LSL of 100. |
| Guidelines | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |
### RiskDPM

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskDPM(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the defective parts per million for cellref or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function.</th>
</tr>
</thead>
</table>
| Examples    | **RiskDPM(A10)** returns the defective parts per million for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.  
**RiskDPM(A10, ,RiskSixSigma(100,120,110,1.5,6))** returns the defective parts per million for the output cell A10, using an LSL of 100 and USL of 120. |
| Guidelines  | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |

### RiskK

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskK(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates a measure of process center for cellref or output name in Sim# optionally using the LSL and USL in the included RiskSixSigma property function.</th>
</tr>
</thead>
</table>
| Examples    | **RiskK(A10)** returns a measure of process center for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.  
**RiskK(A10, ,RiskSixSigma(100,120,110,1.5,6))** returns a measure of process center for the output cell A10, using an LSL of 100 and USL of 120. |
| Guidelines  | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |
### RiskLowerXBound

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskLowerXBound(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, Long Term Shift, Number of Standard Deviations)) returns the lower X-value for a specified number of standard deviations from the mean for cellref or output name in Sim #, optionally using the Number of Standard Deviations in the RiskSixSigma property function.</th>
</tr>
</thead>
</table>
| Examples | RiskLowerXBound(A10) returns the lower X-value for a specified number of standard deviations from the mean for cell A10.  
RiskLowerXBound(A10, , RiskSixSigma(100, 120, 110, 1.5, 6)) returns the lower X-value for -6 standard deviations from the mean for cell A10, using a Number of Standard Deviations of 6. |
| Guidelines | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |

### RiskPNC

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskPNC(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, Long Term Shift, Number of Standard Deviations)) calculates the total probability of defect outside the lower and upper specification limits for cellref or output name in Sim# optionally using the LSL, USL and Long Term Shift in the included RiskSixSigma property function.</th>
</tr>
</thead>
</table>
| Examples | RiskPNC(A10) returns the probability of defect outside the lower and upper specification limits for the output cell A10.  A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.  
RiskPNC(A10, , RiskSixSigma(100, 120, 110, 1.5, 6)) returns the probability of defect outside the lower and upper specification limits for the output cell A10, using an LSL of 100, USL of 120 and LongTerm shift of 1.5. |
| Guidelines | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |
### RiskPNCLower

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskPNCLower(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations)) calculates the probability of defect outside the lower specification limits for cellref or output name in Sim# optionally using the LSL, USL and LongTerm Shift in the included RiskSixSigma property function.</th>
</tr>
</thead>
</table>
| Examples | **RiskPNCLower (A10)** returns the probability of defect outside the lower specification limits for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.  
**RiskPNCLower(A10, ,RiskSixSigma(100,120,110,1.5,6))** returns the probability of defect outside the lower specification limits for the output cell A10, using an LSL of 100, USL of 120 and LongTerm shift of 1.5. |
| Guidelines | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |

### RiskPNCUpper

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskPNCUpper(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations)) calculates the probability of defect outside the upper specification limits for cellref or output name in Sim# optionally using the LSL, USL and LongTerm Shift in the included RiskSixSigma property function.</th>
</tr>
</thead>
</table>
| Examples | **RiskPNCUpper(A10)** returns the probability of defect outside the upper specification limits for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.  
**RiskPNCUpper(A10, ,RiskSixSigma(100,120,110,1.5,6))** returns the probability of defect outside the upper specification limits for the output cell A10, using an LSL of 100, USL of 120 and LongTerm shift of 1.5. |
<p>| Guidelines | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |</p>
<table>
<thead>
<tr>
<th><strong>RiskPPMLower</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description</strong></td>
</tr>
<tr>
<td><strong>Examples</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Guidelines</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>RiskPPMUpper</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description</strong></td>
</tr>
<tr>
<td><strong>Examples</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Guidelines</strong></td>
</tr>
</tbody>
</table>
### RiskSigmaLevel

**Description**

RiskSigmaLevel(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the Process Sigma level for cellref or output name in Sim# optionally using the USL and LSL and Long Term Shift in the included RiskSixSigma property function. (Note: This function assumes that the output is normally distributed and centered within the specification limits.)

**Examples**

- RiskSigmaLevel(A10) returns the Process Sigma level for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.
- RiskSigmaLevel(A10, ,RiskSixSigma(100,120,110,1.5,6)) returns the Process Sigma level for the output cell A10, using an USL of 120, LSL of 100, and a Long Term Shift of 1.5.

**Guidelines**

A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included.
**RiskUpperXBound**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskUpperXBound(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, Long Term Shift, Number of Standard Deviations)) returns the upper X-value for a specified number of standard deviations from the mean for cellref or output name in Sim #, optionally using the Number of Standard Deviations in the RiskSixSigma property function.</th>
</tr>
</thead>
</table>
| Examples | RiskUpperXBound(A10) returns the upper X-value for a specified number of standard deviations from the mean for cell A10.  
RiskUpperXBound(A10, RiskSixSigma(100, 120, 110, 1.5, 6)) returns the upper X-value for -6 standard deviations from the mean for cell A10, using a Number of Standard Deviations of 6. |
| Guidelines | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |

**RiskYV**

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskYV(cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations)) calculates the yield or the percentage of percentage of the process that is free of defects for cellref or output name in Sim# optionally using the LSL, USL and LongTerm Shift in the included RiskSixSigma property function.</th>
</tr>
</thead>
</table>
| Examples | RiskYV(A10) returns the yield or the percentage of the process that is free of defects for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.  
RiskYV(A10, RiskSixSigma(100,120,110,1.5,6)) returns the yield or the percentage of the process that is free of defects for the output cell A10, using an LSL of 100, USL of 120 and LongTerm shift of 1.5. |
| Guidelines | A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included |
## RiskZlower

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskZlower</strong>(cellref or output name, Sim#, RiskSixSigma(LSL,USL, Target,LongTerm Shift,Number of Standard Deviations)) calculates how many standard deviations the Lower Specification Limit is from the mean for cellref or output name in Sim# optionally using the LSL in the included RiskSixSigma property function.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RiskZlower</strong>(A10) returns how many standard deviations the Lower Specification Limit is from the mean for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10.</td>
</tr>
<tr>
<td><strong>RiskZlower</strong>(A10, ,RiskSixSigma(100,120,110,1.5,6)) returns how many standard deviations the Lower Specification Limit is from the mean for the output cell A10, using an LSL of 100.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>A RiskSixSigma property function needs to be entered for cellref or output name, or a RiskSixSigma property function needs to be included</td>
</tr>
</tbody>
</table>
### RiskZMin

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskZMin}(\text{cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations}) ) calculates the minimum of Z-Lower and Z-Upper for cellref or output name in Sim# optionally using the USL and LSL in the included RiskSixSigma property function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskZMin}(\text{A10}) ) returns the minimum of Z-Lower and Z-Upper for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10. ( \text{RiskZMin}(\text{A10, ,RiskSixSigma(100,120,110,1.5,6)}) ) returns the minimum of Z-Lower and Z-Upper for the output cell A10, using a USL of 120 and LSL of 100.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>A RiskSixSigma property function needs to be entered for ( \text{cellref or output name} ), or a RiskSixSigma property function needs to be included</td>
</tr>
</tbody>
</table>

### RiskZUpper

<table>
<thead>
<tr>
<th>Description</th>
<th>( \text{RiskZUpper}(\text{cellref or output name, Sim#, RiskSixSigma(LSL, USL, Target, LongTerm Shift, Number of Standard Deviations}) ) calculates how many standard deviations the Upper Specification Limit is from the mean for cellref or output name in Sim# optionally using the USL in the included RiskSixSigma property function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>( \text{RiskZUpper}(\text{A10}) ) returns how many standard deviations the Upper Specification Limit is from the mean for the output cell A10. A RiskSixSigma property function needs to be entered in the RiskOutput function in Cell A10. ( \text{RiskZUpper}(\text{A10, ,RiskSixSigma(100,120,110,1.5,6)}) ) returns how many standard deviations the Upper Specification Limit is from the mean for the output cell A10, using a USL of 120.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>A RiskSixSigma property function needs to be entered for ( \text{cellref or output name} ), or a RiskSixSigma property function needs to be included</td>
</tr>
</tbody>
</table>
Reference: Supplemental Functions

The following functions return information on the status of a running simulation or correlations used in a simulation.

**RiskCorrectCorrmat**

| Description | **RiskCorrectCorrmat**(correlationMatrixRange,adjustmentWeightsMatrixRange) returns the corrected correlation matrix for the matrix located in correlationMatrixRange using the adjustment weight matrix located in adjustmentWeightsMatrixRange. An invalid matrix specifies inconsistent simultaneous relationships between three or more inputs and must be corrected prior to simulation.

The returned matrix is a valid correlation matrix, that is, all diagonal entries are 1, the off-diagonal entries are in the range -1 to 1, inclusive, and the matrix is positive-definite (the smallest eigenvalue is > 0, and the correlations are consistent). If the adjustmentWeightsMatrixRange was specified, the correlations have been optimized so they are as close as possible to the originally-specified correlations, taking into account the weights. |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskCorrectCorrmat(A1:C3,E1:G3) returns the corrected correlation matrix for the correlation matrix in the range A1:C3, and the adjustment weight matrix in E1:G3</td>
</tr>
</tbody>
</table>
| Guidelines | adjustmentWeightsMatrixRange is an optional argument

This is an array formula which returns an array with the corrected correlation matrix. To enter it

1) Highlight a range with the same number of rows and columns as the original correlation matrix

2) Enter the function

=RiskCorrectCorrmat(CorrelationMatrixRange,AdjustmentWeightsMatrixRange)

3) Press <Ctrl><Shift><Enter> at the same time to enter your formula as an array formula. |
## RiskCurrentIter

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCurrentIter() returns the current iteration number of an executing simulation. No arguments are required.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>None.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

## RiskCurrentSim

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskCurrentSim() returns the current simulation number. No arguments are required.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>None.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>

## RiskStopRun

<table>
<thead>
<tr>
<th>Description</th>
<th>RiskStopRun(cellRef or formula) stops a simulation when the value of cellRef returns TRUE, or the entered formula evaluates to TRUE. Use this function in conjunction with the function RiskConvergenceLevel to stop a simulation when simulation results for cellRef have converged.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>RiskStopRun(A1) stops a simulation when the value of A1 equals TRUE.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>None.</td>
</tr>
</tbody>
</table>
The @RISK function \texttt{RiskResultsGraph} will automatically place a graph of simulation results wherever it is used in a spreadsheet. For example, \texttt{=RiskResultsGraph (A10)} would place a graph of the simulated distribution for A10 directly in your spreadsheet at the function's location at the end of a simulation. Additional optional arguments to \texttt{RiskResultsGraph} allows you to select the type of graph you want to create, its format, scaling and other options.

This function may also be called from the @RISK macro language to generate graphs in Excel in custom @RISK applications.

\textbf{RiskResultsGraph}

<table>
<thead>
<tr>
<th>Description</th>
<th>\texttt{RiskResultsGraph(cellRef or output/input name, locationCellRange,graphType,xlFormat,leftDelimiter, rightDelimiter,xMin,xMax,xScale,title,sim#)} adds a graph of simulation results to a worksheet. The graphs generated are the same as generated in the @RISK-Results Summary Window. Many arguments to this function are optional. If optional arguments are not entered, RiskResultsGraph creates a graph using the current default settings in the @RISK Results Summary Window for any omitted argument.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>\texttt{RiskResultsGraph(A10)} generates a graph of the simulation results for cell A10 as a Excel format graph at the location of the function, using the default graph type (histogram, cumulative ascending or cumulative descending). \texttt{RiskResultsGraph(A10,C10:M30,1,TRUE,1,99)} generates a graph of the simulation results for cell A10 in the range C10:M30 as a histogram in Excel format, and sets the left and right delimiters at the 1% and 99% values, respectively.</td>
</tr>
<tr>
<td>Guidelines</td>
<td>\texttt{cellRef} is any valid Excel cell reference with one or more cells. Either a \texttt{cellRef} or an output/input name argument needs to be included in a RiskResultsGraph function. When \texttt{cellRef} is entered, the results to be graphed depend on the following: If there is a \texttt{RiskOutput} function in \texttt{cellRef}, the simulation results for this output will be graphed. If there is no \texttt{RiskOutput} function in \texttt{cellRef}, but there is a distribution function, RiskResultsGraph will graph the collected samples for this input. If there is no \texttt{RiskOutput}, and no distribution function in \texttt{cellRef}, a RiskOutput function is automatically added and this output is graphed by RiskResultsGraph. If there are multiple cells in \texttt{cellRef}, an overlay graph is created for the simulation results for each cell in \texttt{cellRef}. Each overlay has the same \texttt{graphType}.</td>
</tr>
</tbody>
</table>
*locationCellRange* is any valid Excel cell range. The created graph will be located in and sized according to this cell range.

*graphType* (optional) is one of the following constants:

- 0 for histogram
- 1 for cumulative ascending graph
- 2 for cumulative descending graph
- 3 for tornado graph of regression sensitivity results
- 4 for tornado graph of correlation sensitivity results
- 5 for a summary graph of 1) the output range that includes cellRef or
  2) the results for each cell in cellRef (where cellRef is a multi-cell
  range)
- 6 for a box plot of 1) the output range that includes cellRef or 2) the
  results for each cell in cellRef (where cellRef is a multi-cell range)
- 7 for a graph of a theoretical distribution function
- 8 for a histogram of a simulated input overlaid with its theoretical
  distribution
- 9 for a histogram with a cumulative ascending overlay
- 10 for a histogram with a cumulative descending overlay
- 11 for tornado graph of mapped value sensitivity results
- 12 for a scatter plot graph of the results for each cell in cellRef (where
  cellRef is a multi-cell range)
- 13 for histogram using relative frequency

*xlFormat* (optional) specifies if the graph will be created as an Excel format chart. Enter TRUE for an Excel format chart, or enter FALSE, or leave blank, for an @RISK format chart.

*leftDelimiter* (optional) specifies the location of the left delimiter on the graph in % for histograms and cumulative graphs only. *leftDelimiter* is a value from 0 to 100.

*rightDelimiter* (optional) specifies the location of the right delimiter on the graph in % for histograms and cumulative graphs only. *rightDelimiter* is a value from 0 to 100.

*xMin* (optional) specifies the minimum value for the X axis in unscaled units for histograms and cumulative graphs only.

*xMax* (optional) specifies the maximum value for the X axis in unscaled units for histograms and cumulative graphs only.

*xScale* (optional) specifies the scale factor for the X axis for histograms and cumulative graphs only. *xScale* is an integer value representing the power of 10 used to convert the x-axis values when labeling the axis. For example, a *xScale* of 3 would specify that values would be shown in thousands.

*title* (optional) specifies the graph title shown on the graph. A string in quotes, or cell reference containing the title, can be entered.

*sim#* (optional) specifies simulation number for which results are to be graphed when multiple simulations are run.
Introduction

@RISK 5 Professional and Industrial versions include the @RISK Library. The @RISK Library is a separate database application for sharing @RISK’s input probability distributions and comparing results from different simulations. It uses SQL Server to store @RISK data.

Different users in an organization can access a shared @RISK Library in order to access:

- Common input probability distributions, which have been pre-defined for use in an organization’s risk models
- Simulation results from different users
- An archive of simulations run on different versions of a model

The @RISK Library is accessed by:

- Clicking the Library icon on the @RISK toolbar and choosing the Show @RISK Library command displays the @RISK Library window. This allows current distributions along with stored simulation results to be reviewed. The Command Add Results to Library adds a current simulation result to the Library.

- Clicking the Add Distribution to Library icon in the Define Distribution window to add a probability distribution to the Library. Once a distribution is added, it is available to others who use the Library.

Multiple libraries can be accessed from different SQL servers. You may, for example, wish to keep a local library where you store simulations and distributions for personal use. A different library could be used to share distributions and results among @RISK users in a workgroup or division. A corporate library could hold common distributions for company-wide assumptions such as future interest rate, prices or the like.
The @RISK Library includes two types of stored information for @RISK models – **Distributions** and **Results**. Each of these are shown in tabs in the main @RISK Library window.
Distributions in the @RISK Library

The @RISK Library allows the sharing of probability distributions among different @RISK users. This is done to insure that all @RISK users in an organization use the same, and most current, definition for common risk inputs that are used in different models. By using the same definitions for key inputs, an organization can insure that all models are run using the same common assumptions. This allows results to be compared from model to model.

@RISK automatically updates all library distributions present in a model each time a simulation is run. This is done with the RiskLibrary property function that is present in any input distribution function that is added from the @RISK Library. The RiskLibrary property function includes a special identifier that allows @RISK to fetch the most recent definition of the distribution from the library, changing the function if necessary. For example, if the Corporate Planning Department has updated the distribution for next year’s Oil Price, your model will automatically use this distribution when you resimulate.

Two different methods can be used for adding probability distributions to the @RISK Library:

- Adding from the Define Distribution Window. Any distribution displayed in the Define Distribution window can be added to the @RISK Library. The Add Input to Library icon adds the displayed distribution to the @RISK Library.

- Entering a Distribution Directly in the @RISK Library. Clicking the Add button in the Distributions tab in the @RISK Library allows you to define a new distribution and make it available to users who access your library.

The @RISK Library allows you to enter additional information about a distribution you add. Properties of a Library distribution include:

- Name. The Name of the distribution
- Description. A custom description you can add.
- Function. The functional definition of the distribution. This can be edited at any time by those who have write access to the database.
- Revisions. Tracks the revisions made to any distribution while it is stored in the library.
Distribution functions that include Excel cell references can be added to the @RISK Library; however, this should be done with caution. Typically this would be done only when the library distribution was going to be used locally in the same workbook where it was originally defined. Inserting a library distribution with cell references into a different model may not properly resolve the argument values as the model structure may be different and the specified cell references do not contain the values you expect.

Seeding Library Distributions

Usually a library distribution will contain a RiskSeed property function for seeding its random number stream. This insures that each model in which the distribution is used will get the same sequence of sampled values for the library distribution. This insures that a valid comparison of results from different models which use the library distribution can be made.
Graphing a library distribution is done much like the graphing of input distributions in @RISK’s Define Distribution and Model windows. Clicking the Graph icon at the bottom of the Distributions tab selects the type of graph to display for the selected distributions (i.e., rows) in the list. Dragging an input off of the list to the bottom of the @RISK Library window also generates a graph. Right-clicking on a graph displays the Graph Options dialog where graph settings can be entered. The definition of a library distribution can be changed by clicking the Edit button and using the Argument Panel when a graph of distribution is displayed.
The Distribution Tab columns can be customized to select which statistics and information you want to display on the input distributions in the library. The Columns icon at the bottom of the window displays the Columns For Table dialog.

Library distributions are added to a model in Excel from either the Define Distributions window or the @RISK Library itself.

The Distribution Palette has a tab titled @RISK Library which lists all distributions available in the library. Clicking on one of these distributions selects it and adds it to the displayed cell formula.

To add a distribution to a model in Excel from the Distributions tab in the @RISK Library itself, highlight the distribution you want to add in the Distributions list and click the Add to Cell icon. Then, select the cell in Excel where you wish to place the function.
@RISK automatically updates all library distributions present in a model each time a simulation is run. This is done with the RiskLibrary property function that is present in any input that is added from the @RISK Library. For example:

`=RiskNormal(50000,10000,RiskName("Product Development/2008"),RiskLibrary(5,"8RENDCKN"))`

instructs @RISK to update the definition of this function from the library identified by "8RENDCKN" at the start of the simulation. This identifier links to a unique library on your system. If the library is not available, @RISK will use the current definition in your model (in this case, RiskNormal(50000,10000)).
Results in the @RISK Library

The @RISK Library allows results from different models and simulations to be stored and compared. In the @RISK Library, results from multiple @RISK simulation runs may be active at any time vs. results from only a single simulation run in @RISK in Excel.

Once results are stored in the Library, overlay graphs can be made to compare results from different runs. For example, you may run a simulation using an initial set of parameters, storing that result in the @RISK Library. Then, you could change your model in Excel and re-run the analysis, storing the second result in the library. Overlaying the graphs for outputs from each run will show you how your results have changed.

You may also sample from an output stored in the @RISK Library in a new simulation in Excel. The @RISK Library can place a RiskResample function in Excel that references the data that was collected for the output and stored in the @RISK Library. This is useful for combining the results of many separate models in a new single simulation or portfolio optimization.
Simulation results are stored in the @RISK Library by selecting the **Add Result to Library** command on the Library icon on the @RISK for Excel toolbar. You can select to store a new simulation in the library or overwrite a currently stored simulation.

When a simulation is placed in the Library, the simulation data and associated Excel workbooks are automatically placed in the @RISK Library. By using the **Open Model** icon (the yellow “folder” at the bottom of the Results tab), you can re-open any stored simulation (and the workbooks used in that simulation) in Excel. This allows you to quickly “roll back” to a prior simulation and model.

**Note:** A shortcut for reverting to a prior simulation and and its workbooks in Excel is to right-click on the list in the Results tab and select the Open Model command.
Graphing a simulation result in the library is done much like the graphing of results in @RISK’s Results Summary window. Clicking the Graph icon at the bottom of the Results tab selects the type of graph to display for the selected outputs (i.e., rows) in the list. Dragging a result off of the list to the bottom of the @RISK Library window also generates a graph. Right-clicking on a graph displays the Graph Options dialog where graph settings can be entered.

To overlay different results, drag a result from the list onto an existing graph.
You may sample from an output stored in the @RISK Library in a new simulation in Excel. There are times when you may wish to use output distributions from many different simulations as inputs in a new simulation in Excel. For example, you may wish to create a portfolio optimization model that uses the output distributions from a set of different models to select an optimal mix of projects or investments. Each possible project or investment in the portfolio has an individual simulation associated with it that was stored in the @RISK Library. The portfolio optimization model then references these individual output distributions. It samples from them each iteration it performs while calculating the results for the portfolio as a whole.

The output distribution from each project or investment becomes an input that can be sampled via the RiskResample function. You can place an output in the library into a workbook in Excel using the Add to Model as Resampled Input command. When you do this, the data that was collected and stored for the output becomes the data set that is sampled from during the portfolio simulation. This data is stored in the workbook with portfolio simulation.

The RiskResample function that makes an output into an input distribution has different options for sampling its referenced data set. You may sample the data in order, randomly sample with replacement or randomly sample without replacement. Usually, however, when resampling from simulation outputs, you will use the Order option. This preserves the ordering of iteration data from the stored simulations during the combined simulation.

Preserving the ordering of iteration data from the stored simulations is important when the individual simulations share common input distributions. These common distributions often have a RiskSeed property function that causes them to return the same sample values in the same order each time they are used. Thus, each simulation for an individual project or investment will use the same sampled values for the common distributions in each iteration.

If the Order option is not used, inaccurate combinations of the output values from individual projects or investments could be introduced into the combined simulation. For example, take the case where you were doing a simulation of a portfolio of individual oil and gas projects and Random, not Order, resampling is used. A given iteration could then resample a value from the output distribution of one project where a high oil price was used and then randomly resample a value from the output distribution of a second project where a low oil price was used. This would be a combination that
could not occur and would lead to inaccurate simulation results for the portfolio.

To enter an output from the library as a resampled input:

1) Highlight the output distribution you wish to resample from in the Results tab of the @RISK Library.

2) Click the Add to Model as Resampled Input icon or right-click and select the Add to Model as Resampled Input command.

3) Select the Sampling Method you wish to use - In Order, Random with Replacement or Randomly without Replacement.

4) Select Update at the Start of Each Simulation if you want to update the data for the output at the start of each simulation. If this is done @RISK will check the @RISK Library at the start of each simulation to see if the stored simulation for the output has been updated with more current results. This would happen if you overwrote the original simulation stored in the library with a newer version.

Updating is done with the RiskLibrary property function that is present in a resampled output that is added from the @RISK Library when the Update at the Start of Each Simulation option is selected. For example:

=RiskResample(1,RiskLibraryExtractedData!B1:B100,RiskIsDiscrete(FALSE),RiskLibrary(407,"TB8GKF8C","RiskLibraryLocal"),RiskName("NPV (10%)"))

instructs @RISK to update the data for the output from the library identified by "TB8GKF8C" at the start of the simulation. This identifier links to a unique library on your system. If the library is not available, @RISK will use the data for the output that was stored in

Reference: @RISK Library 641
the workbook the last time the data was updated and the workbook was saved.

5) Select **Graph as Continuous Distribution** if you want the resampled data to be graphed continuously (as you would see when looking at the output distribution and statistics in the stored simulation) vs. a discrete distribution. This is done with a `RiskIsDiscrete(FALSE)` property function entry in the RiskResample function. The RiskResample distribution is a discrete distribution as only values in the referenced data set can be sampled. However, graphing continuously shows graphs in a form that is easier to present to others. **Note: Selecting Graph as Continuous Distribution has no effect on the values resampled or simulation results.**

6) Select the cell in Excel where you wish to place the resampled output.
Technical Notes

The @RISK Library uses Microsoft SQL Server to store saved simulations and workbooks. Accessing an @RISK Library file is the same as accessing any SQL database. Multiple @RISK Library databases may be open at a single time. By clicking the Library icon at the bottom of the @RISK Library window, connections to existing @RISK Library databases can be set up and new databases can be created.

Connecting to an Existing Library

Clicking the Connect button allows you to navigate to a Server where SQL is installed and an @RISK Library database is available. Clicking on a Server name will check that server for available databases.
Clicking the **Create** button allows you to navigate to a Server where SQL is installed. Enter a name for the new library in the **Library Name** field and click **Create**. Once created, the library will be available for storing @RISK distributions and simulation results.

The @RISK Library uses **SQL Server Express** as the platform for storage and retrieval of RiskLibrary functions and simulation results. It is Microsoft’s free database product that is based on SQL Server 2005 technology.

SQL Server Express uses the same database engine as the other versions of SQL Server 2005, but has several limitations including a 1 CPU, 1 GB RAM, and a 4 GB database size limit.

Although SQL Server Express can be used as a server product, @RISK also uses it as a local client data store where the @RISK Library data access functionality does not depend on the network.

SQL Server Express can install and run on multiprocessor machines, but only a single CPU is used at any time. The 4 GB database size limit applies to all data files, however there are no limits to the number of databases that can be attached to the server and @RISK Library users can create or connect to several databases.

Multiple SQL Server 2005 Express installations can coexist on the same machine along with other installations of SQL Server 2000, and SQL Server 2005.
SQL Server Express by default installs as a named instance called SQL EXPRESS. We recommend that you use this instance unless other applications have special configuration needs.

You will notice when connecting to or creating databases, or editing RiskLibrary functions, that there are SQL Server Authentication options. For most users, and for all local instances of SQL Server Express, **Windows Authentication** is probably adequate. Windows Authentication uses your network credentials to connect to SQL Server as a login. When you log onto your workstation your password is authenticated by Windows, and these credentials allow you access SQL Server, as well as other applications on your workstation or network. This does not automatically grant you access to an @Risk Library database, but you should be able to connect to the server.

With SQL Server Authentication, a login name and password are stored inside SQL Server Express and when you attempt to connect using SQL Server Authentication, the login name is checked for a matching one. If a match is found, then the password is checked against the stored value. If this also matches, you are granted access to the server.

SQL Server Authentication will allow you to protect your database by granting or denying permissions to specific users, or user groups. The details of setting and managing these permissions are normally handled by a database or network administrator, and are not included here. Utilizing these will allow you to grant or deny specific permissions to specific users on a database server.

The SA or System Admin account is disabled by default if Windows Authentication is used. Normal users on the machine have almost no privileges on the SQL Server Express instance. A Local administrator on the server must explicitly grant relevant permissions for normal users so that they can use SQL functionality.

**Library Capacity**

In SQL Server Express, a single library database can hold approximately 2000 representative simulations with 10 outputs, 100 inputs and 1000 iterations. Different sized simulations will have different storage requirements. There are no limits to the number of databases that can be attached to the server, and @RISK Library users can create or connect to several databases.
@RISK for Excel includes a powerful API for use in automating @RISK and building custom applications on @RISK using VBA, VB, C or other programming languages. For more information on this programming interface, see the separate help file titled the @RISK for Excel Developers Kit (XDK) Reference distributed with your copy of @RISK.
Appendix A: Sampling Methods

Sampling is used in an @RISK simulation to generate possible values from probability distribution functions. These sets of possible values are then used to evaluate your Excel worksheet. Because of this, sampling is the basis for the hundreds or thousands of “what-if” scenarios @RISK calculates for your worksheet. Each set of samples represents a possible combination of input values which could occur. Choosing a sampling method affects both the quality of your results, and the length of time necessary to simulate your worksheet.

What is Sampling?

Sampling is the process by which values are randomly drawn from input probability distributions. Probability distributions are represented in @RISK by probability distribution functions, and sampling is performed by the @RISK program. Sampling in a simulation is done, repetitively, with one sample drawn every iteration from each input probability distribution. With enough iterations, the sampled values for a probability distribution become distributed in a manner which approximates the known input probability distribution. The statistics of the sampled distribution (mean, standard deviation and higher moments) approximate the true statistics input for the distribution. The graph of the sampled distribution will even look like a graph of the true input distribution.

Statisticians and practitioners have developed several techniques for drawing random samples. The important factor to examine when evaluating sampling techniques is the number of iterations required to accurately recreate an input distribution through sampling. Accurate results for output distributions depend on a complete sampling of input distributions. If one sampling method requires more iterations, and longer simulation runtimes than another to approximate input distributions, it is the less “efficient” method.
The two methods of sampling used in @RISK — Monte Carlo sampling and Latin Hypercube sampling — differ in the number of iterations required until sampled values approximate input distributions. Monte Carlo sampling often requires a large number of samples to approximate an input distribution, especially if the input distribution is highly skewed or has some outcomes of low probability. Latin Hypercube sampling, a new sampling technique used in @RISK, forces the samples drawn to correspond more closely with the input distribution, and thus converges faster on the true statistics of the input distribution.

**Cumulative Distribution**

It is often helpful, when reviewing different sampling methods, to first understand the concept of a cumulative distribution. Any probability distribution may be expressed in cumulative form. A cumulative curve is typically scaled from 0 to 1 on the Y-axis, with Y-axis values representing the cumulative probability up to the corresponding X-axis value.

In the cumulative curve above, the .5 cumulative value is the point of 50% cumulative probability (.5 = 50%). Fifty percent of the values in the distribution fall below this median value and 50% are above. The 0 cumulative value is the minimum value (0% of the values will fall below this point) and the 1.0 cumulative value is the maximum value (100% of the values will fall below this point).
Why is this cumulative curve so important to understanding sampling? The 0 to 1.0 scale of the cumulative curve is the range of the possible random numbers generated during sampling. In a typical Monte Carlo sampling sequence, the computer will generate a random number between 0 and 1 — with any number in the range equally likely to occur. This number is then used to select a value from the cumulative curve. In the example above, the value sampled for the distribution shown would be X1 if a random number of .5 was generated during sampling. Since the shape of the cumulative curve is based on the shape of the input probability distribution, it is more probable that more likely outcomes will be sampled. The more likely outcomes are in the range where the cumulative curve is the “steepest”.

**Monte Carlo Sampling**

Monte Carlo sampling refers to the traditional technique for using random or pseudo-random numbers to sample from a probability distribution. The term Monte Carlo was introduced during World War II, as a code name for the simulation of problems associated with the development of the atomic bomb. Today, Monte Carlo techniques are applied to a wide variety of complex problems involving random behavior. A wide variety of algorithms are available for generating random samples from different types of probability distributions.

Monte Carlo sampling techniques are entirely random — that is, any given sample may fall anywhere within the range of the input distribution. Samples, of course, are more likely to be drawn in areas of the distribution which have higher probabilities of occurrence. In the cumulative distribution shown earlier, each Monte Carlo sample uses a new random number between 0 and 1. With enough iterations, Monte Carlo sampling “recreates” the input distributions through sampling. A problem of clustering, however, arises when a small number of iterations are performed.
In the illustration shown here, each of the 5 samples drawn falls in the middle of the distribution. The values in the outer ranges of the distribution, are not represented in the samples, and thus their impact on your results is not included in your simulation output.

Clustering becomes especially pronounced when a distribution includes low probability outcomes, which could have a major impact on your results. It is important to include the effects of these low probability outcomes. To do this, these outcomes must be sampled, but if their probability is low enough, a small number of Monte Carlo iterations may not sample sufficient quantities of these outcomes to accurately represent their probability. This problem has led to the development of stratified sampling techniques such as the Latin Hypercube sampling used in @RISK.

**Latin Hypercube Sampling**

Latin Hypercube sampling, is a recent development in sampling technology, designed to accurately recreate the input distribution through sampling in fewer iterations when compared with the Monte Carlo method. The key to Latin Hypercube sampling is stratification of the input probability distributions. Stratification divides the cumulative curve into equal intervals on the cumulative probability scale (0 to 1.0). A sample is then randomly taken from each interval or “stratification” of the input distribution. Sampling is forced to represent values in each interval, and thus, is forced to recreate the input probability distribution.
In the illustration above, the cumulative curve has been divided into 5 intervals. During sampling, a sample is drawn from each interval. Compare this to the 5 clustered samples drawn using the Monte Carlo method. With Latin Hypercube, the samples more accurately reflect the distribution of values in the input probability distribution.

The technique being used during Latin Hypercube sampling is “sampling without replacement”. The number of stratifications of the cumulative distribution is equal to the number of iterations performed. In the example above, there were 5 iterations and thus 5 stratifications were made to the cumulative distribution. A sample is taken from each stratification. However, once a sample is taken from a stratification, this stratification is not sampled from again — its value is already represented in the sampled set.

How does sampling within a given stratification occur? In effect, @RISK chooses a stratification for sampling, then randomly chooses value from within the selected stratification.

When using the Latin Hypercube technique to sample from multiple variables, it is important to maintain independence between variables. The values sampled, for one variable, need to be independent of those sampled for another (unless, of course, you explicitly want them correlated). This independence is maintained by randomly selecting the interval to draw a sample from for each variable. In a given iteration, Variable #1 may be sampled from stratification #4, Variable #2 may be sampled from stratification #22, and so on. This preserves randomness and independence, and avoids unwanted correlation between variables.
As a more efficient sampling method, Latin Hypercube offers great benefits in terms of increased sampling efficiency and faster runtimes (due to fewer iterations). These gains are especially noticeable in a PC based simulation environment such as @RISK. Latin Hypercube also aids the analysis of situations, where low probability outcomes are represented in input probability distributions. By forcing the sampling of the simulation, to include the outlying events, Latin Hypercube sampling assures they are accurately represented in your simulation outputs.

When low probability outcomes are very important, it often helps to run an analysis which just simulates the contribution to the output distribution from the low probability events. In this case the model simulates only the occurrence of low probability outcomes — they are set to 100% probability. Through this you will isolate those outcomes and directly study the results they generate.

The concept of convergence is used to test a sampling method. At the point of convergence, the output distributions are stable (additional iterations do not markedly change the shape or statistics of the sampled distribution). The sample mean versus the true mean is typically a measure of convergence, but skewness, percentile probabilities and other statistics are often used as well.

@RISK provides a good environment for testing the speed, at which the two available sampling techniques converge on an input distribution. Run an equal number of iterations, with each of the sampling techniques, while selecting an input distribution @function as a simulation output. Using the built-in Convergence Monitoring capability in @RISK, you see how many iterations it takes the percentiles, mean and standard deviation, to stabilize. It should be evident that Latin Hypercube sampling converges faster on the true distributions when compared with Monte Carlo sampling.

More About Sampling Techniques

The academic and technical literature has addressed both Monte Carlo and Latin Hypercube sampling. Any of the references to simulation in the Recommended Readings give an introduction to Monte Carlo sampling. References which specifically address Latin Hypercube sampling are included in a separate section.
Palisade’s DecisionTools Suite is a complete set of decision analysis solutions for Microsoft Windows. With the introduction of DecisionTools, Palisade brings you a decision-making suite whose components combine to take full advantage of the power of your spreadsheet software.

The DecisionTools Suite

The DecisionTools Suite focuses on providing advanced tools for any decision, from risk analysis, to sensitivity analysis, to distribution fitting. Software packaged with the DecisionTools Suite includes:

- **@RISK** — risk analysis using Monte-Carlo simulation
- **TopRank®** — sensitivity analysis
- **PrecisionTree®** — decision analysis with decision trees and influence diagrams
- **NeuralTools®** — neural networks in Excel
- **Evolver®** — genetic optimization in Excel
- **StatTools®** — statistics in Excel

While the tools listed above can be purchased and used separately, they become more powerful when used together. Analyze historical and fit data for use in an @RISK model, or use TopRank to determine which variables to define in your @RISK model.

This chapter explains many of the ways the components of the DecisionTools suite interact, and how they will make your decision making easier and more effective.

### Note: Palisade also offers a version of @RISK for Microsoft Project. @RISK for Project allows you to run risk analyses on project schedules created in Microsoft Project, the leading software package for project management. Contact Palisade for more information on this exciting implementation of @RISK!
Purchasing Information

All of the software mentioned here, including the DecisionTools Suite, can be purchased directly from Palisade Corporation. To place an order or receive more information, please contact the technical sales department at Palisade Corporation using one of the following methods:

- **Telephone:** *(800) 432-7475* (U.S. only) or *(607) 277-8000*  
  Mon-Fri. from 8:30 AM to 5:00 PM, EST
- **Fax:** *(607) 277-8001*
- **E-mail:** sales@palisade.com
- **Visit us on the Web:** at [http://www.palisade.com](http://www.palisade.com)
- **Or, mail a letter to:**  
  Technical Sales  
  Palisade Corporation  
  798 Cascadilla St  
  Ithaca, NY 14850  
  USA

To contact Palisade Europe:

- **E-mail:** sales@palisade-europe.com.
- **Telephone:** +44 1895 425050 (UK).
- **Fax:** +44 1895 425051 (UK).
- **Or, mail a letter to:**  
  Palisade Europe  
  31 The Green  
  West Drayton  
  Middlesex  
  UB7 7PN  
  United Kingdom

If you want to contact Palisade Asia-Pacific:

- **Email us at** sales@palisade.com.au
- **Telephone us at** + 61 2 9252 5922 (AU).
- **Fax us at** + 61 2 9252 2820 (AU).
- **Mail us a letter to:**  
  Palisade Asia-Pacific Pty Limited  
  Suite 404, Level 4  
  20 Loftus Street  
  Sydney NSW 2000  
  Australia
Palisade’s DecisionTools Case Study

The Excelsior Electronics company currently makes desktop computers. They are working on a laptop computer, the Excelsior 5000, and want to know whether or not the company will profit from this venture. They built a spreadsheet model which spans the next two years, each column representing one month. The model takes into account production costs, marketing, shipping, price per unit, units sold, etc. The bottom line for each month is “Profit”. Excelsior expects some initial setbacks on this venture, but as long as they are not too great and profits are up towards the end of two years, they will go ahead with the E5000.

Run TopRank First, Then @RISK

TopRank is used on the model to find the critical variables. The “Profit” cells are selected as outputs, and an automatic What-if analysis is run. The results quickly show there are five variables (out of many more) that have the most impact on profits: price per unit, marketing costs, build time, price of memory, and price of CPU chips. Excelsior decided to concentrate on these variables.

Next, Assess Probabilities

Distribution functions are needed to replace the five variables in the spreadsheet model. Normal distributions are used for price per unit and build time, based on internal decisions and information from Excelsior’s manufacturing division.

Add Distribution Fitting

Research is done to get weekly price quotes for memory and CPU’s over the past two years. This data is fed into @RISK’s distribution fitting, and distributions are fitted to the data. Confidence level information confirms that the distributions are good fits, and the resulting @RISK distribution functions are pasted into the model.

Simulate with @RISK

Once all the @RISK functions are in place, the “Profit” cells are selected as outputs and a simulation is run. Overall, the results look promising. Although there will be losses initially, there is an 85% chance they will make an acceptable profit, and a 25% chance the venture will generate more revenue than they had initially assumed. The Excelsior 5000 project has been given the go-ahead.

Decide with PrecisionTree

Excelsior Electronics had assumed they would sell, and distribute, the Excelsior 5000 themselves. However they could use various catalogs and computer warehouses to distribute their product. A decision tree model is built using PrecisionTree, taking into account unit prices, sales volume, and other critical factors for direct sales versus catalog sales. A Decision Analysis is run, and PrecisionTree suggests using catalogs and warehouses. Excelsior Electronics puts that plan into full motion.
Introduction to TopRank®

TopRank is the ultimate What-if tool for spreadsheets, from Palisade Corporation. TopRank greatly enhances the standard What-if and data table capabilities found in your spreadsheet. In addition, you can easily step up to powerful risk analysis with its companion package, @RISK.

TopRank and What-if Analysis

TopRank helps you find out which spreadsheet value(s) or variable(s) affects your results the most — an automated What-if or sensitivity analysis. You also can have TopRank, automatically, try any number of values for a variable — a data table — and tell you the results calculated at each value. TopRank also tries all possible combinations of values for a set of variables (a Multi-Way What-if analysis), giving you the results calculated for each combination.

Running a What-if or sensitivity analysis, is a key component of making any decision based on a spreadsheet. This analysis identifies which variables affect your results the most. It shows you those factors you should be most concerned with as you 1) gather more data and refine your model, and 2) manage and implement the situation described by the model.

TopRank is a spreadsheet add-in for Microsoft Excel. It can be used with any pre-existing, or new, spreadsheet. To set up your What-if analyses, TopRank adds new custom “Vary” functions to the spreadsheet function set. These functions specify how the values in your spreadsheet can be varied in a What-if analysis; for example, +10% and -10%, +1000 and -500, or according to a table of values you’ve entered.

TopRank can also run a fully automatic What-if analysis. It uses powerful auditing technology, to find all possible values in your spreadsheet, which could affect your results. It can then change all these possible values automatically, and find out which is most significant in determining your results.
TopRank applications are the same as spreadsheet applications. If you can build your model in a spreadsheet, you can use TopRank to analyze it. Businesses use TopRank to identify the critical factors — price, up front investment amount, sales volume, or overhead — that most affect the success of their new product. Engineers use TopRank to show them the individual product components whose quality most affects final product production rates. A loan officer can have TopRank quickly run a model at any possible interest rate, loan principle amount, and down payment combinations, and review results for each possible scenario. Whether your application is in business, science, engineering, accounting, or another field, TopRank can work for you to identify the critical variables which affect your results.

Modeling Features

As an add-in to Microsoft Excel, TopRank links directly to your spreadsheet to add What-if analysis capabilities. The TopRank system provides all the necessary tools for conducting a What-if analysis on any spreadsheet model. And TopRank works in a style you are familiar with — Excel style menus and functions.

What-if analysis and Data Tables are functions that can be performed directly in your spreadsheet, but only in a manual, unstructured format. Simply changing a cell value in your spreadsheet and calculating a new result is a basic What-if analysis. A Data Table, which gives a result for each combination of two values, can also be built in your spreadsheet.

TopRank, however, performs these tasks automatically and analyzes their results for you. It instantly performs What-ifs on all possible values in your spreadsheet which could affect your result, instead of requiring you to individually change values and recalculate. It then tells you what spreadsheet value is most significant in determining your result.

TopRank also runs data table combinations automatically, without requiring you to set up tables in your spreadsheet. Combine more than two variables in its Multi-Way What-if analysis — you can generate combinations of any number of variables — and rank your combinations by their affect on your results. You can perform these sophisticated and automated analyses quickly, as TopRank keeps track of all the values and combinations it tries, and their results, separate from your spreadsheet. By taking an automated approach, TopRank gives you What-if and Multi-Way What-if results, almost instantly. Even the least experienced modeler can get powerful analysis results.
TopRank defines variations in spreadsheet values using functions. To do this, TopRank has added a set of new functions to the Excel function set, each of which specifies a type of variation for your values. These functions include:

- **Vary** and **AutoVary** functions which, during a What-if analysis, change a spreadsheet value across a + and — range you define.
- **VaryTable** functions which, during a What-if analysis, substitute each of a table of values for a spreadsheet value.

TopRank uses functions to change spreadsheet values during a What-if analysis, and keeps track of the results calculated for each value change. These results are then ranked by the amount of change from the original expected results. Then functions which caused the greatest change are identified as the most critical to the model.

TopRank Pro also includes over 30 probability distribution functions found in @RISK. These functions can be used, along with Vary functions, to describe variation in spreadsheet values.

TopRank functions are entered wherever you want to try different values in a What-if analysis. The functions can be added to any number of cells in a spreadsheet, and can include arguments which are cell references and expressions — providing extreme flexibility in defining variation in value in spreadsheet models.

In addition to adding Vary functions yourself, TopRank can automatically enter Vary functions for you. Use this powerful feature to quickly analyze your spreadsheets, without manually identifying values to vary and typing in functions.

When automatically entering Vary functions, TopRank traces back through your spreadsheet and finds all possible values which could affect the result cell you identify. As it finds a possible value, it substitutes in an “AutoVary” function with the default variation parameters (such as +10% and -10%) you’ve selected. With a set of AutoVary functions inserted, TopRank can then run its What-if analysis, and rank the values which could affect your results by their importance.

With TopRank, you can step through your Vary and AutoVary functions and change the variation each function specifies. As a default you can use a -10% and +10% variation, but for a certain value you may feel that a -20% and +30% change is possible. You can also select to not have a value varied — as in some cases a spreadsheet value is fixed and could never be changed.
During its analysis TopRank individually changes values for each Vary function and recalculates your spreadsheet using each new value. Each time it recalculates, it collects the new value calculated in each result cell. This process of changing value and recalculating is repeated for each Vary and VaryTable function. The number of recalculations performed depends on the number of Vary functions entered, the number of steps (i.e., values across the min-max range) you want TopRank to try for each function, the number of VaryTable functions entered, and the values in each table used.

TopRank ranks all varied values by their impact on each result cell, or output you’ve selected. Impact is defined as the amount of change in the output value that was calculated when the input value was changed. If, for example, the result of your spreadsheet model was 100 prior to changing values, and the result was 150 when an input changed, there is a +50% change in results caused by changing the input.

TopRank results can be view graphically in a Tornado, Spider or Sensitivity graph. These graphs summarize your results to easily show the most important inputs for your results.
Appendix B: Using @RISK with TopRank

What-if analysis is often the first analysis performed on a spreadsheet. Its results lead to a further refinement of the model, additional analyses and ultimately, a final decision based on the best model possible. Risk analysis, a powerful analytical technique available using TopRank’s companion product, @RISK, is often the next analysis performed on a spreadsheet after a What-if analysis.

Moving from What-if to Simulation

A What-if analysis initially identifies what’s important in your model. You can then focus on these important components and better estimate what their values could be. Usually, however, there are several or more of these important uncertain components, and, in reality, they could all vary at the same time. To analyze an uncertain model, such as this, you need risk analysis or Monte Carlo simulation. Risk analysis varies all uncertain inputs simultaneously — just as they do in real life — and builds a range and distribution of the possible results that could occur.

With risk analysis, inputs are described with a probability distribution — such as normal, lognormal, beta or binomial. This is a much more detailed description of the uncertainty present in an input’s value than a simple + or — percentage variation. A probability distribution shows both the range of values possible for an input, and the likelihood of occurrence of any value in the range. Simulation combines these input distributions to generate both a range of possible results from your model, and the likelihood of any result occurring.

The simple + and — change defined by a Vary function in a What-if analysis can be used directly in risk analysis. @RISK actually samples your Vary functions directly in a risk analysis.

The values sampled by @RISK from Vary and VaryTable functions, during a simulation, depend on either distribution argument entered for the function, or the default distribution setting used in TopRank. For example, the TopRank function `RiskVary(100,-10,+10)` using a default distribution setting of Uniform and a default range type of +/- percentage, is sampled like the @RISK distribution `RiskUniform(90,110)`. VaryTable functions from TopRank are sampled as RiskDuniform functions in @RISK.
The Differences Between TopRank and @RISK

TopRank and @RISK share many common features, so it's easy to think that they perform the same functions. In fact, the two programs perform different, but complementary, tasks. Don't ask yourself “Which should I use, @RISK or TopRank?”, ask yourself “Shouldn't I use both?”

The Similarities

Both @RISK and TopRank are add-ins for analysis of models designed in spreadsheets. By using special spreadsheet formulas, both programs explore how uncertainty affects your model, and thus the decisions you make. And, a common user-interface guarantees a smooth transition between the two products: one learning curve instead of two.

The Differences

There are three main areas where @RISK and TopRank differ:

- **Inputs** how uncertainty is defined in your model
- **Calculations** what happens during an analysis
- **Results** what types of answers the analyses provide

**Inputs**

@RISK defines uncertainty in your model using probability distribution functions. These functions define all the possible values an input can have, with a corresponding probability of that value occurring. There are over 30 probability distribution functions available in @RISK.

To define uncertainty in @RISK, you need to assign a distribution function to every value that you think is uncertain. It's up to you, the user, to determine which inputs are uncertain, and which distribution function describes the uncertainty.

TopRank defines uncertainty in your model using Vary functions. Vary functions are simple: they define possible values that an input can have without assigning probabilities to those values. There are only two basic Vary functions in TopRank — Vary and VaryTable.

TopRank can automatically define variable cells in your model every time you select an output. You don't need to know which cells are uncertain or important, TopRank identifies those cells for you.

**Calculations**

@RISK runs a Monte Carlo or Latin Hypercube simulation. For each iteration (or step), every @RISK distribution in the spreadsheet model takes on a new value determined by the probability distribution function. To run a thorough analysis, @RISK needs to run hundreds, sometimes thousands, of iterations.
TopRank runs a single or Multi-Way sensitivity analysis. During the analysis, only one cell (or a small number of cells) varies at a time according to the values defined in the Vary function. With TopRank, only a few iterations are needed to study a large number of uncertain cells.

For each output defined, @RISK produces a probability distribution as an analysis result. The distribution describes which values an output (such as profit) could have, as well as how probable certain outcomes are. For example, @RISK can tell you whether there is a 30% chance that your company will not make a profit next quarter.

For each output defined, TopRank tells you which inputs have the largest effect on the output. The results show how much change you can expect in an output, when a given input changes by a defined amount. For example, TopRank can tell you that your company's profits are most sensitive to sales volume, and that when the sales volume is 1000 units, you will lose $1 million. So, TopRank told you that, to make a profit, you'll need to concentrate on keeping sales volumes high.

The most important difference between the two packages is that @RISK studies how the combined uncertainty of all variables affect the output. TopRank only tells you how an individual input (or a small group of inputs) affects the output. So, while TopRank is faster and easier to use, @RISK provides a more detailed, comprehensive look at the problem. **We strongly recommend using TopRank first to determine which variables are the most important. Then, use @RISK to run a comprehensive analysis of your problem for the best possible results.**

In summary, TopRank tells you what the most important variables are in your model. The results of a TopRank What-if analysis can be used on their own to make better decisions. But, for the most thorough analysis, use TopRank to find the most important variables in your model, then use @RISK to define uncertainty in those variables and run a simulation. TopRank can help you optimize your @RISK simulations by defining uncertainty in only the most important variables, making your simulation faster and more compact.
Introduction to PrecisionTree™

PrecisionTree from Palisade Corporation is a decision analysis add-in to Microsoft Excel. Now you can do something you've never been able to do before — define a decision tree or influence diagram directly in your spreadsheet. PrecisionTree allows you to run a complete decision analysis, without leaving the program where your data is — your spreadsheet!

Why You Need Decision Analysis and PrecisionTree

You might wonder if the decisions you make are suitable for decision analysis. If you are looking for a way to structure your decisions, to make them more organized and easier to explain to others, you should definitely consider using formal decision analysis.

When faced with a complex decision, decision makers must be able to organize the problem efficiently. They have to consider all possible options by analyzing all available information. In addition, they need to present this information to others in a clear, concise format. PrecisionTree allows decision makers to do all this, and more!

But, what exactly does decision analysis allow you to do? As the decision maker, you can clarify options and rewards, describe uncertainty quantitatively, weigh multiple objectives simultaneously, and define risk preferences. All in an Excel spreadsheet.
Modeling Features

As an “add-in” to Microsoft Excel, PrecisionTree “links” directly to Excel to add Decision Analysis capabilities. The PrecisionTree system provides all the necessary tools for setting up and analyzing decision trees and influence diagrams. And PrecisionTree works in a style you are familiar with — Excel-style menus and toolbars.

With PrecisionTree, there's no limit to the size tree you can define. Design a tree which spans multiple worksheets in an Excel workbook! PrecisionTree reduces the tree to an easy-to-understand report right in your current workbook.

PrecisionTree allows you to define, as well as influence diagrams and decision tree nodes, in Excel spreadsheets. Node types offered by PrecisionTree include:

- Chance nodes
- Decision nodes
- End nodes
- Logic nodes
- Reference nodes

Values and probabilities for nodes are placed directly in spreadsheet cells, allowing you to easily enter and edit the definition of your decision models.

PrecisionTree creates both decision trees and influence diagrams. Influence diagrams are excellent for showing the relationship between events and the general structure of a decision clearly and concisely, while decision trees outline the chronological and numerical details of the decision.

In PrecisionTree, all decision model values and probabilities are entered directly in spreadsheet cells, just like other Excel models. PrecisionTree can also link values in a decision model directly to locations you specify in a spreadsheet model. The results of that model are then used as the payoff for each path through the decision tree.

All calculations of payoff happen in “real-time” — that is, as you edit your tree, all payoff and node values are automatically recalculated.

PrecisionTree's decision analyses give you straightforward reports, including statistical reports, risk profiles and policy suggestions.* (*PrecisionTree Pro only). Also, decision analysis can produce more
qualitative results by helping you understand tradeoffs, conflicts of interest, and important objectives.

All analysis results are reported directly in Excel for easy customization, printing and saving. There's no need to learn a whole new set of formatting commands, since all PrecisionTree reports can be modified like any other Excel worksheet or chart.

Have you ever wondered which variables matter most in your decision? If so, you need PrecisionTree's sensitivity analysis options. Perform both one, and two-way sensitivity analyses, and generate Tornado Graphs, spider graphs, strategy region graphs (PrecisionTree Pro only), and more!

For those who need more sophisticated sensitivity analyses, PrecisionTree links directly to TopRank, Palisade Corporation's sensitivity analysis add-in.

Because decision trees can expand as more possible decision options are added, PrecisionTree offers a set of features designed to help you reduce trees to a more manageable size. All nodes can be collapsed, hiding all paths which follow the node from view. A single subtree can be referenced from multiple nodes in other trees, saving the repeated re-entry of the same.

Sometimes you need help in creating a utility function that is used to factor your attitude towards risk into the calculations in your decision models. PrecisionTree contains features which help you identify your attitude towards risk and create your own utility functions.

PrecisionTree offers many advanced analysis options including:

- Utility functions
- Use of multiple worksheets to define trees
- Logic nodes
Using @RISK with PrecisionTree

@RISK is a perfect companion to PrecisionTree. @RISK allows you to 1) quantify the uncertainty that exists in the values and probabilities which define your decision trees, and 2) more accurately describe chance events as a continuous range of possible outcomes. Using this information, @RISK performs a Monte-Carlo simulation on your decision tree, analyzing every possible outcome and graphically illustrating the risks you face.

Using @RISK to Quantify Uncertainty

With @RISK, all uncertain values and probabilities for branches in your decision trees, and supporting spreadsheet models, can be defined with distribution functions. When a branch from a decision or chance node has an uncertain value, for example, this value can be described by an @RISK distribution function. During a normal decision analysis, the expected value of the distribution function will be used as the value for the branch. The expected value for a path in the tree will be calculated using this value.

However, when a simulation is run using @RISK, a sample will be drawn from each distribution function during each iteration of the simulation. The value of the decision tree, and its nodes, will then be recalculated using the new set of samples and the results recorded by @RISK. A range of possible values will then be displayed for the decision tree. Instead of seeing a risk profile with a discrete set of possible outcomes and probabilities, a continuous distribution of possible outcomes is generated by @RISK. You can see the chance of any result occurring.

In decision trees, chance events must be described in terms of discrete outcomes (a chance node with a finite number of outcome branches). But, in real life, many uncertain events are continuous, meaning that any value between a minimum and maximum can occur.

Using @RISK with PrecisionTree, makes modeling continuous events easier, using distribution functions. Also, @RISK functions can make your decision tree smaller and easier to understand!
Methods of Recalculation During a Simulation

Two options are available for recalculation of a decision model during a simulation performed with @RISK. The first option, Expected Values of the Model, causes @RISK to first sample all distribution functions in the model, and supporting spreadsheets each iteration, then recalculates the model using the new values to generate a new expected value. Typically the output from the simulation is the cell containing the expected value of the model. At the end of the run an output distribution, reflecting the possible range of expected values for the model and their relative likelihood of occurrence, is generated.

The second option, Values of One Sampled Path Through the Model, causes @RISK to randomly sample a path through the model each iteration of a simulation. The branch to follow from each chance node is randomly selected, based on the branch probabilities entered. This method does not require that distribution functions be present in the model; however, if they are used a new sample is generated each iteration and used in path value calculations. The output from the simulation is the cell containing the value of the model, such as the value of the root node of the tree. At the end of the run an output distribution reflecting the possible range of outcomes for the model, and their relative likelihood of occurrence, is generated.

Using Probability Distributions in Nodes

Let’s take a look at a chance node in an oil drilling decision tree:

The results of drilling are divided into three discrete outcomes (Dry, Wet, and Soaking). But, in reality, the amount of oil found should be described with a continuous distribution. Suppose the amount of money made from drilling follows a lognormal distribution with a mean of $22900 and a standard deviation of $50000, or the @RISK distribution =RiskLognorm(22900,50000).
To use this function in the oil drilling model, change the chance node to have only one branch, and the value of the branch is defined by the @RISK function. Here’s how the new model should look:

During an @RISK simulation, the RiskLognorm function will return random values for the payoff value of the Results node and PrecisionTree will calculate a new expected value for the tree.

But, what about the decision to Drill or Not Drill? If the expected value of the Drill node changes, the optimum decision could change iteration to iteration. That would imply that we know the outcome of drilling before the decision is made. To avoid this situation, PrecisionTree has an option Decisions Follow Current Optimal Path to force decisions before running an @RISK simulation. Every decision node in the tree will be changed to a forced decision node, which causes each decision node to select the decision that’s optimal when the command is used. This avoids changes in a decision, due to changing a decision tree’s values and probabilities during a risk analysis.

**Using @RISK to Analyze Decision Options**

There may be times when you want to know the outcome of a chance event before making a decision. You want to know the value of perfect information.

Before running a risk analysis, you know the expected value of the Drill or Don’t Drill decision from the value of the Drill Decision node. If you ran a risk analysis on the model without forcing decisions, the return value of the Drill Decision node would reflect the expected value of the decision if you could perfectly predict the future. The difference between the two values is the highest price you should pay (perhaps by running more tests) to find out more information before making the decision.
**Selecting @RISK Outputs**

Running a risk analysis on a decision tree can produce many types of results, depending on the cells in your model you select as outputs. True expected value, the value of perfect information, and path probabilities can be determined.

Select the value of a start node of a tree (or the beginning of any sub-tree) to generate a risk profile from an @RISK simulation. Since @RISK distributions generate a wider range of random variables, the resulting graph will be smoother, and more complete, than the traditional discrete risk profile.
## Appendix C: Glossary

### Glossary of Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition and Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Averse</td>
<td>See risk averse</td>
</tr>
<tr>
<td>Continuous Distribution</td>
<td>A probability distribution where any value between the minimum and maximum is possible (has finite probability). See discrete distribution</td>
</tr>
<tr>
<td>Cumulative Distribution</td>
<td>A cumulative distribution, or a cumulative distribution function, is the set of points, each of which equals the integral of a probability distribution, starting at the minimum value and ending at the associated value of the random variable. See cumulative frequency distribution, probability distribution</td>
</tr>
<tr>
<td>Cumulative Frequency Distribution</td>
<td>A cumulative frequency distribution is the term for the output and the input cumulative distributions of @RISK. A cumulative distribution is constructed by cumulating the frequency (progressively adding bar heights) across the range of a frequency distribution. A cumulative distribution can be an “upwardly sloping” curve, where the distribution describes the probability of a value less than, or equal to, any variable value. Alternatively, the cumulative curve may be a “downwardly sloping” curve, where the distribution describes the probability of a value greater than, or equal to, any variable value. See cumulative distribution</td>
</tr>
<tr>
<td>Dependent Variable</td>
<td>A dependent variable is one that depends in some way on the values of other variables in the model under consideration. In one form, the value of an uncertain dependent variable can be calculated from an equation as a function of other uncertain model variables. Alternatively, the dependent variable may be drawn from a distribution based on the random number which is correlated with a random number used to draw a sample of an independent variable. See independent variable</td>
</tr>
<tr>
<td>Deterministic</td>
<td>The term deterministic indicates that there is no uncertainty associated with a given value or variable. See stochastic, risk</td>
</tr>
</tbody>
</table>
Discrete Distribution
A probability distribution, where only a finite number of discrete values are possible between the minimum and maximum.
See continuous distribution

Event
The term event refers to an outcome or group of outcomes that might result from a given action. For example, if the action is the pitch of a baseball, the possible events might include a hit (with outcomes being either a single, double, triple, or home run), an out, a foul ball, or a ball, etc.

Expected Value
See mean

Frequency Distribution
Frequency distribution is the proper term for the output probability distributions and the input histogram distributions (HISTOGRM) of @RISK. A frequency distribution is constructed from data, by arranging values into classes and representing the frequency of occurrence in any class by the height of the bar. The frequency of occurrence corresponds to probability.

Higher Moments
Higher moments are statistics of a probability distribution. The term generally refers to the skewness and kurtosis, the third and fourth moments respectively. The first and second moments are the mean and the standard deviation, respectively.
See skewness, kurtosis, mean, standard deviation

Independent Variable
An independent variable is one that does not depend, in any way, on the values of any other variable in the model under consideration. The value of an uncertain independent variable is determined by drawing a sample from the appropriate probability distribution. This sample is drawn without regard to any other random sample drawn for any other variable in the model.
See dependent variable

Iteration
An iteration is one recalculation of the user's model during a simulation. A simulation consists of many recalculations or iterations. During each iteration, all uncertain variables are sampled once, according to their probability distributions, and the model is recalculated using these sampled values.
Also known as a simulation trial

Kurtosis
Kurtosis is a measure of the shape of a distribution. Kurtosis indicates how flat, or peaked, the distribution is. The higher the kurtosis value, the more peaked the distribution.
See skewness
**Latin Hypercube**
Latin Hypercube is a relatively new stratified sampling technique used in simulation modeling. Stratified sampling techniques, as opposed to Monte Carlo type techniques, tend to force convergence of a sampled distribution in fewer samples.

*See Monte Carlo*

**Mean**
The mean of a set of values is the sum of all the values in the set, divided by the total number of values in the set.

*Synonym: expected value*

**Monte Carlo**
Monte Carlo refers to the traditional method of sampling random variables in simulation modeling. Samples are chosen completely randomly across the range of the distribution, thus necessitating large numbers of samples for convergence for highly skewed or long-tailed distributions.

*See Latin Hypercube*

**Most Likely Value**
The most likely value, or mode, is the value that occurs most often in a set of values. In a histogram and a result distribution, it is the center value in the class or bar with the highest probability.

**Objective Risk**
Objective risk, or objective probability, refers to a probability value or distribution that is determined by “objective” evidence or accepted theory. The probabilities associated with an objective risk are known with certainty.

*See subjective risk*

**Percentile**
A percentile is an increment of the values in a data set. Percentiles divide the data into 100 equal parts, each containing one percent of the total values. The 60th percentile, for example, is the value in the data set for which 60% of the values are below it and 40% are above.

**Preference**
Preference refers to an individual’s choices where many attributes of a decision or object are considered. Risk is an important consideration in personal preference.

*See risk averse*

**Probability**
Probability is a measure of how likely a value or event is to occur. It can be measured from simulation data, as frequency, by calculating the number of occurrences of the value or event, divided by the total number of occurrences. This calculation returns a value between 0 and 1 which then can be converted to percentage by multiplying by 100.

*See frequency distribution, probability distribution*
| **Probability Distribution** | A probability distribution, or probability density function, is the proper statistical term for a frequency distribution constructed from an infinitely large set of values where the class size is infinitesimally small.  
*See frequency distribution*
|---|
| **Random Number Generator** | A random number generator is an algorithm for choosing random numbers, typically in the range of 0 to 1. These random numbers are equivalent to samples drawn from a uniform distribution with a minimum of 0 and a maximum of 1. Such random numbers are the basis for other routines that convert them into samples drawn from specific distribution types.  
*See random sample, seed*
|---|
| **Random Sample** | A random sample is a value that has been chosen from a probability distribution describing a random variable. Such a sample is drawn randomly according to a sampling “algorithm”. The frequency distribution constructed from a large number of random samples, drawn by such an algorithm, will closely approximate the probability distribution for which the algorithm was designed.
|---|
| **Range** | The range is the absolute difference between the maximum and minimum values in a set of values. The range is the simplest measure of the dispersion or “risk” of a distribution.
|---|
| **@RISK** | @RISK (pronounced “at risk”) is the name of the Excel Add-In for Risk Analysis described in this User’s Guide.
|---|
| **Risk** | The term risk refers to uncertainty or variability in the outcome of some event or decision. In many cases the range of possible outcomes can include some that are perceived as a loss, or undesirable, along with others that are perceived as a gain or desirable. The range of outcomes is often associated with levels of probability of occurrence.
|---|
| **Risk Analysis** | Risk Analysis is a general term used to describe any method used to study and understand the risk inherent to a situation of interest. Methods can be quantitative and/or qualitative in nature. @RISK uses a quantitative technique, generally referred to as simulation.  
*See simulation*
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Risk Averse</strong></td>
</tr>
</tbody>
</table>
**Sample**  
*See random sample*

**Seed**  
The seed is a number that initializes the selection of numbers by a random number generator. Given the same seed, a random number generator will generate the same series of random numbers each time a simulation is run.  
*See random number generator*

**Simulation**  
Simulation is a technique whereby a model, such as an Excel worksheet, is calculated many times with different input values with the intent of getting a complete representation of all possible scenarios that might occur in an uncertain situation.

**Skewness**  
Skewness is a measure of the shape of a distribution. Skewness indicates the degree of asymmetry in a distribution. Skewed distributions have more values to one side of the peak or most likely value — one tail is much longer than the other. A skewness of 0 indicates a symmetric distribution, while a negative skewness means the distribution is skewed to the left. Positive skewness indicates a skew to the right.  
*See kurtosis*

**Standard Deviation**  
The standard deviation is a measure of how widely dispersed the values are in a distribution. It equals the square root of the variance.  
*See variance*

**Stochastic**  
Stochastic is a synonym for uncertain, risky.  
*See risk, deterministic*

**Subjective Risk**  
Subjective risk, or subjective probability, is a probability value or distribution determined by an individual's best estimate based on personal knowledge, expertise, and experience. New information often causes changes in such estimates. Reasonable individuals may disagree on such estimates.  
*See objective risk*

**Summary Graph**  
A Summary Graph is an output graphic in @RISK that presents simulation results for a range of cells in an Excel worksheet. The Summary Graph takes the underlying distributions for each cell, and summarizes them by showing the trend of the means, and two measures on either side of the means. These two measures default to the 10th and 90th percentile trends.

**Trial**  
Trial is another term for iteration.  
*See iteration*
**Truncation**

Truncation is the process by which a user chooses a minimum-maximum range for a random variable that differs from the range indicated by the distribution type of the variable. A truncated distribution has a smaller range than the untruncated distribution, because the truncation minimum is greater than the distribution minimum and/or the truncation maximum is less than the distribution maximum.

**Uncertainty**

*See risk*

**Variable**

A variable is a basic model component that can take on more than one value. If the value that actually will occur is not known with certainty, the variable is considered uncertain. A variable, certain or uncertain, may be either dependent or independent.

*See dependent variable, independent variable*

**Variance**

The variance is a measure of how widely dispersed the values are in a distribution, and thus is an indication of the “risk” of the distribution. It is calculated as the average of the squared deviations about the mean. The variance gives disproportionate weight to “outliers”, values that are far away from the mean. The variance is the square of the standard deviation.
Appendix D: Recommended Readings

Readings by Category

The @RISK User’s Guide has given you a start on understanding the concepts of Risk Analysis and simulation. If you're interested in finding out more about the Risk Analysis technique, and the theory behind it, here are some books and articles which examine various areas in the Risk Analysis field.

**Introduction to Risk Analysis**

If you are new to Risk Analysis or if you would just like some more background information on the technique, the following books and articles might be helpful:


**Distribution Fitting**

If you are interested in finding out more about distribution fitting, consult any of these books:


**Distribution Functions**

To learn more about the distribution functions used by @RISK’s BestFit distribution fitting software, refer to the following book:


**Technical References to Simulation and Monte Carlo Techniques**

If you would like a more in depth examination of simulation, sampling techniques and statistical theory, the following books may be useful:


Technical References to Latin Hypercube Sampling Techniques

If you are interested in the relatively new technique of Latin Hypercube sampling, the following sources might be helpful:


Examples and Case Studies Using Risk Analysis

If you would like to examine case studies showing the use of Risk Analysis in real life situations, see the following:


*Nersesian, Roy L. @RISK Bank Credit: Roy L. Nersesian, 1998.


* These titles can be purchased through Palisade Corporation. Call (800-432-7475 or 607-277-8000), fax (607-277-8001), or write to order or request further information on these and other titles relevant to risk analysis. The Palisade Technical Sales Department can also be reached by e-mail at sales@palisade.com or on the Web at http://www.palisade.com.
Index

@RISK Library
  Distributions ........................................................................................... 631
  Results in Library ................................................................................... 637
  Seeding Distributions ............................................................................. 632
  SQL Server ............................................................................................. 643
  Updating Distributions ........................................................................... 635
  @RISK Library .......................................................................................... 629

A

About Command ........................................................................................ 407
Add Output Command................................................................................ 234
Add-In, @RISK.................................................................................... 41, 217
  Toolbar ............................................................................207, 211, 213, 214
Adjustment Weights ................................................................................... 257
Advanced Sensitivity Analysis Command ......................................... 343, 344
Alternate Parameters................................................................................... 226
Anderson-Darling (A-D) Statistic....................................................... 191, 286
Application Settings Command.................................................................. 395
Authorization.............................................................................................. 407
Authorization Command ............................................................................ 407
Auto-Stop .............................................................................................. 120

C

Check Matrix Consistency Command .................................................... 256
Chi-Squared
  Statistic .............................................................................................. 190, 286
Circular References ................................................................................... 301
Collect Distribution Samples................................................................. 312
Compatibility .......................................................................................... 15
Convergence Monitor ............................................................................ 119
Correlation.......................................................................................... 109, 152
Adding .............................................................................................. 260
Adjustment Weights ............................................................................. 257
Checking Matrix Consistency .............................................................................. 256
Coefficients ......................................................................................................... 248
Instances, Multiple .......................................................................................... 251, 586
Rank Order ......................................................................................................... 261, 588
Correlations ....................................................................................................... 247
Critical Values ................................................................................................. 192
Cumulative Descending Percentiles . See Percentiles, Cumulative Descending

D

Data Command .................................................................................................... 370
Dates in @RISK Functions .................................................................................. 449
DecisionTools
   Suite .................................................................................................................. 7, 653
Define Correlations Command .......................................................................... 247
Define Distribution Command .......................................................................... 219
Define Distribution Window ............................................................................ 47, 52, 56, 95, 108, 220
   Linking to Fits ............................................................................................... 196, 275
   Showing .......................................................................................................... 219
Delimiters .......................................................................................................... See Graphs, Delimiters
Detailed Statistics Command ............................................................................ 367
Distribution
   Drawing .......................................................................................................... 296
Functions .......................................................................................................... 94, 443, 680
Shifting ............................................................................................................. 460, 594
Truncating ......................................................................................................... 596
Distribution Artist Command ............................................................................ 295
Distribution Fitting .......................................................................................... See Fitting

E

Example Models .................................................................................................. 137–71, 681
CLAIMS.XLS .................................................................................................... 147
CORRMAT.XLS ............................................................................................... 151
DEP.XLS .......................................................................................................... 151
DISCRETE.XLS ............................................................................................... 145
ERROR.XLS ..................................................................................................... 149
HIPPO.XLS ..................................................................................................... 155
NCAA.XLS ...................................................................................................... 169
RATE.XLS ....................................................................................................... 141
SENSIM.XLS .................................................................................................. 153
VAR.XLS ......................................................................................................... 165
VARIABLE.XLS ............................................................................................. 143
Excel
   Graphs ......................................................................................................... See Graphs, Excel Format
   Hiding on Start of Simulation ........................................................................ 305
   Reports ......................................................................................................... See Reports, Excel

Readings by Category
### F

**Filters**
- Input Data ................................................................. 179, 278
- Result ............................................................................... 382

**Fitting** ........................................................................... 111–12, 173–96, 680
- Algorithms ....................................................................... 183
- Continuous Distributions ............................................... 181
- Cumulative Data ................................................................ 179
- Density Data ..................................................................... 178
- Discrete Distributions ..................................................... 181
- Domain Limits .................................................................. 182, 280
- Estimated Parameters .................................................... 181, 279
- Goodness-of-Fit Tests ..................................................... 190
- Input Data ....................................................................... 177, 276, 279, 282, 290
- Predefined Distributions ............................................... 181, 281
- Sample Data ...................................................................... 177
- Selecting Distributions to Fit ......................................... 181

### G

**Goal Seek** ......................................................................... 321
**Graphs** ........................................................................... 124–30
- Box-Whisker Plot .............................................................. 338
- Delimiters ......................................................................... 126, 221, 414
- Excel Format ...................................................................... 195
- Fit Comparison Graph ..................................................... 187, 288
- Formatting ......................................................................... 127
- Overlays ........................................................................... 125, 415, 416
- P-P Graph ......................................................................... 188, 289
- Q-Q Graph ......................................................................... 188, 289
- Scatter Plots .................................................................... 425
- Summary Graph .............................................................. 127, 429
- Tornado Graph ................................................................. 132, 134, 161, 381, 422, 424

### H

**Histograms** ....................................................................... See Graphs, Histograms

### I

**Icons**
- @RISK ............................................................................... 207
- Desktop ........................................................................... 8
- Input Data Options Command ........................................... 276, 279, 282, 290

*Inputs*
Adding .................................................................................................... 105
Collecting Distribution Samples ..................................................... 312, 584
Listing ..................................................................................................... 114
Locking................................................................................................... 593
Naming ........................................................................... 230, 237, 264, 593
Properties ........................................................................................ 230, 237
Insert Function Command ............................................................. 241
Insert Row/Column Command .................................................... 253
Installation Instructions........................................................................ 7–8
Instance Commands ............................................................................... 251
Instances, Multiple........................................................................ See Correlation, Instances
Iteration................................................................................................. 99, 300

K

Kolmogorov-Smirnov (K-S) Statistic ................................................. 191, 286

L

Latin-Hypercube Sampling ........................................................... 308, 650, 681
Least Squares, Method of ................................................................. 185
Library Commands .............................................................................. 405

M

Macros
VBA Control of @RISK ................................................................. 314, 601–24
Maximum Likelihood Estimators (MLEs).......................................... 183
Menus
Help Menu (Model Window) ........................................................... 407
Model Menu (@RISK Add-in) ............................................................ 219
Results Menu (@RISK Add-in) .......................................................... 359
Simulate Menu (@RISK Add-in) ....................................................... 275, 299, 317
Model Window ................................................................................... 114, 262
Modeling............................................................................................... 137–71
Chance Events ..................................................................................... 145
Dependency Relationships ................................................................. 151
Insurance Claims ................................................................................. 147
Interest Rates ....................................................................................... 141
NCAA Tournament ............................................................................ 169
New Product Launch ........................................................................... 155
Oil Wells ............................................................................................... 147
Random Trends ................................................................................... 142
Random Walks.................................................................................... 142
Time-Dependent Variability .............................................................. 143
Uncertainty Around a Fixed Trend.................................................... 149
Value-At-Risk (VAR) .................................................................................. 165
Monte-Carlo Sampling ............................................................................. 308, 649, 680
Multiple CPUs ............................................................................................ 302

O

Opening @RISK Simulations .......................................................................... 400, 403
Outputs
Adding ........................................................................................................... 96, 234
Listing ............................................................................................................ 114
Naming ........................................................................................................... 235, 264

P

Palisade Corporation .................................................................................. 5, 654
Parameters
Alternate ..................................................................................................... 226, 446
Variable ...................................................................................................... 151
Pause on Error ............................................................................................. 306
Percentiles
Calculating Targets ..................................................................................... 123, 292, 368
Cumulative Descending ........................................................................... 368, 448
Spreadsheet Function .................................................................................. 606, 611
PrecisionTree ............................................................................................... 653, 655, 665–72
Property Functions ...................................................................................... See Risk Functions, Property
P-Values ....................................................................................................... 192

Q

Quick Reports .......................................................................................... See Reports, Quick

R

Regression .................................................................................................... 374
Report Settings Command ......................................................................... 275, 294, 359
Reports
Excel ............................................................................................................. 135
Quick .............................................................................................................. 385
Settings ......................................................................................................... 275, 294, 359
Template Sheet .............................................................................................. 136, 386, 453
Results Summary Window ........................................................................... 122, 360
Risk Functions ............................................................................................ 443
Arguments ................................................................................................... 449
Arrays ............................................................................................................ 451
Dates .............................................................................................................. 449
Listing in @RISK .......................................................................................... 114
Property Functions ......................................................... 446, 583–98
RiskBeta ........................................................................... 468
RiskBetaGeneral .......................................................... 470
RiskBetaGeneralAlt ....................................................... 473
RiskBetaSubj ............................................................... 474
RiskBinomial ................................................................. 477
RiskCategory ............................................................... 583
RiskChiSq ................................................................. 479
RiskCollect ................................................................. 312, 584
RiskCompound .......................................................... 481
RiskConvergence ......................................................... 585
RiskConvergenceLevel .................................................. 603
RiskCorrectCorrmat ....................................................... 625
RiskCorrel ................................................................. 603
RiskCorrmat ............................................................. 260, 586
RiskCumul ................................................................. 482
RiskCurrentIter .......................................................... 453, 626
RiskCurrentSim .......................................................... 453, 626
RiskData ................................................................. 461, 604
RiskDepC ................................................................. 588
RiskDiscrete ............................................................. 145, 488
RiskDUniform .......................................................... 491
RiskErf ................................................................. 494
RiskErlang ................................................................. 496
RiskExpon ................................................................. 498
RiskExponAlt ........................................................... 500
RiskExtValue ............................................................ 500
RiskExtValueAlt ........................................................ 502
RiskFit ................................................................. 590
RiskGamma ................................................................. 503
RiskGammaAlt .......................................................... 505
RiskGeneral .............................................................. 506
RiskGeomet .............................................................. 509
RiskHistogrm ............................................................ 511
RiskHypergeo .......................................................... 514
RiskIndepC .............................................................. 591
RiskIntUniform .......................................................... 517
RiskInvgauss ............................................................ 519
RiskInvgaussAlt ........................................................ 521
RiskIsDate ............................................................... 592
RiskIsDiscrete .......................................................... 591
RiskJohnsonMoments ................................................ 522
RiskJohnsonSB ........................................................ 524
RiskJohnsonSU ........................................................ 526
RiskKurtosis ............................................................. 604
RiskLibrary .............................................................. 592
RiskLock ................................................................. 593
RiskLogistic .............................................................. 529
RiskLogisticAlt .......................................................... 531
<table>
<thead>
<tr>
<th>Function</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>RiskTheoMode</td>
<td>610</td>
</tr>
<tr>
<td>RiskTheoPercentile</td>
<td>611</td>
</tr>
<tr>
<td>RiskTheoRange</td>
<td>611</td>
</tr>
<tr>
<td>RiskTheoSkewness</td>
<td>611</td>
</tr>
<tr>
<td>RiskTheoStdDev</td>
<td>612</td>
</tr>
<tr>
<td>RiskTheoTarget</td>
<td>612</td>
</tr>
<tr>
<td>RiskTheoVariance</td>
<td>612</td>
</tr>
<tr>
<td>RiskXtoP</td>
<td>612</td>
</tr>
<tr>
<td>RiskTriang</td>
<td>571</td>
</tr>
<tr>
<td>RiskTriangAlt</td>
<td>574</td>
</tr>
<tr>
<td>RiskTrigen</td>
<td>574</td>
</tr>
<tr>
<td>RiskTruncate</td>
<td>596</td>
</tr>
<tr>
<td>RiskTruncateP</td>
<td>596</td>
</tr>
<tr>
<td>RiskUniform</td>
<td>575</td>
</tr>
<tr>
<td>RiskUniformAlt</td>
<td>577</td>
</tr>
<tr>
<td>RiskUnits</td>
<td>597</td>
</tr>
<tr>
<td>RiskVariance</td>
<td>608</td>
</tr>
<tr>
<td>RiskWeibull</td>
<td>578</td>
</tr>
<tr>
<td>RiskWeibullAlt</td>
<td>581</td>
</tr>
<tr>
<td>RiskXtoP</td>
<td>608</td>
</tr>
<tr>
<td>Shifting</td>
<td>460, 594</td>
</tr>
<tr>
<td>Statistic Functions</td>
<td>45, 452, 601–24, 601–24</td>
</tr>
<tr>
<td>Table of</td>
<td>455–65</td>
</tr>
<tr>
<td>Truncating</td>
<td>596</td>
</tr>
<tr>
<td>Risk Six Sigma Functions</td>
<td></td>
</tr>
<tr>
<td>RiskCp</td>
<td>614</td>
</tr>
<tr>
<td>RiskCpk</td>
<td>615</td>
</tr>
<tr>
<td>RiskCpkLower</td>
<td>616</td>
</tr>
<tr>
<td>RiskCpkUpper</td>
<td>616</td>
</tr>
<tr>
<td>RiskCpm</td>
<td>614</td>
</tr>
<tr>
<td>RiskDPM</td>
<td>617</td>
</tr>
<tr>
<td>RiskK</td>
<td>617</td>
</tr>
<tr>
<td>RiskLowerXBound</td>
<td>618</td>
</tr>
<tr>
<td>RiskPNC</td>
<td>618</td>
</tr>
<tr>
<td>RiskPNCLower</td>
<td>619</td>
</tr>
<tr>
<td>RiskPNCLUpper</td>
<td>619</td>
</tr>
<tr>
<td>RiskPPMLower</td>
<td>620</td>
</tr>
<tr>
<td>RiskPPMUpper</td>
<td>620</td>
</tr>
<tr>
<td>RiskSigmaLevel</td>
<td>621</td>
</tr>
<tr>
<td>RiskUpperXBound</td>
<td>622</td>
</tr>
<tr>
<td>RiskYV</td>
<td>622</td>
</tr>
<tr>
<td>RiskZLower</td>
<td>623</td>
</tr>
<tr>
<td>RiskZMin</td>
<td>624</td>
</tr>
<tr>
<td>RiskZUpper</td>
<td>624</td>
</tr>
<tr>
<td>Root-Mean Squared Error (RMSErr)</td>
<td>192, 286</td>
</tr>
</tbody>
</table>
Index

S

Saving @RISK Simulations ................................................................. 403
Scenario Analysis ................................................................. 103, 133, 161
Scenarios Command ................................................................. 377, 385
Seed, Random ................................................................................. 302
Sensitivities Command ................................................................. 373
Sensitivity Analysis
  Advanced .................................................................................. 343–58, 343–58
  Standard .................................................................................. 102, 131, 373
Show Expanded Toolbar Command ................................................. 399
Simulation
  Multiple .............................................................................. 153, 301, 311, 412, 434
  Settings ............................................................................... 116, 299
  Starting .............................................................................. 118, 317
  Stopping ............................................................................. 120
Simulation Settings Command ......................................................... 299
Smart Sensitivity Analysis ................................................................. 373
Standard Recalc ............................................................................. 308
Start Simulation Command ............................................................. 317
Statistics
  Anderson-Darling (A-D) ............................................................ 191, 286
  Chi-Squared ........................................................................ 190, 286
  Detailed ............................................................................... 367
  Fitting ............................................................................... 190
  Kolmogorov-Smirnov (K-S). ..................................................... 191, 286
  Root-Mean Squared Error (RMSErr) .................................. 192, 286
Stress Analysis ............................................................................. 329–41
Stress Analysis Command ............................................................... 329
Student Version ............................................................................. 6
System Requirements ..................................................................... 6

T

Technical Support ........................................................................... 4–6
Template Sheet .............................................................. See Reports, Template Sheet
Toolbars
  @RISK Add-in ..................................................................... 207, 211, 213, 214, 399
  Expanded vs Collapsed ................................................................ 399
  TopRank .................................................................................. 653, 655, 657–63
  True EV ............................................................................... 303, 390
  Truncation ............................................................................ 596
  Tutorial ............................................................................... 13, 93

U

Uninstalling @RISK .......................................................................... 7
Upgrade Information ................................................................. 37–90

V

VBA Control of @RISK ................................................................. 314, 601–24

W

Windows
  Data Window ........................................................................... 370
  Define Distribution Window ................................................ See Define Distribution Window
  Detailed Statistics Window ...................................................... 367
  Fit Results Window ................................................................. 285, 292
  Fit Summary Window .............................................................. 292
  Model Window ........................................................................ See Model Window
  Results Window ...................................................................... See Results Window
  Scenario Analysis Window ....................................................... 377, 385
  Sensitivity Analysis Window ................................................... 373