Modeling VaR of Swaps

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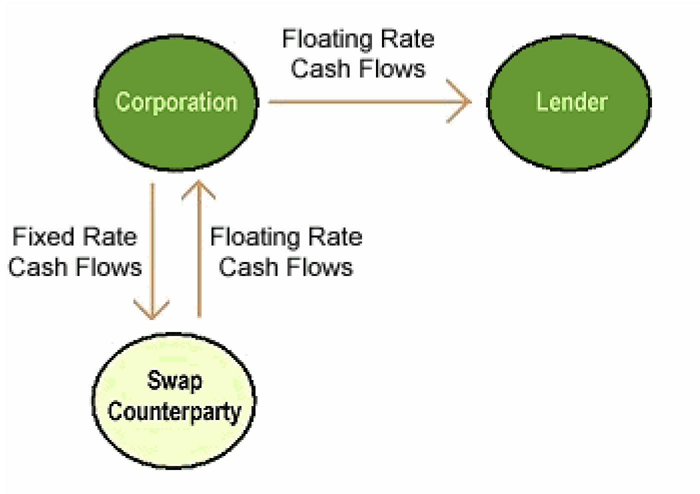
Modeling VaR of Swaps

@Risk© application

Overview of Presentation

- Swaps
  - Interest Rate Swap
    - Structure and Uses
  - The Swap Curve
  - Valuation of IR Swap – Bond Viewpoint & as PV of Forward Rate
  - VaR analysis
  - @Risk Simulation to find VaR for swaps
  - Implications

Nature of Swaps



A swap is an agreement to exchange cash flows at specified future times according to certain specified rules

Interest Rate Swap

An agreement between two parties to exchange a series of interest payments, one for another.

Typically:

- One series @ floating rate
  - (LIBOR, T-Bill, Federal Funds)
- Other series @ fixed rate
  - (Based on LT rates: T-Bond)

Interest Rate Swap

Notional Principal

- **Size of Payment**

Example 8% on 10m quarterly

Amortization = $(1/4) \times 0.08 \times 10\text{m} = 200\text{k}$

Parties in Swap:

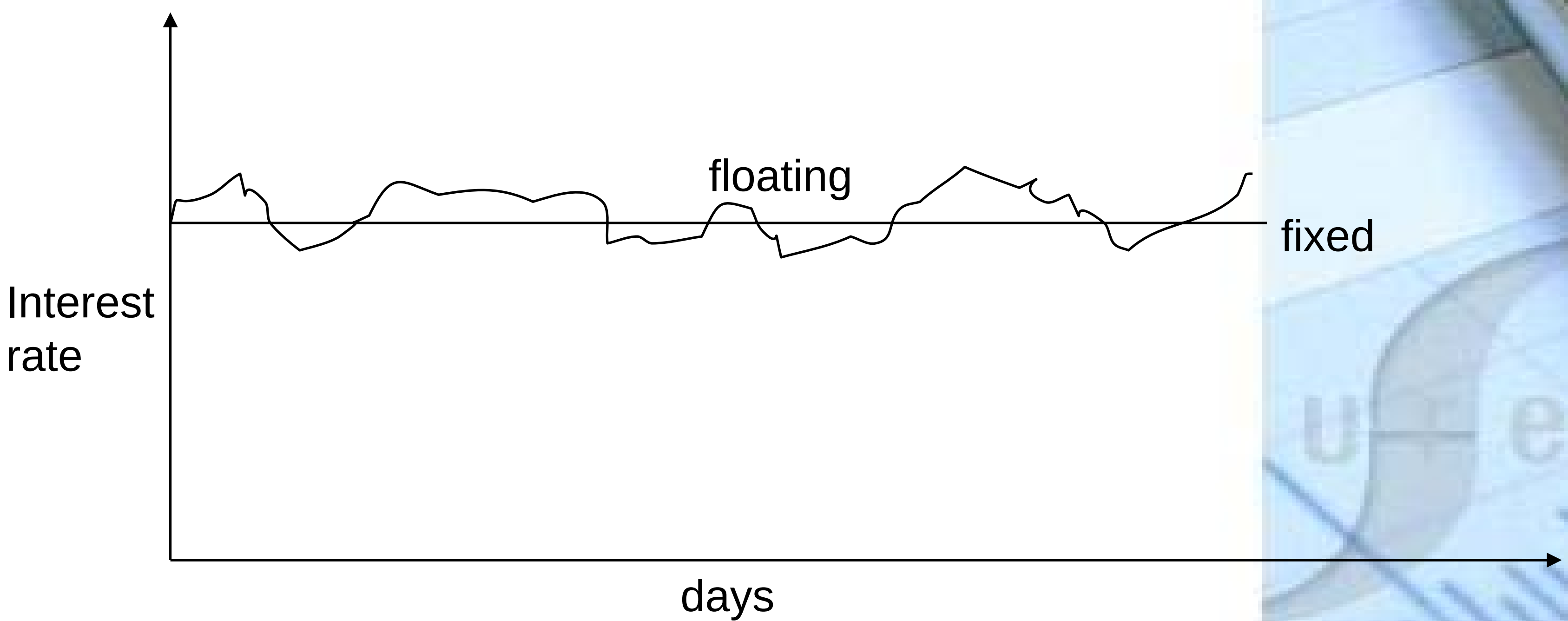
- Fixed-rate Payer
- Floating-Rate Payer

Interest Rate Swap

Why Swaps Work?

People have opposite positions

Interest rate swap – swapping fixed for floating or vice- versa



Interest Rate Swap

- Swaps can be used for:
  - Hedging against interest rates
- Major players:
  - Citigroup, JP Morgan, bankers Trust, First Chicago etc.

Interest Rate Swap

Why Swaps Work?

- Market access

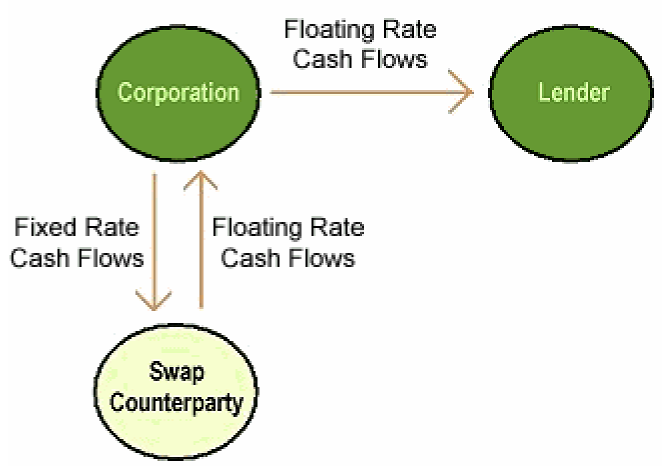
- Cheap to eliminate risk

    Arose from: S&L exposure in late 1970s

    Lent long @ fixed rate

    Took ST loan@ variable rates

Interest Rate Swap example

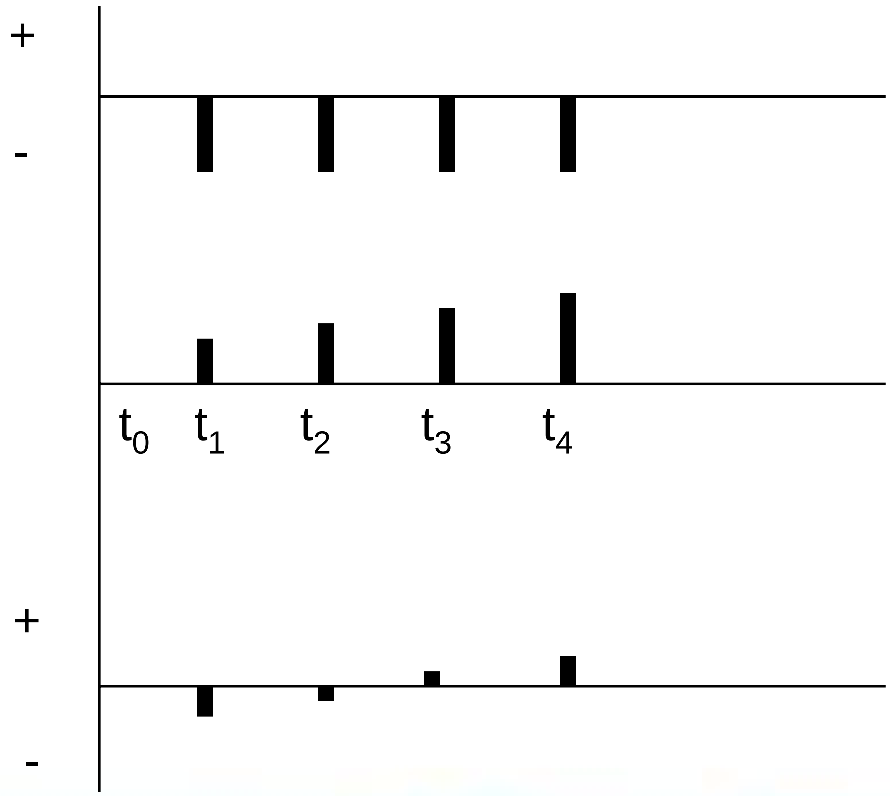


An agreement by Intel to receive 6-month LIBOR & pay a fixed rate of 5% per annum every 6 months for two years on a notional principal of $10 million

Cash Flows to Intel

Date	LIBOR Rate	FLOATING Cash Flow	FIXED Cash Flow	Net Cash Flow
Sept 2, 2005	4.2%			
March 2, 2006	4.8%	+2.10	–2.50	–0.40
Sept 2, 2006	5.3%	+2.40	–2.50	–0.10
March 2, 2007	5.5%	+2.65	–2.50	+0.15
Sept 2, 2007	5.4%	+2.75	–2.50	+0.25

Cash Flow Diagram of Swap



- Pay Fixed at 5%
- Receive Floating at LIBOR
  - Payment at t_i, fixed at t_{i-1}
- Net Cash exchanged

Typical Uses of an Interest Rate Swap

- Converting a liability from
  - fixed rate to floating rate
  - floating rate to fixed rate

- Converting an investment from
  - fixed rate to floating rate
  - floating rate to fixed rate

Six-month LIBOR is a short-term AA borrowing rate

Swap Curve – March 01, 2007

<HELP> for explanation. P181c Govt SWPM

Options | New Deal | Save Deal | View | SWAP MANAGER

Deal Counterparty SWAP CNTRPARTY Ticker / SWAP Series Deal # DETAIL

Curve # 23 Bid USD Swaps(30/360,S/A) EXPORT TO EXCEL Mty Par Cpn 4.97100 Stub Reset 5.34750

#	Mty/Term	RATE	DISCNT	#	Mty/Term	RATE	DISCNT	#	Mty/Term	RATE	DISCNT
1	1 DY	5.30000	0.999853	13	8 MO	5.29563	0.965214	25	9 YR	5.07500	0.635968
2	2 DY	5.30000	0.999706	14	9 MO	5.28125	0.961222	26	10 YR	5.09900	0.603168
3	1 WK	5.30850	0.998969	15	10 MO	5.26313	0.956911	27	11 YR	5.11900	0.572023
4	2 WK	5.32000	0.997935	16	11 MO	5.24875	0.953167	28	12 YR	5.14100	0.541944
5	3 WK	5.32000	0.996906	17	1 YR	5.23250	0.949490	29	15 YR	5.19600	0.460071
6	1 MO	5.32000	0.995440	18	2 YR	5.01700	0.905794	30	20 YR	5.23900	0.351612
7	2 MO	5.33750	0.990746	19	3 YR	4.95200	0.863735	31	25 YR	5.25000	0.270300
8	3 MO	5.34750	0.986518	20	4 YR	4.94800	0.822423	32	30 YR	5.24800	0.208967
9	4 MO	5.34000	0.982225	21	5 YR	4.97100	0.782433	33	40 YR	5.23000	0.127284
10	5 MO	5.33188	0.977700	22	6 YR	4.99600	0.743709	34	50 YR	5.16100	0.085468
11	6 MO	5.32813	0.973489	23	7 YR	5.02100	0.706496				
12	7 MO	5.31125	0.969394	24	8 YR	5.04700	0.670684				

DAYTYPE/FREQ CONVENTIONS

Cash Rates ACT/360
Swap Rates 30I/360 (S
Rate Source Standard

INTERPOLATION METHOD

Piecewise Linear (Simple)

GLOBAL CHANGE FIELDS

From 1 To 34 Shift 0.00

Valuation Curve 03/01/07 Valuation 03/05/07 All Values in USD

Market Value	10,000,000.00	DV01	4,375.59
Accrued	0.00		
Market Value	-10,000,000.00	DV01	-252.10
Accrued	-0.00		

Net Principal 0.00 Par Cpn 4.97100
Accrued 0.00 Premium 0.00000 DV01 4,123.49
Market Value 0.00 Unwind PV 0.00 Refresh

Main | Curves | Cashflow | Risk | Horizon

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2007 Bloomberg L.P.
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The Comparative Advantage of Swaps

- Ability of Counterparties to take advantage of their comparative advantage in the Debt Markets
- Borrowers with different credit quality will have comparative advantage in fixed or floating Markets

- As a result a company may
  - Borrow Fixed when it wants Floating
  - Borrow Floating when it wants Fixed
  - …..It can Adjust the result through the Swap Market

Valuation of swaps in terms of Bonds

Notional, yields, floating and fixed payments are known

Effective swaps to be computed

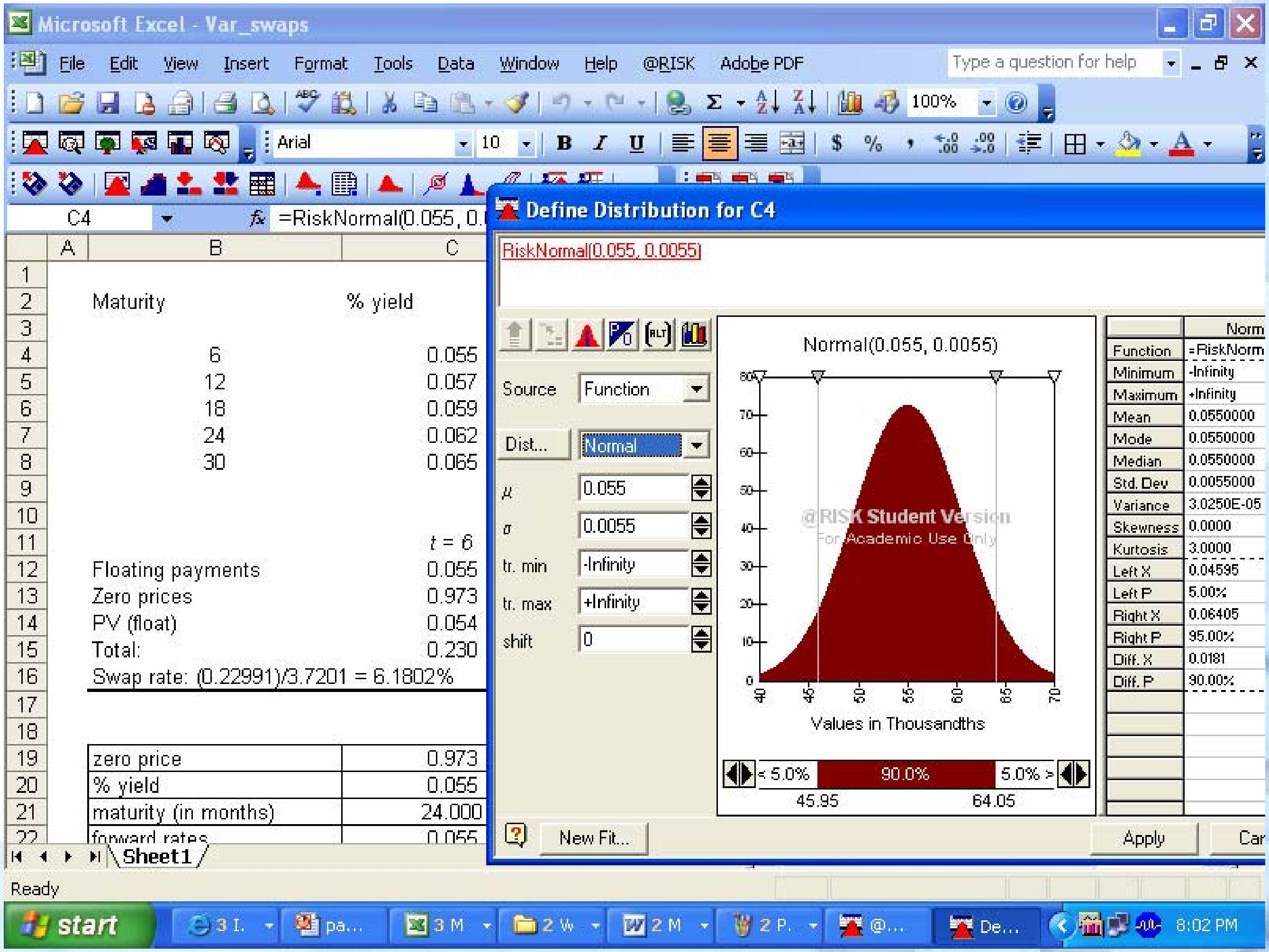
Procedure….

discounting to present values

$$Dp = r_6 p_0$$

At date t = 6, the floating leg will pay r6% which is discounted back to t = 0 by using the six-month zero price at t = 0, which is p0.

Volatility in yield



forward rates

$$f_0(6,12) \quad = 2\left(\frac{\left[1+\frac{f_0(0,12)}{2}\right]^2}{\left[1+\frac{f_0(0,6)}{2}\right]}-1\right)$$

At date t = 0, $f_0(0,12)$ six month forward rate that will prevail from t = 6 to t = 12

present value at t = 0 of receiving at t = 12

$$PV_{12} = f_0(6{,}12) * P_{12}$$

As floating leg will reset to at t = 6 the counter party will actually pay this amount at t = 12.

Set PV of fixed equal to PV of floating

$$x = \frac{\sum_{o}^{T} PV_{t,t+1}}{\sum_{t}^{T} PV_t}$$

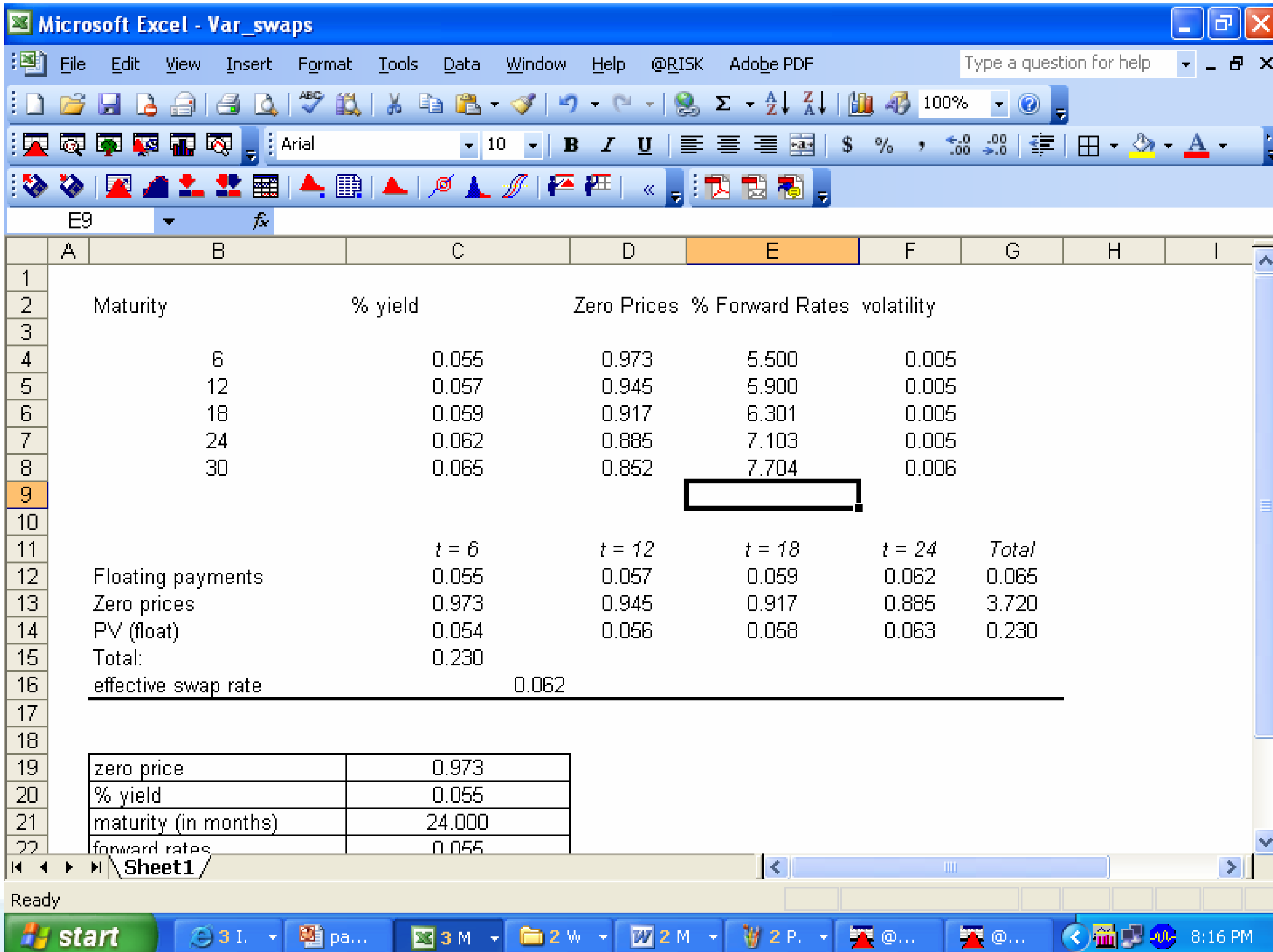
x = effective swap rate

PV_t = present value of all floating payments

t= time period (maturity)

$PV_{t,t+1}$ = present value of fixed leg payments

@Risk model file



	B	C	D	E	F	G
2	Maturity	% yield	Zero Prices	% Forward Rates	volatility	
4	6	0.055	0.973	5.500	0.005	
5	12	0.057	0.945	5.900	0.005	
6	18	0.059	0.917	6.301	0.005	
7	24	0.062	0.885	7.103	0.005	
8	30	0.065	0.852	7.704	0.006	
11		*t = 6*	*t = 12*	*t = 18*	*t = 24*	*Total*
12	Floating payments	0.055	0.057	0.059	0.062	0.065
13	Zero prices	0.973	0.945	0.917	0.885	3.720
14	PV (float)	0.054	0.056	0.058	0.063	0.230
15	Total:	0.230				
16	effective swap rate	0.062				
19	zero price	0.973				
20	% yield	0.055				
21	maturity (in months)	24.000				
22	forward rates	0.055				

Case illustration

Maturity	Yield	Zero Prices	Forward Rates
6	0.0550	0.96754	5.5000
12	0.0570	0.9543	5.9002
18	5.6700	0.9333	6.3006
24	6.0500	0.9124	7.1026
30	6.2100	0.8876	7.7044

Valuation of Swaps

	t = 6	*t = 12*	*t = 18*	*t = 24*	*Sum*
Floating payments	5.50%	5.90%	6.30%	7.10%	
Zero prices	0.9732	0.9453	0.9165	0.8850	3.7201
PV (float)	0.0535	0.0558	0.0577	0.0629	0.2299
Total:	0.2299	**Swap rate: (0.22991)/3.7201 = 6.1802%**			

Value at Risk

Measuring risk in swaps

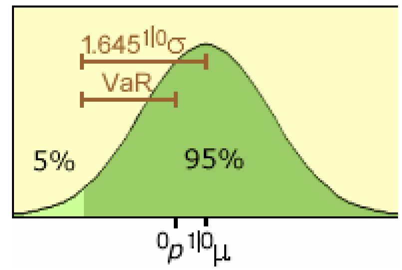
Value at Risk

- Quantifies risk which captures the odds of losing money

  - ‘What is the probability that an investor will lose money in a worst case scenario’ ?
  - ‘How much (amount) can be lost in a worst case scenario?’

VaR – Monte Carlo Simulation

- …it has a confidence level through which the analyst can estimate VaR with a certain degree of confidence
- An investor in a swap transaction has a potential to lose money on his transaction but what is the most that investor can (with a given confidence level) expect to lose over the next time period?



$$VaR = 1.645\sigma + (p - \mu)$$

Based on the fact that the 5%-quantile of a normal distribution always occurs 1.645 standard deviations below its mean

Simulation to find VaR

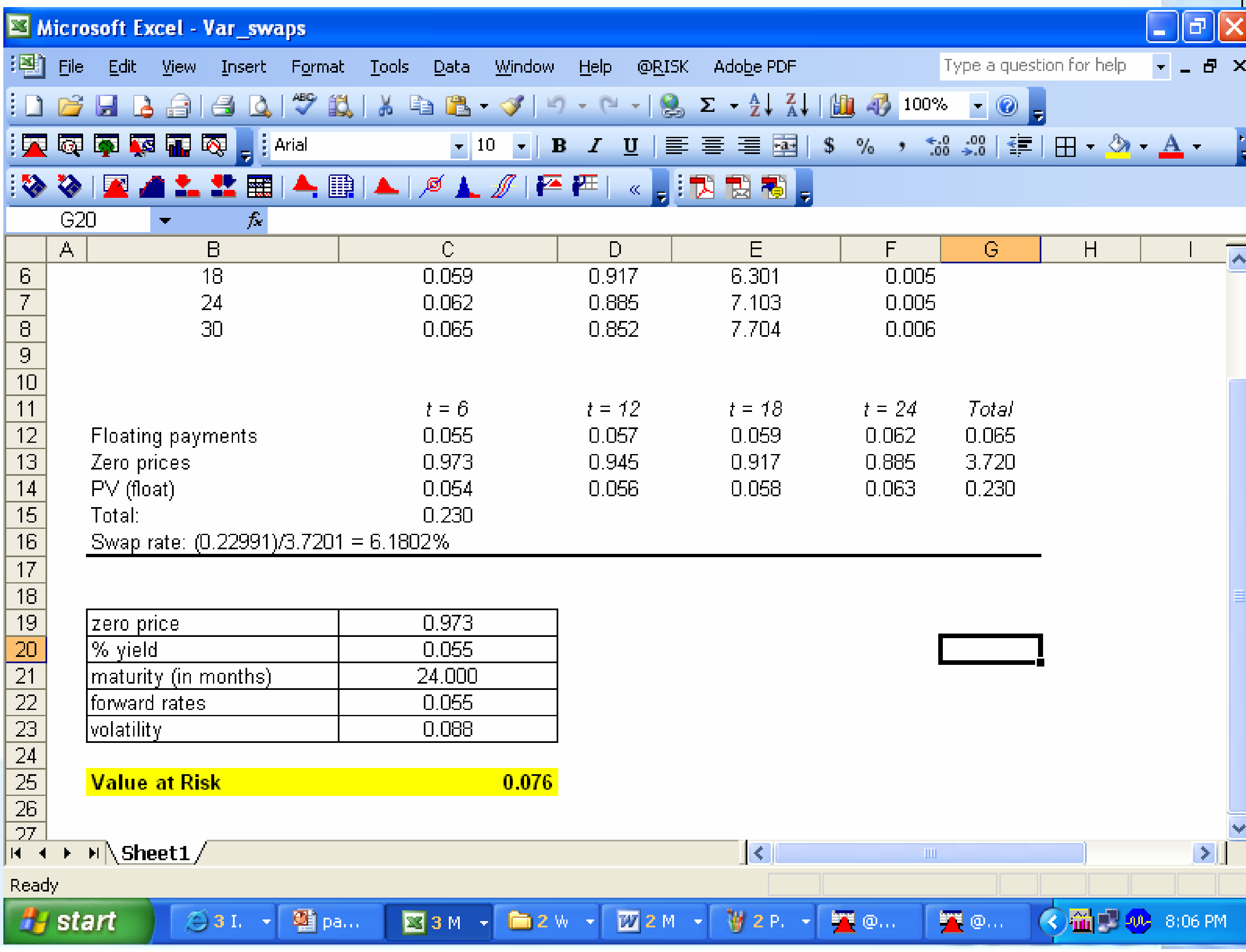
- Simulation used to generate multiple interest rate scenarios and compute the value that could be potentially lost

- …..Defined by three basic elements – confidence level, time period and the value that could be potentially lost

Simulation through @Risk

- Simulation model developed for future bond yields
- multiple trials though the model
- Simulation length was kept to 100
- Replicated the simulation 50 times

VaR at 7.56 % of notional amount



Microsoft Excel - Var_swaps

	A	B	C	D	E	F	G
6		18	0.059	0.917	6.301	0.005	
7		24	0.062	0.885	7.103	0.005	
8		30	0.065	0.852	7.704	0.006	
9							
10							
11			*t = 6*	*t = 12*	*t = 18*	*t = 24*	*Total*
12		Floating payments	0.055	0.057	0.059	0.062	0.065
13		Zero prices	0.973	0.945	0.917	0.885	3.720
14		PV (float)	0.054	0.056	0.058	0.063	0.230
15		Total:	0.230				
16		Swap rate: (0.22991)/3.7201 = 6.1802%					

zero price	0.973
% yield	0.055
maturity (in months)	24.000
forward rates	0.055
volatility	0.088

Value at Risk **0.076**

Sheet1

Ready

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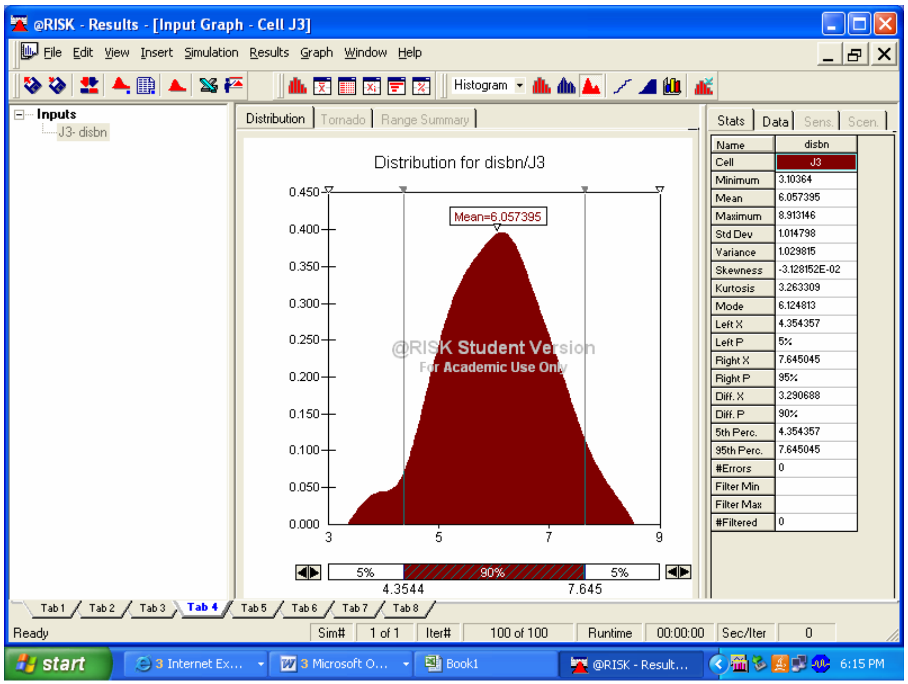
@Risk output



@RISK - Results - [Input Graph - Cell J3]
File Edit View Insert Simulation Results Graph Window Help
Histogram
Inputs
J3- disbn
Distribution
Tornado
Range Summary
Distribution for disbn/J3
Mean=6.057395
@RISK Student Version
For Academic Use Only
0.450
0.400
0.350
0.300
0.250
0.200
0.150
0.100
0.050
0.000
3
5
7
9
5%
90%
5%
4.3544
7.645
Stats
Data
Sens.
Scen.
Name disbn
Cell J3
Minimum 3.10364
Mean 6.057395
Maximum 8.913146
Std Dev 1.014798
Variance 1.029815
Skewness -3.128152E-02
Kurtosis 3.263309
Mode 6.124813
Left X 4.354357
Left P 5%
Right X 7.645045
Right P 95%
Diff. X 3.290688
Diff. P 90%
5th Perc. 4.354357
95th Perc. 7.645045
#Errors 0
Filter Min
Filter Max
#Filtered 0
Tab 1
Tab 2
Tab 3
Tab 4
Tab 5
Tab 6
Tab 7
Tab 8
Ready
Sim#
1 of 1
Iter#
100 of 100
Runtime
00:00:00
Sec/Iter
0
start
6:15 PM

Conclusions

- A spreadsheet model to value interest rates
- VaR at 7.56% of notional amount indicates that the investor is likely (5% chances) to lose this % of its invested amount
- An easy to use model which can be modified to do similar analysis including credit and currency risk

Questions..